Electricity Markets: Design and Optimisation

Thesis Proposal

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1 Introduction

Electricity has a major impact on the social and economic developments of nations around the world since it is an essential ingredient of modern society. The modern society depends on constant accessibility of this commodity in order to maintain the present way of living. We have got used to the supply of electricity and we take it for granted that whenever we switch on a power point we are able to access the electricity. In principle, electricity is a commodity that can be traded like all other commodities. The only difference is that with our present technology this commodity cannot be stored in any feasible way and as consumers we believe that we are entitled to buy the commodity whenever we want to. Since it cannot be stored the electricity has to be balanced such that supply always equals consumption, where both the supply and the consumption are uncertain. This makes it a unique and interesting commodity with its very own characteristics.

For many decades the traditional way of supplying electricity was highly dependent on fossil fuels such as coal and oil. This has formed the structure of the electricity market and even today fossil fuels are a very big part of the global supply chain for electricity. However, fossil fuels have a high climate impact and they are limited resources. Therefore, there is a pressing need for efficient renewable energy resources to substitute the fossil fuels resources. In Northern Europe wind energy is one of the most promising large-scale renewables that can replace some of the conventional supply. One drawback is, though, that the wind is fluctuating and it creates fluctuating energy. Some of these fluctuations can be met by planned conventional electricity production from thermal plants but only to the extent that the wind speed forecasts made beforehand are correct. If they are incorrect, the imbalances must be taken care of through reserves. Thereby, the electricity system security depends on accessibility of reserves.

With the liberalisation and deregulation of the European electricity markets these markets have developed and gone though some major changes. Given a political wish for more renewable energy in the European systems the systems have to change further and this gives rise to major challenges. Today Denmark has a 20% share of renewable energy in its electricity system but the Danish Government wants to have a 50% share of the traditional electricity consumption supplied from wind energy by 2020. The European energy targets state that Europe should have an overall 20% share of renewable energy in the total system by year 2020 (this is one of the 20-20-20 targets). This target will be reached by letting each member country have its own target depending on the feasibility of installing renewables in its system. Until now the European renewable energy targets stated that Denmark should have 30% of the energy coming from renewables, but the Government has apparently raised this target. With the expectation of 50% wind energy in 2020 Denmark has to figure out if it has enough reserves to cope with the fluctuations, if it has to invest in further reserves or if it has to handle the challenges created by wind energy in another way. The important issue is if there are enough reserves to smoothen the imbalances. This should be analysed with both unit commitment models as well as intra-hour models.

Increased focus on cross-border collaboration could be one way of coping with the future challenges arising from the increased share of wind energy. With closer collaboration exchanging opposite imbalances and sharing reserves would be a possibility. Since collaboration seemingly would be beneficial it is necessary that the analysis models can handle these relationships in order to fully illustrate the complexity of the problem.

Our aim with this project is to develop models that can analyse electricity systems to support decision makers in electricity markets when they have to make decisions about further developments of the systems.

The structure of this thesis proposal

In the following, we will introduce Energinet.dk since they are one of the big collaborators of this project. Then we will give a small introduction to some of the main topics of the PhD project. Our proposals for research topics are given in sections 2, 3 and 4. Finally, the PhD workplan is presented in section 5.

1.1 Energinet.dk

With the liberalisation of the energy markets in Europe it was decided that the production and the transmission of energy should no longer be operated by the same operators. This was the reason for the establishment of Energinet.dk in 2005. Energinet.dk is the transmission system operator (TSO) in Denmark and this means that they are responsible for the safety and the maintenance as well as the development of the transmission cables. It is a non-profit enterprise owned by the Danish Climate and Energy Ministry and it is paid for by the Danish consumers through their electricity bills.

The main task for a TSO is the maintenance of security which means that they have to supervise the system at all times and make sure that the electricity is always balanced such that supply equals consumption. How a TSO does it depends on the market structure, the predetermined product structure for reserves, which agreements the TSO has with the production plants and restrictions given by governments. Since numerous factors influence the way in which the task of balancing the energy can be done, the TSOs do not do it in the same way. The main reason for this, though, is the market structures of the different countries but it is difficult to find an overview of the electricity systems in Europe and how they are operated. Therefore we started off by writing a paper on this topic in order to get the overview ourselves. The paper is written in co-operation with Nina K. Detlefsen and can be found in Appendix A.

The tasks described above also apply to Energinet.dk and for this reason they need to be sure that they can handle the planned increase in renewable energy. Therefore, they are presently analysing the Danish system in order to see if the system can cope with the amounts of fluctuating energy to be incorporated in the system in the future. Doing this process they discovered that they need a tool to analyse the system on a intra-hour basis and that their models should be able to describe the stochastic properties of wind energy.

As the PhD project is being conducted in close collaboration with Energinet.dk, we will look into some of the challenges presently facing Energinet.dk. We will mainly focus on the problematics of balancing the electricity. For us to do so we need to have a good understanding of stochastic programming, since we have to deal with the stochastics of wind energy. We also need to have a good understanding of unit commitment models since they will be closely related to intra-hour models and often an intra-hour model will get information about hourly values from a unit commitment model. Finally, we need to know which research has be done within the field of intra-hour modelling. Therefore, we will introduce these three topics in the next couple of subsections.

1.2 Stochastic programming

Stochastic programming is a way of modelling and solving problems where uncertain data are represented as random variables. Therefore, this tool is very useful when modelling electricity systems with high percentage of wind energy. As a field it has inherited its modelling approaches and solving techniques from its deterministic counterpart: mathematical programming. The topic of stochastic programming was first introduced by Dantzig [31] as a linear program with recourse for sequential decision-making under uncertainty. A lot of research has been carried out over the years in this area and various formulations have been given. This is especially due to the fact that there are numerous challenges when it comes to solving large-scale mixed-integer and pure integer programs in mathematical programming and when you add the dimension of uncertainty these challenges only get bigger. Efficient solution techniques have still not been developed and this indicates that the topic of stochastic programming will continue to be an interesting topic in the years to come from a researcher's point of view.

In this section we will focus on the class of stochastic programming problems referred to as two-stage stochastic linear programs with fixed recourse. Here the decision-maker has to make his decisions based on the partial information available at the time of the decision. The decision-maker implements the first-stage variables, called "here-and-now" variables, without knowing the actual outcome, and when the uncertainty has been revealed, the decision-maker has the opportunity to take further action and choose the second-stage variables called "wait-and-see" variables. His objective is to minimise the total expected cost. A stochastic two-stage program with fixed recourse can be formulated as shown below.

$$\min_{x}\{c^t x + \mathrm{E}_{\xi}[\mathcal{Q}(x,\xi(\omega))] : Ax \leq b, x \geq 0\}, \tag{1}$$

where

$$\begin{aligned} \mathcal{Q}(x,\xi(\omega)) = \min \quad & d(\omega)^t y && (2)\\ \text{s.t.} \quad & Wy = h(\omega) - T(\omega)x && (3)\\ & y \geq 0. && (4) \end{aligned}$$

Here x denotes the first-stage variables, y denotes the second-stage variables, and c and d denote the costs of the first and second stage, respectively. h is the requirement vector, A is the first-stage coefficient matrix, W is called the recourse matrix, and T is the technology matrix.

If the uncertainty is discrete, the stochastic two-stage program can be formulated as a linear program with a number of scenarios each having a probability. This way of formulating the program is called the deterministic equivalent problem and can be seen in (5)-(8), where π_u denotes the probability for scenario u, and $\mathcal{U}$ is the set of scenarios. For further details on the deterministic equivalent problem see Wets [50].

$$\begin{aligned}
\min \quad & c^t x + \sum_{u \in \mathcal{U}} \pi_u d_u^t y && (5)\\
\text{s.t.} \quad & Ax \leq b && (6)\\
& Wy = h_u - T_u x \quad \forall u \in \mathcal{U} && (7)\\
& x \geq 0, y \geq 0 && (8)
\end{aligned}$$

To model real-life settings, the number of scenarios needed will often be extremely high and therefore the problem, even though linear, will be a large-scale problem which will need Monte-Carlo simulations to reduce the number of scenarios and/or special-structure algorithms like Dantzig and Wolfe [32] or Benders [28] decomposition. A very well-known algorithm for solving the deterministic equivalent problem built on the ideas from Benders decomposition, is the L-shaped algorithm by Slyke and Wets [46]. This algorithm can also be found in the book "Introduction to Stochastic Programming" by Birge and Louveaux [29], which is a primer when it comes to learning stochastic programming.

Decomposition methods have proven to be the most efficient algorithms when it comes to stochastic programming and a lot of research has therefore been done in this area. Generally, all of these new decomposition methods look at special cases and structures. Take, for example, the latest paper on this topic by Sen and Sherali [45] where they consider the class of problems in which the second-stage subproblem(s) may impose integer restrictions on some variables. They discuss alternative decomposition methods for solving the second-stage integer subproblems using branch-and-cut methods. However, the L-shaped algorithm is still the most general and most cited algorithm.

With a high share of wind energy, stochastic programming is an important tool for modelling electricity systems for analytical purposes. As discussed earlier, this is due to the fact that the fluctuation of wind potentially creates major imbalances in the electricity system at the time of operation. However, when researchers make/model programs within the field of energy they often do not look at how we can solve the programs efficiently. They are often interested in formulating the program and then they use a tool like GAMS to solve it. If it is not solvable they just make the program smaller or divide it into smaller parts. Take for example the unit commitment model WILMAR by Weber et al. [49], which is a stochastic model and relatively complex but still they solve it in GAMS. We wonder if they would get better results if looking at solution methods. In our opinion the complexity of electricity systems and the use of tools like stochastic programming make it necessary to look into solution methods such that in the future we can get better models for describing our electricity systems.

1.3 Unit commitment models

The problem of unit commitment involves finding the least-cost dispatch of available generation resources to meet the electrical demand. Traditionally, day-ahead unit commitment problems involve two sets of decisions that have to be determined. The first decision is to determine the status of the generators for the next day; which ones are off and which ones are on. The plan for which ones to use has to meet the physical restrictions of the generators. The second decision involves the determination of the production level. More generally, a traditional unit commitment model tries to minimise the total cost of running generators subject to constraints on power balance, minimum unit on and off time, minimum and maximum generation limits, ramp-up and ramp-down rates as well as start-up and shut-down cost.

A survey of modern unit commitment models as well as mathematical formulations and general background knowledge of research and developments within the field can be found in Padhy [44]. Two new and interesting unit commitment models that are not mentioned in the survey paper are Zhao and Zeng [51] and Weber et al. [49].

Zhao and Zeng [51] (their paper has not yet been published but is available online) try to exploit the structure of the decisions by making a two-stage stochastic program which can be used to solve the day-ahead unit commitment problem. In the first stage the status of the generators has to be determined. In the second stage the production level is decided in relation to the unknown factor of wind energy. Another very well-known unit commitment model is WILMAR (*Wind Power Integration in Liberalised Electricity Markets*). This is a model that analyses the integration of wind power in a large liberalised electricity system such as the Danish, Finnish, German, Norwegian or Swedish system. A detailed description of the model can be found in Weber et al. [49]. It consists of two modules: the Scenario Tree Tool (STT) and the Scheduling Model (SM). The first one builds a scenario tree and the second one minimises the expected operational cost of meeting load and reserve demands subject to all modelled constraints taking into account all different paths of the scenario tree.

In general, unit commitment models are used for analysing electricity systems in order to see how the system will react to different policies. But unit commitment models are not designed to analyse the systems within the hour and they are too computationally complex to run on a shorter time-resolution than one hour. Therefore, intra-hour modelling is a very interesting topic to look into.

1.4 Balancing models

Even small deviations in wind speed can entail rather large deviations in the power production and this can only be analysed properly with intra-hour models. Another very interesting topic that justifies the development of intra-hour models is ramping. Ramping on interconnectors is getting more and more interesting as electricity is traded across countries since different ramping speed and ramping rules on interconnectors create imbalances in countries that have a lot of transit electricity.

Little research has taken place within this topic yet, but as more and more researchers find it necessary for this topic to be investigated, more research is being done in this area. The way of modelling intra-hour models is closely related to the topic of unit commitment models, since many of the same things have to be considered. Therefore, some of the same types of constraints appear in both types of models or at least one could get inspiration from unit commitment models when making intra-hour models.

Our literature review revealed some papers on intra-hour models. A model for estimating the socio-economic outcome of an integrated Northern European power market can be found in Doorman and Jaehnert [34]. The paper focuses on a northern market since its focus is on hydro power resources. The time resolution is 15 minutes and the model can be used to illustrate how resources or reserves should be used optimally across regions. The model uses historical data for the wind power production and the system error. Lindgren and Söder [39] present a multi-area optimisation model that takes uncertain wind power forecasts into account. The model re-optimises each time a new wind power forecast is available. The focus of the model is on minimising the real-time balancing cost by concentrating on which bids of regulating power to call and when to call them. One of the first models or tools presented that was able to do something like Lindgren and Söder [39] can be found in Bakken et al. [25] where the Stepwise Power Flow model is presented. This is a regular modified AC power flow algorithm that runs in five minute time steps. Banakar et al. [26] make a simulation study that also takes wind into consideration, investigating the impact of minute to minute wind generation on the system operation. However, as pointed out in Olsson et al. [43], this study does not have a stochastic representation. All models mentioned do to some extent take the market into consideration. Olsson et al. [43], on the other hand, do not take the market into consideration. In this paper they develop models based on stochastic differential equations that describe the balance in continuous time. These models can be used to evaluate the impact of wind energy on the real-time balancing of the system.

A common drawback of all these models is that neither of them can handle the systems you find in reality since their assumptions do not meet real life requirements. Therefore, it makes sense to look further into this topic in order to see if we can model the actuality more closely. For example, none of these models take ramping into consideration, although it is an essential element. This is the reason why we are in the process of developing such a model - the working paper can be seen in Appendix B and it is joint work with Ditte M. Heide-Jørgensen, Trine K. Boomsma and Nina K. Detlefsen.

* * *

Having introduced our research subject above the proposals consitituting the project will be found on the following pages. Our review of interesting literature found in this section will be repeated in both of the working papers, since it is relevant information in all three places.

2 Proposal I: An optimisation model for balancing power

There are various types of reserves which TSOs can use when balancing the energy in real time. Often, the reserves are categorised into automatic and manual reserves. The automatic reserves are automatically activated when the frequency deviates from a given set point value. Manual reserves are manually activated by the TSO in the area that causes the frequency change. These are normally used to release the automatic reserves. In Denmark the TSO also uses manual reserves to smoothen out the expected imbalances beforehand. They do so by activating the reserves based on forecasts. This reduces the expected imbalances and often results in reduced need for automatic reserves. Since automatic reserves are more expensive than manual reserves this is beneficial from an economic point of view.

In this way the use of balancing power can be optimised, and this optimisation is the foundation for our intra-hour model. Our model is described in a working paper that can be found in Appendix B. Until now we have mainly focused on describing a mathematical model for intra-hour balancing. Next we will implement the model and make some analyses of it.

2.1 The model

Our model, OPTIBA, optimises the use of manual reserves based on predicted imbalances. It takes the next two hours into consideration, and if deemed beneficial it activates balancing power taking into account ramping constraints and a minimum activation period. The model can be used to analyse electricity systems intra-hour and will be useful if different scenarios for future electricity systems have to be examined.

New information about wind power forecasts and outages is essential for the model and continual new information is the reason for implementing the model as a rolling planning model. By rolling planning we mean that with a two hour horizon the model will be repeated for each hour where relevant information is passed on between the runs of the model.

2.2 Implementation

We will start by implementing the model in the modelling language GAMS in order to see how efficiently and fast it runs. The likely result will be the conclusion that the model needs efficient solution methods, but for now it will be good enough just to implement it in GAMS since it can give us a picture of the efficiency. The implementation can also help us understand how robust the model is to changes in wind power production. The main reason for implementing the intra-hour model in GAMS is that we would like it to communicate with the unit commitment model WILMAR by Weber et al. [49], and since WILMAR is implemented in GAMS it would be preferable also to implement our model in GAMS.

2.3 Future work

First we will implement it, then we will run it on some small data sets.

In the future we will implement it such that it works together with WILMAR. It should be implemented in a way that allows information to be fed back to the unit commitment model if we have started any power plants. Figure 1 depicts the expected interaction.

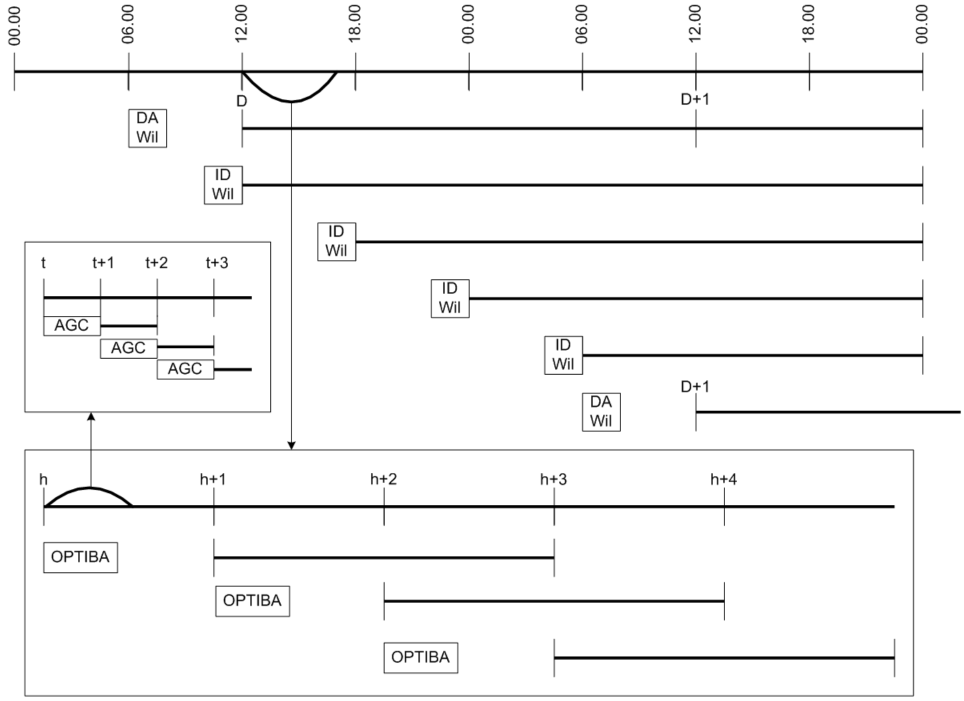

Figure 1: How OPTIBA will interact with other models.

The figure shows how WILMAR, OPTIBA and a model for Automatic Generation Control (AGC) will run over a time horizon of two days. The time line is shown in the top of the figure. Each time we run a model we have a box with the name of the model and a line next to it. The line illustrates the running period of the model. First, the figure shows the runs of the unit commitment model. We will run WILMAR with day-ahead settings every 24 hours and in between we will run it with intraday settings every sixth hour. Between each intraday run we will run our intra-hour model; here we will run it six times with rolling planning. Each run of OPTIBA will have a time horizon of two hours but it will be repeated every hour. If we had a model for automatic generation control, then we would run this model between each run of the intra-hour model with a time step of t minutes. We believe that such a model would be beneficial but will leave it to others to develop it.

Each run of WILMAR resembles either the day-head or the intraday market. The positions of each participant in the market are decided upon here. Afterwards, with the run of OPTIBA we imitate what is done in the control room. The AGC model should then imitate the use of automatic reserves. If all these models are brought to communicate with each other at a later stage, we will have a model that reflects Danish real-life production very closely.

When finished, this interaction between the models can be used for analysing the electricity system when different scenarios have to be examined. Then, for example, the Danish TSO could look at how the Danish system would react on 50% of the electricity coming from wind power in 2020.

3 Proposal II

The basis for this paper will be the model described in Proposal I. In this paper we will simplify the model, but make it stochastic. The simplification of the model will mainly consist in not allowing the model to start up any units; this way we will not have as many binary variables as before. In relation to the stochastics of the model, there are at least two stochastic parts of the model, namely the demand and the wind power. Wind power is the most uncertain factor when balancing energy, and therefore we have chosen to make that part stochastic (and assume that the demand is known). Given that we have access to reliable data for wind power gathered over the past many years we may be able to construct realistic probability functions or scenarios for describing the wind power in a stochastic setting.

We propose to construct a stochastic two-stage mathematical programming model where start-up of power plants has to be decided upon in the first stage, and the amount of balancing power has to be decided upon in the second stage. The decision about the amount of balancing power is closely related to the amount of wind power and this is the reason for having that decision in the second stage.

4 Proposal III

In the third paper we will look into solution methods for both the deterministic and the stochastic model. Until now not much research has been carried out in this area, maybe due to the fact that it is mostly engineers who have proposed mathematical models for analyzing purposes within the field of energy.

The first model, described in Proposal I, is expected to be implemented in GAMS. Keeping the complexity of the two models in mind it may be possible to improve their efficiencies by developing and applying purpose-specific solution methods. One possibility may be to see if there are specific structures in the models which can be exploited. Often, when considering large scale mathematical programs, decomposition methods such as the L-shaped method are necessary to solve the models to optimality. We also expect to implement and test the developed algorithms on a number of distinct scenarios.

5 Workplan

My PhD project is expected to follow the standard three-year duration of a PhD scholarship. When divided into six modules, one for each semester (spring and fall), then the following outline describes when the different PhD activities are expected to take place.

S11 Investigation of the relevant literature with focus on stochastic programming and the application to the electricity markets was made. I spent two months at Energinet.dk where I obtained knowledge of the Danish electricity market and the problems Energinet.dk faces when designing markets. Together with Nina K. Detlefsen I wrote a small paper for the Ackerman Conference that was held in Aarhus on 25th and 26th of October 2011.

F11 I presented a paper on metaheuristics at OR 2011 Zurich: International Conference on Operations Research. I also attended the Ackerman Conference. I started working on the first paper on modelling of balancing markets and this working paper has been presented at two different workshops.

S12 The working paper on modelling of balancing markets is expected to be finished and the second paper is expected to be started. I plan to have a change of environment for three months, where I will visit professor Asgeir Tomasgard in Trondheim. I also hope there will be time for me to visit Energinet.dk for a month or two as well as time tovisit Kim Allan Andersen in Greenland.

F12 My teaching obligations are expected to be completed. The second paper is expected to be finished.

S13 The third paper is expected to be started.

F13 The completion of the third paper and my PhD thesis.

Of course various conferences and summer schools should be attended. A time schedule of the most important obligations of my PhD can be seen in Table 1. First, there is a schedule of when I plan to work on the different papers. Next, it shows when I will fulfil my teaching obligations and when I will take courses as well as when I will make my change of environment.

Table 1: Time table

	S11	F11	S12	F12	S13	F13
Ackerman Paper	X	X	X	X	X	X
Paper I		X	X			
Paper II			X	X		
Paper III					X	X
Teaching		X		X		
Courses	X	X	X	X		
Change of environment			X			

Teaching

Regarding my teaching obligations I have been a teaching assistant in Management Science Models and I have graded exams for Mathematics and for Management Science Models. So far, I have covered a total of 309.5 out of 570 hours of teaching.

I plan to teach the rest in the fall of 2012. Most of the hours will be in Management Science models.

Courses

Which courses I have taken and which courses I plan to take can be seen in Table 2.

Table 2: Courses

Organiser (Time)	Title	ECTS
AU (S11)	Reviewing papers on OR applications in logistics/SCM/OM	5.0
NTNU (S11)	Managing uncertainty in energy infrastructure investments	7.5
AU (F11)	Branch and Bound and CPLEX implementations	5.0
CET DTU (F11)	Electricity Market Design and Operation	2.5
AU (S12)	Metaheuristics	5.0
AU (E12)	Electricity Markets	5.0

Change of enviroment

Institution: NTNU - Trondheim (Norwegian University of Science and Technology)
Contact: Asgeir Tomasgard
Professor in managerial economics
Department of Industrial Economics and Technology Management
Period: April 2012 - June 2012

Presentations

OR 2011 Zurich - International Conference on Operation Research

Working paper: Iterated local search and record-to-record travel applied to the fixed charge transportation problem.

10th International Workshop on Large-Scale Integration of Wind Power into Power Systems as well as on Transmission Networks for Offshore Wind Power Plants (also known as the Ackerman Conference)

Working paper: Operational management of intraday and balancing markets - a survey.

SimBa Workshop

Working paper: An optimisation model for balancing power.

Appendices

A Operational management of intraday and balancing markets - a survey

This is joint work with Nina K. Detlefsen. N. D. works in Systems Analysis at Energinet.dk, 7000 Fredericia, Denmark (e-mail: nid@energinet.dk).

The paper has been presented at the *10th International Workshop on Large-Scale Integration of Wind Power into Power Systems as well as on Transmission Networks for Offshore Wind Power Plants* and it has been published in the proceedings of the workshop.

The workshop link: `http://www.windintegrationworkshop.org/`

Operational management of intraday and balancing markets - a survey

Jeanne Andersen and Nina K. Detlefsen

Abstract

Recent years have seen increased focus on the challenges that will arise as electricity has to be balanced and markets have to be integrated across countries and regions in the future. Close cooperation is necessary to meet those challenges. In order to evaluate the benefits of cross-border balancing, it is essential to understand the mechanisms and operations of the European markets and how they interact. Therefore, this paper aims at describing the European electricity markets: day-ahead markets and markets with shorter time spans. We present the most promising theoretical models found in the literature with applications for balancing. The purpose of this is to identify the tools necessary to analyse future balancing of power systems.

Keywords: balancing, cross-border collaboration, market design, reserves.

A.1 Introduction

The European renewable energy targets state that renewable energy should comprise a substantial percentage of the total energy production in each European member state in the near future. Integration of more wind energy will be a major contributor to reaching this target. However, the fluctuation of wind is known to create significant imbalances in the electricity system at the time of operation and this will lead to a need for more balancing power [37]. If closer collaboration is established across the borders of the European countries and regions, the amount of balancing power can be reduced. A straightforward example of beneficial cross-border collaboration would be to exchange opposite imbalances across borders since this would reduce the need for balancing power in the regions. We will provide an example of this later in this paper.

Most of the European TSOs (Transmission System Operators) operate according to the N-1 security criterion as a minimum. The N-1 criterion states that the TSOs have to ensure that they have enough reserves to at least cover an outage of the largest generator, transmission line, transformer or reactor [23]. In principle, each electricity system has to have the necessary reserves available within its own system, but with collaboration the opportunity of buying reserves across the borders can arise. In some cases, this can result in reduced prices [19]. However, in order to make collaboration beneficial and in some places even possible, it is essential that the balancing markets become more harmonised across the regions [23]. Knowledge about the market design and operations of other regions would be enlightening and necessary for this process to happen.

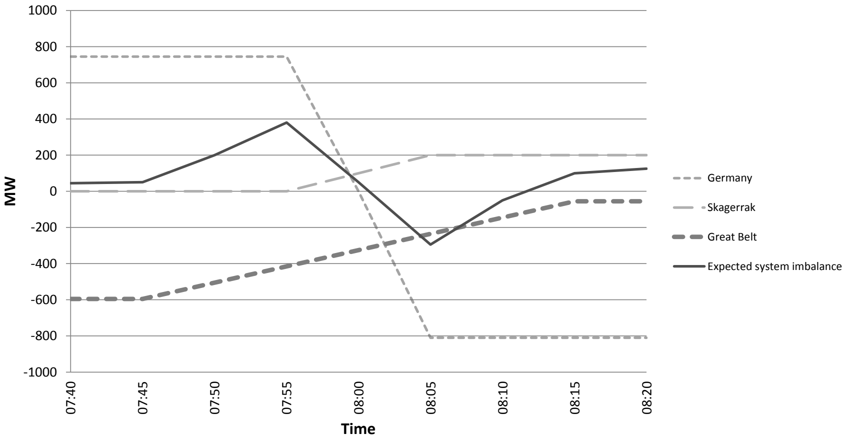

Figure 2: The scheduled amount of MW on the interconnectors as per 9 October 2009. The expected imbalance is also shown.

But closer collaboration has its downsides. With closer collaboration some countries will experience an extreme amount of transit electricity in their systems which can create major imbalances. Imbalances may occur around hour shifts if the flow direction changes and if the individual interconnectors have different ramping speeds. Located between Central Europe and the Nordic countries, Denmark very often has a lot of transit power in its system today since the ramping speed (MW/min) differs between the two directions. Figure 2 is an example of the influence that this transit power has on the Danish system, showing the scheduled amount of MW on some of the interconnectors into Denmark West on 9 October 2009. Only the schedules for the hour shift around 8 am are shown. As it can be seen in the figure, ramping as well as the different ramping speeds between Germany and the Nordic countries are a major source for creating imbalances in the Danish system. On the interconnector from Denmark to Germany the ramping starts five minutes before the hour shift and has to be completed five minutes after the hour shift. By contrast the ramping on interconnectors between the Nordic countries can start 15 minutes before the hour shift and the ramping has to be completed 15 minutes after the hour shift. In Figure 2 there is no exchange of electricity from 7 am to 8 am on the Skagerrak interconnector and therefore in this case the ramping first starts five minutes before the hour shift. The figure shows a bigger difference in the expected system imbalance than the interconnectors actually create. This is simply due to the fact that the expected system imbalance is for the overall system and there are factors apart from the interconnectors influence the system.

This phenomenon with transit electricity and ramping around the shift of the hour along with the fluctuation of renewables creates a need for tools that can analyse the system balance intra-hour. Furthermore, with an increased share of wind power the production characteristics and capabilities are expected to change [33], [36]. Hence, it is important that the tools can model the technical capabilities of the production, the load fluctuations and ramping in order to determine which challenges the electricity systems will face in the future. Mathematical programming is a good tool for analysing the electricity systems. Concerning intra-hour modelling, not much has been published in the literature. Most of the models concentrate on modelling the problems concerned with one hour to years ahead, often with an hourly resolution. An example of this is the WILMAR (Wind Power Integration in Liberalized Electricity Markets) model [49]. This stochastic model can be used to analyse how an electricity system will react to large-scale integration of wind energy. Specifically, it can give information about the impact that increased wind generation will have on reserve needs, power plant operation and system costs in the long run. The time resolution is one hour and it is not an adequate tool to clearly illustrate the problematic aspects of fluctuating wind energy and different ramping speeds when the energy has to be balanced within the hour of operation.

Regarding intra-hour models, some publications can be found in the literature, though. A model for estimating the socio-economic outcome of an integrated Northern European regulating power market can be found in [34]. The paper focuses on a Northern market since its focus is on hydro power resources. The time resolution is 15 minutes and the model can be used to illustrate how resources or reserves should be used optimally across regions. The model uses historical data for the wind power production and the system error. [39] presents a multi-area optimisation model that takes uncertain wind power forecasts into account, but it does not consider ramping. The model re-optimises each time a new wind power forecast is available. The focus of the model is on minimising the real-time balancing cost by concentrating on which bids of regulating power to call and when to call them. [25] presents the Stepwise Power Flow model which is one of the first models or tools that is somewhat similar to [39]. This is a regular modified AC power flow algorithm that runs in five minute time steps. Reference [26] is a simulation study that also considers wind energy production and investigates the impact of minute to minute wind generation on the system operation. However, as pointed out in [43], this study does not have a stochastic representation. To some extent all the mentioned models do take the market into consideration, but if only the balance without the market has to be considered, [43] would be relevant. In this paper they develop models based on stochastic differential equations which describe the balance in continuous time. These models can be used to evaluate the impact of wind energy on the real-time balancing of the system.

The challenges posed by the handling of wind energy and other renewables will make intra-hour balancing a very interesting topic in the years to come. Even though models have been developed on this topic there is a need for more research within the area to really clarify all the needed aspects. Together with the Technical University of Denmark the Danish TSO, Energinet.dk, is actually in the process of developing a tool called SimBa (Simulation of the Balance). When finished, SimBa can be used for calculating/simulating the regulation cost related to intra-hour balancing, taking wind production, other fluctuating production and consumption units into account. The model will be based on the Danish balancing principle and therefore, at first, this tool will only apply to other countries with similar balancing mechanisms.

Besides intra-hour models for analysing the system other tools are important when balancing a system with a large share of wind energy. It is essential to have good wind power forecasting tools in order to predict the wind power production and reduce the system imbalances. Reference [48] shows that the total system forecast error asymptotically converges to the wind forecast error as the proportion of wind capacity installed in the system increases. Thereby, they show that with large wind power capacities the wind forecast error becomes more important and this again stresses the fact that it is essential to have good wind power forecasting models. A system with a high share of wind power depends on the accuracy of the models since this will affect the level of reserves needed to make the system secure. Many TSOs or other balance responsible parties use more than one wind power production model and more than one meteorological model to forecast the wind production since it has been shown that combining models reduces the forecast error [42]. For an overview of some wind power forecasting and prediction models see [38] and [35].

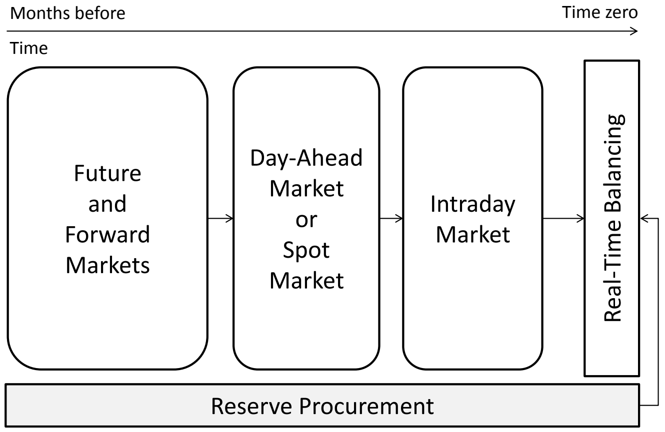

Figure 3: Market structure.

In the next couple of sections we will go through some of the market operations and mechanisms around Europe. We do so in order to illustrate where we are today and how much collaboration is going on. For an overview of the most common market structures in Europe see Figure 3. The figure shows the sequence of market openings. First, the future and the forward markets open and here trading can take place many months before the hour of operation. As the name suggests, the day-ahead market opens the day before the energy has to be delivered. After gate closure of the day-ahead market the intraday market opens and it closes just before the hour of operation. Reserve procurement is independent of these markets and therefore it is handled separately. Market coupling on day-ahead markets in Europe will be presented in section A.3, which shows that this is where most of the integration between markets in Europe has taken place. Then the intraday and balancing markets will be reviewed in sections A.4 and A.5, respectively. Finally, we conclude in section A.6. But first we will give a small example to illustrate the benefits of cross-border collaboration.

A.2 Example of beneficial cross-border collaboration

Let us take a fictitious example and assume that we have two countries with an interconnector. There are no capacity limits and no restrictions attached to the interconnector. The historical imbalances of the two systems, if no regulating power had been activated, on a given day can be seen in Figure 4.

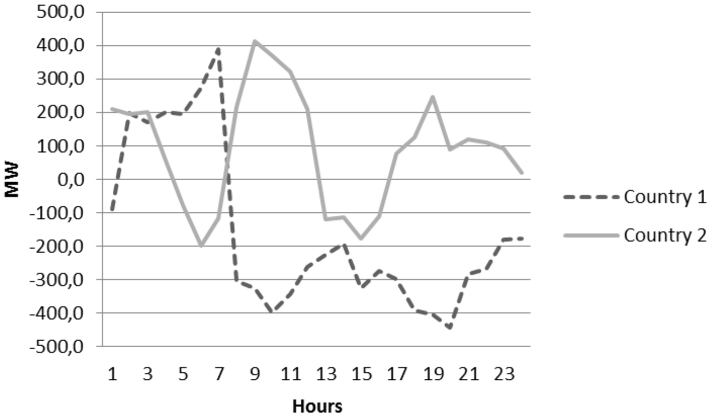

Figure 4: 24-hour graph of the imbalances of two countries.

The gain expressed in MW by exchanging opposite imbalances between the two countries can be calculated as follows: let us assume that we have two continuous functions f and h. Both should be functions of time $t \in [0, T]$ and they should illustrate the imbalances of the two systems at time t. Define a new function g, which is the amount of MW exchanged at time t. $g(t)$ can then be calculated using the following formula.

$$g(t) = \begin{cases} 0 & \text{if } f(t) \cdot h((t) \geq 0 \\ \min\{|f(t)|, |h(t)|\} & \text{otherwise} \end{cases} \tag{9}$$

$g(t)$ is calculated numerically and is a function greater or equal to zero since exchanging imbalances is a benefit that can be expressed in MW for both parties. This exchange results in an overall reduction in the imbalances for both countries.

The total amount of MW exchanged, G, can then be calculated using the formula below.

$$G = \int_{t=0}^{T} g(t)\,\mathrm{dt} \tag{10}$$

Figure 5 on the next page shows the imbalances of the two systems after the exchanges have taken place. The figure clearly illustrates that the imbalances of the two systems are reduced noticeable. The exchange of opposite imbalances is beneficial for both countries since otherwise

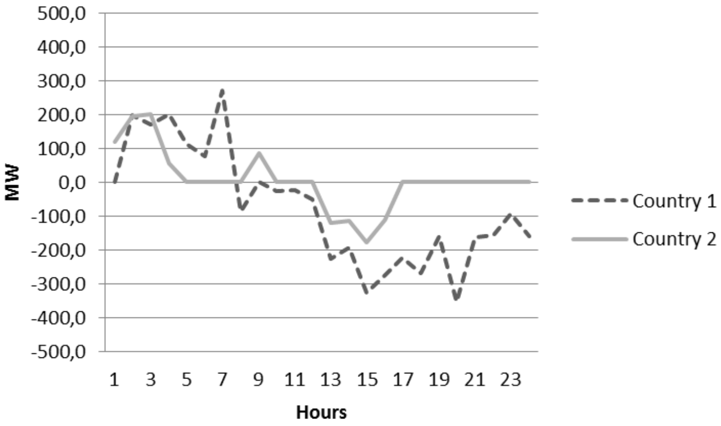

Figure 5: The imbalances of the two countries after opposite imbalances have been traded.

each country should buy regulating power. If capacity restrictions had to be taken into account formula (9) and formula (10) should be a bit different. In that case there would be an upper limit on how much power that could be exchanged and the resulting advantage would have been reduced. [23], on the other hand, provides an example illustrating the opportunity to buy reserves on either side of an interconnector where the capacity restrictions and the market mechanisms are taken into account.

A.3 Day-ahead markets and market coupling

Market coupling is a way of integrating day-ahead markets and thereby reducing costs and ultimately increasing social welfare in various areas and countries. As described in [41] and [1], market coupling is an implicit auction involving two or more power exchanges. It is an implicit auction since transmission capacity is implicitly included in the electricity trading. One result of implementing market coupling is that the prices between the areas of the power exchanges are identical if there is sufficient capacity on the interconnectors. This means that the electricity can flow from the low-price area to the high-price area. Another result is increased liquidity in the markets. In the same category as market coupling is market splitting. Market splitting is also an implicit auction, but it is handled by a single power exchange.

Market splitting is implemented today between the Nordic countries including Estonia. The Nordic countries were among the first in Europe to implement this form of collaboration. In 2006 market coupling on interconnectors between France-Belgium and Belgium-Netherlands was introduced on the day-ahead market [2]. This market coupling was called the Tri-Lateral market Coupling (TLC) and was expanded in 2007 when Luxembourg and Germany together with the members of TLC signed a Memorandum of Understanding (MoU). In 2010 Luxembourg and Germany/Austria were included in TLC which thereby became the Central Western Europe Market Coupling (CWE). At the same time a tight volume coupling between Germany and Denmark was introduced. With this coupling the CWE market, consisting of Belgium, the

Netherlands, France, Luxemburg and Germany/Austria, was linked with the Nordic market [3]. At the end of 2010 the Polish power exchange PolPX was also coupled to the Nordic market via the interconnector, SwePol Link, between Sweden and Poland [4]. In April 2011 market coupling between the Netherlands and Great Britain via the BritNed cable was established [5].

The described market couplings have joined and connected the day-ahead markets of a large number of countries in Europe, but market coupling between just two countries has also taken place. Take, for example, the Iberian market (MIBEL) that joins Spain and Portugal. This market was launched in 2007. The Iberian market is a single day-ahead market with a market splitting mechanism in case of congestion [6]. Another example can be found on the border between Italy and Slovenia where a bilateral market started at the beginning of 2011 [7]. Today market coupling is also in place between the Czech and the Slovak Republics and this market coupling is planned to be taken further by integrating the Czech and Slovakian markets with the Hungarian market in 2012 [8].

This process of integration and harmonisation of day-ahead markets through market coupling will continue in the future; more projects are already under development.

A.4 Intraday markets

Intraday markets are used for buying or selling energy in the case unforeseen events occur after the gate closure of the day-ahead markets. In this manner the participants can reduce their imbalances for which they would otherwise be held responsible. In the case of wind power, for instance, the closer to the hour of operation the better the wind forecasts which could be a reason for trading on the intraday market.

On intraday markets there are different ways in which trading can take place. Some of these are: Over The Counter (OTC), Power eXchange (PX) continuous trade and PX sessions [22]. OTC is a term used for transactions concluded directly between two traders or through a dealer network. PX continuous trade is anonymous trade made through a system operated by a central exchange. The bids are taken on a first come, first served basis and the products typically have to be standard products in order to make a transaction possible. Trading using PX sessions means that the market is cleared one or more times during the intraday period. It resembles the day-ahead market since there is a gate opening and a gate closure for every session within the intraday period. When cross-border intraday capacity is to be allocated, it can be done in several ways: first come, first served; pro rata and auctions [22]. When operating under the first come, first served principle, the market participants have to inform their local TSO on how much of the remaining capacity they are interested in. The TSO then verifies whether the capacity is available and the participants are allocated capacity in the order in which the requests arrive. Pro rata, on the other hand, is a way of allocating the capacity to the market participants at a number of predefined gate closures in proportion to the requested amounts. The latter way uses auctions which are managed according to the normal principles of explicit auctions. Further information about these principles and when/how they are used can be found in [22].

Across Europe the technicalities and liquidities of intra-day markets differ. In Great Britain and Ireland, for example, they do not have the market structure shown in Figure 3. Great Britain does not distinguish between day-ahead and intraday markets. Instead they have a single market, called the spot market, where half hour bids are traded. This market closes one hour prior to the time of operation [9], [10]. In Ireland, including Northern Ireland, they do not yet have an intraday market, but they are in the process of developing one which should be ready in 2012 [24]. The market structure of Great Britain and Ireland differs from the market structure within the Nord Pool Spot that has a structure similar to the one illustrated in Figure 3. On the intraday market, Elbas, players can trade products with an hourly resolution in order to balance their positions when the day-ahead market, Elspot, is closed. Its trading period starts two hours after the gate closure of Elspot and ends one hour prior to the hour of operation. Elbas is a continuous cross-border intraday market of the form PX continuous trade, where cross-border capacity is implicitly allocated using market splitting between the Nordic countries and tight volume coupling on the Kontek interconnector between Denmark and Germany. Elbas covers the Nordic countries, Estonia and Germany via Kontek. At the beginning of 2011 an integrated cross-border intraday market was implemented between APX-ENDEX and Belpex in the Netherlands and Belgium, respectively, using the Elbas technology. In the future they will collaborate with Nord Pool Spot in order to establish an integrated cross-border intraday market [11]. Another example of intraday market collaboration is in the Iberian market that joins Spain and Portugal.

A.5 Reserve and balancing markets

Often the following distinctions are made when talking about reserves: primary control, secondary control and tertiary reserve [18]. The first two belong to the category of frequency control and the last one is a slower manual reserve. Primary control is automatically activated when the frequency deviates from the set point value by a predetermined amount in order to maintain the balance between demand and generation in the network. In the former area of Nordel and UCTE primary control has to be fully operational within 30 seconds after a disturbance has occurred. Secondary control is only used in the former UCTE area and it is used to restore primary control. It is activated 30 seconds after a disturbance and has to be fully operational within 15 minutes. The tertiary reserves have slower response times and are manually (or sometimes automatically) activated. In both the former Nordel area and the former UCTE area the tertiary reserves have to be fully activated within 15 minutes after activation. It restores the primary and secondary controls. In Denmark the Danish TSO tries to predict anticipated imbalances and activates the manual reserves in order to minimise the use of the automatic - and more expensive - one.

Ireland has frequency control that consists of a primary operating reserve, a secondary operating reserve and a tertiary operating reserve 1. In the category of manual reserves they have a tertiary operating reserve 2 and a replacement reserve [12]. The primary operating reserve is the first one to be activated and it has to be available from 5 to 15 seconds after an event that causes the frequency to drop. Then the secondary operating reserve is activated and this one has to be fully available from the 15th second and for further 75 seconds. The tertiary operating reserve 1 then restores the secondary reserve by being fully available 90 seconds after the event and for further 210 seconds. The manually activated tertiary operating reserve 2 restores the tertiary operating reserve 1. It has to be fully available from 5 to 20 minutes and then the replacement reserve takes over. This reserve is a longer term resource that has to be able to be in full operation for 4 hours. Further details of the system of Ireland can be found in [12].

In Great Britain they control the frequency by means of three different responses: primary response, secondary response and high frequency response [13], [21]. The first two responses are automatically activated if there is a decrease in the frequency. Primary response has to be in full operation within 10 seconds and for further 20 seconds. Secondary response has to be fully available from 30 seconds after the drop in frequency and sustainable for at least 30 minutes. High frequency response, on the other hand, is automatically activated when an increase in the frequency in the system is registered. This response has to be fully available from 10 seconds after the event and then remain operational for as long as needed. In Great Britain where they also have manually activated tertiary reserves, they have four different reserves: a contingency reserve, a short term operating reserve, demand management and a fast reserve [13], [21]. The contingency reserve covers longer term plant losses as well as demand forecasting errors. It is divided into start up and hot standby services. These services have to be available in the system in order to get access to additional generation that would not otherwise be available and which could not have been made available due to their technical characteristics and associated lead times. The short term operating reserve is used for short term generation losses and demand forecasting errors. It has to be available not later than 4 hours after activation and has to be able to operate for at least 2 hours when instructed to do so. Demand management is a bilateral agreement that ensures a reduction in active power from demand sites if necessary. The last reserve is the fast reserve and this is required for the maintenance of system frequency. It has to be able to cover sudden and unpredictable frequency changes and therefore, it has to be provided within 2 minutes of notice and to be sustainable for at least 15 minutes. For more details on the British reserves see [13] and [21]. For further information about the technicalities of the balance management of each country in Europe in general see [18], [14] and [27].

These different technicality requirements for the frequency and the tertiary reserves in various countries make it difficult to have fully integrated markets across borders. There are different ways in which cross-border balancing or trading can take place, though, and they require different degrees of harmonisation between the markets. The different strategies are listed below and can be found in [47] and [20]:

- *No Trading.* Each area procures its own reserves in its own area based on its own requirements. No trading takes place.

- *TSO-TSO trading.* Each TSO has to have contracts on reserves within its own area based on its own requirements, but the TSOs are allowed to trade with each other on a voluntary basis. A variation is that Balance Responsible Parties (BRP) are allowed to trade with TSOs.

- *Cross-border reserve trading.* The TSO can procure some of its need for reserves through a balancing market that allows offers from other control areas.
- *Sharing of reserve capacity.* TSOs can agree on sharing a common reserve to which the individual TSO each supplies a share. None of the TSOs has the exclusive right to use the reserve. The reserve is activated according to a merit list when transmission capacity is available.
- *One regional control area.* The control areas are joined into one global area, where only one party is responsible for maintaining the balance.

With its implemented NOIS system the Nordic region is a good example of integration of balancing markets among European countries. The NOIS system belongs to the cross-border reserve trading category and is a list containing all up and down regulation offers in the Nordic countries. In order to activate an offer in a neighbouring control zone of the TSO, this TSO must contact the TSO responsible for the neighbouring area and if there are no bottlenecks in the system, the offer is activated. When an offer is activated, it should be available within 15 minutes. This Nordic market has not yet been coupled with the German balancing market due to dependencies in the capacity market. German TSOs are only allowed to balance their control areas using balancing power that has been contracted and these contracts include transmission capacity that is reserved beforehand either by TSOs or market participants. In Germany the four TSOs (EnBW Transportnetze AG, TenneT GmbH, Amprion GmbH and 50Hetz Transmission GmbH) use a common platform, called regelleistung.net, to buy their balancing power [15]. This is a daily or weekly tender auction, where the participants get the exclusive rights of providing the reserves. Each TSO in Germany may need to have an amount of control reserve available within its own control area, in order to ensure the security of the system. Cross-border collaboration also takes place on the interconnector between France and Great Britain. Here they use a tool called BALIT to manage the collaboration. BALIT enables RTE (the TSO in France) and National Grid (the TSO in Great Britain) to exchange offers up until one hour before delivery for one hour balancing products [16]. Similar cross-border balancing arrangements are in place across the Moyle interconnector between Ireland and Great Britain [17]. In France the market players from neighbouring countries are allowed to participate in the French balancing market; this is not a common market since trading is only in one direction. France does not have agreements with other TSOs except from National Grid.

A.6 Conclusion

When an increasing amount of renewable power is put into the grid the balancing of the system becomes a challenge that calls for tools for analysing the system on an intra-hour basis, and closer market integration will be needed especially in the sense of balancing markets. As can be seen in this paper, there is cross-border collaboration on day-ahead, intraday and balancing markets, although most of the integration has happened on the day-ahead market. The technicality requirements for the reserves of the individual electricity systems still do not conform to any agreed standard and make collaboration across borders difficult.

A.7 Acknowledgement

We gratefully appreciate the comments that we have received from various people at Energinet.dk who work in this field.

B An optimisation model for balancing power

This is joint work with Ditte Mølgård Heide-Jørgensen, Trine Krogh Boomsma and Nina Kildegaard Detlefsen. D. H.-J. is a PhD student at Risø DTU (email: dihj@risoe.dtu.dk). T. B. is Senior Scientist at Risø DTU (e-mail: trkr@risoe.dtu.dk). N. D. is employed by Energinet.dk, 7000 Fredericia, Denmark (e-mail: nid@energinet.dk).

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The workshop link: `http://www.risoe.dtu.dk/Conferences/VES_Workshop/workshop_six.aspx`

An optimisation model for balancing power

Jeanne Andersen, Ditte Mølgård Heide-Jørgensen,
Trine Krogh Boomsma and Nina Kildegaard Detlefsen

Abstract

Until recently, the modelling of energy system operations has mainly focused on hourly management. However, with the introduction of renewables such as wind power, fluctuations within the hour result in costs that are undetectable at the hourly level. Further intra-hour variations are caused by factors such as ramping on interconnection lines due to the increased transmission distances. Therefore, we introduce a model that can be used for analysing the electricity systems within the hour. It is formulated as a multi-area model and is capable of taking ramping into account. The model aims at reducing imbalances by activating manual reserves based on forecasts. In this way, total cost of reserves can be reduced since the manual reserves are cheaper than automatic reserves.

Keywords: balancing, balancing power, market design, optimisation.

B.1 Introduction

For many decades the energy supply was dependent on fossil fuels such as coal and oil. This has formed the structure of the energy market and even today fossil fuels make up a very big part of the global supply chain for energy. However, fossil fuels have a high climate impact and they are limited resources. Much focus has therefore been directed at sustainable energy development over the last decade. Sustainable energy development focuses mainly on three major aspects: energy savings on the demand side, efficiency improvements in the energy production and replacement of fossil fuels by various sources of renewable energy [40]. To achieve these aims in the energy systems the design of the systems has to be rethought and flexible energy technologies have to be implemented. In order to be able to do this, analyses of future scenarios have to be made. A most relevant scenario to examine is the impact that renewable energy such as solar or wind energy will have on the electricity system. They are expected to make up a substantial share of the energy production in the future but these resources are fluctuating and they create significant imbalances in the electricity system at the time of operation. Since the electricity has to be balanced, meaning that demand equals supply, these fluctuations will, presumably, lead to a need for more balancing power [37]. In order to have access to balancing power TSOs (Transmission System Operators) have to have some amount of reserves of electricity ready for activation. These reserves can be categorised into three types: primary, secondary and tertiary reserves. The first two are automatically activated reserves, whereas the tertiary reserve is manually activated. Primary reserve is activated when the frequency drops below a certain threshold. The secondary reserve is then activated in order to release the primary reserve. Finally, the manual reserve is activated to release the secondary reserve.

In this paper we will concentrate on the manually activated reserves and aim to optimise the use of these. The automatic reserves are the more expensive ones so it would be beneficial to let the manual reserve cover a greater share of the imbalances and thereby reduce cost. The manual reserves can be used more efficiently if activated at an earlier stage when major imbalances are expected. Hopefully, the need for automatic reserves can be reduced in this way.

Until now most of the research regarding modelling of electricity systems for analysing purposes has concentrated on unit commitment models, see for example Weber et al. [49], Cerisola et al. [30] or the overview by Padhy [44]. But with increased use of renewables in the electricity systems the modelling of reserve handling becomes an interesting topic. We need models that can describe the systems using time scales markedly below the hourly resolution used by the unit commitment models. Regarding intra-hour models, some publications can be found in the literature. A model for estimating the socio-economic outcome of an integrated Northern European power market can be found in Doorman and Jaehnert [34]. The paper focuses on a northern market and its focus is on hydro power resources. The time resolution is 15 minutes and the model can be used to illustrate how resources or reserves should be used optimally across regions. The model uses historical data for the wind power production and the system error. Lindgren and Söder [39] present a multi-area optimisation model that takes uncertain wind power forecasts into account. The model re-optimises each time a new wind power forecast is available. The focus of the model is on minimising the real-time balancing cost by concentrating on which bids of regulating power to call and when to call them. Bakken et al. [25] presented the Stepwise Power Flow model which is an earlier model capable of doing something like Lindgren and Söder [39]. The SPF model is a regular modified AC power flow algorithm that runs in five minute time steps. Banakar et al. [26] make a simulation study that also takes wind into consideration, investigating the impact of minute to minute wind generation on the system operation. However, as pointed out in Olsson et al. [43], this study does not have a stochastic representation. All models mentioned do to some extent take the market into consideration. Olsson et al. [43], on the other hand, do not take the market into consideration. In this paper they develop models based on stochastic differential equations that describe the balance in continuous time. These models can be used to evaluate the impact of wind energy on the real-time balancing of the system.

One common drawback of all these models is that neither of these models can handle the systems you find in reality since their assumptions are inadequate. One example is that all of these models do not take ramping into account, which is an essential element. For instance, different ramping speeds on interconnectors can create major imbalances in some countries with a lot of transit electricity. Therefore, it is highly relevant to look further into the topic of intra-hour modelling in order to see if we can model reality more closely.

Our model aims to optimise the use of balancing power. It does so by activating balancing power in response to predicted imbalances in order to reduce the actual imbalances that will occur. Therefore, the model has to be able to estimate the imbalances. For it to do so it collects information from the modules described in Section B.2 and displayed in Figure 6. It calculates and activates bids based on spare capacity on power plants and a minimum activation period. It is formulated as a multi-area model that takes ramping into account; both on power plants, on balancing power and on interconnectors. After activating balancing power based on forecasts the actual imbalances hopefully get smaller, but they will not disappear totally. The model optimises on the basis of cost of manual reserves and automatic reserves and therefore it will find a trade-off between these.

B.2 Optimisation of intra-hour balancing

Our modelling framework for optimisation of intra-hour balancing consists of a number of linked modules and is called the OPTImisation of intra-hour BAlancing model (OPTIBA). In the following we will explain the function of the modules in detail. In particular, we will describe each module of the model and how the modules interact. The modules can be seen in Figure 6.

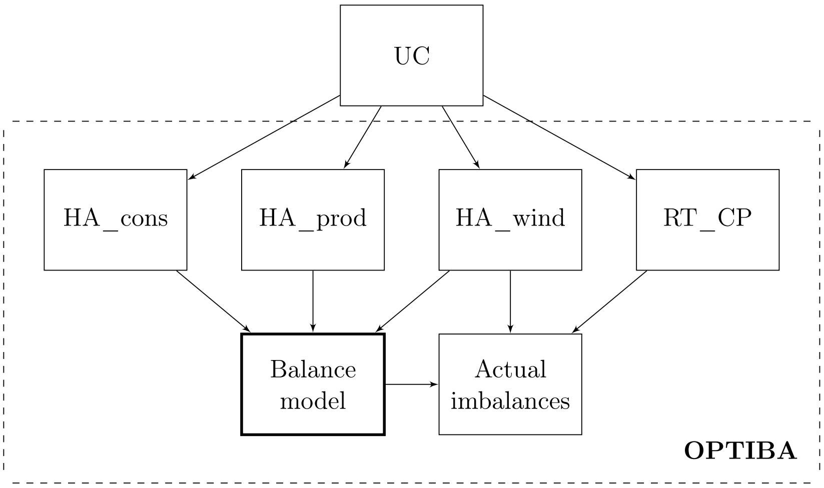


Figure 6: A diagram showing the modules of OPTIBA.

The first module, **UC**, has to collect information from a unit commitment model and here we will use WILMAR from Weber et al. [49]. This first module collects information about predicted consumption, wind power forecasts, production schedules from each electricity production plant, the bottlenecks in the transmission systems, import, export and unforeseen outages from the unit commitment model. All information given has an hourly resolution.

For intra-hour balancing we assume that the information is available in τ-minutes intervals. We construct this information in the modules: HA_cons, HA_prod and HA_wind. In real life, however, the Danish TSO has requested the power plants themselves to provide this information at five minute schedules and the consumption and the wind energy are based on forecasts.

The module, **HA_cons**, calculates schedules of τ-minute consumption based on updated hour-ahead consumption. This module receives schedules of expected consumption on an hourly basis from the UC module, updates it and converts the hourly schedules into τ-minute interval schedules describing the expected consumption in MWh for each interval. We apply a spline function between any two consecutive values of the hour-ahead consumption to convert the expected consumption.

The module **HA_prod** calculates updated hour-ahead production, import and export schedules on a t-minute resolution taking ramping into account. This module gets production schedules as well as information about import and export from the UC module. It then refines the schedules such that they are split into τ-minute intervals describing the expected production, import and export in MWh for each interval. This refinement is done by adjusting the hourly schedule of production, import and export according to ramp rates, both for the power plants and for the interconnection lines. For the remaining part of the hour the production, import and export are treated as constant. The ramping takes place in the first τ^{ramp} minutes of the operating hour (final ramping to the power level within this hour) as well as the last τ^{ramp} minutes of the hour (initial ramping towards the level of the coming hour). The reason for refining the production in this way is that the bids are hourly and there is no further information that could justify another way of doing it.

Updated information on forecasted and actual wind is simulated in the module **HA_wind**. The values are based on wind power forecasts from the UC model. First, the actual values are calculated which is done by modelling a day ahead wind power forecast error using a simple autoregressive process. This process is estimated by use of the maximum likelihood estimation. For simplicity the hourly forecasted values from the UC and the errors are converted to τ-minute interval values by means of linear interpolation. Subtracting the errors from the forecast gives the "actual" values for the wind. Then hour-ahead wind forecast errors are modelled using a persistence model on the actual values. These errors are added to the forecast in order to get the updated hour-ahead wind power production forecast for each τ-minute interval of a given hour.

The module **Balance model** optimises the use of balancing power. The method used for finding the optimal combination of activating balancing power based on forecasts contra leaving the imbalances to real-time balancing is described in detail and as a mathematical model in Section B.3.

Actual consumption and production are the contents of the **RT_CW** module (Real-Time Consumption and Production). The consumption as well as the production are assumed to be perfectly forecasted at the moment. Therefore, we get the information from the HA_cons and HA_prod modules. Information about outages from the UC module is also stored in this module.

Actual system imbalances are calculated in the module **Actual imbalances**. This is done by combining the actual production, the actual wind power and the actual consumption from HA_wind and RT_CP with the balancing power activated in the balance model. The reason for having this module is the possibility of quantifying the need for automatic reserves. The actual imbalance then has to be handled by the automatic reserves in the operating hour.

B.3 The balance model

The model is discreet. It optimises over a time horizon $T = \{1, \dots, t^{\text{end}}\}$. Since we design the model for rolling planning we also need a time period t_0, which is the period just before the start of the optimisation period of the model. In addition to the set T, we have subsets T^h, which consists of time periods in T that belongs to hour $h \in H$.

The objective function

Let $b_a^{+,t}$ and $b_a^{-,t}$ be two non-negative variables that describe the imbalances in a time period $t \in T$ in area $a \in A$. Furthermore, let $b_a^{+,t}$ denote the surplus and let $b_a^{-,t}$ denote the shortage of electricity in time period t, note that only one of them can be positive at any given time. When we have the expected imbalances, we can activate balancing power in order to minimise them. We can activate balancing power on a unit $u \in U_a$, $a \in A$, in time period $t \in T$, which is done by the two non-negative variables $\Delta p_u^{\text{up},t}$ and $\Delta p_u^{\text{down},t}$. Activation of balancing power on a unit entails some costs that depend on the cost structure of the unit. This is illustrated by the two parameters $c_u^{\text{up},t}$ and $c_u^{\text{down},t}$. The imbalances that occur real-time also entail a cost and therefore we let $\bar{c}_{\text{auto}}^t$ denote the expected average cost for the primary and secondary reserves at time $t \in T$.

The objective function (11) minimises the expected imbalance costs and the costs of balancing power. The reason for minimising the imbalance costs is that we would like the imbalance costs reduced as much as possible and the tertiary reserves are much cheaper than primary and secondary reserves. Thereby, the model finds a trade-off between costs of manual and automatic reserves. If we chose only to minimise the cost of activating tertiary reserves then the program would not choose to activate any since the cheapest option would be to do nothing. The last term of the minimisation sum is the cost of starting up additional units within the time horizon of the model. Here $c_u^{\text{start},t}$ is a parameter describing the cost of starting up a unit $u \in U_a$, $a \in A$ and $y_u^{\text{start},t}$ is a non-negative variable that indicates if unit u is in its start-up phase.

$$\min \quad \sum_{t \in T} \sum_{a \in A} \Big(\bar{c}_{\text{auto}}^t (b_a^{+,t} + b_a^{-,t}) + \sum_{u \in U_a} \big(c_u^{\text{up},t} \Delta p_u^{\text{up},t} + c_u^{\text{down},t} \Delta p_u^{\text{down},t} + y_u^{\text{start}} c_u^{\text{start}} \big) \Big) \tag{11}$$

The above objective function must be minimised subject to a number of constraints that will be described below.

The balance constraint

Constraint (12) describes the expected balance in each area $a \in A$, in which we have included the possibility of activating balancing power. Let p_u^t be a parameter that refers to the expected production on a unit $u \in U_a$, $a \in A$, in time period $t \in T$. The wind production in time period $t \in T$ in area $a \in A$ is denoted w_a^t and the demand is denoted d_a^t. Balancing power can either come from a unit in the balancing area or from interconnectors in the form of a change in import or export. The flow on an interconnector is denoted by the parameter $l_{aa'}^t$, $a, a' \in A$. The parameter can be positive or negative depending on the direction of the flow on the interconnector. If balancing power needed in area a is activated in another area it will result in a change in the flow on the interconnector between the areas. This change is denoted by the variable $\Delta l_{aa'}^{t}$. The balance constraint can be seen below.

$$\begin{aligned} \sum_{u \in U_a} (p_u^t + \Delta p_u^{\text{up},t} - \Delta p_u^{\text{down},t}) + w_a^t - d_a^t \\ + \sum_{a' \in A} (l_{aa'}^t + \Delta l_{aa'}^t) = b_a^{+,t} - b_a^{-,t} \quad (\forall t \in T, a \in A) \end{aligned} \tag{12}$$

Start-up of power plants

To ensure that a unit only provides balancing power when online we have a binary variable $y_u^{\text{on},t}$ that tells us whether unit u is online in time period t. It is 1 if it is online and 0 otherwise. Furthermore we have the non-negative variable $y_u^{\text{start},t}$ that tells us whether the unit is in its start-up phase, which is the first τ^{su} time periods after it has been turned on. To ensure this we let $\delta^{\text{su}}(t) = \min\{t - t_0, \tau^{\text{su}}\}$, since then the following constraint will be adequate. $\delta^{\text{su}}(t)$ is constructed such that the constraint also applies to the first τ^{su} time periods of the model's time horizon.

$$y_u^{\text{on},t} - y_u^{\text{on},t-\delta^{\text{su}}(t)} \leq y_u^{\text{start},t} \quad (\forall t \in T, u \in U_a, a \in A) \tag{13}$$

Since $y_u^{\text{start},t}$ is part of the cost function which is minimised it will always be either 0 or 1 due to the constraint above.

Each unit has a minimum uptime which means that it must be online for a minimum number of time periods after start-up. This is handled in constraint (14) below, where $\delta^{\text{up}}(t) = \min\{\tau^{\text{up}}, t^{\text{end}} - t\}$. Here $\delta^{\text{up}}(t)$ is constructed such that the constraint applies to the last τ^{up} time periods of the model's time horizon, where τ^{up} is the minimum uptime of the unit.

$$\sum_{s=t}^{t+\delta^{\text{up}}(t)-1} y_u^{\text{on},s} \geq \delta^{\text{up}}(t)(y_u^{\text{on},t} - y_u^{\text{on},t-1}) \quad (\forall t \in T, u \in U_a, a \in A) \tag{14}$$

It is implicitly assumed that $\tau^{\text{up}} \geq \tau^{\text{su}}$.

Balancing power as bids

The next two constraints determine the maximum deliverable balancing power of each unit. Constraint (15) ensures that the amount of up-regulation activated by each unit in any given time period is greater than or equal to a lower limit $p^{\min}$. This lower limit represents the minimum size of a bid for regulating power that can be provided to TSOs. This term is multiplied by a binary variable x_u^t, $t \in T$ and $u \in U_a, a \in A$, that is 1 if unit u has an activated bid in time period t and 0 otherwise. The constraint also ensures that the amount of up-regulation is smaller than or equal to the amount by which the unit will reach its capacity limit $\bar{p}_u$.

Constraint (16), on the other hand, ensures that the amount of down-regulation offered by each unit is smaller than or equal to the maximum allowable decrease in production level for the unit. $\underline{p}_u^t$ is the lowest production level that the unit can handle. Down-regulation bids for a unit also have to be greater than the predetermined amount $p^{\min}$.

Both constraints have a binary variable that makes sure that the regulation amount is zero if the unit is turned off.

$$p^{\min} x_u^t \leq \Delta p_u^{\text{up},t} \leq \left(\bar{p}_u - \max_{t' \in T^h} p_u^{t'} \right) y_u^{\text{on},t} \qquad (\forall u \in U_a, a \in A, t \in T^h, h \in H) \tag{15}$$

$$p^{\min} x_u^t \leq \Delta p_u^{\text{down},t} \leq \left(\min_{t' \in T^h} p_u^{t'} - \underline{p}_u^t \right) y_u^{\text{on},t} \qquad (\forall u \in U_a, a \in A, t \in T^h, h \in H) \tag{16}$$

The bids are looked upon on an hourly basis since in reality units offer only the amount of regulating power they can provide throughout an entire hour.

Interconnectors

The next two constraints ensure that we stay within the upper and lower limits for the total flow through the interconnectors. $\bar{l}_{aa'}^t$ is the upper limit and $\underline{l}_{aa'}^t$ is the lower limit in time period $t \in T$ between the areas a and a', $a, a' \in A$. These limits depend on the areas, therefore $\bar{l}_{aa'}^t = -\underline{l}_{a'a}^t$. We also ensure that if the flow is positive in one direction, then the same amount of electricity but with negative sign is flowing in the other direction. Constraint (19) makes sure that we only can use an interconnector for transporting balancing power if we are allowed to do so. Therefore, $\alpha_{aa'}$ is set to 1 if the exchange of regulating power is allowed on the interconnector and 0 otherwise. M is a sufficiently large number; we could choose M to be equal to $\max\{\bar{l}_{aa'}^t, -\underline{l}_{aa'}^t\}$.

$$\underline{l}_{aa'}^t \leq l_{aa'}^t + \Delta l_{aa'}^t \leq \bar{l}_{aa'}^t \qquad (\forall t \in T, a, a' \in A) \tag{17}$$

$$\Delta l_{aa'}^t = -\Delta l_{a'a}^t \qquad (\forall t \in T, a, a' \in A) \tag{18}$$

$$\Delta l_{aa'}^t \leq M \alpha_{aa'} \qquad (\forall t \in T, a, a' \in A) \tag{19}$$

Ramping

The next constraints relate to ramping. Since we can activate balancing power on units in the balancing area and from other areas, we have ramping on interconnectors as well as units.

Ramping on interconnectors is described in constraint (20) below. Here $R_{aa'}^t$ defines maximum ramping in one time period on the interconnector between area a and a', where $a, a' \in A$.

$$-R_{aa'}^t \leq \Delta l_{aa'}^{t+1} - \Delta l_{aa'}^t \leq R_{aa'}^t \quad (\forall t \in \{t_0, 1, \ldots, t^{\text{end}}\}, a, a' \in A) \tag{20}$$

In order for the units to deliver the desired amount of balancing power the units have to ramp their production in order to reach the desired level. Each unit can or is willing to ramp their production to the desired level in a certain amount of time periods called τ^{ramp}. They can either ramp up, RU_u^t, or they can ramp down, RD_u^t. In the start-up phase the unit has a start-up ramp rate, consisting of SU_u^t. The ramp rates depend on the time, $t \in T$, since the unit may already be ramping to reach a new production level and therefore cannot ramp as fast as when in steady production state. The constraints (21)-(24) below make sure that the ramping cannot exceed the ramping limit attached to the unit $u \in U_a$, $a \in A$. The first two constraints are applied when we ramp towards a bid and the last two are used when ramping back to the normal production level. Each ramp rate is multiplied by a binary variable, either $x_u^{\mathrm{ramp},t}$ or $x_u^{\mathrm{rd},t}$, that indicates if the unit is ramping towards a bid or ramping back from a bid.

$$\begin{aligned}
&\Delta p_u^{\mathrm{up},t+1} \leq \Delta p_u^{\mathrm{up},t} + x_u^{\mathrm{ramp},t}\mathrm{RU}_u^t + y_u^{\mathrm{start},t}\mathrm{SU}_u^t && (\forall u \in U_a, a \in A, t = \{t_0, 1, \ldots, t^{\mathrm{end}}-1\}) && (21)\\
&\Delta p_u^{\mathrm{down},t+1} \leq \Delta p_u^{\mathrm{down},t} + x_u^{\mathrm{ramp},t}\mathrm{RD}_u^t && (\forall u \in U_a, a \in A, t = \{t_0, 1, \ldots, t^{\mathrm{end}}-1\}) && (22)\\
&\Delta p_u^{\mathrm{up},t} - x_u^{\mathrm{rd},t}\mathrm{RD}_u^t \leq \Delta p_u^{\mathrm{up},t+1} && (\forall u \in U_a, a \in A, t = \{t_0, 1, \ldots, t^{\mathrm{end}}-1\}) && (23)\\
&\Delta p_u^{\mathrm{down},t} - x_u^{\mathrm{rd},t}\mathrm{RU}_u^t + y_u^{\mathrm{start},t}\mathrm{SU}_u^t \leq \Delta p_u^{\mathrm{down},t+1} && (\forall u \in U_a, a \in A, t = \{t_0, 1, \ldots, t^{\mathrm{end}}-1\}) && (24)
\end{aligned}$$

Minimum activation time

When an up or down regulating bid has been accepted the model must be forced to keep this bid running for the minimum activation time called τ^{res}. To do this two groups of constraints are needed. First the bid variable, x_u^t or $x_u^{\mathrm{ramp},t}$, must be forced to be strictly positive (and hence 1) in this period and secondly the value of the bid, i.e. the bid size, $\Delta p_u^{\mathrm{up},t}$ or $\Delta p_u^{\mathrm{down},t}$, must be kept at the bid level for the minimum activation time period. The first is ensured by the inequalities below.

$$x_u^t + x_u^{\mathrm{ramp},t} \leq 1 \qquad (\forall u \in U_a, a \in A, t \in \{t_0, 1, \ldots, t^{\mathrm{end}}\}) \tag{25}$$

$$\sum_{s=t}^{t+\delta^{\mathrm{end}}(t)-1} (x_u^s + x_u^{\mathrm{ramp},s}) \geq \delta^{\mathrm{end}}(t) x_u^{\mathrm{ramp},t-1} \qquad (\forall u \in U_a, a \in A, t \in \{2, \ldots, t^{\mathrm{end}}\}) \tag{26}$$

Where $\delta^{\mathrm{end}}(t) = \min\{t^{\mathrm{end}} - t, \tau^{\mathrm{res}}\}$. The latter constraint shows that if a bid has been activated and the plant is currently ramping towards the bid value, the plant has to either keep the ramping towards the bid value, i.e. $x_u^{\mathrm{ramp},s} = 1$, keep the bid activated, i.e. $x_u^s = 1$, or accept a new bid by letting $x_u^{\mathrm{ramp},s} = 1$ after x_u^s has been 1 for some time periods.

If a new bid is accepted before the old bid has finished, the bid size variable, $\Delta p_u^{\mathrm{up},t}$ or $\Delta p_u^{\mathrm{down},t}$, is also updated. We introduce the non-negative variable, $x_u^{\mathrm{oldbid},t}$, to keep track of any possible running old bids that have not exceeded the minimum activation time yet. It tells whether bids have been activated since τ^{res} time periods ago. It is defined by the following constraint.

$$x_u^{\mathrm{oldbid},t} \leq x_u^{\mathrm{ramp},s} + x_u^s \quad (\forall u \in U_a, a \in A, t \in T, s = t - \tau^{\mathrm{res}}, \ldots, t-1) \tag{27}$$

Furthermore we need to know the bid level of any old bid. This information is stored in a positive variable $\Delta p_{\mathrm{old},u}^{\mathrm{up},t}$ (similarly for down-regulation $\Delta p_{\mathrm{old},u}^{\mathrm{down},t}$). It states the minimum level of the bids that have been activated since τ^{res} time periods ago. The term $Mx_u^{\mathrm{rd},s}$ is included to ensure that we do not take $\Delta p_u^{\mathrm{up},s}$ into consideration when it is ramping back from a bid. Here M is a sufficiently large number.

$$\begin{aligned}
&\Delta p_{\mathrm{old},u}^{\mathrm{up},t} \leq \Delta p_u^{\mathrm{up},s} + Mx_u^{\mathrm{rd},s} && (\forall u \in U_a, a \in A, t \in T, s = t - \tau^{\mathrm{res}}, \ldots, t-1) && (28)\\
&\Delta p_{\mathrm{old},u}^{\mathrm{down},t} \leq \Delta p_u^{\mathrm{down},s} + Mx_u^{\mathrm{rd},s} && (\forall u \in U_a, a \in A, t \in T, s = t - \tau^{\mathrm{res}}, \ldots, t-1) && (29)
\end{aligned}$$

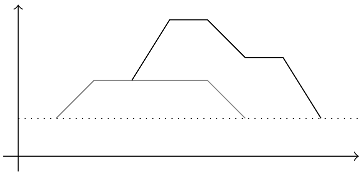
Furthermore we need to know the maximum size of activated regulating power on the unit over the last τ^{res} periods.

$$\Delta p_{\max,u}^{\text{up},t} \geq \Delta p_u^{\text{up},s} \quad (\forall u \in U_a, a \in A, t \in T, s = t - \tau^{\text{res}}, \ldots, t-1) \tag{30}$$

$$\Delta p_{\max,u}^{\text{down},t} \geq \Delta p_u^{\text{down},s} \quad (\forall u \in U_a, a \in A, t \in T, s = t - \tau^{\text{res}}, \ldots, t-1) \tag{31}$$

Note that in constraints (27)-(31) above it is assumed that the bid activation and the bid value are stored from a previous run of the model in the variables with negative t values.

When a bid is activated the bid value variable is forced to be larger than or equal to the bid value of the previous time period. However, if an old bid is running after the minimum activation time, we can subtract the bid value for this bid in the constraint, since it is allowed to ramp back to the initial level from this bid. An example of an up-regulating bid is shown in the figure below. The gray bid is the old bid and the black bid is the current bid. Even though the total production level at the plant is not kept at the same level, the "actual" bid value of the current bid (i.e. the difference between the current bid value and the level at which the plant would have produced if the current bid had not been activated) is kept the same for the whole bid activation period.



In the following constraints M is again a sufficiently large number. The first bid level constraint for up-regulating bids is given below.

$$-M(1 - x_u^{\text{oldbid},t}) \leq \Delta p_u^{\text{up},t} - \Delta p_u^{\text{up},t-1} + \Delta p_{\text{old},u}^{\text{up},t-1} \quad (\forall u \in U_a, a \in A, t \in T) \tag{32}$$

The same holds for down-regulating bids.

$$-M(1 - x_u^{\text{oldbid},t}) \leq \Delta p_u^{\text{down},t} - \Delta p_u^{\text{down},t-1} + \Delta p_{\text{old},u}^{\text{down},t-1} \quad (\forall u \in U_a, a \in A, t \in T) \tag{33}$$

Note that these constraints will force the old bid variables (the bid activation and the bid level) to be as large as possible when it is beneficial to ramp back from the bid.

Furthermore, we have to ensure that we do not ramp below the bids still in force, therefore we have the following constraints.

$$-M(1 - x_u^{\text{oldbid},t}) \leq \Delta p_u^{\text{up},t} - \Delta p_{\max,u}^{\text{up},t} + \Delta p_{\text{old},u}^{\text{up},t} \quad (\forall u \in U_a, a \in A, t \in T) \tag{34}$$

$$-M(1 - x_u^{\text{oldbid},t}) \leq \Delta p_u^{\text{down},t} - \Delta p_{\max,u}^{\text{down},t} + \Delta p_{\text{old},u}^{\text{down},t} \quad (\forall u \in U_a, a \in A, t \in T) \tag{35}$$

If there are no old bids the two constraints below ensure that the value of the bid variables is kept at the bid level or above, see the constraints below.

$$-Mx_u^{\text{oldbid},t} \leq \Delta p_u^{\text{up},t+1} - \Delta p_u^{\text{up},t} \quad (\forall u \in U_a, a \in A, t \in T) \tag{36}$$

$$-Mx_u^{\text{oldbid},t} \leq \Delta p_u^{\text{down},t+1} - \Delta p_u^{\text{down},t} \quad (\forall u \in U_a, a \in A, t \in T) \tag{37}$$

Information between the runs of the model

We have not completed this section as we want to implement the model for one period first. When the model works, we will investigate how to pass the information between the runs of the model.

Nomenclature

$\alpha_{aa'}$	A parameter that is set to 1 if exchange of regulating power is allowed on the interconnector and 0 otherwise
$\bar{c}^t_{\text{auto}}$	Denotes the expected average cost for the primary and secondary reserves at time $t \in T$
$\bar{l}^t_{aa'}$	The upper limit on the interconnector in time period $t \in T$
$\bar{p}_u$	The capacity limit of a unit $u \in U_a$, $a \in A$
$\Delta p^{\text{down},t}_{\text{old},u}$	A positive variable that holds the lowest down regulating bid value from the last τ^{res} time periods
$\Delta p^{\text{up},t}_{\text{old},u}$	A positive variable that holds the lowest up regulating bid value from the last τ^{res} time periods
$\Delta l^t_{aa'}$	The change in the flow on the interconnector between the areas a and a', $a, a' \in A$
$\Delta p^{\text{down},t}_{\text{max},u}$	The largest amount of down-regulating power activated in the last τ^{res} time periods
$\Delta p^{\text{up},t}_{\text{max},u}$	The largest amount of up-regulating power activated in the last τ^{res} time periods
$\Delta p^{\text{down},t}_u$	A non-negative variable that describes the amount of down regulating power activated in period $t \in T$ on unit $u \in U_a$, $a \in A$
$\Delta p^{\text{up},t}_u$	A non-negative variable that describes the amount of up regulating power activated in period $t \in T$ on unit $u \in U_a$, $a \in A$
τ^{ramp}	The amount of time periods in which a unit is allowed to ramp
τ^{res}	The minimum activation time of a bid
τ^{su}	Number of time periods in which a unit is in start-up phase after activation
τ^{up}	Minimum uptime of the unit
SU^t_u	Ramp rate for a unit in its start-up phase
$\underline{l}^t_{aa'}$	The lower limit on the interconnector in time period $t \in T$
$\underline{p}^t_u$	The lowest production level manageable for the unit $u \in U_a$, $a \in A$
A	A set describing the balancing areas
$b^{+,t}_a$	A non-negative variable denoting the surplus of electricity in time period t
$b^{-,t}_a$	A non-negative variable denoting the shortage of electricity in time period t

$c_u^{\text{down},t}$	A parameter describing the cost structure of unit $u \in U_a$, $a \in A$, when down-regulating in time period $t \in T$
$c_u^{\text{start},t}$	A parameter describing the cost of starting up a unit $u \in U_a$, $a \in A$
$c_u^{\text{up},t}$	A parameter describing the cost structure of unit $u \in U_a$, $a \in A$, when up-regulating in time period $t \in T$
d_a^t	The demand in time period $t \in T$ in area $a \in A$
$l_{aa'}^t$	A parameter describing the flow on the interconnector between the areas a and a', $a, a' \in A$
$p^{\min}$	The minimum size of a bid for regulating power that can be provided to TSOs
p_u^t	A parameter that refers to the expected production on a unit $u \in U_a$, $a \in A$, in time period $t \in T$
$R_{aa'}^t$	The ramp rate on the interconnector at time $t \in T$
T	The time periods of the model, $T = \{1, \ldots, t^{\text{end}}\}$
T^h	Consists of time periods in T that belong to the hour $h \in H$
t_0	The period just before the start of the optimisation period of the model
U_a	A set of units belonging to area $a \in A$
w_a^t	The wind production in time period $t \in T$ in area $a \in A$
$x_u^{\text{rd},t}$	A binary variable telling if we are ramping from a bid
x_u^t	A binary variable that is 1 if unit $u \in U_a$, $a \in A$, has an activated bid in time period $t \in T$ and 0 otherwise
$x_u^{\text{oldbid},t}$	Positive variable that takes only binary values. It is 1 if a bid has exceeded its minimum uptime and 0 otherwise
$x_u^{\text{ramp},t}$	A binary variable telling whether a unit, u, is ramping towards a bid in a given time period, t
$y_u^{\text{on},t}$	A binary variable telling if unit $u \in U_a$, $a \in A$, is online in time period $t \in T$
$y_u^{\text{start},t}$	A non-negative variable indicating if unit u is in its start-up phase
RD_u^t	Ramp down rate for a unit $u \in U_a$, $a \in A$, in time period $t \in T$
RU_u^t	Ramp up rate for a unit $u \in U_a$, $a \in A$, in time period $t \in T$

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