The Inflation Response to Government Spending Shocks: 
A Fiscal Price Puzzle?

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Abstract

This paper provides empirical evidence that prices decline significantly and persistently in response to a positive government spending shock. This result stands out across a wide variety of specifications of our Structural Vector Autoregression (SVAR) model and for different price indices. The decline in prices is accompanied by an increase in output and private consumption, as found in most of the existing literature, as well as an increase in Total Factor Productivity. These findings are hard to reconcile with standard New Keynesian models with exogenous productivity, which typically generate higher prices and a drop in consumption following a fiscal expansion. We show that the introduction of variable technology utilization can enable an otherwise standard New Keynesian model to match our empirical findings. Variable technology utilization allows firms to accommodate an increase in demand by adopting new technology into the production process. The resulting increase in measured productivity leads to a decline in prices and an increase in consumption.

Keywords: Government Spending Shocks, Fiscal Policy, Business-cycle Comovement, DSGE Modelling, Endogenous Productivity.


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1 Introduction

The macroeconomic effects of changes in government spending have received widespread attention in the economics profession, not least since the onset of The Great Recession in 2007. Following the tradition of Blanchard and Perotti (2002), a large literature has employed Structural Vector Autoregressive (SVAR) models to characterize the empirical effects of government spending shocks on GDP, private consumption, and a range of other macroeconomic variables (e.g., Ramey, 2011a). However, the response of inflation to government spending shocks has typically received limited attention in the empirical literature. Nonetheless, the conventional wisdom is that increases in government spending are inflationary. Indeed, this idea plays an important role in the transmission of fiscal policy shocks in several theoretical models, including the textbook New Keynesian model. A prominent example is the effectiveness of government spending shocks when the nominal interest rate is at the zero lower bound. The finding of a large fiscal multiplier under these circumstances relies entirely on the ability of higher government spending to drive up (expected) inflation and thus reduce the real interest rate (e.g., Christiano et al., 2011).

In this paper, we study the effects of government spending shocks on prices in the U.S. economy using an SVAR approach. Our main finding is that prices decline significantly and persistently in response to an increase in government spending. This result stands out across a variety of specifications of our empirical model, as well as across different price indices and identification strategies. Importantly, the drop in prices coexists with the increase in output and private consumption found in most of the existing literature (e.g., Blanchard and Perotti, 2002; and Galí et al., 2007). Moreover, we observe an increase in Total Factor Productivity (TFP). We show that an otherwise standard New Keynesian model augmented with variable technology utilization can account for these empirical findings.

The existing evidence on the response of prices to government spending shocks is rather mixed, as illustrated in Table 1. Some previous studies, including Fatas and Mihov (2001b) and Mountford and Uhlig (2009), have also reported a decline in prices in response to a fiscal expansion in the US. However, other authors—for example, Edelberg et al. (1999) and Caldara and Kamps (2007)—report that prices increase, while yet others—e.g., Nakamura and Steinsson (2014)—find an insignificant response. Perotti (2004) finds mixed evidence of the response of inflation in the US and four other OECD countries, but concludes that there is little evidence in support of the common perception that government spending shocks are inflationary.

1 Another example emerges from open-economy models: The fiscal multiplier is typically found to be smaller in countries with floating exchange rates (as compared to countries with a currency peg), as they will experience a tightening of monetary policy to combat the rise in inflation assumed to follow an increase in government spending (e.g., Corsetti et al., 2013).

2 As seen in Table 1, some studies report the response of the price level, and others that of the inflation
prices at all, and most of the authors who do find evidence of a decline in inflation do not attempt to provide a structural explanation for it.\textsuperscript{3}

<table>
<thead>
<tr>
<th>Fiscal Policy Study</th>
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<tr>
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<td>Fatas and Mihov (2001a)</td>
<td>Prices are insignificant</td>
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<tr>
<td>Fatas and Mihov (2001b)</td>
<td>Prices decline</td>
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<td>Ricco et al. (2017)</td>
<td>Inflation declines or is insignificant</td>
</tr>
</tbody>
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Notes: In Edelberg et al. (1999), the GDP deflator increases, while the CPI index first increases and then declines. In Fatas and Mihov (2001b) and Canzoneri et al. (2002), the decline in inflation is barely significant. All studies use U.S. data, though Perotti (2004) and Canova and Pappa (2007) also report evidence from other OECD countries and from Euro Area countries, respectively.

Our empirical findings are hard to reconcile with traditional accounts of the transmission mechanism of fiscal policy. To provide a structural interpretation of our results, we therefore propose a version of the New Keynesian model featuring time-varying adoption of new technology into the production process, as in recent work by Anzoategui et al. (2017) and Bianchi et al. (2017). In our model, firms decide on the extent to which they utilize the available technology level. In response to an increase in government spending, firms rate, but this cannot explain the different findings in the literature. While we use the price level in all our estimations, none of our findings depend on this choice.

\textsuperscript{3}Canova and Pappa (2007) offer a discussion of potential explanations for a decline in prices after a fiscal expansion, of which they find some evidence, but they stop short of proposing a theoretical model.
find it optimal to raise the utilization rate of technology in order to meet the increase in aggregate demand, despite the costs associated with a higher utilization rate. An increase in technology utilization raises measured productivity, in line with the empirical evidence we present. Provided this mechanism is sufficiently powerful, it dominates the upward pressure on marginal costs stemming from higher wages, thus ensuring that marginal costs decline in equilibrium. This paves the way for firms to reduce their prices, generating the desired decline in inflation. In turn, this induces the central bank to reduce the nominal interest rate, in line with what we observe in our SVAR evidence. This leads to a drop in the real interest rate, facilitating an increase in consumption.

The theoretical model is deliberately simple in order to allow for an analytical solution. As in the basic New Keynesian model, combining a standard consumption Euler equation with a version of the Taylor rule for monetary policy results in a negative relationship between consumption and inflation. In our model, an increase in government spending shifts the economy down along this consolidated Euler equation, resulting in a decline in inflation and an increase in consumption, in line with the data. We provide an analytical characterization of the parameter requirements for our model to generate these findings, and show that a range of parameters always exists for which this is the case. We also show that a calibrated version of our model can account for the dynamic effects of government spending shocks in the data for reasonable parameter values. Finally, we introduce capital formation into the model and estimate the key parameters using indirect inference, thus confirming our findings in a more realistic model environment.

Incidentally, the textbook version of the New Keynesian model typically features a negative comovement between inflation and private consumption conditional on a shock to government spending, but of the opposite sign than what we find in the data: inflation increases and consumption declines after a positive government spending shock. The response of consumption has received widespread attention in the theoretical literature, with several authors proposing mechanisms to obtain an increase in consumption. However, most of these seem to hold little promise for producing a decline in inflation. For example, the introduction of rule-of-thumb households by Galí et al. (2007) drives up aggregate demand but has no direct effects on the supply side. Allowing for non-separable utility in consumption and leisure, as in Monacelli and Perotti (2008) and Bilbiie (2011), induces consumption and labor supply to increase in tandem, provided consumption and leisure are substitutes. However, as shown by Bilbiie (2011), the demand-side effects still dominate, leading to a rise in inflation.

Correspondingly, while theoretical models exist that may potentially be able to generate a decline in inflation after a fiscal expansion, these are generally not consistent with a contemporaneous increase in consumption. In the New Keynesian model, there are essentially
three ways to bring about a drop in inflation in response to a government spending shock: a drop in the markup, a drop in the wage rate, or an increase in productivity. A countercyclical markup is the hallmark of the so-called deep habits model of Ravn et al. (2006), who show that this mechanism can even generate an increase in consumption after a government spending shock in their flexible-price model. However, Jacob (2015) demonstrates that the deep habits model performs quite differently in sticky-price environments: while it may indeed generate a decline in inflation, this occurs alongside a decline in consumption.4 A drop in the wage rate may be obtained in the presence of a sufficiently strong increase in labor supply in response to the reduction in permanent income associated with higher government spending (Baxter and King, 1993). Besides requiring an implausibly large Frisch elasticity of labor supply, however, a declining wage makes it very unlikely to observe an increase in consumption, as shown, e.g., by Monacelli and Perotti (2008).5 Altogether, these considerations lead us to focus on endogenous changes in the level of productivity as a more promising avenue for matching the empirical evidence.

We contribute to an emerging literature studying endogenous changes in productivity over the business cycle. We build directly on the work of Bianchi et al. (2017), who propose an endogenous growth model capturing both business-cycle fluctuations and long-term growth. In their model, endogenous variations in TFP can arise due to variable technology utilization or R&D investments in “knowledge capital”. At business-cycle frequencies, they find that variations in technology utilization account for the bulk of fluctuations in TFP, whereas the accumulation of knowledge capital is important for long-term growth. While the model of Bianchi et al. (2017) features endogenous technological progress through vertical innovations, as in Aghion and Howitt (1992), other authors have employed horizontal innovations featuring increasing returns to specialization as in Romer (1990) and Comin and Gertler (2006). A prominent recent example is the paper by Anzoategui et al. (2017), who find that most of the observed decline in TFP during the Great Recession can be attributed to endogenous factors, primarily a decline in the intensity of technology adoption. Moran and Queralto (2018) use a similar model to study the link between monetary policy shocks and endogenous movements in technology after establishing that a monetary expansion leads to an increase in TFP in the data. More generally, several recent papers have pointed to various types of demand shocks as potentially important drivers of fluctuations in TFP over the business cycle. Notable examples are Bai et al. (2017) and Benigno and Fornaro (2017). However, none of these papers study the connection between endoge-

4 In a nutshell, price stickiness erodes the ability of firms to reduce their markup as desired under deep habits, thus impeding the increase in real wages necessary to drive up consumption.

5 An alternative way to obtain a decline in the wage rate is to allow for a sufficiently strong reaction of monetary policy to output, as shown by Linnemann and Schabert (2003). Aside from the fact that an increase in the nominal interest rate is in contrast to our empirical evidence, this approach has the disadvantage of leading to an even larger drop in consumption.
nous productivity and fiscal policy. In this respect, two existing studies are more closely related to our paper. Aghion et al. (2014) find that systematic, countercyclical fiscal policy can have positive long-term effects on productivity growth. To rationalize this finding, they devise a model in which countercyclical fiscal policy leads to a reduction in business-cycle volatility, which in turn facilitates investments in productivity-enhancing long-term projects, such as R&D investments. D’Alessandro and Fella (2017) propose a business-cycle model with learning-by-doing, and show that it can generate positive responses of private consumption, the real wage, and TFP to a government spending shock, in line with their empirical evidence. While this mechanism may potentially complement the one we propose, their full-fledged model with capital generates an increase in inflation on impact, in contrast with the empirical results we present.6

Finally, our findings are reminiscent of the so-called “price puzzle” of monetary policy (Sims, 1992). Upon confirming that our results do not suffer from common types of misspecification that have been proposed in this regard (notably, the drop in prices is confirmed when commodity prices are included in the VAR model), we propose a structural explanation sharing some features with the cost channel of monetary policy proposed by Barth and Ramey (2002) to account for the monetary price puzzle. Both mechanisms rely on supply-side effects to produce a reduction in marginal costs in response to an expansionary demand shock. Our results are also related to the puzzling behavior of the real exchange rate in connection with fiscal policy in open economies. Kim and Roubini (2008), Monacelli and Perotti (2010), and Ravn et al. (2012) all find that the real exchange rate depreciates in response to an expansionary government spending shock, i.e. that domestic prices decline relative to foreign (exchange-rate-adjusted) prices.

The rest of the paper is structured as follows. We present our empirical exercises and results in Section 2. Our model of variable technology utilization is outlined in Section 3, while Section 4 is devoted to studying its properties analytically. We present numerical model simulations in Section 5. In Section 6 we augment our baseline model with capital formation and estimate the parameters of the model. Finally, Section 7 concludes.

2 Fiscal Policy and the Price Level: Empirical Evidence

In this section, we set up a Structural VAR model for the U.S. economy to investigate the effects of government spending shocks on key macroeconomic variables. As a first step, following the tradition of Blanchard and Perotti (2002), we identify spending shocks through a standard Cholesky decomposition with government spending ordered first. Second, to

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6In addition, their proposed mechanism may not square well with the finding of Bianchi et al. (2017) that accumulation of knowledge capital plays virtually no role for business-cycle fluctuations.
account for anticipated changes in fiscal policy, we use the forecast errors of government spending computed by Auerbach and Gorodnichenko (2012) to identify shocks to government spending. To check the robustness of our results, we consider a vast number of alternative specifications of our VAR model, as well as alternative identification schemes.

We estimate the following quarterly VAR model on U.S. data:

\[
X_t = a_0 + a_1 t + a_2 t^2 + B^{-1} A(L) X_{t-1} + B^{-1} e_t, \tag{1}
\]

where \(X_t\) is the vector of endogenous variables, \(e_t\) is a vector of i.i.d. structural shocks with unit variance, \(A(L)\) comprises the coefficients on the lagged endogenous variables, \(L\) is the lag operator and \(B\) comprises the coefficients on the contemporaneous endogenous variables. We include linear and quadratic time trends, as in Blanchard and Perotti (2002). We follow most of the literature and use 4 lags as our baseline. Our data sample covers the period 1960:Q1-2017:Q2. We use the following variables in our analysis: Real government expenditure and investment \((G_t)\), real GDP \((Y_t)\), real private consumption \((C_t)\), real tax revenue (tax receipts less current transfers, interest payments and subsidies) \((T_t)\), the Personal Consumption Expenditures (PCE) price index \((P_t)\), the nominal interest rate on 3-month treasury bills \((R_t)\), and Total Factor Productivity \((A_t)\). \(^7\) All variables except \(R_t\) are in logs, and the variables \(G_t, Y_t, C_t\) and \(T_t\) are measured in real per-capita terms. \(T_t\) is converted into real terms using the GDP deflator. We use the TFP measure of Fernald (2014). \(^8\) Appendix A contains a detailed description of the data.

2.1 The Cholesky Decomposition

Under the Cholesky identification scheme, the model contains the following variables:

\[
X_t = [G_t \ Y_t \ C_t \ T_t \ P_t \ R_t \ A_t]'.
\]

Following the approach of Blanchard and Perotti (2002), we assume that the structural shocks to government spending can be recovered from the estimated residuals \(B^{-1} e_t\) in (1) by imposing that the matrix \(B\) is lower triangular. This implies that government spending does not respond to any other variable within-quarter, but affects other variables within the same quarter. Intuitively, the assumption is motivated by decision lags in fiscal policy. By the time policymakers realize that a shock has hit the economy and implement an appropriate policy response, at least one quarter would have passed.\(^9\) The ordering of the

\(^7\) All results are robust if we use non-durable consumption instead of total consumption.

\(^8\) We use the non-utilization-adjusted TFP measure as our baseline. All results are robust to using the utilization-based measure instead.

\(^9\) This implies that the within-period elasticity of real government spending to a change in prices is assumed to be zero. In the absence of perfect indexation of government spending, this assumption may not...
remaining variables is such that real variables (with the exception of TFP) are determined before nominal ones, and that monetary policymakers are assumed to be able to observe and react to changes in output and prices within-period. Our findings are robust to different orderings of the variables.

Figure 1 shows the impulse-response functions to a positive government spending shock normalized to 1 percent, along with 68 percent bootstrapped confidence bands, obtained using the delta method with 2,000 replications. All responses are denoted in percent, except for the interest rate, where the response is reported in basis points. Following a fiscal expansion, output and consumption increase persistently, in line with most of the empirical literature.\(^\text{10}\) Prices display a strongly significant and very persistent decline. The price level drops by around 0.3 percent at the peak. The implied annualized inflation rate drops by slightly more than 25 basis points at its trough 2 quarters after the shock. TFP also increases significantly, in line with the evidence reported by Bachmann and Sims (2012) for the US, and by Afonso and Sousa (2012) for four OECD countries including the US. Finally, the short-term nominal interest rate drops by around 20 basis points, and tax revenues decline.

2.2 Controlling for Fiscal Foresight: Baseline VAR model

A common criticism of the Cholesky identification strategy employed above is that changes in fiscal policy are—at least to some extent—anticipated by economic agents, as discussed by Ramey (2011a), among others. In this case, it is not possible to recover a structural shock to fiscal policy using the identification strategy of Blanchard and Perotti (2002). To account for this, we consider an identification scheme that controls for fiscal foresight. Following Auerbach and Gorodnichenko (2012), we identify an unanticipated government spending shock as an innovation to the forecast error of the growth rate of government spending. The vector of endogenous variables becomes:

\[
X_t = \begin{bmatrix} FE_t & G_t & Y_t & C_t & T_t & P_t & R_t & A_t \end{bmatrix}',
\]

where \(FE_t\) is the implied forecast error of the survey-based forecasts of the growth rate of government spending, which we obtain from Auerbach and Gorodnichenko (2012). In order to recover an unanticipated government spending shock, we order \(FE_t\) first in the system. Perotti (2004) suggests that the within-period elasticity of government spending to a change in prices might be as high as \(-0.5\). We have verified that our findings are robust to this choice.

\(^{10}\)The implied government spending multiplier on output can be found by multiplying the reported output response by the inverse of the sample average of the ratio of government spending to output, which is 0.245. This implies an impact multiplier of 1.02, well within the range of available estimates reported in the survey of Ramey (2011b) of 0.8 – 1.5.
Figure 1: The dynamic effects of a shock to government spending. VAR model with Cholesky identification scheme. The black line denotes the estimated response, while the grey areas represent 68 percent confidence bands.
The data sample covers the period 1966:Q3-2010:Q3, for which the forecast errors of Auerbach and Gorodnichenko (2012) are available. Figure 2 shows the effects of a government spending shock under this identification scheme. After controlling for fiscal foresight, most of the results are qualitatively similar to those presented above, thus confirming our main findings. A fiscal expansion generates an increase in output, consumption, and productivity, as well as a decline in prices and the interest rate. In this case, the price level declines by around 0.5 percent, while the drop in the implied annualized inflation rate reaches almost 50 basis points at its trough 6 quarters after the shock. In contrast to the previous subsection, tax revenues now increase. Given the common criticism of the Cholesky identification scheme, as well as the somewhat counterintuitive decline in tax revenues observed in Figure 1, we use the forecast error specification as our baseline model in the remaining part of the paper.

2.3 Robustness

We consider a series of alternative specifications of our baseline VAR model with forecast error identification to check the robustness of our results. These include a) using alternative price indices, b) including commodity prices in the VAR, c) using an alternative productivity measure, d) excluding TFP from the baseline VAR. The results are reported in Appendix C. Figure C.1 shows the impulse responses when the PCE price index is replaced by, respectively, the GDP deflator, the CPI index, and the core PCE index. All of these display a clear decline. In general, the results are very similar across the different price indices, confirming our main findings. Figure C.2 displays the results from the next set of specifications. First, we include a measure of commodity prices in the VAR model. Sims (1992) showed that prices increase on impact in response to a tightening of monetary policy; the so-called “price puzzle", but that this counterintuitive response could be alleviated by including commodity prices in the VAR model. Intuitively, commodity prices may contain signals of future price changes observed by central bankers, but not by an econometrician excluding commodity prices from her model. While this argument appears less appealing in the case of fiscal policy, we check the robustness of our results when commodity prices are included. The first column of Figure C.2 shows that our results are confirmed in this

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11In Appendix B, we perform an impulse-response decomposition along the lines of Kilian and Lewis (2011). We find that the decline in the PCE index can be explained by its own lagged values and the fiscal policy shock itself, whereas the drop in the interest rate plays virtually no role. This rules out any concerns that the drop in prices could simply be a response to the drop in the interest rate, and thus merely a manifestation of the price puzzle of Sims (1992).

12Another popular explanation of the price puzzle of monetary policy is proposed by Giordani (2004), who suggests using a measure of the output gap in the VAR model instead of output. We have confirmed that our results are virtually unaffected by this choice.
Figure 2: The dynamic effects of a shock to government spending. Estimates obtained using the identification scheme based on forecast errors. The black line denotes the estimated response, while the grey areas represent 68 percent confidence bands.
Next, we show that an alternative measure of productivity (the log of real output per hour in the nonfarm sector) responds similarly to TFP. Lastly, we address the potential concern that any of our initial results could be driven by the inclusion of TFP in the VAR model. All results are confirmed when TFP is excluded from the baseline model.

Additional robustness checks reported in Appendix C include: e) alternative ordering of variables, f) changing the lag length, g) excluding the quadratic time trend. We also perform a complete set of robustness checks based on the Cholesky decomposition as in Section 2.1. The qualitative findings presented above are not altered by any of these changes.

2.4 Alternative Identification Strategies

We finally consider two alternative identification schemes. First, as an alternative to the forecast-error approach of Auerbach and Gorodnichenko (2012), we use the defense news shocks constructed by Ramey (2011a) to control for anticipated changes in government spending. Based on news sources, this data series seeks to identify surprise build-ups in U.S. military spending. We replace the forecast error, $FE_t$, with the news shock variable, $NS_t$, in our model and estimate the model with data covering the period 1960:Q1-2013:Q4; the sample for which the news series is available. The effects of an innovation to the news shock variable are reported in the left column of Figure 3. Prices still display a significant and persistent decline. The increase in consumption is less pronounced and barely significant, not unlike the findings of Ramey (2011a), while productivity still increases. Thus, our main findings remain unaltered.

Second, we use sign restrictions to identify fiscal policy shocks. We use a VAR model featuring the same variables used for the Cholesky decomposition in Section 2.1. We identify a government spending shock as a shock that pushes up government spending and output on impact and in the following three quarters. We obtain 500,000 realizations that satisfy our identifying assumptions, and then report the median along with the 16th and 84th percentile. The results are shown in the right column of Figure 3. Using this approach, the price index declines, although it is not statistically significant. The responses of the remaining variables are largely in line with our previous findings. Altogether, the evidence reported in Figure 3 appears to confirm the puzzling behavior of prices in response to a government spending shock.

13 The commodity price itself displays a decline in response to a government spending shock (not shown).
14 In Appendix B we confirm that the inclusion of commodity prices does indeed alleviate the price puzzle of monetary policy very substantially.
15 We follow the Bayesian approach of Mountford and Uhlig (2009).
Figure 3: The dynamic effects of a shock to government spending. Left column: Estimates obtained using the identification scheme based on the defense news shocks of Ramey (2011a). The black line denotes the estimated response, while the grey areas represent 68 percent confidence bands. Right column: Estimates obtained using sign restrictions. The black line denotes the estimated response, while the grey areas represent 68 percent credible sets.
3 The Model

We consider a version of the baseline New Keynesian model without capital, as in Galí (2015). A representative household works, saves, consumes, and owns the firms in the economy. The production side consists of an intermediate goods sector operating under imperfect competition and subject to price rigidities, and a perfectly competitive final goods sector. A central bank conducts monetary policy, and a fiscal authority makes decisions about changes in government spending. A key feature of the model is the presence of variable utilization of the available technology level, as in Bianchi et al. (2017).16

3.1 The Household

The representative household maximizes expected discounted lifetime utility $E_0 \sum_{t=0}^{\infty} \beta^t U_t$, where the period utility function is given by:

$$U_t = \begin{cases} 
\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{\psi N_t^{1+\varphi}}{1+\varphi}, & \sigma \neq 1, \\
\log C_t - \frac{\psi N_t^{1+\varphi}}{1+\varphi}, & \sigma = 1,
\end{cases}$$

with $C_t$ and $N_t$ denoting non-durable consumption and labor. $\beta \in (0,1)$ is the discount factor, $\sigma > 0$ is the coefficient of relative risk aversion, $\varphi > 0$ is the inverse of the Frisch elasticity of labor supply, and $\psi > 0$ is the weight of labor disutility. Utility maximization is subject to the following budget constraint:

$$C_t + \frac{R_{t-1} b_{t-1}}{\pi_t} = w_t N_t + b_t + d_t - t_t,$$

where $\pi_t \equiv \frac{P_t}{P_{t-1}}$ is the rate of inflation in the price of consumption goods $P_t$, $b_t$ denotes one-period risk-free bonds at the nominal interest rate $R_t$, $w_t$ is the real wage, $d_t$ is real profits from firms, and $t_t$ is a lump-sum tax. The household chooses $C_t$, $N_t$, and $b_t$, and the associated first-order conditions can be stated as:

$$\psi N_t^{\varphi} = w_t C_t^{-\sigma}, \quad (2)$$

$$C_t^{-\sigma} = \beta E_t \frac{R_t C_{t+1}^{-\sigma}}{\pi_{t+1}}. \quad (3)$$

16The model of Bianchi et al. (2017) features endogenous variations in TFP due to variable technology adoption and R&D investments in “knowledge capital”. Given our focus on the business-cycle effects of changes in fiscal policy, we abstract from the latter, as Bianchi et al. (2017) find that it plays virtually no role at business-cycle frequencies.
3.2 Final Goods Producers

There is a perfectly competitive sector of final goods producers, who purchase goods from different intermediate goods producers, bundle them together, and sell them to the household or the government. Final goods producers have the following production function:

\[ Y_t = \left( \int_0^1 Y_t(i) \frac{1}{\varepsilon} di \right)^{\frac{1}{1-\varepsilon}}, \quad \varepsilon > 1, \]

where \( Y_t \) is aggregate production of the final good, and \( Y_t(i) \) denotes the amount produced by individual firm \( i \) in the intermediate goods sector. The cost-minimization problem of the representative final goods firm gives rise to the following demand for intermediate good \( i \):

\[ Y_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\varepsilon} Y_t, \quad (4) \]

where \( P_t(i) \) is the price of good \( i \), and where \( \varepsilon \) thus represents the elasticity of substitution between different intermediate goods.

3.3 Intermediate Goods Producers

There is monopolistic competition in the intermediate goods sector. Individual firm \( i \) produces according to the following production function:

\[ Y_{it} = V_{it} N_{it}^{1-\alpha}, \quad (5) \]

where \( 0 \leq \alpha < 1 \), so as to allow for decreasing or constant returns to scale in labor. \( N_{it} \) is the amount of labor hired by firm \( i \), and \( V_{it} \) is the level of utilized technology. In turn, this is given by:

\[ V_{it} = u_{it} A_t, \quad (6) \]

where \( u_{it} \) denotes the firm-specific utilization rate, and \( A_t \) is the economy-wide and exogenous level of technology, which grows deterministically at the rate \( \lambda_A \geq 0 \):

\[ A_t = (1 + \lambda_A) A_{t-1}. \quad (7) \]

We let each firm decide on the rate at which it wishes to utilize the available technology in society. As in Bianchi et al. (2017), technology utilization may be interpreted as a measure of the capacity of the firm to adopt new knowledge or inventions into the production setup. As new inventions arrive, each firm needs to exert an effort to internalize this new technology. By endogenizing the rate of technology adoption, we allow firms to choose when to make
this effort, subject to an adjustment cost whenever $u_{it}$ differs from its steady-state level $u$. We thus assume that it is costly for a firm to fully adopt new inventions into their production process as they arrive, for example because employees must be trained in using the new technology. We let the function $z(u_{it})$ denote the adjustment costs associated with the choice of $u_{it}$. As in Bianchi et al. (2017), this function satisfies $z(u) = 0$, i.e., adjustment costs are zero in steady state. We also require $z'(u) > 0$ and $z''(\cdot) > 0$. Further, in line with the literature on variable utilization of capital (e.g., Christiano et al., 2005), we assume that $u = 1$. As we shall see, this choice pins down $z'(1)$. The curvature parameter $z''(\cdot)$ measures how quickly adjustment costs rise with changes in the rate of technology utilization.\footnote{The only characteristic of the function $z$ affecting the steady state is $z'(1)$. Moreover, as in Christiano et al. (2005), only the ratio $z''(\cdot)/z'(1)$ affects the dynamics of our model outside steady state.}

Each firm chooses labor inputs $N_{it}$ and technology utilization $u_{it}$ so as to minimize its costs subject to (5). This gives rise to the following first-order conditions:

$$w_t = (1 - \alpha) mc_{it} \frac{Y_{it}}{N_{it}};$$

$$z'(u_{it}) = mc_{it} \frac{Y_{it}}{u_{it}};$$

where $mc_{it}$ is the multiplier associated with (5), and represents the real marginal cost of production. (8) equates the real wage to the marginal product of labor, while (9) states that the marginal cost of higher utilization, given by the increase in adjustment costs $z'(u_{it})$, must equal the marginal product of a higher utilization rate. The utilization rate of technology affects the marginal cost in two ways: On the one hand, a higher rate of utilization allows the firm to increase production for given inputs of labor, effectively working like an increase in productivity. On the other hand, higher utilization is costly. If the former effect is sufficiently strong, a higher utilization rate reduces the marginal cost. In response to a government spending shock, this effect may even be strong enough to overcome the increase in the wage rate, thus paving the way for an equilibrium decline in the marginal cost and, as a consequence, inflation.

When setting their price, intermediate goods firms are subject to a nominal rigidity in the form of quadratic price adjustment costs, as in Rotemberg (1982). Adjustment costs $\Upsilon_{it}$ are scaled by nominal output, and take the following form:

$$\Upsilon_{it} = \gamma \left( \frac{P_{it}}{P_{it-1}} - 1 \right)^2 P_t Y_t,$$

where $\gamma > 0$ measures how costly it is to change prices. Firm $i$ sets its price so as to
maximize profits, and this problem can be written in real terms as:

\[
\max_{P^i_t} \mathbb{E}_0 \sum_{t=0}^{\infty} q_{t,t+1} \left[ \left( \frac{P_{t}}{P_{t}^{i}} - mc_{it} \right) Y_{it} - z(u_{it}) - Y_{it} \right],
\]

subject to the demand function (4). Here, \( q_{t,t+1} \equiv \beta \frac{\lambda_{t+1}}{\lambda_{t}} \) is the stochastic discount factor of the household, with \( \lambda_{t} \) denoting the marginal utility of consumption. Upon deriving the first-order condition, we impose a symmetric equilibrium in which all firms charge the same price, allowing us to state the optimality condition as:

\[
1 - \varepsilon + \varepsilon mc_{t} = \gamma (\pi_{t} - 1) \pi_{t} - \gamma E_{t} \frac{q_{t,t+2}}{q_{t,t+1}} \frac{(\pi_{t+1} - 1)}{\pi_{t+1}} \frac{Y_{t+1}}{Y_{t}},
\]

This condition can be written on log-linearized form as a New Keynesian Phillips Curve.

### 3.4 Monetary and Fiscal Policy

Fiscal policy is assumed to follow a balanced-budget rule:

\[
g_{t} = t_{t},
\]

where government spending, \( g_{t} \), satisfies:

\[
\log g_{t} = (1 - \rho_{G}) g + \rho_{G} \log g_{t-1} + \varepsilon^{G}_{t},
\]

with the innovation \( \varepsilon^{G}_{t} \) following an i.i.d. normal process with standard deviation \( \sigma_{G} \), and where \( g \) denotes government spending in steady state, while \( \rho_{G} \geq 0 \) is the persistence of the shock.

The monetary policy rule is specified as follows:

\[
\frac{i_{t}}{\bar{i}} = \left( \frac{\pi_{t}}{\bar{\pi}} \right)^{\phi_{\pi}} \left( \frac{Y_{t}}{\bar{Y}} \right)^{\phi_{y}},
\]

where \( \phi_{\pi} > 1 \) and \( \phi_{y} \geq 0 \) denote the policy responses to inflation and output deviations from steady state, respectively, with \( \pi \) and \( Y \) denoting the steady-state levels of inflation and output.

### 3.5 Market Clearing

Bonds are in zero net supply:

\[
b_{t} = 0.
\]
The labor market clears when:

\[ \int_0^1 N_i di = N_t. \]  \(15\)

Finally, goods market clearing requires:

\[ Y_t - z(u_{it}) - \Upsilon_{it} = C_t + g_t. \]  \(16\)

When solving the model, we detrend all variables to eliminate the trend growth in the level of technology. Considering only symmetric equilibria in which all firms make the same decisions allows us to discard subscript \(i\)’s. We then log-linearize the equilibrium conditions around the non-stochastic steady state of the model, which is described in Appendix D.1. The log-linearized equilibrium conditions are presented in Appendix D.2.

4 Analytics of the Model

To build intuition on the ability of the model to reproduce our empirical findings—in particular, a decline in inflation and an increase in consumption in response to expansionary fiscal policy—we find it useful to offer some analytical insights. To this end, we make the following simplifying assumptions, all of which are regularly encountered in the existing business-cycle literature: We assume constant returns to scale in production \((\alpha = 0)\), log utility in consumption \((\sigma = 1)\), unitary (inverse) Frisch elasticity of labor supply \((\varphi = 1)\), no monetary policy reaction to the output gap \((\phi_y = 0)\), and a constant level of technology \((A_t = 1, \forall t)\). Under these conditions, the log-linearized version of the model can be reduced to two equations in consumption and inflation (plus an exogenous process for government spending), as we show in Appendix D.3. In fact, letting \(\tilde{x}_t\) denote the (log) deviation of a generic variable \(x_t\) from its steady-state value \(x\), these two equations can be stated as (see Appendix D.3 for details):

\[
\begin{align*}
-\frac{C_t}{C_{t+1}} &= E_t \left(-\frac{C_{t+1}}{C_{t+1}} + \phi \pi_t - \pi_{t+1} \right), \quad \text{(EE)} \\
\tilde{\pi}_t &= \beta E_t \tilde{\pi}_{t+1} + a \hat{C}_t - b \hat{g}_t, \quad \text{(NKPC)}
\end{align*}
\]

where \(a, b\) are functions of the deep parameters of the model. We provide necessary and sufficient conditions below for \(a\) and \(b\) to be strictly positive. (EE) simply combines the household’s Euler equation with the monetary policy rule, while (NKPC) emerges by substitution of the remaining equilibrium conditions into the New Keynesian Phillips Curve. In Figure 4, we provide a graphical representation of the model (EE)-(NKPC) in \((\hat{C}_t, \tilde{\pi}_t)\)-space. (EE) can be represented by a downward-sloping line (this can be seen most clearly in the case of non-persistent shocks, in which case \(E_t \hat{C}_{t+1} = E_t \tilde{\pi}_{t+1} = 0\)), whereas (NKPC)
Figure 4: The effects of a positive government spending shock. The NKPC’-curve refers to our baseline model, while the NKPC”-curve refers to the basic New Keynesian model without variable technology utilization.

implies an upward-sloping relationship between the two variables. Starting from the steady state of the model, indicated by the intersection of the curves EE and NKPC in Figure 4, a positive shock to government spending ($g_t > 0$) shifts the NKPC-curve down, leaving the EE curve unaffected. As shown by the curve labelled NKPC’ in Figure 4, an increase in government spending thus leads to a drop in inflation and an increase in consumption, in line with the empirical evidence of Section 2.

We proceed by deriving a closed-form solution of the model, as well as an analytical characterization of the conditions for a unique and determinate solution. We do this under the simplifying assumption that shocks to government spending have no persistence ($\rho_G = 0$). As we show in Appendix D.3 and D.4, the following statements are warranted:

**Proposition 1** The model has a determinate solution (and the parameter $a$ is strictly positive) if and only if the curvature of the cost function associated with changes in the utilization rate of technology is above the following threshold:

$$z''(\cdot) > z'(1) \frac{mc - \frac{q}{Y}}{2 - \frac{q}{Y}}. \quad (17)$$

**Proposition 2** If the model has a unique and determinate solution, it features a decline in inflation along with an increase in consumption on impact in response to a positive shock to government spending (and a strictly positive value of the parameter $b$) if and only if the
curvature of the cost function is below the following threshold:

\[ z''(\cdot) < z'(1). \]

**Proof.** See Appendix D.3 and D.4. □

Note that the steady-state value of \( mc \) is given by \( mc = \frac{z_{00}}{\xi} < 1 \). This means that there always exists a range of values for \( z''(\cdot) \) for which both (17) and (18) are satisfied. Our baseline calibration of the next section implies \( mc = \frac{5}{6}, \frac{g}{\nu} = 0.245 \), and \( z'(1) = 0.21 \). These values produce an admissible range of \( z''(\cdot) \in [0.07, 0.21] \). For all values within this range, the model has a determinate equilibrium featuring a drop in inflation and an increase in consumption on impact.

We can explain these requirements as follows: (18) requires that the curvature \( z''(\cdot) \) cannot be too large. If \( z''(\cdot) \) is very high, changes to the utilization rate are very costly, so firms will be hesitant to make such changes. In the limiting case of \( z''(\cdot) \to \infty \), firms will choose to never adjust the utilization rate, which will therefore remain constant, exactly as in a model without an endogenous utilization rate. Indeed, we show in Appendix D.5 that for \( z''(\cdot) \to \infty \), the analytical solution to our model collapses to that of a basic New Keynesian model, and that the latter always implies an increase in inflation—driven by the upward movement in the wage rate—along with a decline in consumption when \( \hat{g}_t \) increases. Graphically, this implies that the NKPC-curve is shifted up, as illustrated by the curve labelled NKPC" in Figure 4.\(^{18}\) To overturn this, and ensure a positive value of \( b \) and a downward shift in the NKPC-curve in Figure 4, it is crucial that the utilization rate is sufficiently responsive, which in turn requires a limited cost of adjusting it.

Conversely, (17) provides a lower bound on the adjustment cost, effectively entailing that the rate of technology utilization cannot be too responsive. If this condition is not met, the model does not have a determinate solution. Intuitively, if the costs associated with changing the utilization rate are sufficiently small, the optimal utilization rate may tend to infinity in response to an expansionary shock. Thus, the adjustment cost function needs to display a certain degree of curvature for the costs to increase sufficiently with the utilization rate and contain the movements in the latter. In terms of the graphical representation in Figure 4, (17) ensures an upward-sloping NKPC-curve, which is necessary for the model to have a determinate equilibrium.\(^{19}\)

\(^{18}\)The basic New Keynesian model—subject to the same parameter restrictions as our model—features an Euler equation identical to \( EE \), and a rewritten New Keynesian Phillips Curve of the same form as \( NKPC \), but where the coefficient in front of \( \hat{g}_t \) is strictly negative. See Appendix D.5 for details.

\(^{19}\)Interestingly, this type of constraint does not arise in models featuring variable utilization of the capital stock. In those models, adjustment costs will be tied to the rental rate of capital in equilibrium: capital producers will never find it optimal to raise the utilization rate to a level at which the associated adjustment costs outweigh the rental rate earned on utilized capital (see, e.g., Smets and Wouters, 2007). In our setup, instead, the utilization decision regards a production “factor” which is intrinsically free to use; the level of
The analysis above establishes some general conditions under which our model is able to generate impact multipliers in line with the empirical evidence from Section 2. Effectively, our mechanism works much like an increase in the level of technology—in fact, it produces an increase in measured TFP ($V_t$), as we shall see below. The decline in marginal costs induces firms to reduce their prices, thus generating a decline in inflation. The central bank responds by reducing the nominal and real interest rate, thus facilitating an increase in consumption. In fact, in our simple environment, this is necessary and sufficient to generate an increase in private consumption. Again, this can be seen most easily in the case of non-persistent shocks, in which case (EE) simply states that consumption equals minus the nominal (and real) interest rate (in deviations from steady state). More generally, these insights carry over to the next section, where we lift some of the simplifying assumptions made in this section and resort to numerical analyses of the quantitative implications of our model.\footnote{A final insight can be obtained from the simple model above: If we were to introduce a monetary policy shock into the model, the shock would appear in (EE), but not in (NKPC). A contractionary monetary policy shock would shift the EE-curve down along the NKPC-curve, generating a decline in inflation. In other words, our model is not able to account for the “price puzzle” of monetary policy (Sims, 1992). The reason is that a government spending shock affects natural (i.e., flexible-price) output via its effect on labor supply, and thus exerts a direct effect on inflation, whereas a monetary policy shock leaves natural output unaffected.}

5 Dynamic Effects of a Government Spending Shock

In this section, we use model simulations to study the effects of a government spending shock beyond the quarter in which the shock hits the economy. To this end, we assign realistic values to all parameters of the model, and study the implied impulse responses. We also offer a set of sensitivity checks regarding certain key parameters.

5.1 Calibration

The baseline calibration of the model is as follows: We set $\beta = 0.99$, implying an annualized real interest rate of 4% in steady state. We set the deterministic growth rate of technology to $\lambda_A = 0$ for simplicity. The coefficient of relative risk aversion is set to $\sigma = 2$, in line with microeconometric estimates (see, e.g., Attanasio and Weber, 1995). As in Christiano et al. (2005), we maintain the assumption from the previous section of an (inverse) Frisch elasticity of labor supply of unity; $\varphi = 1$. This value represents a compromise between microeconometric studies—where 1 can be regarded as an upper bound; see Chetty et al. (2011)—and macroeconomic models, where values above 1 are not uncommon (see, e.g., technology in society. The only cost of utilizing technology comes from the adjustment costs, motivating the presence of a lower bound on these.
Hall, 2009). The weight on disutility of labor hours in the utility function, $\psi$, is calibrated so that $N = 1/4$ (this only affects the scale of the economy). On the production side, we follow most of the literature and set $\varepsilon = 6$, implying a steady-state markup of 20 percent. We maintain the assumption of constant returns to scale ($\alpha = 0$) in our baseline analysis, and then study the case of decreasing returns to scale in Section 5.3.3. The adjustment cost associated with price changes is calibrated so that a given price is changed, on average, every 3 quarters, consistent with microeconometric evidence reported by Nakamura and Steinsson (2008). Given the other parameters, this implies a value of $\gamma = 29.41$.

Regarding the policy-related parameters, we follow most of the literature in setting the steady-state inflation rate to zero. The response of monetary policy to movements in inflation is set to a standard value of $\phi_{\pi} = 1.5$ (see, e.g., Taylor, 1993). We initially set the output response to zero, $\phi_y = 0$, and then “switch on” this reaction when studying the role of monetary policy in Section 5.3.2. The persistence of government spending shocks is set to $\rho_G = 0.9$, in line with Gali et al. (2007). The ratio of government spending to output in the model matches the sample average in the data for the period 1960-2017, which equals $\frac{g}{Y} = 0.245$.

Finally, we need to specify and parametrize the functional form of the adjustment cost associated with changes in the technology utilization rate. We assume that adjustment costs are given by:

$$ z(u_t) = \chi_1 (u_t - u) + \frac{\chi_2}{2} (u_t - u)^2, \tag{19} $$

where $\chi_1, \chi_2 > 0$, and where $u = 1$ again denotes the steady-state level of $u_t$. This implies that $z'(u_t) = \chi_1 + \chi_2 (u_t - u)$. As already described, we calibrate the value of $z'(1) = \chi_1$ to ensure that the rate of utilization equals 1 in steady state. This returns a value of $\chi_1 = 0.21$. The curvature parameter $z''(\cdot) = \chi_2$ is harder to pin down. Conditional on our baseline calibration of all other parameters, the admissible range of values for this parameter established analytically in Section 4 changes slightly: For any value of $\chi_2 \in [0.03, 0.21]$, we obtain impact effects of inflation and consumption in line with the data. In the simulations below, we pick a baseline value of $\chi_2 = 0.1$, while our robustness checks shed more light on the quantitative importance of this parameter.

### 5.2 Impulse-Response Analysis

Given our baseline calibration, Figure 5 displays the impulse responses of the model to a government spending shock of 1 percent (solid blue lines), along with the responses of a basic New Keynesian model without variable technology utilization (dashed red lines). As the

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21This functional form satisfies the requirements stated by Bianchi et al. (2017) and is consistent with the standard specification of capital adjustment costs in the literature; e.g. in Christiano et al. (2005).
figure illustrates, our baseline model implies an increase in the rate of technology utilization in response to the shock. This is sufficient to generate a decline in marginal costs, despite the increase in the wage rate. As a consequence, inflation drops. This leads to a reduction in the nominal interest rate, reducing also the real rate. In line with the intuition traced out in the previous section, consumption therefore increases, in turn amplifying the increase in total output. The negative response of the nominal interest rate is in line with the empirical evidence from Section 2. Also in line with the data, we observe an increase in “Measured TFP” as given by the utilized technology level, $V_t$. In the absence of exogenous technology shocks, this variable moves one-for-one with the utilization rate. In contrast, measured TFP remains constant in the model without variable technology utilization. In that case marginal costs increase in response to the shock, generating an increase in inflation and the nominal interest rate, and a drop in consumption, in contrast to our empirical evidence. The government spending multiplier on total value added (defined as the sum of private and public consumption) is substantially higher in our baseline model (1.30) than in the model without variable technology utilization (0.75).

5.3 Sensitivity Analysis

This subsection explores the robustness of our findings with respect to some of our key parameter values and modeling choices.

5.3.1 Movements in the Technology Utilization Rate

Given the uncertainty surrounding the cost of changing the rate of technology utilization, it is worth pointing out that we do not require dramatic changes in the utilization rate to obtain a decline in inflation: Under our baseline calibration, the utilization rate increases by less than 0.5 percent; somewhat less than the increase in output. This is similar to the behavior of the utilization rate of capital in Christiano et al. (2005), which moves slightly less than 1-for-1 with output in the data and in their model. To shed some additional light on the robustness of our findings, the dotted green lines in Figure 5 show the corresponding impulse responses after changing the value of $\chi_2$ from 0.1 to 0.15. In this case, the utilization rate increases only by around 0.3 percent on impact. Yet, inflation and consumption still behave in accordance with the empirical evidence, but now display much smaller changes. This shows that even relatively small movements in the utilization rate are sufficient to obtain the desired responses from the model. While data on technology utilization is not readily

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22 Additionally, the positive response of the wage rate, as well as that of labor hours (not reported), while not included in our VAR model, are in line with existing empirical evidence. Among others, Gali et al. (2007) and Andres et al. (2015) report that wages and hours both rise in response to an increase in government spending.
available, Bianchi et al. (2017) argue that their model-implied rate of technology utilization is closely correlated with data on the software expenditures of firms; one potential measure of technology adoption. We have verified that this correlation also emerges conditional on a government spending shock: When we include software expenditures in our baseline VAR model of Section 2, we observe a significant increase in this variable after an increase in government spending (see Figures C.7-C.8 in Appendix C).

5.3.2 The Role of Monetary Policy

The stance of monetary policy plays a key role in the transmission of fiscal policy. At the heart of the negative relationship between inflation and consumption implied by our baseline model are movements in the real interest rate: In the simplified model version studied in Section 4, consumption increases if and only if the central bank engineers a decline in the real interest rate upon observing a drop in inflation. This, in turn, requires a sufficiently strong reaction of the nominal policy rate to a given change in inflation. When we allow for a monetary policy reaction to output fluctuations, this direct link between consumption and inflation breaks down. In terms of the graphical representation in Figure 4, the (EE)-curve becomes steeper and is shifted down in response to a positive government spending shock.
With this in mind, we should expect a smaller increase—or even a decline—in consumption, and a larger drop in inflation. Figure 6 confirms this intuition: The dashed red lines report impulse responses from a version of our model featuring a non-zero policy reaction to output, where we set $\phi_y = 0.125$ (0.5 divided by 4) in accordance with the original proposal of Taylor (1993). In this case, we observe a very small increase in consumption, but a much larger decline in inflation, as compared to our baseline model (solid blue lines).

We can elaborate further by characterizing numerically the requirements that monetary policy must meet in order for our model to match, at least from a qualitative viewpoint, the empirical evidence. Figure 7 shows the behavior of our model as a function of the parameters in the monetary policy rule (13), keeping all other parameters at their baseline calibration. For low values of $\phi_\pi$ and $\phi_y$, as illustrated by the blue area, the model does not have a unique and stable equilibrium given our baseline calibration. As also shown analytically in Appendix D.4, a version of the Taylor principle of standard New Keynesian models holds up in our model: To ensure a unique and stable solution, monetary policy must be sufficiently responsive to movements in inflation. When this condition is satisfied, the

\footnote{For a given combination of $\phi_\pi$ and $\phi_y$, there may exist different combinations of the other parameters of the model (in particular $\chi_2$) for which a unique, stable solution is restored, cf. the discussion in Section 4.}
ratio $\frac{\delta \gamma}{\phi_y}$ must be sufficiently high to ensure that the model produces the desired responses. The green area indicates combinations of policy parameters for which the model produces an increase in consumption and a decline in inflation on impact, while the yellow area indicates combinations where either of these does not obtain. The black dot denotes our baseline calibration. For relatively high values of $\phi_y$, the decline in inflation associated with an increase in government spending leads to a reduction in the nominal and real interest rate, and thus an increase in consumption. Given the empirical evidence presented in Section 2, this case appears to be the most realistic.

### 5.3.3 Decreasing Returns to Scale

So far in our analysis, we have assumed a constant-returns-to-scale technology in the intermediate goods sector. This assumption facilitates a decline in inflation. If instead there are decreasing returns to scale ($\alpha > 0$), a given increase in production requires a larger increase in labor inputs, thus driving up marginal costs, which—all else equal—makes it harder to observe a decline in marginal costs in equilibrium. It is therefore important to verify that our proposed mechanism can reproduce the empirical evidence even in the case of decreasing returns to scale. To this end, the dotted green lines in Figure 6 show impulse responses from our model under the assumption that $\alpha = 0.25$, as in Galí (2015), while the
solid blue lines display our benchmark model for comparison. As can be seen, our main findings are confirmed, as the model is still able to generate a drop in inflation alongside an increase in consumption. However, from a quantitative viewpoint, the movements in these variables are somewhat smaller than those observed in our baseline model, reflecting that our mechanism of variable technology adoption has less quantitative bite in this case.

6 An Estimated Model with Capital Formation

The final step of our analysis is to evaluate the quantitative performance of our proposed mechanism in an estimated model of the U.S. economy. To this end, we augment the model with physical capital formation in order to make it more realistic and thus appropriate for estimation. We also introduce habit formation in consumption to enable our model to reproduce the hump-shaped response of private consumption observed in our empirical analysis. In this section, we first describe the details of these model extensions, and then turn to the estimation of the model.

6.1 A Model with Capital and Habit Formation

We assume that the capital stock is owned by the household and rented to intermediate goods producers in each period. This means that the household makes the choices related to capital accumulation, while the firms choose how much capital to employ in production. The law of motion for capital is given by:

\[ K_t = (1 - \delta) K_{t-1} + \left[ 1 - \frac{\kappa}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right) \right]^2 I_t, \]  

where \( K_t \) and \( I_t \) denote the stock of capital and the investment in new capital, \( 0 \leq \delta < 1 \) is the rate at which capital depreciates, while \( \kappa > 0 \) denotes quadratic investment adjustment costs. The budget constraint of the household now incorporates investment expenses and rental income from capital (with \( r_t^K \) denoting the rental rate):

\[ C_t + I_t + \frac{R_{t-1}b_{t-1}}{\pi_t} = w_t N_t + r_t^K K_{t-1} + b_t + d_t - t_t. \]  

\(^{24}\)In this experiment, we have again set \( \phi_y = 0 \) as in the baseline model. The calibrated parameters \( \psi, \gamma, \) and \( \chi_1 \) are automatically adjusted so as to ensure that our calibration targets are maintained.
In the presence of internal habit formation in consumption, the utility function of the representative household becomes:

\[ E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{(C_t - \theta C_{t-1})^{1-\sigma}}{1 - \sigma} - \frac{\psi N_t^{1+\varphi}}{1 + \varphi} \right], \quad \sigma \neq 1, \]

where \( 0 \leq \theta < 1 \) is the degree of habit formation. The first-order conditions for the choices of consumption, labor, and bond holdings can be stated as:

\[ \lambda_t = (C_t - \theta C_{t-1})^{-\sigma} - \beta \theta (E_t C_{t+1} - \theta C_t)^{-\sigma}, \quad (22) \]

\[ \frac{\psi N_t^\varphi}{w_t} = \lambda_t, \quad (23) \]

\[ \lambda_t = \beta E_t \frac{R_t \lambda_{t+1}}{\pi_{t+1}}, \quad (24) \]

with \( \lambda_t \) denoting the multiplier associated with (21). We can use (22) to substitute for \( \lambda_t \) in (23) and (24), thus obtaining two conditions to replace (2) and (3) in our extended model. In addition, the household now also chooses investment \( I_t \) and capital \( K_t \), subject to (20) and (21). The relevant first-order conditions can be written as:

\[ 1 = Q_t \left( 1 - \frac{\kappa}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 \right) \right) \right) \]

\[ + \beta E_t \frac{Q_t \lambda_{t+1} \lambda_t}{\lambda_t} \kappa \left( \frac{I_{t+1}}{I_t} - 1 \right) \frac{I_{t+1}^2}{I_t^2}, \quad (25) \]

\[ \beta E_t \lambda_{t+1} \left[ r_t^K + (1 - \delta) E_t Q_{t+1} \right] = \lambda_t Q_t, \quad (26) \]

where \( Q_t \) denotes the price of installed capital (in units of consumption), which is given by \( Q_t = \frac{\mu_t}{\lambda_t} \), where \( \mu_t \) is the multiplier associated with (20). Note that (24) and (26) can be combined to yield the following arbitrage condition relating the expected real return on capital to that on bonds:

\[ \frac{r_t^K + (1 - \delta) E_t Q_{t+1}}{Q_t} = \frac{R_t}{E_t \pi_{t+1}}. \]

Production is now given by:

\[ Y_{it} = V_{it} N_{it}^{1-\alpha} K_{it-1}^{\alpha}, \quad (27) \]

and the cost-minimization problem of each intermediate goods producer yields the following first-order condition for the demand for capital:

\[ r_t^K = \alpha E_t m c_{it+1} \frac{E_t Y_{it+1}}{K_{it}}, \quad (28) \]
while the first-order conditions for labor and for technology utilization are still given by (8) and (9). The aggregate resource constraint now reads:

$$Y_t = C_t + I_t + g_t + z(u_t) + \frac{\gamma}{2} (\pi_t - 1)^2 Y_t.$$  

(29)

This completes the description of our extended model. We present the steady state and the log-linearization of this model in Appendix D.6.

### 6.2 Estimation Strategy

The model is estimated using indirect inference. Following Christiano et al. (2005) among others, we estimate (a subset of) the parameters of the model by matching the model-implied impulse responses to a government spending shock to the empirical responses presented in Figure 2. To this end, we first split the parameters into two groups. $\omega_1 = \{\alpha, \beta, \varepsilon, \chi_1, \psi, \frac{\sigma}{\tau}\}$ contains the parameters that we choose to calibrate. We maintain the same parameter values and calibration targets as described in Section 5.1, as well as $\alpha = 0.25$ as in Section 5.3.3. We then collect in $\omega_2 = \{\gamma, \delta, \theta, \kappa, \rho_G, \sigma, \varphi, \phi_0, \phi_y, \chi_2\}$ the parameters to be estimated. Let $\Lambda(\omega_2)$ denote the model-implied impulse responses, which are functions of the parameters, while $\hat{\Lambda}$ denotes the corresponding empirical estimates from our VAR model. We obtain the vector of parameter estimates $\hat{\omega}_2$ by solving:

$$\hat{\omega}_2 = \arg \min_{\omega_2} \left( \Lambda(\omega_2) - \hat{\Lambda} \right)^T W \left( \Lambda(\omega_2) - \hat{\Lambda} \right).$$  

(30)

The weighting matrix $W$ is a diagonal matrix with the sample variances of the VAR-based impulse responses along the diagonal. Effectively, this means that we are attaching higher weights to those impulse responses that are estimated most precisely. We match impulse responses for the seven variables reported in Figure 2 plus investment, which we now include in our structural VAR model, using the responses during the first 20 quarters after the shock. In addition to the intervals over which they are defined, we impose few bounds on the estimated parameters, as discussed in the next subsection.

### 6.3 Estimation Results

We report the estimated parameter values in Table 2, as well as the associated standard errors, which are computed using an application of the delta method, as described, e.g., in Hamilton (1994). We first note that all parameters take on values that are generally in

---

25Since the vector of estimated parameters includes both the parameters in the monetary policy rule ($\phi_a$ and $\phi_y$) and the curvature of the utilization cost function ($\chi_2$), our estimation procedure sometimes draws parameter vectors for which the model has no determinate solution. To circumvent this problem, we introduce a penalty function that drives the procedure away from such cases.
line with the existing literature. The estimated cost of adjusting prices implies an average lifetime of a given price of around $4\frac{1}{2}$ quarters. The depreciation rate of capital and the degree of habit formation are similar to those found in most studies. The estimated investment adjustment cost parameter is modest, although available estimates of this parameter display substantial variation. The persistence of the government spending shock is relatively high. Regarding the estimate of $\varphi$, we note that it almost reaches the lower bound of $1/5$ that we impose in order to avoid a Frisch elasticity of labor supply above 5. As discussed in Section 5.1, while this value is too high according to microeconometric studies, it is not uncommon in the Real Business Cycle literature. The estimated coefficient of risk aversion is relatively low in order to facilitate a sizable increase in consumption in response to the observed drop in the interest rate. The parameters of the monetary policy rule imply a predominance of inflation over output gap stabilization. The estimate of $\phi_y$ is almost driven to its lower bound of zero in order to enable the model to produce an increase in private consumption, cf. Section 5.3.2. Finally, given its central role in our model, the estimated value of $\chi_2$ is of particular interest. We obtain a parameter estimate of $\chi_2 = 0.457$. This value is somewhat higher than those considered so far. Note, however, that the introduction of capital changes the admissible range of values of $\chi_2$ for which the model has a determinate solution featuring a decline in inflation and an increase in consumption on impact. Given the remaining parameter estimates, this result obtains for all values of $\chi_2 \in [0.457; 0.875]$, with the estimation procedure selecting a value of $\chi_2$ at the lower bound of this range in order to generate a substantial increase in consumption.

We turn next to the estimated impulse-response functions from the model, which are reported in Figure 8 alongside their empirical counterparts from the VAR model. Several things are noteworthy. First, the estimated DSGE model matches the responses of all variables qualitatively. Second, the quantitative performance of the model is satisfactory for most variables. The model-implied increase in consumption and the decline of the interest rate fall short of our VAR evidence. With the exception of net tax revenues, the remaining variables are generally in line with the data.

---

26 With decreasing returns to labor inputs, the mapping between the value of $\gamma$ and the duration of a given price is different from the one in our basic model with constant returns to scale calibrated in Section 5.1. See also Galí (2015).

27 The model-implied impulse response of the price level is computed as the cumulative sum of the response of inflation.

28 This suggests that our proposed mechanism might successfully combine with existing methods to generate a more sizable increase in private consumption, such as the rule-of-thumb households of Galí et al. (2007).

29 The response of taxes is substantially smaller than in the VAR model. However, matching the response of tax revenues would require a more thorough treatment of public finances than warranted by our assumption of a balanced government budget each period. In the context of our estimation, the wide confidence band of the VAR-based response of taxes implies that this variable receives a low weight in the estimation procedure.
### Table 2: Parameter Estimates

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\gamma)</td>
<td>232.495</td>
</tr>
<tr>
<td>(\delta)</td>
<td>0.034</td>
</tr>
<tr>
<td>(\theta)</td>
<td>0.582</td>
</tr>
<tr>
<td>(\kappa)</td>
<td>1.930</td>
</tr>
<tr>
<td>(\rho_G)</td>
<td>0.962</td>
</tr>
<tr>
<td>(\sigma)</td>
<td>0.555</td>
</tr>
<tr>
<td>(\varphi)</td>
<td>0.211</td>
</tr>
<tr>
<td>(\phi_{\pi})</td>
<td>1.276</td>
</tr>
<tr>
<td>(\phi_y)</td>
<td>0.010</td>
</tr>
<tr>
<td>(\chi_2)</td>
<td>0.457</td>
</tr>
</tbody>
</table>

Notes: We report standard errors in brackets, obtained using the delta method.

In particular, the response of the price level is usually within the estimated confidence band from the VAR model. This confirms the ability of our model to generate a drop in prices of a realistic magnitude. The DSGE model slightly overestimates the increases in output and TFP in the first year after the shock. Notably, we observe an increase in investment in the DSGE model as well as in the VAR model. In the former, this finding is driven by increases in labor supply and technology utilization in combination with the reduction in the nominal interest rate. Finally, while the response of technology utilization is not matched to any empirical counterpart, we note that the utilization rate again increases somewhat less than 1-for-1 with output, as discussed in Section 5.3.1.

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30Regarding the VAR evidence, Furlanetto (2011) points out that the empirical literature has produced no firm consensus on the response of investment to government spending shocks, and that the response tends to differ across identification schemes. Indeed, when we include investment in our Cholesky VAR model, the response becomes negative.
Figure 8: Impulse responses to a government spending shock of 1 percent. Solid black lines: estimated VAR model using the identification based on forecast errors. Grey areas: 68 percent confidence bands from the VAR model. Dotted blue lines: estimated DSGE model.
7 Conclusion

The assumption that increases in government spending are inflationary is a key building block of many theoretical accounts of fiscal policy. However, this paper presents empirical evidence that inflation tends to drop in response to increases in government spending in the U.S. economy. This result is robust across a wide range of empirical specifications. It emerges alongside the increase in output and private consumption documented in previous studies, as well as an increase in TFP.

To account for our results, we propose a model of variable technology utilization in the spirit of Bianchi et al. (2017). We show that the model can replicate the observed response of prices and other key macroeconomic variables to a government spending shock. Our proposed mechanism may be one of several candidates that can help explain this set of findings. In this respect, our analytical and graphical analysis has sketched some general requirements that other candidate explanations must satisfy. In particular, accounting for potential supply-side effects of fiscal policy seems crucial. We think of this as a fruitful avenue for future research.
References


Appendices

A  The Data

All data used in the baseline specification of our SVAR model—with the exception of total factor productivity (TFP) and the forecast errors of Auerbach and Gorodnichenko (2012)—are taken from Federal Reserve Economic Data (FRED). The series are described in detail below with series names in FRED indicated in brackets:

- $G_t$: Government consumption expenditure and gross investment (GCECE1, seasonally adjusted, Chained 2009 $).
- $Y_t$: Real GDP (GDPC1, seasonally adjusted, Chained 2009 $).
- $C_t$: Real Personal Consumption Expenditures (PCECC96).
- $T_t$: Government current tax receipts (W054RC1Q027SBEA, seasonally adjusted) - Government current transfer receipts (A084RC1Q027SBEA, seasonally adjusted) - Government interest payments (A180RC1Q027SBEA, seasonally adjusted) - Government subsidies (GDISUBS, seasonally adjusted). We convert from nominal to real terms using the GDP deflator (see below).
- $P_t$: Personal Consumption Expenditures Price Index (PCECTPI, seasonally adjusted, 2009=100).
- $R_t$: Nominal interest rate on 3-month Treasury Bills (TB3MS).
- $A_t$: Raw Total Factor Productivity series constructed by the Federal Reserve Bank of San Francisco based on the methodology of Fernald (2014).31

The first four series are converted to per capita terms using the Census Bureau Civilian Population (All Ages) estimates, which we collect from the FRED database (POP). We take logs of all variables except the interest rate, $R_t$.

In addition, we use the following series from the FRED database for the robustness checks:

- CPI index: Consumer Price Index for All Urban Consumers: All Items (CPIAUCSL, seasonally adjusted, 2009=100).
- PCE Core index: Personal Consumption Expenditures Excluding Food and Energy Price Index (PCEPILFE, seasonally adjusted, 2009=100).
- GDP deflator index: Gross Domestic Product: Implicit Price Deflator (GDPDEF, seasonally adjusted, 2009=100).
- Commodity price index: Producer Price Index for All Commodities (PPIACO, not seasonally adjusted, 2009=100).
- Productivity: Real Output per Hour of All Persons in the Nonfarm Business Sector (OPHNFB, seasonally adjusted, 2009=100).
- Investment: Nonresidential Real Private Fixed Investment, Quantity Index (obtained directly from the Bureau of Economic Analysis, NIPA tables, Table 5.3.3., line 2).

Finally, we use the following two series of “narrative” shocks to government spending:

31The data can be collected from https://www.frbsf.org/economic-research/indicators-data/total-factor-productivity-tpf/
$FE_t$: Forecast error of government spending, computed as the difference between forecasts (obtained from the Greenbook data of the Federal Reserve Board combined with the Survey of Professional Forecasters) and the actual, first-release data for the growth rate of government spending. We obtain the series directly from Auerbach and Gorodnichenko (2012).

$NS_t$: Defense news shocks series constructed by Ramey (2011a) and obtained from the webpage of Valerie Ramey.
B Additional Empirical Evidence

This appendix contains supportive evidence and details of some of the results reported in the main text.

B.1 Impulse-Response Decomposition

We perform the decomposition of impulse responses along the lines of Kilian and Lewis (2011). The idea is that the value of any impulse response function at horizon \( h \geq 0 \) can be written as the sum of the response to its own lagged values and the responses to lagged values of the other variables in the system. Consider again our baseline VAR model studied in Section 2.2, where we now abstract from the constant and trends included in the model, as these do not affect the impulse responses directly. We can then rewrite (1) as:

\[
B X_t = A_1 X_{t-1} + \ldots + A_p X_{t-p} + \epsilon_t.
\] (B.1)

Observe that this system can be restated as:

\[
X_t = C X_t + A_1 X_{t-1} + \ldots + A_p X_{t-p} + \epsilon_t,
\] (B.2)

where the matrix \( C \) is strictly lower triangular, i.e., it has zeros on the diagonal. This expression is useful in defining the coefficient matrix \( D \equiv [C \ A_1 \ ... \ A_p] \).

Conditional on a government spending shock at time \( t = 0 \), we can then state the contribution of variable \( j \) to the horizon \( h \)-value of variable \( k \) in the VAR, denoted \( \Gamma_{h,j,k} \), as follows:

\[
\Gamma_{h,j,k} = \min(p,h) \sum_{s=0}^{\min(p,h)} D_{k,s} \Xi_{j,k,h-s,m}
\]

where \( N \) is the number of variables in the VAR, and with \( \Xi_{j,k,h-s,m} \) denoting the \((j,k)\)-entry in the matrix of impulse responses at horizon \( h - m \).

Consider the decomposition of the response of the PCE index shown in Figure B.1. The solid blue line represents the original impulse response of this variable, as reported in Figure 2. As the figure illustrates, this response is overwhelmingly accounted for by lagged values of the price level itself, although the shock to government spending exerts a direct effect in the first few quarters. Notably, there is no indication of prices reacting to the decline of the interest rate. This excludes that our results could simply be a manifestation of the monetary policy price puzzle of Sims (1992).
Figure B.1: Impulse-response decomposition of the response of prices in the baseline VAR model with identification based on forecast errors. The line “Contribution from G” is the sum of the contribution from lagged values of government spending and lagged values of the forecast error of government spending.

B.2 The Effect of Commodity Prices

In this appendix, we show that the inclusion of commodity prices in the VAR model significantly changes the response of prices to a monetary policy shock, but not to a fiscal policy shock. For the sake of clarity, we do this using a smaller VAR model featuring only a subset of the variables considered in the main text. In Figure B.2 we report the impulse responses to a government spending shock identified using forecast errors in a VAR model of the following variables: \( X_t = \begin{bmatrix} F E_t & G_t & Y_t & P_t & PC_t \end{bmatrix}' \), where \( PC_t \) denotes the commodity price index. We also show impulse responses from the same model without commodity prices. In Figure B.3, we report the corresponding impulse responses based on the Cholesky identification scheme from a VAR model of the same set of variables. The message from these exercises is clear, confirming our findings from Section 2.3 that the inclusion of commodity prices in the VAR model does not alter our findings regarding the response of prices.

For comparison, Figure B.4 reports the impulse responses to a monetary policy shock from a VAR model of the following variables: \( X_t = \begin{bmatrix} Y_t & P_t & PC_t & R_t \end{bmatrix}' \). The shock is identified using the Cholesky identification scheme with the nominal interest rate ordered after output, prices, and commodity prices, as in Christiano et al. (2005). In this case, consumer prices increase significantly and persistently in response to a monetary policy tightening when commodity prices are not included. However, with commodity prices in the model, the initial increase in prices quickly turns into a significant and persistent decline. We thus confirm that the introduction of commodity prices is able to resolve the price puzzle of monetary policy, but does not have major effects on the response of prices to fiscal policy shocks.
Figure B.2: The dynamic effects of a shock to government spending with (blue lines) and without (red lines) commodity prices in the VAR model. Estimates obtained using the identification scheme based on forecast errors. Dashed blue and dotted red lines represent 68 percent confidence intervals from the models with and without commodity prices, respectively.

Figure B.3: The dynamic effects of a shock to government spending with (blue lines) and without (red lines) commodity prices in the VAR model. Estimates obtained using the Cholesky identification scheme. Dashed blue and dotted red lines represent 68 percent confidence intervals from the models with and without commodity prices, respectively.
Figure B.4: The dynamic effects of a shock to the monetary policy interest rate with (blue lines) and without (red lines) commodity prices in the VAR model. Estimates obtained using the Cholesky identification scheme. Dashed blue and dotted red lines represent 68 percent confidence intervals from the models with and without commodity prices, respectively.

### C Robustness Checks

This appendix contains a set of robustness checks. The first three figures report robustness checks from our baseline VAR model with identification based on forecast errors. The next three figures show the same set of robustness checks for the VAR model with identification based on a Cholesky decomposition. Finally, we present the results obtained when we add software expenditures to our baseline VAR model in Figures C.7-C.8.

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32 The alternative ordering used in some of the robustness checks below is the following: \( \mathbf{X}_t = [ F_{E_t} \ G_t \ T_t \ Y_t \ C_t \ P_t \ A_t \ R_t ]' \). We have experimented with other alternative orderings, in particular regarding the placement of the price index. This did not lead to any changes in our results.
Figure C.1: The dynamic effects of a shock to government spending. Robustness checks: Different price indices: GDP deflator (first column), CPI index (second column), PCE core price index (third column). Estimates obtained using the identification scheme based on forecast errors. The black line denotes the estimated response, while the grey areas represent 68 percent confidence bands.
Figure C.2: The dynamic effects of a shock to government spending. Robustness checks: Including commodity prices (first column), alternative productivity measure (second column), excluding TFP from the baseline model (third column). Estimates obtained using the identification scheme based on forecast errors. The black line denotes the estimated response, while the grey areas represent 68 percent confidence bands.
Figure C.3: The dynamic effects of a shock to government spending. Robustness checks: Alternative ordering of variables (first column), model with 6 lags instead of 4 (second column), model excluding the quadratic trend (third column). Estimates obtained using the identification scheme based on forecast errors. The black line denotes the estimated response, while the grey areas represent 68 percent confidence bands.
Figure C.4: The dynamic effects of a shock to government spending. Robustness checks: Different price indices: GDP deflator (first column), CPI index (second column), PCE core price index (third column). Estimates obtained using the identification scheme based on Cholesky identification scheme. The black line denotes the estimated response, while the grey areas represent 68 percent confidence bands.
Figure C.5: The dynamic effects of a shock to government spending. Robustness checks: Including commodity prices (first column), alternative productivity measure (second column), excluding TFP from the baseline model (third column). Estimates obtained using the identification scheme based on Cholesky identification scheme. The black line denotes the estimated response, while the grey areas represent 68 percent confidence bands.
Figure C.6: The dynamic effects of a shock to government spending. Robustness checks: Alternative ordering of variables (first column), model with 6 lags instead of 4 (second column), model excluding the quadratic trend (third column). Estimates obtained using the Cholesky identification scheme. The black line denotes the estimated response, while the grey areas represent 68 percent confidence bands.
Figure C.7: The dynamic effects of a shock to government spending. Model augmented with software expenditure. Estimates obtained using the identification scheme based on forecast errors. The black line denotes the estimated response, while the grey areas represent 68 percent confidence bands.
Figure C.8: The dynamic effects of a shock to government spending. Model augmented with software expenditure. Estimates obtained using the Cholesky identification scheme. The black line denotes the estimated response, while the grey areas represent 68 percent confidence bands.
D The Model

This appendix presents the details of our model of variable technology utilization. We impose the functional form of \( z(u_t) \) proposed in (19) throughout the appendix.

D.1 The Steady State

As usual, the steady-state interest rate is pinned down by the inverse of the household’s discount factor: \( R = 1/\beta \). From the optimal price setting of intermediate goods firms (10), we obtain \( mc = \frac{z - 1}{z} \). From the goods market clearing condition (16), we get:

\[
\frac{C}{Y} = 1 - \frac{g}{Y},
\]

where \( \frac{g}{Y} \) is determined exogenously. Steady-state production is pinned down from (5):

\[
Y = uA N^{1-\alpha},
\]

where \( A \) is exogenous, \( u \) is fixed at 1 in steady state, and \( N \) is fixed at \( N = 0.25 \). Combining labor supply (2) and labor demand (8), and using the production function, we can find the value of \( \psi \) that ensures this:

\[
\psi N^\sigma = C_t^{-\sigma} (1 - \alpha) mc \frac{Y}{N} \Leftrightarrow \\
\psi N^\sigma = C_t^{-\sigma} (1 - \alpha) mcuA N^{-\alpha} \Leftrightarrow \\
\psi = \frac{(1 - \alpha) mcuA}{N^{\sigma+\alpha} C^\sigma}.
\]

Finally, to ensure that the utilization rate equals 1 in steady state, we rewrite (9) to get:

\[
z'(1) = mc \frac{Y}{u} \Leftrightarrow \\
\chi_1 = mc \frac{Y}{u},
\]

which pins down the required value of \( \chi_1 \). This completes the characterization of the steady state.

D.2 Log-linearized Model

Before simulating the model, we log-linearize it around the non-stochastic steady state. Letting \( \hat{x}_t \) denote the log deviation of a generic variable \( x_t \) from its steady-state value \( x \), we obtain the following set of log-linearized equilibrium conditions:

\[
\varphi \hat{N}_t = -\sigma \hat{C}_t + \hat{w}_t, \\
- \sigma \hat{C}_t = E_t \left( -\sigma \hat{C}_{t+1} + \hat{R}_t - \hat{\pi}_{t+1} \right), \\
\hat{Y}_t = \hat{u}_t + \hat{A}_t + (1 - \alpha) \hat{N}_t, \\
\frac{C}{\bar{Y} \hat{C}_t} = \hat{Y}_t - mc \hat{u}_t - \frac{g}{\bar{Y} \hat{g}_t}, \\
\frac{\chi_2}{\chi_1} \hat{u}_t = \hat{m} \hat{c}_t + \hat{Y}_t - \hat{u}_t,
\]

(1) (2) (3) (4) (5)
\[ \tilde{mc}_t = \tilde{w}_t - \tilde{u}_t - \tilde{A}_t + \alpha \tilde{N}_t, \]  
(D.6)
\[ \tilde{\pi}_t = \beta E_t \tilde{\pi}_{t+1} + \frac{\varepsilon - 1}{\gamma} \tilde{mc}_t, \]  
(D.7)
\[ \hat{R}_t = \phi_x \tilde{\pi}_t + \phi_y \tilde{Y}_t, \]  
(D.8)
\[ \hat{g}_t = \rho_g \hat{g}_{t-1} + \varepsilon_t^g, \]  
(D.9)
\[ \hat{A}_t = \rho_A \hat{A}_{t-1} + \varepsilon_t^A. \]  
(D.10)

We thus have a system of 10 equations in 10 variables: \( \tilde{Y}_t, \tilde{C}_t, \hat{g}_t, \tilde{\pi}_t, \tilde{mc}_t, \tilde{u}_t, \tilde{w}_t, \tilde{A}_t, \tilde{N}_t, \hat{R}_t. \)

### D.3 Analytical Solution

As described in the main text, we derive the analytical solution to the model under the following simplifying assumptions: No technology shocks \( \tilde{A}_t = 0 \), constant returns to scale in production \( \sigma = 0 \), log utility in consumption \( \sigma = 1 \), unitary (inverse) Frisch elasticity of labor supply \( \varphi = 1 \), and no monetary policy reaction to the output gap \( \phi_y = 0 \). Under these assumptions, it is straightforward to verify that (D.2) and (D.8) can be combined to obtain the Euler equation presented in Section 4:

\[ -\tilde{C}_t = E_t \left( -\tilde{C}_{t+1} + \phi_x \tilde{\pi}_t - \tilde{\pi}_{t+1} \right). \]  
(D.11)

To arrive at the New Keynesian Phillips Curve studied in Section 4, we begin by combining (D.1) and (D.3) to obtain:

\[ \tilde{Y}_t = \tilde{u}_t - \tilde{C}_t + \tilde{w}_t, \]

This expression can be inserted twice, into (D.4) and (D.5), to obtain:

\[ \begin{align*}
\frac{C}{Y} \tilde{C}_t &= \tilde{u}_t - \tilde{C}_t + \tilde{w}_t - mc \tilde{u}_t - \frac{q}{Y} \hat{g}_t \Leftrightarrow \\
\tilde{w}_t &= \left(1 + \frac{C}{Y}\right) \tilde{C}_t - \left(1 - mc\right) \tilde{u}_t + \frac{q}{Y} \hat{g}_t, 
\end{align*} \]  
(D.12)

and

\[ \begin{align*}
\frac{\chi_2}{\chi_1} \tilde{u}_t &= \tilde{mc}_t + \tilde{u}_t - \tilde{C}_t + \tilde{w}_t - \tilde{u}_t \Leftrightarrow \\
\frac{\chi_2}{\chi_1} \tilde{u}_t &= \tilde{w}_t - \tilde{u}_t - \tilde{C}_t + \tilde{w}_t \Leftrightarrow \\
2\tilde{w}_t &= \left(\frac{\chi_2}{\chi_1} + 1\right) \tilde{u}_t + \tilde{C}_t, 
\end{align*} \]

where the second-to-last line uses (D.6). We can combine these two expressions:

\[ \begin{align*}
\begin{align*}
\left(1 + \frac{C}{Y}\right) \tilde{C}_t - \left(1 - mc\right) \tilde{u}_t + \frac{q}{Y} \hat{g}_t &= \frac{\chi_2}{\chi_1} + 1 \Leftrightarrow \\
\tilde{u}_t &= \frac{\chi_2}{\chi_1} + 2 \left(1 - mc\right) \tilde{C}_t + \frac{2q}{Y} \hat{g}_t. 
\end{align*} \]  
(D.13)
We are now ready to insert into the original New Keynesian Phillips Curve (D.7), using first (D.6):
\[
\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \frac{\varepsilon - 1}{\gamma} \hat{mc}_t \\
\pi_t = \beta E_t \hat{\pi}_{t+1} + \frac{\varepsilon - 1}{\gamma} (\hat{\omega}_t - \hat{\omega}_t),
\]
and then inserting from (D.12):
\[
\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \frac{\varepsilon - 1}{\gamma} \left( \left( 1 + \frac{C}{Y} \right) \hat{C}_t - (2 - mc) \hat{\omega}_t + \frac{g}{Y} \hat{g}_t \right),
\]
where we can insert from (D.13) to get:
\[
\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \frac{\varepsilon - 1}{\gamma} \left( \left( 1 + \frac{C}{Y} \right) \hat{C}_t + \frac{g}{Y} \hat{g}_t \right) - \frac{\varepsilon - 1}{\gamma} (2 - mc) \left[ \frac{\frac{\frac{\xi_1}{\xi} + 1 + \left( \frac{\frac{\xi_1}{\xi} - 1}{\xi} \frac{C}{Y} - mc \right)}{\frac{\frac{\xi_1}{\xi} + 1 + 2(1 - mc) \hat{\omega}_t}} \hat{C}_t + \frac{\frac{\frac{\xi_1}{\xi} - 1}{\frac{\frac{\xi_1}{\xi} + 3 - 2mc}}}{\frac{\frac{\xi_1}{\xi} + 1 + 2(1 - mc) \hat{\omega}_t}} \frac{\frac{\frac{\xi_1}{\xi} + 1 + 2(1 - mc) \hat{\omega}_t}}{\frac{\frac{\xi_1}{\xi} + 1 + 2(1 - mc) \hat{\omega}_t}} \right],
\]
which can be rewritten as:
\[
\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \frac{\varepsilon - 1}{\gamma} \left( \left( 1 + \frac{C}{Y} \right) \hat{C}_t + \frac{g}{Y} \hat{g}_t \right) - \frac{\varepsilon - 1}{\gamma} (2 - mc) \left[ \frac{\frac{\frac{\xi_1}{\xi} + 1 + \left( \frac{\frac{\xi_1}{\xi} - 1}{\xi} \frac{C}{Y} - mc \right)}{\frac{\frac{\xi_1}{\xi} + 1 + 2(1 - mc) \hat{\omega}_t}} \hat{C}_t + \frac{\frac{\frac{\xi_1}{\xi} - 1}{\frac{\frac{\xi_1}{\xi} + 3 - 2mc}}}{\frac{\frac{\xi_1}{\xi} + 1 + 2(1 - mc) \hat{\omega}_t}} \frac{\frac{\frac{\xi_1}{\xi} + 1 + 2(1 - mc) \hat{\omega}_t}}{\frac{\frac{\xi_1}{\xi} + 1 + 2(1 - mc) \hat{\omega}_t}} \right],
\]
\[
\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + a \hat{C}_t - b \hat{g}_t,
\]
(D.14)
after defining
\[
a = \frac{\varepsilon - 1}{\gamma} \left( \frac{\frac{\xi_1}{\xi} + 1 + \left( \frac{\frac{\xi_1}{\xi} - 1}{\xi} \frac{C}{Y} - mc \right)}{\frac{\frac{\xi_1}{\xi} + 1 + 2(1 - mc) \hat{\omega}_t}} \right),
\]
(D.15)
\[
b = \frac{\varepsilon - 1}{\gamma} \left( \frac{\frac{\xi_1}{\xi} + 1 + \left( \frac{\frac{\xi_1}{\xi} - 1}{\xi} \frac{C}{Y} - mc \right)}{\frac{\frac{\xi_1}{\xi} + 1 + 2(1 - mc) \hat{\omega}_t}} \right).
\]
(D.16)
Equations (D.11) and (D.14) can be combined with (D.9) to obtain 3 equations in \( \hat{\pi}_t \), \( \hat{C}_t \), and \( \hat{g}_t \). We can solve this system analytically using the method of undetermined coefficients. For expository simplicity, we assume that the shock to government spending has no persistence \( (\rho_G = 0) \). We conjecture that the solutions for \( \hat{\pi}_t \), \( \hat{C}_t \), and \( \hat{g}_t \) take the form:
\[
\hat{C}_t = \Psi \hat{g}_t,
\]
\[
\hat{\pi}_t = \Phi \hat{g}_t,
\]
where the coefficients \( \Psi \) and \( \Phi \) are yet to be determined. Inserting these conjectured solutions into (D.11) and (D.14), we obtain:
\[
-\hat{C}_t = E_t \left( -\hat{C}_{t+1} + \phi_x \hat{\pi}_t - \hat{\pi}_{t+1} \right) \equiv -\Psi \hat{g}_t = E_t \left( -\Psi \hat{g}_{t+1} + \phi_x \Phi \hat{g}_t - \Phi \hat{g}_{t+1} \right) \equiv \Psi = -\phi_x \Phi,
\]
54
where we have used that \( \text{E}_t \hat{g}_{t+1} = 0 \) when shocks have no persistence. Further, we get:

\[
\hat{\pi}_t = \beta \text{E}_t \hat{\pi}_{t+1} + \frac{\varepsilon - 1}{\gamma} \left( \frac{\chi_x + 1 + \left( \frac{\chi_y}{\gamma} - 1 \right) \frac{\chi_y}{\gamma} - mc}{\frac{\chi_x}{\gamma} + 3 - 2mc} \hat{C}_t + \frac{\varepsilon - 1}{\gamma} \left( \frac{\chi_x}{\gamma} + 3 - 2mc \hat{g}_t \right) \right) \]

\[
\Phi \hat{g}_t = \beta \text{E}_t \Phi \hat{g}_{t+1} + \frac{\varepsilon - 1}{\gamma} \left( \frac{\chi_x + 1 + \left( \frac{\chi_y}{\gamma} - 1 \right) \frac{\chi_y}{\gamma} - mc}{\frac{\chi_x}{\gamma} + 3 - 2mc} \Psi \hat{g}_t + \frac{\varepsilon - 1}{\gamma} \left( \frac{\chi_x}{\gamma} + 3 - 2mc \hat{g}_t \right) \right) \]

\[
\Phi = \frac{\varepsilon - 1}{\gamma} \left( \frac{\chi_x + 1 + \left( \frac{\chi_y}{\gamma} - 1 \right) \frac{\chi_y}{\gamma} - mc}{\frac{\chi_x}{\gamma} + 3 - 2mc} \right) + \frac{\varepsilon - 1}{\gamma} \left( \frac{\chi_x}{\gamma} + 3 - 2mc \right) \]

Combining these two expressions yields:

\[
\Phi = \frac{\varepsilon - 1}{\gamma} \left( \frac{\chi_x + 1 + \left( \frac{\chi_y}{\gamma} - 1 \right) \frac{\chi_y}{\gamma} - mc}{\frac{\chi_x}{\gamma} + 3 - 2mc} \right) \Phi + \frac{\varepsilon - 1}{\gamma} \left( \frac{\chi_x}{\gamma} + 3 - 2mc \right) \]

\[
\Phi \left[ 1 + \frac{\varepsilon - 1}{\gamma} \phi \left( \frac{\chi_x + 1 + \left( \frac{\chi_y}{\gamma} - 1 \right) \frac{\chi_y}{\gamma} - mc}{\frac{\chi_x}{\gamma} + 3 - 2mc} \right) \right] = \frac{\varepsilon - 1}{\gamma} \frac{\chi_x}{\gamma} + 3 - 2mc \]

\[
\Phi \left[ \frac{\varepsilon - 1}{\gamma} \phi \left( \frac{\chi_x + 1 + \left( \frac{\chi_y}{\gamma} - 1 \right) \frac{\chi_y}{\gamma} - mc}{\frac{\chi_x}{\gamma} + 3 - 2mc} \right) \right] = \frac{\varepsilon - 1}{\gamma} \frac{\chi_x}{\gamma} + 3 - 2mc \]

and then:

\[
\Psi = -\phi \Phi \Rightarrow \]

\[
\Psi = -\phi \left( \frac{\varepsilon - 1}{\gamma} \frac{\chi_x}{\gamma} + 1 + \left( \frac{\chi_y}{\gamma} - 1 \right) \frac{\chi_y}{\gamma} - mc \right) + \frac{\chi_x}{\gamma} + 3 - 2mc \]

so that the solution is:

\[
\hat{C}_t = -\phi \left( \frac{\varepsilon - 1}{\gamma} \frac{\chi_x}{\gamma} + 1 + \left( \frac{\chi_y}{\gamma} - 1 \right) \frac{\chi_y}{\gamma} - mc \right) + \frac{\chi_x}{\gamma} + 3 - 2mc \hat{g}_t, \tag{D.17} \]

\[
\hat{\pi}_t = -\phi \left( \frac{\varepsilon - 1}{\gamma} \frac{\chi_x}{\gamma} + 1 + \left( \frac{\chi_y}{\gamma} - 1 \right) \frac{\chi_y}{\gamma} - mc \right) + \frac{\chi_x}{\gamma} + 3 - 2mc \hat{g}_t. \tag{D.18} \]

This confirms the form of our conjectured solution, and provides us with closed-form expressions of how consumption and inflation react to a government spending shock on impact. To establish the
sign of these coefficients, we first note that the denominator is positive whenever:

\[
\frac{\varepsilon - 1}{\gamma} \phi_\pi \left[ \frac{\chi_2}{\chi_1} + 1 + \left( \frac{\chi_2}{\chi_1} - 1 \right) \frac{C}{Y} - mc \right] + \frac{\chi_2}{\chi_1} + 3 - 2mc > 0 \iff \\
\frac{\chi_2}{\chi_1} \left[ 1 + \frac{\varepsilon - 1}{\gamma} \phi_\pi \left( 1 + \frac{C}{Y} \right) \right] > mc \left( 2 + \phi_\pi \frac{\varepsilon - 1}{\gamma} \right) - 3 - \frac{\varepsilon - 1}{\gamma} \phi_\pi \left( 1 - \frac{C}{Y} \right) \iff \\
\chi_2 > \frac{mc \left( 2 + \phi_\pi \frac{\varepsilon - 1}{\gamma} \right) - 3 - \frac{\varepsilon - 1}{\gamma} \phi_\pi \left( 1 - \frac{C}{Y} \right)}{1 + \frac{\varepsilon - 1}{\gamma} \phi_\pi \left( 1 + \frac{C}{Y} \right)}. \tag{D.19}
\]

This is a lower bound on \( \chi_2 \). We show below that this condition is always satisfied when the model has a unique and determinate solution. We therefore obtain a decline in inflation and an increase in consumption if and only if the numerators in both expressions are negative:

\[
\phi_\pi \frac{\varepsilon - 1}{\gamma} \frac{g}{Y} \left( \frac{\chi_2}{\chi_1} - 1 \right) < 0 \iff \\
\left( \frac{\chi_2}{\chi_1} - 1 \right) < 0 \iff \\
\chi_2 < \chi_1. \tag{D.20}
\]

This is the condition stated in Proposition 2 in the main text. However, to complete the proof, the next subsection derives the conditions for the model to have a unique and stable equilibrium.

### D.4 Equilibrium Determinacy and Uniqueness

The system consisting of (D.11) and (D.14) has two non-predetermined variables. This implies that a necessary and sufficient condition for the model to have a unique and determinate equilibrium is that both eigenvalues of the characteristic polynomial should be inside the unit circle. To write up the characteristic polynomial, we first restate the system on matrix form. After some algebra, we arrive at the following expression:

\[
\begin{bmatrix}
\hat{C}_t \\
\hat{\pi}_t
\end{bmatrix} = \Omega \begin{bmatrix}
\gamma & (1 - \beta \phi_\pi) \gamma \\
\Gamma & \Gamma + \beta \gamma
\end{bmatrix} \begin{bmatrix}
E_t \hat{C}_{t+1} \\
E_t \hat{\pi}_{t+1}
\end{bmatrix} + \Omega \Xi \begin{bmatrix}
-\phi_\pi \\
1
\end{bmatrix} g_t \iff \\
\begin{bmatrix}
\hat{C}_t \\
\hat{\pi}_t
\end{bmatrix} = A_0 \begin{bmatrix}
E_t \hat{C}_{t+1} \\
E_t \hat{\pi}_{t+1}
\end{bmatrix} + B_0 g_t, \quad A_0 \equiv \Omega \begin{bmatrix}
\gamma & (1 - \beta \phi_\pi) \gamma \\
\Gamma & \Gamma + \beta \gamma
\end{bmatrix}, \quad B_0 \equiv \Omega \Xi \begin{bmatrix}
\phi_\pi \\
1
\end{bmatrix},
\]

where we have defined:

\[
\Omega \equiv \frac{1}{\frac{\chi_2}{\chi_1} + 3 - 2mc + \phi_\pi \frac{\varepsilon - 1}{\gamma} \left( \frac{\chi_2}{\chi_1} + 1 + \left( \frac{\chi_2}{\chi_1} - 1 \right) \frac{C}{Y} - mc \right)},
\]

\[
\Gamma \equiv \frac{\varepsilon - 1}{\gamma} \left( \frac{\chi_2}{\chi_1} + 1 + \left( \frac{\chi_2}{\chi_1} - 1 \right) \frac{C}{Y} - mc \right),
\]

\[
\Xi \equiv \frac{\varepsilon - 1}{\gamma} \left( \frac{\chi_2}{\chi_1} - 1 \right) g, 
\]

\[
\Upsilon \equiv \left( \frac{\chi_2}{\chi_1} + 3 - 2mc \right). 
\]
The characteristic polynomial is then:

$$|A_0 - \lambda I| = 0 \Leftrightarrow \begin{vmatrix} \frac{\chi_2}{\chi_1} + 3 - 2mc & \frac{\phi_x}{\gamma} \left(1 - \beta \phi_x\right) \frac{\chi_2}{\chi_1} - mc \end{vmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0 \Leftrightarrow$$

$$0 = \begin{bmatrix} \frac{\chi_2}{\chi_1} + 3 - 2mc + \frac{\phi_x}{\gamma} \left(1 - \beta \phi_x\right) \frac{\chi_2}{\chi_1} - mc \\ \frac{\chi_2}{\chi_1} + 1 + \left(\frac{\chi_2}{\chi_1} - 1\right) \frac{mc}{\gamma} \end{bmatrix} - \lambda \begin{bmatrix} \frac{\chi_2}{\chi_1} + 3 - 2mc + \frac{\phi_x}{\gamma} \left(1 - \beta \phi_x\right) \frac{\chi_2}{\chi_1} - mc \\ \frac{\chi_2}{\chi_1} + 1 + \left(\frac{\chi_2}{\chi_1} - 1\right) \frac{mc}{\gamma} \end{bmatrix} + \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

After some tedious algebra, we are able to restate the implied second-order polynomial as:

$$0 = \lambda^2 + a_1 \lambda + a_0,$$

where we have defined:

$$a_1 \equiv - \left(1 + \beta \right) \left(\frac{\chi_2}{\chi_1} + 3 - 2mc\right) + \frac{\phi_x}{\gamma} \left[\frac{\chi_2}{\chi_1} + 1 + \left(\frac{\chi_2}{\chi_1} - 1\right) \frac{mc}{\gamma} - mc\right],$$

$$a_0 \equiv \beta \left(\frac{\chi_2}{\chi_1} + 3 - 2mc\right) + \frac{\phi_x}{\gamma} \left[\frac{\chi_2}{\chi_1} + 1 + \left(\frac{\chi_2}{\chi_1} - 1\right) \frac{mc}{\gamma} - mc\right].$$

We know from, e.g., LaSalle (1986) that both eigenvalues are inside the unit circle if and only if both of the following conditions are satisfied:

$$|a_0| < 1,$$  \hspace{1cm} (D.21)

$$|a_1| < 1 + a_0.$$  \hspace{1cm} (D.22)

We can check these in turn. The first condition yields:

$$\frac{\beta \left(\frac{\chi_2}{\chi_1} + 3 - 2mc\right) + \frac{\phi_x}{\gamma} \left[\frac{\chi_2}{\chi_1} + 1 + \left(\frac{\chi_2}{\chi_1} - 1\right) \frac{mc}{\gamma} - mc\right]}{\frac{\chi_2}{\chi_1} + 3 - 2mc} < 1.$$

Since $\beta < 1$ and the bracket in the numerator is always positive, the denominator will be larger than the numerator (and thus, the inequality satisfied) as long as the second term in the denominator is non-negative:

$$\frac{\phi_x}{\gamma} \left(1 - \frac{1}{2}\frac{\chi_2}{\chi_1} - 1\right) \frac{mc}{\gamma} - mc > 0 \Leftrightarrow$$

$$\frac{\chi_2}{\chi_1} + 1 + \left(\frac{\chi_2}{\chi_1} - 1\right) \left(1 - \frac{g}{Y}\right) - mc > 0 \Leftrightarrow$$

$$\frac{\chi_2}{\chi_1} \left(1 + \left(1 - \frac{g}{Y}\right)ight) > mc + \left(1 - \frac{g}{Y}\right) - 1 \Leftrightarrow$$

$$\frac{\chi_2}{\chi_1} > \frac{mc - \frac{g}{Y}}{2 - \frac{g}{Y}}.$$  \hspace{1cm} (D.23)
This is the condition stated in Proposition 1 in the main text, providing another lower bound on \( \chi_2 \). We can verify that this is the relevant, binding lower bound on \( \chi_2 \) by showing that this expression is strictly larger than the one implied by (D.19):

\[
\chi_1 \frac{mc - \frac{g}{Y}}{2 - \frac{g}{Y}} > \chi_1 \frac{mc (2 + \phi_\pi \frac{\varepsilon - 1}{\gamma}) - 3 - \frac{\varepsilon - 1}{\gamma} \phi_\pi (1 - \frac{C}{Y})}{1 + \frac{\varepsilon - 1}{\gamma} \phi_\pi (1 + \frac{C}{Y})} \iff
\]

\[
\left( mc - \frac{g}{Y} \right) \left[ 1 + \frac{\varepsilon - 1}{\gamma} \phi_\pi \left( 1 + \frac{C}{Y} \right) \right] > \left( 2 - \frac{g}{Y} \right) \left[ mc \left( 2 + \phi_\pi \frac{\varepsilon - 1}{\gamma} \right) - 3 - \frac{\varepsilon - 1}{\gamma} \phi_\pi \left( 1 - \frac{C}{Y} \right) \right] \iff
\]

\[
6 > \frac{2g}{Y} (2 - mc) + 3mc,
\]

where the last step follows from some simple but tedious algebra. The right-hand side is maximized when \( \phi_\pi \) reaches its upper bound of 1 and \( mc \) reaches its upper bound of 1 (when \( \varepsilon \to \infty \)). In this case, the right-hand side approaches 5. We can thus conclude that this condition is always satisfied, so that the binding lower bound on \( \chi_2 \) is given from (D.23).

Consider now the second necessary and sufficient condition for a unique and determinate equilibrium, (D.22), which yields:

\[
|a_1| < 1 + a_0 \iff
\]

\[
- \left( 1 + \beta \left( \frac{x_2}{x_1} + 3 - 2mc \right) + \frac{\varepsilon - 1}{\gamma} \left[ \frac{x_2}{x_1} + 1 + \left( \frac{x_2}{x_1} - 1 \right) \frac{C}{Y} - mc \right] \right) < \]

\[
\frac{\beta \left( \frac{x_2}{x_1} + 3 - 2mc \right)}{\left( \frac{x_2}{x_1} + 3 - 2mc \right) + \frac{\phi_\pi (\varepsilon - 1)}{\gamma} \left[ \frac{x_2}{x_1} + 1 + \left( \frac{x_2}{x_1} - 1 \right) \frac{C}{Y} - mc \right]} \iff
\]

\[
\frac{\beta \left( \frac{x_2}{x_1} + 3 - 2mc \right) + \frac{\phi_\pi (\varepsilon - 1)}{\gamma} \left[ \frac{x_2}{x_1} + 1 + \left( \frac{x_2}{x_1} - 1 \right) \frac{C}{Y} - mc \right]}{\left( \frac{x_2}{x_1} + 3 - 2mc \right) + \frac{\phi_\pi (\varepsilon - 1)}{\gamma} \left[ \frac{x_2}{x_1} + 1 + \left( \frac{x_2}{x_1} - 1 \right) \frac{C}{Y} - mc \right]}.
\]

We saw above that the last term in the denominator is positive, and we have established that also the first term is positive, so we can cancel out the denominators:

\[
- \left[ (1 + \beta) \left( \frac{x_2}{x_1} + 3 - 2mc \right) + \frac{\varepsilon - 1}{\gamma} \left( \frac{x_2}{x_1} + 1 + \left( \frac{x_2}{x_1} - 1 \right) \frac{C}{Y} - mc \right) \right] <
\]

\[
(1 + \beta) \left( \frac{x_2}{x_1} + 3 - 2mc \right) + \frac{\phi_\pi (\varepsilon - 1)}{\gamma} \left[ \frac{x_2}{x_1} + 1 + \left( \frac{x_2}{x_1} - 1 \right) \frac{C}{Y} - mc \right].
\]

Using the same insights, we conclude that all terms on the left-hand side must be positive, so taking absolute values yields:

\[
(1 + \beta) \left( \frac{x_2}{x_1} + 3 - 2mc \right) + \frac{\varepsilon - 1}{\gamma} \left( \frac{x_2}{x_1} + 1 + \left( \frac{x_2}{x_1} - 1 \right) \frac{C}{Y} - mc \right) <
\]

\[
(1 + \beta) \left( \frac{x_2}{x_1} + 3 - 2mc \right) + \frac{\phi_\pi (\varepsilon - 1)}{\gamma} \left[ \frac{x_2}{x_1} + 1 + \left( \frac{x_2}{x_1} - 1 \right) \frac{C}{Y} - mc \right] \iff
\]

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\[
\frac{\varepsilon - 1}{\gamma} \left[ \frac{x_2}{x_1} + 1 + \left( \frac{x_2}{x_1} - 1 \right) \frac{C}{Y} - mc \right] < \frac{\phi_\pi (\varepsilon - 1)}{\gamma} \left[ \frac{x_2}{x_1} + 1 + \left( \frac{x_2}{x_1} - 1 \right) \frac{C}{Y} - mc \right] \iff \\
\phi_\pi > 1,
\]
which is just the well-known Taylor-principle (in the absence of a monetary policy reaction to output, as assumed above). This condition is satisfied by assumption, as we have assumed \( \phi_\pi > 1 \) already in the main text.

To sum up, we have established that the model has a unique and determinate solution if and only if conditions (D.23) and (D.24) are satisfied, and that when this is the case, the solution features an increase in consumption and a decline in inflation if and only if condition (D.20) holds. This completes the proof of Propositions 1 and 2 in the main text.

As a final note, recall our graphical representation of equations (D.11) and (D.14) in Section 4. Given the definition of the parameters \( a \) and \( b \) in (D.15) and (D.16), it is easy to verify that the condition for the parameter \( a \) to be positive, and thus for the rewritten New Keynesian Phillips Curve (D.14) to be upward-sloping, is identical to the condition in (D.23). Likewise, it can be easily verified that the parameter \( b \) is positive, so that a government spending shock shifts this curve down, if and only if the condition given by (D.20) is satisfied.

### D.5 Detour: The Basic New Keynesian Model

In this subsection, we derive the solution to a model version without variable technology utilization. Incidentally, in this case the model collapses to the basic New Keynesian model, as presented, e.g., in Galí (2015), augmented with government spending. For comparison, we make the same assumptions as in the simplified version of our baseline model: No technology shocks (\( \Delta_t = 0 \)), constant returns to scale in production (\( \alpha = 0 \)), log utility in consumption (\( \sigma = 1 \)), unitary (inverse) Frisch elasticity of labor supply (\( \phi = 1 \)), no monetary policy reaction to the output gap (\( \phi_y = 0 \)), and no persistence in fiscal policy shocks (\( \rho_G = 0 \)). Under these assumptions, the basic New Keynesian model is given by the following set of equations:

\[
\begin{align*}
\tilde{N}_t &= -\tilde{C}_t + \tilde{\omega}_t, \\
-\tilde{C}_t &= E_t \left( -\tilde{C}_{t+1} + \tilde{R}_t - \tilde{\pi}_{t+1} \right), \\
\tilde{Y}_t &= \tilde{N}_t, \\
\frac{C}{Y} \tilde{C}_t &= \tilde{Y}_t - g \tilde{\omega}_t, \\
\tilde{mc}_t &= \tilde{\omega}_t, \\
\tilde{\pi}_t &= \beta E_t \tilde{\pi}_{t+1} + \frac{\varepsilon - 1}{\gamma} \tilde{mc}_t, \\
\tilde{R}_t &= \phi_{\pi} \tilde{\pi}_t,
\end{align*}
\]

plus an exogenous process for \( \tilde{\omega}_t \). We can combine these equations to obtain:\(^{33}\)

\[
\begin{align*}
\tilde{\pi}_t &= \beta E_t \tilde{\pi}_{t+1} + \frac{\varepsilon - 1}{\gamma} \left( \tilde{C}_t \left( 1 + \frac{C}{Y} \right) + g \tilde{\omega}_t \right), \\
-\tilde{C}_t &= E_t \left( -\tilde{C}_{t+1} + \phi_{\pi} \tilde{\pi}_t - \tilde{\pi}_{t+1} \right).
\end{align*}
\]

\(^{33}\)Having carefully described the analytical solution to our baseline model above, we do not present any intermediate steps in this subsection, but simply state the results.
From these expressions, it follows directly—as argued in Section 4—that this model version also implies a downward-sloping Euler equation in \((\hat{C}_t, \hat{\pi}_t)\)-space, and an upward-sloping NKPC-curve. Importantly, a positive shock to government spending shifts the NKPC-curve up, unlike our model of variable technology utilization, see Figure 4. Following the same steps as in the preceding subsections, we can derive the solution to this model, which is given by:

\[
\begin{align*}
\hat{C}_t &= - \frac{\phi \zeta}{\gamma} \frac{\varphi}{\gamma} \left( 1 - \frac{\chi_1}{\chi_2} \right) + 1 + \hat{g}_t, \\
\hat{\pi}_t &= \frac{\varepsilon \phi}{\gamma} \left( 1 - \frac{\chi_1}{\chi_2} \right) + 1 + \hat{g}_t.
\end{align*}
\]

Both the numerator and denominator of both expressions are necessarily positive. An increase in \(\hat{g}_t\) thus leads to an increase in inflation and a decline in consumption in this model, in contrast to our baseline model studied above.

Finally, we can verify that the solution to our baseline model collapses to that of the simple New Keynesian model when the adjustment costs associated with changes in technology utilization become sufficiently high. This can be seen by rewriting the solution given by (D.17) and (D.18) as:

\[
\begin{align*}
\hat{C}_t &= - \frac{\phi \zeta}{\gamma} \frac{\varphi}{\gamma} \left[ 1 + \frac{\chi_1}{\chi_2} + \left( 1 - \frac{\chi_1}{\chi_2} \right) \frac{C}{Y} - \frac{\chi_1}{\chi_2} mc \right] + 1 + (3 - 2mc) \frac{\chi_1}{\chi_2} \hat{g}_t, \\
\hat{\pi}_t &= \frac{\varepsilon \phi}{\gamma} \left[ 1 + \frac{\chi_1}{\chi_2} + \left( 1 - \frac{\chi_1}{\chi_2} \right) \frac{C}{Y} - \frac{\chi_1}{\chi_2} mc \right] + 1 + (3 - 2mc) \frac{\chi_1}{\chi_2} \hat{g}_t,
\end{align*}
\]

and letting \(\chi_2 \to \infty\), in which case these expressions collapse to those presented in (D.25) and (D.26).

### D.6 The Model with Capital and Habits

To summarize, the introduction of capital introduces four new equations in the variables \(I_t, K_t, Q_t,\) and \(r^K\). These are equations (20), (25), (26), and (28). The steady-state versions of these equations give rise to the following relationships:

\[
\begin{align*}
I &= \delta K, \\
Q &= 1, \\
r^K &= \frac{1}{\beta} + \delta - 1, \\
K = \frac{\alpha mc}{r^K}.
\end{align*}
\]

In addition, several of the previous expressions are modified. Having pinned down the capital-output ratio, we can write the production level as:

\[
Y = \left[ uAN^{1-\alpha} \left( \frac{K}{Y} \right)^{\alpha} \right]^{1/\gamma}.
\]
from which we can then back out the steady-state level of capital and, in turn, investment. The ratio of consumption to output then follows from the goods market clearing condition, and reads:

$$\frac{C}{Y} = 1 - \frac{g}{Y} - \frac{\delta K}{Y}.$$  

The condition for labor market equilibrium is modified by the presence of capital formation and consumption habits, which leads to the following expression for $\psi$:

$$\psi = \frac{(1 - \beta \theta) (1 - \alpha) m_c u A K^\alpha}{(C (1 - \theta))^{\nu} N^{\nu + \alpha}}.$$  

The steady-state level of $\lambda$ is:

$$\lambda = \frac{(1 - \beta \theta)}{(C (1 - \theta))^{\alpha}}.$$  

The log-linearized versions of the new equations are:

$$\bar{K}_t = (1 - \delta) \bar{K}_{t-1} + \delta \bar{l}_t,$$  

$$\bar{Q}_t = (1 + \beta) \tilde{\kappa} \bar{I}_t - \beta \kappa E_t \tilde{\iota}_{t+1} - \kappa \bar{I}_{t-1},$$  

$$E_t \bar{\lambda}_{t+1} + \beta \tilde{r}_t K^\alpha + \beta (1 - \delta) E_t \tilde{Q}_{t+1} = \bar{\lambda}_t + \bar{Q}_t,$$  

$$\tilde{r}_t^K = E_t \left( m_c \tilde{\lambda}_{t+1} + \tilde{\lambda}_{t+1} - \bar{K}_t \right),$$  

while we have the following log-linearized first-order conditions for the household:

$$\varphi \tilde{N}_t = \bar{\lambda}_t + \tilde{w}_t,$$  

$$\bar{\lambda}_t = E_t \left( \bar{\lambda}_{t+1} + \bar{R}_t - \bar{\pi}_{t+1} \right),$$  

$$\sigma \beta \theta E_t \tilde{C}_{t+1} - \sigma (1 + \beta \theta^2) \bar{C}_t + \sigma \theta \bar{C}_{t-1} = (1 - \theta) (1 - \beta \theta) \bar{\lambda}_t.$$  

(D.27)  

(D.28)  

(D.29)  

(D.30)  

(D.31)  

(D.32)  

(D.33)