### International Bond Risk Premia

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### Abstract

We extend Cochrane and Piazzesi (2005, CP) to international bond markets by constructing forecasting factors for bond excess returns across different countries. While the international evidence for predictability is weak using Fama and Bliss (1987) regressions, we document that local CP factors have significant predictive power. We also construct a global CP factor and provide evidence that it predicts bond returns with high  $R^2$ s across countries. The local and global factors are jointly significant when included as regressors, which suggests that variation in bond excess returns are driven by country-specific factors and a common global factor. Shocks to US bond risk premia seem to be particularly important determinants for international bond premia. Motivated by these results, we estimate a parsimonious no-arbitrage affine term structure model in which risk premia are driven by one local and one global CP factor. We find that the local factor acts like a slope factor and the global factor as an interest rate level factor.

Keywords: Affine model, local and global factors, time-varying expected returns. JEL Classification Numbers: E43, G12, G15.

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### 1 Introduction

The expectation hypothesis of interest rates states that bond risk premia are constant over time. However, ample evidence suggests that risk premia in bond markets do vary over time. For example, Fama and Bliss (1987, FB) and Campbell and Shiller (1991, CS) show that US bond excess returns are predictable using the forward-spot rate differential and the slope of the yield curve. A steep yield curve has historically predicted lower future long yields and positive excess returns on long bonds over short bonds. Cochrane and Piazzesi (2005, CP) establish even stronger evidence for predictability when more information from the yield curve is incorporated. Using five forward rates as predictors, they document significantly higher  $R^2$  compared to the commonly used FB or CS regressions.

We extend the setup of CP to an international setting and construct local CP factors for Germany, Switzerland, the UK, and the US for the period January 1976 to December 2007. The local factors are shown to have significant forecasting power for bond excess returns while FB regressions show weak or no evidence of predictability for countries outside the US. Next, we construct a global CP factor and show that it predicts bond returns with similar or higher explanatory power compared to local CP factors. The local and global CP factors are jointly significant when included as regressors and increase the explanatory power even further. Our results suggest that there exists a common global return-forecasting factor that predicts bond returns across countries and that bond risk premia are driven by both a country-specific factor and a common global factor. Motivated by this finding, we propose and estimate a parsimonious no-arbitrage affine term structure model in which risk premia for each country vary with the local and global CP factor. Shocks to the CP factors and to the level of interest rates are found to be significantly priced across all four countries. Our estimation results suggest that international bond risk premia are driven by a local slope factor and a global interest rate level factor. The local CP factors are positively associated with the slope of local yield curves and with the fourth principal component of local yields and are shown to be positively correlated over the sample period. Correlations are higher during the second half of the sample period compared to the first half which indicates an increasing comovement of international bond risk premia over time. The global CP factor is computed as a GDP-weighted average of the local CP factors and is positively associated with the level and slope of local term structures. The global factor is close to perfectly correlated with the US CP factor. The fact that the global factor predicts bond returns with high  $R^2$  for countries outside the US indicates that shocks to US bond risk premia are important determinants for international bond premia.

Our evidence of predictable bond returns across countries stands in contrast to the existing literature which finds weak or no evidence of predictability internationally. For example, Hardouvelis (1994) and Bekaert and Hodrick (2001) find it hard to reject the expectation hypothesis for countries outside the US. In contrast, we show that both a local and global CP factor predict returns significantly in countries for which FB regressions finds no or weak evidence of predictability. Flamini and Veronesi (2008) document similar results using a common return forecasting factor. In a recent paper, Kessler and Scherer (2009) also construct CP factors across countries and find significant forecasting power. However, the focus of their paper is different from ours as they are mainly interested in evaluating different trading strategies. Our finding that bond returns are governed by a country-specific and a global factor is related to Dahlquist (1995), who find that variations in forward term premia are to a great extent captured by the shape of domestic and world term structures, and Driessen et al. (2003), who find that a world interest rate level factor accounts for nearly half of the variation in bond returns. Furthermore, Perignon et al. (2007) find that US bond returns share only one common factor with German and Japanese bond returns which they link to changes in the level of interest rates. Ilmanen (1995) also examines the predictability of international bond returns and find that global factors predict bond returns across countries.

Our work is also related to Cochrane and Piazzesi (2008), who estimate an affine model on US data using the local CP factor plus three latent variables. Only level shocks are assumed to be priced in their model where risk premia vary with the CP factor. Our estimations show that not only level shocks are priced but also shocks to the CP factor itself. Koijen et al. (2009) find that the CP factor is able to price the cross-section of US stock returns.

Several equilibrium models have been put forward to explain the mechanics of timevarying bond risk premia, linking macroeconomic variables to changing expected excess returns. For example, Brandt and Wang (2003), Wachter (2006), and Buraschi and Jiltsov (2007) all build on the habit-formation model of Campbell and Cochrane (1999) and show that it can generate rejections of the expectation hypothesis. Bansal and Shaliastovich (2008) and Hasseltoft (2008) build on the long-run risk model of Bansal and Yaron (2004) and argue that changing bond risk premia are driven by time-varying volatility of consumption growth. Ludvigson and Ng (2008) provide empirical evidence that macro factors do predict bond returns. By using common factors from a large set of macro variables, they document  $R^2$ s up to 26% when predicting US bond excess returns. They find that including the CP factor increases the  $R^2$ s up to over 40% with all coefficients being statistically significant. Our paper is also related to work on term structure models such as Dai and Singleton (2000), Duffee (2002), and Dai and Singleton (2002). Diebold et al. (2008) build on Nelson and Siegel (1987) and document the existence of global yield curve factors which appear to be linked to global macroeconomic factors such as inflation and real activity. Related is also the literature on global factors in other asset markets. For example, Harvey (1991), Campbell and Hamao (1992), and Ferson and Harvey (1993) use global risk factors to predict international stock returns while Backus et al. (2001) and Lustig et al. (2009) address the forward premium puzzle using affine models including country-specific and common factors.

Our paper proceeds as follows. In Section 2 we describe the data, present summary statistics, and provide the key results related to predictability regressions of bond returns.

In Section 3 we propose an affine term structure model with local and global factors. We present the results of estimating these models in Section 4, and discuss implications for yields in terms of yield loadings, impulse response functions, and variance decompositions. In Section 5 we discuss how the affine model can be linked to structural models and outline future research in light of our results. We conclude in Section 6.

### 2 Predictability of bond returns

### 2.1 Data

Our data set covers monthly zero-coupon interest rates for Germany, Switzerland, United Kingdom, and United States and spans the time period January 1976 to December 2007. One-to-five year zero-coupon yields for Germany are collected from Bundesbank, Swiss yields are derived from forward rates up to December 2003 after which yields from the Swiss National Bank are used, yields for the UK are retrieved from Bank of England, while yields for the US are collected from the Fama-Bliss discount bond file in CRSP. The one-month interbank rate, collected from Datastream, is used as short rate for Germany and Switzerland. For the UK, the one-month interbank rate is used until February 1997 and then one-month yields from Bank of England. The Fama one-month yield from CRSP is used for the US. Quarterly data on GDP, computed using purchasing power parity, is retrieved for each country from Datastream. As the GDP data are quarterly, the weights applied to the monthly CP factors are constant within each quarter. Table 1 presents summary statistics for yields across countries. Yield curves tend to be upward sloping on average while yields on short-maturity bonds tend to be more volatile than yields on long-maturity bonds. Yield levels are positively correlated across countries with correlations being higher among yields on longer-term bonds. Annual bond excess returns on 2-5 year bonds are also positively correlated across countries as indicated in Table 2.

### 2.2 Constructing local and global Cochrane-Piazzesi factors

We construct local CP factors as in Cochrane and Piazzesi (2005) for each country c in our sample. Define the annual return on a n-period bond in excess of the one-year yield as  $rx_{c,t+1}^n = p_{c,t+1}^{n-1} - p_{c,t}^n - y_{c,t}^1$ , where p denotes the log bond price and y denotes the log yield, computed as  $y_{c,t}^n = -p_{c,t}^n/n$ . Define the one-year forward rate between periods n-1 and n as the differential in log bond prices,  $f_{c,t}^n = p_{c,t}^{n-1} - p_{c,t}^n$ . A CP factor is constructed by regressing average excess returns across maturity at each time t on the one-year yield and four forward rates:

$$\overline{rx}_{c,t+1} = \gamma_{c,0} + \gamma_{c,1}y_{c,t}^1 + \gamma_{c,2}f_{c,t}^2 + \gamma_{c,3}f_{c,t}^3 + \gamma_{c,4}f_{c,t}^4 + \gamma_{c,5}f_{c,t}^5 + \bar{\epsilon}_{c,t+1}, \tag{1}$$

where  $\overline{rx}_{c,t+1} = \sum_{n=2}^{5} rx_{c,t+1}^{n}/4$ . Let the right hand side variables, including the constant term, for each country be collected in the vector  $f_{c,t}$  and let the corresponding estimated coefficients be collected in the vector  $\hat{\gamma}_{c}$ . A local CP factor  $CP_{c,t}$  is then given by  $\hat{\gamma}'_{c}f_{c,t}$ .<sup>1</sup>

We construct a global CP factor defined as the GDP-weighted average of each local CP factor at time t. That is:

$$GCP_t = \sum_{c=1}^{C} w_{c,t} CP_{c,t},\tag{2}$$

where  $w_{c,t} = GDP_{c,t} / \sum_{c=1}^{C} GDP_{c,t}$ , and where C = 4. The average weights over the sample period is 0.70 for US, 0.12 for UK, 0.16 for Germany, and 0.02 for Switzerland. Our sizeweighted global risk factor is hence dominated by the US.<sup>2</sup>

<sup>&</sup>lt;sup>1</sup>Cochrane and Piazzesi (2005) find the  $\gamma$ s to form a tent-shaped pattern. We find a similar shape for the US, using the same data source as CP but for a different sample period. The shapes are different for the remaining countries. Dai et al. (2004) emphasize that different ways of smoothing yield curves give rise to different patterns. Yields that are choppy and less smoothed produce patterns that are more similar to tents. While the US yields that we use are unsmoothed Fama-Bliss yields, yields for the remaining countries are smoothed by each country's central bank. Hence, the patterns are different. However, including only the one-year yield, the three-year forward rate, and the five-year forward rate on the right hand side produces tent shapes also for smoothed yields without changing the dynamics of the CP factor to any great extent.

<sup>&</sup>lt;sup>2</sup>We have considered alternative ways of constructing a global CP factor; for example, we have elaborated with an equal-weighted factor and a factor given by the first principal component of the covariance matrix of local CP factors. Our main result that bond risk premia are determined by both a local and a global factor remains.

Table 3 presents correlations of the local CP factors as well as the global CP factor. While the US factor is only weakly positively correlated with the others, the European factors display higher correlations among each other. Correlations are higher for the second half of the sample period with correlations in excess of 0.5. This suggests that international bond risk premia have become more correlated over time. This can also be seen in Figure 1, which plots the four local CP factors. The table also shows that the US factor and the global factor are almost perfectly correlated, while correlations are lower than 0.5 for the other countries. Figure 2 plots the global factor together with NBER contractions. The global factor tends to increase during US recessions, indicating that it is counter cyclical and closely related to US economic conditions.

### 2.3 Predictability regressions

We start by running Fama and Bliss (1987) regressions for each country. We regress excess returns on a n-period bond onto a constant and the forward rate-spot rate differential:

$$rx_{c,t+1}^{n} = a_{c}^{n} + b_{c}^{n}(f_{c,t}^{n} - y_{c,t}^{1}) + \epsilon_{c,t+1}^{n},$$
(3)

where  $a_c^n$  and  $b_c^n$  are parameters and  $\epsilon_{c,t+1}^n$  is an error term. Table 4 displays the results. Consistent with earlier evidence in the literature, we find that a positive forward-spot rate spread predicts US returns positively with  $R^2$ s ranging between 5% and 13%. Slope coefficients for maturities of two to four years are statistically significant at the 1% level, while the coefficient for the five-year bond is statistically significant at the 10% level. However, none of the predictability coefficients for UK and Germany are statistically different from zero while for Switzerland only slope coefficients for the two- and three-year bonds are significant at conventional levels. The explanatory power of the regressions are lower than for the US. This finding goes in line with existing evidence that the expectation hypothesis is more difficult to reject for countries outside the US.

Next, we predict bond returns using our constructed local CP factors and run the following regression for each country:

$$rx_{c,t+1}^{n} = b_{c,CP}^{n}CP_{c,t} + \epsilon_{c,t+1}^{n}.$$
(4)

Table 4 presents also these results. Predictability coefficients are all highly significant across the four countries. The explanatory power is higher for the US compared to the other countries. However, the  $R^2$  is substantially higher for the CP regressions than for the earlier FB regressions. For countries in which the FB regressions pointed to no or weak evidence of predictability, the CP regressions suggest that international bond risk premia are indeed predictable. This is likely due to the fact that CP regressions make use of more information from the yield curve, compared to the FB regressions.

To put the explanatory power of the local CP factors further in context, we contrast the results with the ones using the first three principal components of yield levels to predict returns. It is common in the term-structure literature to summarize the information in yields using these components as they explain virtually all of the variation in yields. See, for example, Litterman and Scheinkman (1991). The first three components are often labeled level, slope, and curvature. We do a principal component analysis of yield levels for each country.<sup>3</sup> We then run the following regression for each country:

$$rx_{c,t+1}^n = a_c^n + b_{c,Level}^n Level_{c,t} + b_{c,Slope}^n Slope_{c,t} + b_{c,Curvature}^n Curvature_{c,t} + \epsilon_{c,t+1}^n.$$
(5)

The results from these regressions are presented in Table 5. Judging from the statistical

<sup>&</sup>lt;sup>3</sup>The principal component (PC) analysis is done through an eigenvalue decomposition of the variancecovariance matrix of demeaned yield levels. The first PC accounts for 97.9-98.9% of the yield variance across countries, the second accounts for 1.0-1.9% of the variance, while the third only accounts for 0.02-0.12% of the variance.

significance of the coefficients, the slope and curvature factors seem important for predicting returns. Furthermore, the explanatory power is higher than for the FB regressions for all countries. However, the  $R^2$  are all lower compared to using the local CP factors with the exception of Switzerland, where the explanatory power of the two regressions are similar.

To sum up the results so far, the local CP factors all predict bond returns with significantly higher  $R^2$  than the commonly used FB regressions and they seem to contain more information than the first three principal components, with the possible exception of Switzerland.

Based on our earlier discussion of international bond risk premia being positively correlated, we investigate whether there exists a common global factor that predicts returns for each country. Using our constructed global CP factor, GCP, we predict excess returns by running the following regression:

$$rx_{c,t+1}^n = b_{c,GCP}^n GCP_t + \epsilon_{c,t+1}^n.$$

$$\tag{6}$$

Table 6 presents the results. Interestingly, the  $R^2$  is higher for the European countries compared to using the local CP factors. The explanatory power is, however, less for the US. Since the global factor is highly correlated with the US factor, our results suggests that shocks to US bond risk premia have great predictive power for bond returns outside the US. The lower  $R^2$  for the US signifies that incorporating information from other countries is less important for predicting US bond returns.<sup>4</sup>

Having established that both a local and global CP factor predict returns significantly with high  $R^2$  we include the local and global factors jointly and run the following regression:

$$rx_{c,t+1}^{n} = b_{c,CP}^{n}CP_{c,t} + b_{c,GCP}^{n}GCP_{t} + \epsilon_{c,t+1}^{n}.$$
(7)

<sup>&</sup>lt;sup>4</sup>Running the predictability regression using the US factor confirms the importance of US risk premia for predicting international bond risk premia.

These results are also presented in Table 6. The results for US suffer from multicollinearity which makes the individual regression coefficients insignificant. However, p-values from Wald tests suggest that the coefficients are jointly significant. Both coefficients are individually and jointly significant for the other three countries. The  $R^2$  are also higher compared to the individual regressions. The joint significance of the coefficients suggests that bond risk premia are driven by both global and local factors.

### 3 An affine model with local and global factors

Motivated by our finding that international bond risk premia seem to be driven by a common global factor as well as a country-specific factor, we explore in this section how CP factors drive expected excess returns. We are interested in finding out how shocks affect yields and whether there are differences across countries. We do so by estimating a parsimonious no-arbitrage term structure model for each country. The model consists of three factors for countries outside the US: The local CP factor, the global CP factor, plus the first principal component of yields which is related to the level of yields. The fact that the US factor and the global factor are close to perfectly correlated renders inference problems, so we instead choose to estimate a two factor model for the US consisting of the global CP factor and the first principal component of yields. The level component is orthogonalized with respect to the CP factors by regressing yields of maturities one-five year on a constant and the CP factors. The level factor is then the first principal component of the residuals.<sup>5</sup> Following the results from our predictive regressions, we assume that risk premia are only driven by the local and global CP factors. The level factors lower pricing errors and serve as country-specific interest rate factors that are not priced. Including more factors such as slope and curvature factors

<sup>&</sup>lt;sup>5</sup>That is, the principal components are computed using yields of maturities one-to-five years as to match the maturities used in the predictability regressions. Yields on one-month bonds are merely used in the affine model to pin down the short end of the term structure.

naturally leads to lower pricing errors, as discussed in Section 4.2 on robustness. However, including incremental factors do not change our main results so we choose instead parsimony.

### 3.1 Setup of the model

The model is described for one country with K state variables. For simplicity, we suppress the country subscript c. Assume that the vector of state variables follows:

$$X_t = \mu + \rho X_{t-1} + \eta_t, \tag{8}$$

where  $\eta_t \sim N(0, \Sigma)$ , and  $X, \mu$ , and  $\eta$  are  $K \times 1$  vectors, and  $\rho$  and  $\Sigma$  are  $K \times K$  matrices. The state vector contains  $CP_{c,t}, GCP_t$ , and  $Level_{c,t}$  for countries outside the US, and  $GCP_t$ and  $Level_{c,t}$  for the US. Assume that the one-month yield follows:

$$r_t = \delta_0 + \delta_1' X_t,\tag{9}$$

where  $\delta_0$  is a scalar and  $\delta_1$  is a  $K \times 1$  vector. The discount factor is specified as an exponential affine function of the three factors:

$$M_{t+1} = \exp\left(-\delta_0 - \delta_1' X_t - \lambda_t' \eta_{t+1} - \frac{1}{2} \lambda_t' \Sigma \lambda_t\right),\tag{10}$$

where  $\lambda_t$  are the time-varying market prices of risk. The process for  $\lambda_t$  is assumed to be affine:  $\lambda_t = \lambda_0 + \lambda_1 X_t$ , where  $\lambda_0$  is a  $K \times 1$  vector and  $\lambda_1$  is a  $K \times K$  matrix. The price of an asset satisfies standard no-arbitrage conditions, such that bond prices can be computed as:  $P_t^{n+1} = E_t(M_{t+1}P_{t+1}^n)$ . Bond prices become exponential affine functions of the state variables:  $P_t^n = \exp(A_n + B'_n X_t)$ , where  $A_n$  is a scalar and  $B_n$  is a  $K \times 1$  vector. A and B satisfy:

$$A_{n+1} = A_n + B'_n \mu^* + \frac{1}{2} B'_n \Sigma B_n - \delta_0, \qquad (11)$$

$$B_{n+1}' = B_n' \rho^* - \delta_1', \tag{12}$$

where  $A_0 = B_0 = 0$  and  $\mu^* = \mu - \Sigma \lambda_0$  and  $\rho^* = \rho - \Sigma \lambda_1$  are the mean vector and transition matrix under the risk neutral measure. The continuously compounded yield  $y_t^n$  is given by:  $y_t^n = -\ln(P_t^n)/n = -A_n/n - B'_n X_t/n$ . Model yields are subject to constant second moments since the state vector is assumed to be homoscedastic. This is counterfactual to data but simplifies our analysis. Expected log excess return on a *n*-period bond over the short rate is given by:

$$E_t(rx_{t+1}^n) = -Cov_t(m_{t+1}, rx_{t+1}^n) - \frac{1}{2}Var_t(rx_{t+1}^n),$$
(13)

where  $rx_{t+1}^n = p_{t+1}^{n-1} - p_t^n - y_t^1$  denotes the log excess return, p denotes the log bond price, m denotes the log discount factor, and where the variance term is a Jensen's inequality term. Recognizing that the covariance term can be written as:

$$-Cov_t(m_{t+1}, rx_{t+1}^n) = Cov_t(\eta_{t+1}, p_{t+1}^{n-1})\lambda_t$$

$$= B'_{n-1}\Sigma\lambda_t,$$
(14)

and that the variance term can be written as:

$$\frac{1}{2}Var_t(rx_{t+1}^n) = \frac{1}{2}B'_{n-1}\Sigma B_{n-1}, \qquad (15)$$

the log excess return can be written as:

$$E_t(rx_{t+1}^n) = B'_{n-1}\Sigma\lambda_0 + B'_{n-1}\Sigma\lambda_1X_t - \frac{1}{2}B'_{n-1}\Sigma B_{n-1}.$$
 (16)

Risk premia vary over time due to the time-varying market price of risk,  $\lambda_t$ , rather than through time-varying volatility of the state vector and are equal to zero when  $\lambda_0 = 0$  and  $\lambda_1 = 0$ , ignoring the Jensen's inequality term. Equation (16) shows that  $\lambda_1$  governs the price of the market risk that is time-varying. The sign of the time-varying part of the risk premium depends on the sign of this market price of risk and on the product of yield loadings and the variance-covariance matrix  $B'_{n-1}\Sigma$ . The usual intuition holds: the risk premium is positive if a positive shock to the state variables raises the pricing kernel while lowering bond prices as it implies low excess returns in bad times. As a result, the bond is considered risky by the investor who accordingly demands a positive risk premium for holding the asset.

### **3.2** Impulse responses and variance decompositions

Impulse response functions and variance decompositions are useful for analyzing the impact of economic shocks on yields and to gauge the relative importance of shocks for the variance of yields. See, for example, Hamilton (1994) for details. Starting with impulse response functions, write the state dynamics in vector  $MA(\infty)$  form:

$$X_t = \sum_{i=0}^{\infty} \Psi_i \eta_{t-i}.$$
(17)

As the state dynamics are given by a VAR(1) process,  $\Psi_i = \rho^i$ . Shocks are orthogonalized using a Cholesky decomposition of the variance-covariance matrix  $\Sigma$ , which returns the lower triangular matrix P where  $PP' = \Sigma$ . Define a new shock vector  $v_t$  as  $P^{-1}\eta_t$ , so that  $\eta_t = v_t P$ . Then  $E(v_t) = 0$  and  $E(v_t v'_t) = I_K$ . Then redefine (17) as :

$$X_t = \sum_{i=0}^{\infty} \Psi_i P v_{t-i}.$$
(18)

Impulse responses can now be interpreted as the response of the system to a one standard deviation shock. Considering that yields are linear functions of the state variables,  $y_t^n = -A_n/n - B'_n X_t/n$ , they can be written as:

$$y_t^n = -\frac{A_n}{n} - \sum_{i=0}^{\infty} \frac{B'_n}{n} \Psi_i P v_{t-i}.$$
 (19)

Hence,  $-\frac{B'_n}{n}\Psi_i P_j$  is the impulse response for a *n*-period yield at a horizon of *i* months given a one standard deviation shock to state variable *j* at time zero, were  $P_j$  is the *j*th column of *P*.

The variance of yields is decomposed as follows. Using the vector  $MA(\infty)$  form of the state dynamics, the error in forecasting the state VAR s periods ahead can be written as:

$$X_{t+s} - \hat{X}_{t+s|t} = \sum_{i=0}^{s-1} \Psi_i \eta_{t+s-i}.$$
 (20)

Using (19), the s-period forecast error of the yield on an n-maturity bond can be written as:

$$y_{t+s}^n - \hat{y}_{t+s|t}^n = -\sum_{i=0}^{s-1} \frac{B'_n}{n} \Psi_i \eta_{t+s-i} = -\sum_{i=0}^{s-1} \frac{B'_n}{n} \Psi_i P v_{t+s-i}.$$
 (21)

Then the mean squared error, MSE, of the forecast is:

$$MSE = E[(y_{t+s}^{n} - \hat{y}_{t+s|t}^{n})(y_{t+s}^{n} - \hat{y}_{t+s|t}^{n})'] =$$

$$= \frac{B'_{n}}{n} \sum \frac{B_{n}}{n} + \frac{B'_{n}}{n} \Psi_{1} \sum \Psi'_{1} \frac{B_{n}}{n} + \dots + \frac{B'_{n}}{n} \Psi_{s-1} \sum \Psi'_{s-1} \frac{B_{n}}{n},$$
(22)

since  $Var(v_t) = I$ . As we are interested in the contribution of shocks to each one of the K state variables, (22) can be rewritten as:

$$MSE = \sum_{j=1}^{K} \left[ \frac{B'_n}{n} P_j P'_j \frac{B_n}{n} + \frac{B'_n}{n} \Psi_1 P_j P'_j \Psi'_1 \frac{B_n}{n} + \dots + \frac{B'_n}{n} \Psi_{s-1} P_j P'_j \Psi'_{s-1} \frac{B_n}{n} \right],$$
(23)

using the fact that  $Var(v_{j,t}) = 1$  and where  $v_{j,t}$  denotes the *j*th element in the *v* vector and where  $p_j$  denotes the *j*th column in matrix *P*. The relative contribution of a shock to the *j*th state variable for the variance of an *n*-period yield and for a horizon of *s* months is therefore:

$$\frac{\frac{B'_n}{n}P_jP'_j\frac{B_n}{n} + \frac{B'_n}{n}\Psi_1P_jP'_j\Psi'_1\frac{B_n}{n} + \dots + \frac{B'_n}{n}\Psi_{s-1}P_jP'_j\Psi'_{s-1}\frac{B_n}{n}}{MSE}.$$
(24)

### 4 Estimation

We estimate in a first step the risk-neutral dynamics of the state variables directly from observed yields. We then estimate the market prices of risk in  $\lambda_1$  in a second step such that the model matches the slope coefficients of the in-sample predictability regressions that includes the local and global CP factors jointly.

The risk-neutral dynamics of the state variables is estimated by matching model-implied yields with observed yields. All state variables are demeaned prior to estimation, that is  $\mu = 0$ . The condition  $\mu^* = -\Sigma\lambda_0$  is imposed in the estimation to make sure that the model reproduces state variables with a sample mean of zero. We use an estimate of  $\Sigma$  from an OLS estimation of the state dynamics in Equation (8). We estimate  $\lambda_0, \rho^*, \delta_0$ , and  $\delta_1$  with the generalized method of moments (GMM) framework of Hansen (1982), using the identity matrix as weighting matrix. The sample counterpart of the following moment condition is used:

$$E\left(\nu_t \otimes z_t\right) = 0,\tag{25}$$

where  $\nu_t$  is a vector of yield errors with a typical element given by  $y_t^{n,data} - y_t^{n,model}$ , where we consider bonds with maturities one month, and one to five years. Vector  $z_t$  contains a constant and the state variables. For countries outside the US,  $z_t = [1, CP_{c,t}, GCP_t, Level_{c,t}]$ , while  $z_t = [1, GCP_t, Level_{c,t}]$  for the US. In total there are 16 parameters to estimate for countries outside the US, consisting of  $\delta_0$ , the three elements in  $\delta_1$ , the three elements in  $\lambda_0$ , and the nine elements in  $\rho^*$ . The number of moment conditions are 24 since  $\nu_t$  has dimension  $6 \times 1$ . For the US, there are nine parameters to estimate and 18 moment conditions. The risk-neutral dynamics of the state variables are restricted to be stationary throughout the estimations by requiring the eigenvalues of  $\rho^*$  to lie inside the unit circle.

Parameters in  $\lambda_1$  are estimated in a second step which provides an understanding of how shocks to each factor are priced. Based on results from our predictive regressions, we restrict  $\lambda_1$  so that risk premia in the model only are driven by the local and global CP factors. We therefore set the column in  $\lambda_1$  that refers to level shocks equal to zero. We also impose restrictions such that each CP factor only price shocks to itself, in addition to level shocks. This is done for simplicity and relaxing the restrictions does not change our results. This means that:

$$\lambda_{1} = \begin{pmatrix} \lambda_{11} & 0 & 0 \\ 0 & \lambda_{22} & 0 \\ \lambda_{31} & \lambda_{32} & 0 \end{pmatrix},$$
(26)

for countries outside the US while the corresponding matrix for the US is:

$$\lambda_1 = \begin{pmatrix} \lambda_{11} & 0 \\ \lambda_{21} & 0 \end{pmatrix}, \tag{27}$$

since only the global CP factor is assumed to drive risk premia for the US market. Based on our regressions, expected excess returns can be written as  $E_t(rx_{c,t+1}^n) = b_{c,CP}^n CP_{c,t} + b_{c,GCP}^n GCP_t$  for n = 2, 3, 4, 5. The estimated slope coefficients are therefore  $4 \times 1$  vectors. The corresponding expression for model-implied log excess returns are as in Equation (16).<sup>6</sup> We have estimated loadings B, the variance-covariance matrix  $\Sigma$ , and  $\lambda_0$  from the first-step

<sup>&</sup>lt;sup>6</sup>Since we have demeaned the CP factors for estimation purposes, we are matching the in-sample slope coefficients obtained using demeaned CP factors. They are very similar to the ones reported in Table 6.

so the only unknown parameters are the  $\lambda_1$  parameters. We estimate these by matching estimated expected returns in data with model-implied expected returns. For the US, we would like the model to match the global CP regression in Table 6. Let the 4 × 1 vector  $\epsilon_t$  denote the difference  $E_t(rx_{t+1})^{data} - E_t(rx_{t+1})^{model}$ . We form 16 moments conditions and estimate four parameters in  $\lambda_1$  for countries outside the US and form 12 moment conditions and estimate two parameters in  $\lambda_1$  for the US. We estimate the system with GMM using the identity matrix as weighting matrix. The moment conditions are:

$$E\left(\epsilon_t \otimes z_t\right) = 0,\tag{28}$$

where  $z_t = [1, CP_{c,t}, GCP_t, Level_{c,t}]$  for Germany, Switzerland, and the UK while  $z_t = [1, GCP_t, Level_{c,t}]$  for the US. Given the earlier estimated  $\rho^*$ , we impose stationarity on the implied physical dynamics of the state variables by requiring the eigenvalues of  $\rho = \rho^* + \Sigma \lambda_1$  to lie inside the unit circle.

### 4.1 Estimation results

The estimation results for each country are reported in Table 7 which consists of parameter estimates and standard errors. All but one element of the  $\lambda_1$  matrices across countries are statistically significant which indicates that shocks to all three state variables are priced and that the local and global CP factor are significant drivers of risk premia. All significant estimates of  $\lambda_1$  are negative which means that positive shocks to the state variables raise the pricing kernel. Whether this give rise to positive or negative risk premia depends on the sign of the yield loadings and on the variance-covariance matrix of the shocks. Figure 3 shows estimated yield loadings across countries. First, yields load positively on the level factor with loadings on the short end being somewhat higher. Second, the local CP factor takes the form of a slope factor in all countries which is consistent with local CP factors being highly positively correlated with the second principal component in each country.<sup>7</sup> In the UK, however, the slope is less pronounced and the CP factor is more similar to a level factor. This is in line with the UK factor being highly positively correlated with the first principal component of yields. The yield loadings of the global factor have similar shapes as the level factor in each country, which is consistent with the global factor being positively correlated with the first principal component. The global factor acts as a combination of a slope and curvature factor in the US which of course is a result of the global factor being dominated by the US factor.

Pricing errors of the model are reported in Table 8 and are lowest for Switzerland with a root mean squared error (RMSE) of 0.22% and highest for the UK with a RMSE of 0.50%. The pricing error for US of 0.36% seems reasonable considering we estimate a two-factor model. The variation of pricing errors is highest for the one-month yield which is known to be difficult to model. In the next sub-section, we discuss the effect of including the second and third principal component as additional factors.

To sum up our estimation results, we find that shocks to all state variables are priced and that local and global CP factors are significant drivers of risk premia. While the local CP factors have yield loadings that are similar to slope factors, the global factor is more similar to a level factor. The predictive power of the local CP factor suggests that a steeper and more curved term structure imply higher expected excess return while the predictive ability of the global factor implies that there exists a global level factor that drives international bond risk premia.

<sup>&</sup>lt;sup>7</sup>The correlations between local CP factors and local slope factors are 0.83, 0.88, and 0.40 for Germany, Switzerland, and the UK respectively. The corresponding correlations between local CP factors and local level factors are 0.35, 0.32, and 0.57. Correlations in Germany, Switzerland, the UK, and the US between the global factor and local level and slope factors are 0.36, 0.44, 0.33, and 0.32 for the level factor and 0.30, 0.29, 0.26, and 0.73 for the slope factor.

### 4.2 Robustness

We have chosen to use only the first principal component of yields in our affine model. However, it is common in the literature to also use a slope and curvature factor in addition to a level factor. Here we discuss how the inclusion of two additional factors affects our results.<sup>8</sup>

Including the first three principal components in addition to the global and local CP factors produces a RMSE for Germany, Switzerland, and the UK of 0.17%, 0.22%, and 0.21% compared to 0.29%, 0.22%, and 0.50% in the original specification. Hence, the pricing error for both the UK and Germany are reduced while the pricing error for Switzerland is unchanged. Even though more variables are added and shocks to the slope and curvature factor also are priced, the GCP and local CP factors still retain their status as level and slope factors respectively. Adding two more factors to the US specification of GCP plus a level factor lowers the RMSE to 0.30% which is somewhat lower than the original 0.35%. The reduction in pricing error is not dramatic since the original specification is close to a level factor plus a slope factor due to the highly positive correlation of 0.73 between GCP and the US slope factor. Including a second and third principal component does not change the slope-like shape of GCP yield loadings for US yields.

Hence, including a second and third principal component lowers pricing errors but it does not change the main message of the paper: The local CP and global CP factors act as slope and level factors and are important for pricing shocks, and determining risk premia in the economy. Motivated by this conclusion, we choose parsimony and focus on affine models with only two and three factors.

 $<sup>^{8}{\</sup>rm The}$  numerical robustness results are not reported in tables and figures for brevity but are available in full form upon request.

### 4.3 Impulse responses and variance decompositions

Figure 4 depicts impulse response functions for yields on one-month and five-year bonds, given a one standard deviation shock to the state variables. In Germany, positive shocks to the level factor and the local CP factor raise short-maturity yields both in the short and long run while long-maturity yields also increase except for very long horizons where the effect of the shocks turns negative. Shocks to the German factor immediately increase the slope of the yield curve by 44 basis points after which the slope decreases, reaching zero two years after the initial shock, and then becomes negative. It is evident that the global factor only has a small impact on long yields while the effect on short yields is larger and negative, leading to a steepening of the yield curve. The figure shows that the impulse responses do not settle down after ten years. This is since the  $\rho$  matrix for Germany contains an eigenvalue very close to one, resulting in shocks that lasts for a very long time. For Switzerland, it is evident that the global factor again has little effect on yields as the impulse responses are close to zero throughout the horizons. In contrast, positive shocks to the local CP factor lower short yields while raising long yields initially, indicating that the CP factor acts as a slope factor. The yield curve steepens initially by 50 basis points after which the slope decreases and reaches zero less than two years after the initial shock. In the UK, positive shocks to local and global CP factors have an initial effect on short-maturity yields that is negative but small while the long-run response of the short yield is virtually the same for the two shocks. However, shocks to the local CP factor have a stronger effect on long-maturity yields, raising the five-year yield by 28 basis points initially. As a result, positive innovations to the local CP factor lead to an initial steepening of the yield curve of 30 basis points after which the curve gradually flattens. The slope effect due to the local CP factor is only five basis points two years after the shock and reaches zero three years after the initial shock. In the US, the global factor acts as a slope factor as it lowers short yields and raises long yields, producing an initial slope of 56 basis points. The yield curve flattens subsequently with a slope of only five basis points after one and a half years. An eigenvalue very close to one for the US  $\rho$  matrix results in impulse response functions that decay very slowly towards zero.

The reason why a shock to the local or global CP factors can have an initial negative effect on yields even though their yield loadings may be strictly positive is the negative correlation between shocks to CP factors and the level factor. For Germany, the correlation of shocks between the level factor and the local and global CP factors are -0.33 and -0.74 respectively. For Switzerland, the corresponding correlations are -0.14 and -0.83. The correlations for the UK are -0.67 and -0.43. In the US, the correlation between shocks to GCP and the level factor is -0.79. The negative correlations also imply that a shock to the GCP factor has less of an impact on yields than the GCP yield loadings suggest since positive GCP shocks are accompanied by offsetting negative level shocks.

To sum up, an increase in local CP factors leads to an initial steepening of yield curves which lasts between one and two years while shocks to the global factor has a muted impact on yields except for the German one-month yield over very long horizons. The former effect is consistent with the positive correlation between local CP factors and the corresponding slope factor for each country. The results are robust to the ordering of the state variables, which otherwise is known to impact the results (see, for example, Bikbov and Chernov, 2008, for a discussion).

Table 9 shows results from the variance decomposition, illustrating the contribution of each shock to the variance of yield forecast errors. In Germany, the local CP factor contributes with 39% and 37% of the short and long-yield variance respectively, for a one-month horizon. Its impact on long-run variance is similar for long yields but drops down to 12% for the short yield. The global factor is not important for determining variance in yields as its largest share of variance is only 6%. In Switzerland, the impact of the local CP factor increases further as it accounts for over half of the variance of five-year yields and between 30% and 49% of the variance in one-month yields. The GCP factor has virtually zero im-

pact on the variance of yields, underlining its little importance for determining the dynamics of yield levels. In the UK, the local CP factor is more important for the variance of long yields than short yields, accounting for 31% of the variance of long yields over a horizon of one month. Shocks to the GCP factor have a rather limited impact on the variance. For example, it accounts for 15% of the long-run variance in one-month yields. In the US, the global factor accounts for half of the short-term variance in short yields while its impact on five-year yields is tiny. As is commonly found in the literature, the bulk of the variance across countries is accounted for by the level factor. Our results are again robust to the ordering of state variables.

The results suggest that the global factor is not important for the dynamics of yield levels as it contributes very little to the variance of yields, except for the US where it is important for the variance of short-maturity yields. In contrast, the local CP factors account for a sizeable part of the forecast error variance and most notably so for Germany and Switzerland.

### 5 Where does the CP factor come from?

The ability of the CP factor to predict returns is intriguing and naturally raises the question of where it is coming from. The literature is still silent on what the CP factor actually represents. Cochrane and Piazzesi (2005) show that the US factor is correlated with business cycles, high in troughs and low in peaks. However, we still do not know exactly what type of information the CP factor captures. The natural starting point would be to consider the link between macroeconomic conditions and the CP factor. We know from asset pricing theory that risk premia should be positive on average for assets whose return covary positively with investors' well being. Furthermore, risk premia have been found to vary over time in a counter-cyclical fashion (e.g., Fama and French, 1989). Using the intuition from consumption-based models, bond risk premia are positive on average if inflation is counter-cyclical since nominal bonds then have low payoffs in bad times. To get timevariation in risk premia, one option is to consider time-varying macroeconomic volatility. For example, Bansal and Shaliastovich (2008) and Hasseltoft (2008) build on the long-run risk model of Bansal and Yaron (2004) and show that time-varying volatility of consumption growth induces time variation in bond risk premia. Using a similar model, Hasseltoft (2009) shows that also inflation volatility is an important determinant for changes in bond risk premia. Using the habit-formation model of Campbell and Cochrane (1999), Brandt and Wang (2003) and Wachter (2006) show that variation in the consumption surplus ratio induces time variation in bond risk premia. These theoretical models suggest a tight link between macroeconomic variables and risk premia. This would imply a link between the CP factor and the macroeconomy.

The reason the predictive power of the CP factor had gone unnoticed until Cochrane and Piazzesi (2005) is that it is common to focus on the first three principal component of yields which account for virtually all of the variation in yields. Even though the CP factors are highly positively correlated with the second principal component, it is also positively associated with the fourth principal component which explains a negligible part of yield variations but which has considerable forecasting power. Duffee (2008) discusses how a factor can have zero effect on current yields but be important for bond risk premia. Since yields of any maturity can be written as the sum of expected future short yields and a risk premium, such a factor must have offsetting effects on these two components. Duffee (2008) estimates a five-factor term structure model and uncovers a factor that has a negligible effect on current yields but contains substantial information about expected future short yields and expected excess bond returns. He finds the factor to be negatively associated with surveybased expected future short yields and positively associated with bond risk premia. The factor is also found to be negatively associated with industrial production, consistent with counter-cyclical risk premia. Ludvigson and Ng (2008) document using US data that the CP factor contains incremental information beyond macroeconomic variables such as inflation and real output.

The mystery of the CP factor remains. We intend to explore, in a global context, where it is coming from. Results from our affine model suggest that macro variables which affect the level and slope of the yield curve also drive risk premia. Natural candidates are inflation, real output, macroeconomic volatility/uncertainty, and monetary policy. We are also interested in understanding the nature of the global factor. Our results seem to suggest that global macro variables have predictive power across countries. A model including both local and global macro variables is likely to match the evidence of predictability. The close relation between the US factor and the global factor suggest that US macro variables or US monetary policy has implications for global bond risk premia. We would like to explore also this aspect in the future.

### 6 Conclusion

We find that bond excess returns outside the US are predictable using locally constructed forecasting factors as in Cochrane and Piazzesi (2005). The explanatory power is significantly higher than when using Fama and Bliss (1987) regressions. We also provide evidence that a global CP factor, closely related to US bond risk premia, has considerable forecasting power for international bond returns. Furthermore, the local and global CP factors are jointly significant, indicating that bond risk premia are driven by both country-specific and global factors.

Having established the predictive power of international CP factors, we propose and estimate a parsimonious no-arbitrage term structure model in which risk premia are assumed to be driven by one local and one global CP factor. The estimation reveals that the local CP factors act as a slope factor while the global factor is similar to a level factor. Hence, risk premia across countries seem to be driven by a local slope factor and a world interest rate level factor.

It is still considered a mystery where the CP factors are coming from. We hope to shed further light on the link between the macroeconomy and the CP factors in the future. Specifically, we think it is worthwhile exploring how macro variables such as inflation, real output, macroeconomic uncertainty, and monetary policy are related to the CP factors across countries, while also exploring the link between the global factor and the world economy. Our results suggest that US macro variables have considerable forecasting power for international bond risk premia.

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	Maturity	Mean	Std.Dev.	1.	2.	3.	4.	5.	6.	7.	%	9.	10.
Germany	1 month	5.15	2.50	1.00	0.97	0.94	0.91	0.88	0.86	1.00	0.88	0.69	0.61
	$1  \mathrm{year}$	5.33	2.35	0.97	1.00	0.99	0.97	0.95	0.93	1.00	0.90	0.76	0.70
	$2  \mathrm{years}$	5.57	2.20	0.94	0.99	1.00	0.99	0.98	0.97	1.00	0.89	0.82	0.75
	3 years	5.80	2.10	0.91	0.97	0.99	1.00	1.00	0.99	1.00	0.90	0.85	0.79
	4 years	5.98	2.01	0.88	0.95	0.98	1.00	1.00	1.00	1.00	0.91	0.87	0.81
	5  years	6.12	1.94	0.86	0.93	0.97	0.99	1.00	1.00	1.00	0.91	0.88	0.83
Switzerland	$1 \mod h$	3.44	2.57	1.00	0.98	0.96	0.93	0.90	0.88	0.88	1.00	0.65	0.47
	1 year	3.75	2.30	0.98	1.00	0.99	0.97	0.95	0.93	0.90	1.00	0.69	0.53
	$2  \mathrm{years}$	3.93	2.05	0.96	0.99	1.00	0.99	0.98	0.97	0.89	1.00	0.71	0.59
	3 years	4.12	1.89	0.93	0.97	0.99	1.00	1.00	0.99	0.90	1.00	0.75	0.64
	4 years	4.29	1.78	0.90	0.95	0.98	1.00	1.00	1.00	0.91	1.00	0.77	0.67
	$5  \mathrm{years}$	4.41	1.69	0.88	0.93	0.97	0.99	1.00	1.00	0.91	1.00	0.78	0.69
UK	1 month	8.39	3.52	1.00	0.98	0.95	0.93	0.91	0.89	0.69	0.65	1.00	0.75
	1 year	8.20	3.11	0.98	1.00	0.99	0.97	0.96	0.95	0.76	0.69	1.00	0.82
	$2  \mathrm{years}$	8.32	3.02	0.95	0.99	1.00	1.00	0.99	0.98	0.82	0.71	1.00	0.85
	3 years	8.41	2.99	0.93	0.97	1.00	1.00	1.00	0.99	0.85	0.75	1.00	0.86
	4 years	8.48	2.99	0.91	0.96	0.99	1.00	1.00	1.00	0.87	0.77	1.00	0.87
	5  years	8.54	3.01	0.89	0.95	0.98	0.99	1.00	1.00	0.88	0.78	1.00	0.87
SU	1 month	5.70	2.94	1.00	0.98	0.95	0.93	0.92	0.90	0.61	0.47	0.75	1.00
	$1 \mathrm{year}$	6.45	3.08	0.98	1.00	0.99	0.98	0.97	0.96	0.70	0.53	0.82	1.00
	$2  \mathrm{years}$	6.73	2.98	0.95	0.99	1.00	1.00	0.99	0.98	0.75	0.59	0.85	1.00
	3 years	6.92	2.88	0.93	0.98	1.00	1.00	1.00	0.99	0.79	0.64	0.86	1.00
	4 years	7.09	2.80	0.92	0.97	0.99	1.00	1.00	1.00	0.81	0.67	0.87	1.00
	5  years	7.19	2.74	0.90	0.96	0.98	0.99	1.00	1.00	0.83	0.69	0.87	1.00
The table presents mea five years. The sample	The table presents means and five years. The sample period	nd standai od is Janua	us and standard deviations of yields on zero-coupon bonds with maturities between one month and period is January 1976 to December 2007. Columns 1-10 present correlations: Columns 1-6 between	of yields December	s on zei 2007.	coup Colum	on bor ns 1-1(	ids wit prese	h matu nt corre	ro-coupon bonds with maturities between one month and Columns 1-10 present correlations: Columns 1-6 between	veen oi Jolumn	ne mon s 1-6 b	th and etween
yields of bon	yields of bonds with different maturities; columns 7-10 between yields of bonds with same maturity across countries	t maturiti	es; columns 7	7-10  betw	reen yie	lds of	ponds v	with sa	me mat	urity acro	inob sse	ntries.	

Table 1: Summary Statistics

	u			'		4.	с	5	:		5		-T-T-	-7-1	10.	Т. Т.	10.	-10.
Germany	2	1.	1.00	0.98	0.96	0.93	0.81	0.81	0.81	0.81	0.68	0.67	0.67	0.67	0.62	0.61	0.61	0.61
	3 C	2.	0.98	1.00	0.99	0.98	0.79	0.80	0.81	0.82	0.70	0.70	0.70	0.71	0.63	0.63	0.63	0.63
	4	ю.	0.96	0.99	1.00	1.00	0.76	0.79	0.81	0.82	0.70	0.71	0.72	0.72	0.63	0.63	0.64	0.64
	ß	4.	0.93	0.98	1.00	1.00	0.75	0.78	0.80	0.81	0.69	0.71	0.72	0.73	0.63	0.63	0.64	0.65
Switzerland	2	5.	0.81	0.79	0.76	0.75	1.00	0.97	0.95	0.94	0.64	0.63	0.62	0.61	0.43	0.45	0.46	0.46
	3 C	6.	0.81	0.80	0.79	0.78	0.97	1.00	0.99	0.98	0.66	0.67	0.67	0.67	0.46	0.49	0.50	0.51
	4	7.	0.81	0.81	0.81	0.80	0.95	0.99	1.00	0.99	0.67	0.68	0.68	0.68	0.46	0.48	0.50	0.51
	5	%	0.81	0.82	0.82	0.81	0.94	0.98	0.99	1.00	0.67	0.70	0.70	0.70	0.48	0.50	0.52	0.53
UK	2	9.	0.68	0.70	0.70	0.69	0.64	0.66	0.67	0.67	1.00	0.98	0.95	0.92	0.43	0.43	0.42	0.42
	n	10.	0.67	0.70	0.71	0.71	0.63	0.67	0.68	0.70	0.98	1.00	0.99	0.98	0.50	0.51	0.51	0.51
	4	11.	0.67	0.70	0.72	0.72	0.62	0.67	0.68	0.70	0.95	0.99	1.00	1.00	0.54	0.55	0.55	0.55
	5	12.	0.67	0.71	0.72	0.73	0.61	0.67	0.68	0.70	0.92	0.98	1.00	1.00	0.55	0.57	0.57	0.57
$\operatorname{SO}$	2	13.	0.62	0.63	0.63	0.63	0.43	0.46	0.46	0.48	0.43	0.50	0.54	0.55	1.00	0.99	0.97	0.95
	က	14.	0.61	0.63	0.63	0.63	0.45	0.49	0.48	0.50	0.43	0.51	0.55	0.57	0.99	1.00	0.99	0.98
	4	15.	0.61	0.63	0.64	0.64	0.46	0.50	0.50	0.52	0.42	0.51	0.55	0.57	0.97	0.99	1.00	0.99
	5	16.	0.61	0.63	0.64	0.65	0.46	0.51	0.51	0.53	0.42	0.51	0.55	0.57	0.95	0.98	0.99	1.00

 Table 2: Correlations between Excess Returns

	Germany	Switzerland	UK	US	Global
1976:01-2007:12					
Germany	1.00	0.63	0.25	0.32	0.46
Switzerland	0.63	1.00	0.55	0.27	0.41
UK	0.25	0.55	1.00	0.12	0.27
US	0.32	0.27	0.12	1.00	0.98
Global	0.46	0.41	0.27	0.98	1.00
$\underline{1976:01-1991:06}$					
Germany	1.00	0.35	-0.15	0.23	0.31
Switzerland	0.35	1.00	0.39	0.12	0.21
UK	-0.15	0.39	1.00	-0.10	0.01
US	0.23	0.12	-0.10	1.00	0.99
Global	0.31	0.21	0.01	0.99	1.00
1991:07-2007:12					
Germany	1.00	0.82	0.63	0.38	0.53
Switzerland	0.82	1.00	0.75	0.53	0.65
UK	0.63	0.75	1.00	0.64	0.73
US	0.38	0.53	0.64	1.00	0.98
Global	0.53	0.65	0.73	0.98	1.00

Table 3: Correlations between Local and Global CP Factors

The table presents correlations between local CP factors for Germany, Switzerland, the UK, and the US, and the global CP factor based on data for the full sample period (1976:01–2007:12) and two sub-sample periods (1976:01–1991:06 and 1991:07–2007:12).

	n	$a_c^n$	$b_c^n$	$\mathbb{R}^2$	$b_{c,CP}^n$	$R^2$
Germany	2	0.35	0.37	0.03	0.42	0.12
		(0.33)	(0.42)		(0.09)	
	3	0.50	0.61	0.05	0.85	0.15
		(0.70)	(0.50)		(0.17)	
	4	0.59	0.74	0.05	1.21	0.17
		(1.03)	(0.57)		(0.24)	
	5	0.69	0.81	0.05	1.52	0.17
		(1.32)	(0.63)		(0.30)	
Switzerland	2	0.16	0.61	0.09	0.44	0.16
		(0.28)	(0.24)		(0.12)	
	3	0.43	0.57	0.04	0.86	0.18
		(0.58)	(0.34)		(0.21)	
	4	0.66	0.58	0.03	1.23	0.19
		(0.88)	(0.46)		(0.28)	
	5	0.93	0.54	0.02	1.47	0.18
		(1.21)	(0.61)		(0.34)	
UK	2	0.33	0.38	0.03	0.42	0.14
		(0.27)	(0.27)		(0.14)	
	3	0.61	0.49	0.03	0.85	0.18
		(0.46)	(0.36)		(0.25)	
	4	0.90	0.47	0.02	1.22	0.19
		(0.64)	(0.45)		(0.37)	
	5	1.24	0.41	0.01	1.51	0.18
		(0.82)	(0.50)		(0.47)	
US	2	0.12	0.87	0.10	0.46	0.29
		(0.34)	(0.30)		(0.06)	
	3	-0.03	1.18	0.13	0.87	0.32
		(0.61)	(0.35)		(0.11)	
	4	-0.15	1.33	0.13	1.24	0.34
		(0.90)	(0.45)		(0.16)	
	5	0.37	0.97	0.05	1.42	0.31
		(1.21)	(0.57)		(0.21)	

Table 4: Fama-Bliss and Cochrane-Piazzesi Regressions

The table presents results from Fama-Bliss (1987) and Cochrane-Piazzesi (2005) regressions, corresponding to regression equations (3) and (4). The sample period is January 1976 to December 2007. Point estimates with Newey and West (1987) standard errors, accounting for conditional heteroscedasticity and serial correlation up to twelve lags, in parentheses are reported together with adjusted R-squares.

	n	$a_c^n$	$b_{c,Level}^n$	$b_{c,Slope}^{n}$	$b_{c,Curvature}^n$	$\mathbb{R}^2$
Germany	2	0.51	5.70	70.51	36.22	0.10
		(0.23)	(4.91)	(34.06)	(187.21)	
	3	1.02	10.54	148.99	119.16	0.12
		(0.43)	(8.51)	(64.86)	(343.41)	
	4	1.41	13.05	226.42	242.61	0.13
		(0.59)	(11.54)	(91.19)	(466.14)	
	5	1.73	14.30	299.69	386.75	0.14
		(0.74)	(14.32)	(113.78)	(569.16)	
Switzerland	2	0.36	4.85	104.12	-278.45	0.18
		(0.25)	(5.38)	(27.25)	(93.77)	
	3	0.80	11.00	199.53	-334.49	0.17
		(0.44)	(10.50)	(45.91)	(179.17)	
	4	1.17	15.11	300.69	-295.05	0.18
		(0.59)	(14.33)	(61.66)	(254.32)	
	5	1.44	20.17	350.13	-266.93	0.17
		(0.73)	(17.62)	(75.21)	(321.04)	
UK	2	0.40	5.89	47.24	249.49	0.10
		(0.25)	(3.61)	(34.78)	(135.69)	
	3	0.76	12.03	76.52	510.69	0.11
		(0.44)	(6.71)	(63.27)	(239.08)	
	4	1.08	17.89	104.79	656.49	0.11
		(0.61)	(9.85)	(89.48)	(338.73)	
	5	1.40	23.08	135.13	679.22	0.10
		(0.77)	(12.70)	(112.79)	(429.02)	
US	2	0.58	6.72	115.34	342.28	0.22
		(0.26)	(4.00)	(32.64)	(110.37)	
	3	0.94	9.13	223.22	685.48	0.21
		(0.48)	(7.62)	(63.93)	(202.86)	
	4	1.28	10.64	337.00	950.36	0.23
		(0.64)	(10.43)	(87.76)	(277.73)	
	5	1.34	11.61	420.57	1099.99	0.23
		(0.77)	(12.69)	(106.45)	(334.09)	

Table 5: Level, Slope, and Curvature Regressions

The table presents results from principal-component regressions, corresponding to regression equation (5). The sample period is January 1976 to December 2007. The first three principal components of the yield covariance matrix are referred to as level, slope, and curvature. Point estimates with Newey and West (1987) standard errors, accounting for conditional heteroscedasticity and serial correlation up to twelve lags, in parentheses are reported together with adjusted R-squares.

	n	$b_{c,CP}^n$	$\mathbb{R}^2$	$b_{c,GCP}^n$	$\mathbb{R}^2$	$b_{c,CP}^n$	$b_{c,GCP}^n$	$\mathbb{R}^2$	Wald
Germany	2	0.42	0.12	0.46	0.28	0.13	0.39	0.29	[0.00]
		(0.09)		(0.05)		(0.10)	(0.06)		
	3	0.85	0.15	0.86	0.27	0.35	0.67	0.29	[0.00]
		(0.17)		(0.10)		(0.20)	(0.11)		
	4	1.21	0.17	1.19	0.25	0.56	0.88	0.29	[0.00]
		(0.24)		(0.15)		(0.29)	(0.17)		
	5	1.52	0.17	1.46	0.24	0.73	1.06	0.28	[0.00]
		(0.30)		(0.19)		(0.37)	(0.21)		
Switzerland	2	0.44	0.16	0.41	0.20	0.25	0.28	0.24	[0.00]
		(0.12)		(0.09)		(0.13)	(0.10)		
	3	0.86	0.18	0.81	0.22	0.48	0.56	0.27	[0.00]
		(0.21)		(0.17)		(0.23)	(0.19)		
	4	1.23	0.19	1.14	0.23	0.71	0.78	0.28	[0.00]
		(0.28)		(0.24)		(0.32)	(0.27)		
	5	1.47	0.18	1.40	0.23	0.80	0.99	0.28	[0.00]
		(0.34)		(0.29)		(0.38)	(0.34)		
UK	2	0.42	0.14	0.40	0.16	0.28	0.28	0.23	[0.00]
		(0.14)		(0.11)		(0.12)	(0.10)		
	3	0.85	0.18	0.78	0.19	0.58	0.53	0.27	[0.00]
		(0.25)		(0.19)		(0.22)	(0.16)		
	4	1.22	0.19	1.14	0.21	0.81	0.80	0.29	[0.00]
		(0.37)		(0.27)		(0.32)	(0.22)		
	5	1.51	0.18	1.49	0.22	0.95	1.09	0.29	[0.00]
		(0.47)		(0.32)		(0.40)	(0.27)		
US	2	0.46	0.29	0.57	0.29	0.26	0.26	0.29	[0.00]
		(0.06)		(0.08)		(0.35)	(0.47)		
	3	0.87	0.32	1.06	0.30	0.85	0.03	0.32	[0.00]
		(0.11)		(0.15)		(0.61)	(0.81)		
	4	1.24	0.34	1.50	0.32	1.30	-0.08	0.34	[0.00]
		(0.16)		(0.22)		(0.79)	(1.04)		
	5	1.42	0.31	1.72	0.29	1.55	-0.15	0.31	[0.00]
		(0.21)		(0.27)		(1.01)	(1.32)		

Table 6: Local and Global Cochrane-Piazzesi Regressions

The table presents results from local and global Cochrane-Piazzesi (2005) regressions, corresponding to regression equations (4), (6), and (7). The sample period is January 1976 to December 2007. Point estimates with Newey and West (1987) standard errors, accounting for conditional heteroscedasticity and serial correlation up to twelve lags, in parentheses are reported together with adjusted R-squares. P-values from Wald tests of joint significance are given in square brackets.

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Table 7

	00	$o_1$		d		$\lambda_0$		$\lambda_1$	
Germany	0.435	-0.007	0.994	-0.019	-0.018	-1.296	-0.270	0	0
	(0.007)	(0.005)	(0.052)	(0.045)	(0.063)	(3.651)	(0.058)		
		0.032	0.255	0.650	-0.342	-3.158	0	-0.699	0
		(0.003)	(0.927)	(0.798)	(1.057)	(8.879)		(0.217)	
		0.046	-0.110	0.236	1.229	-0.695	0.043	-0.343	0
		(0.001)	(0.657)	(0.559)	(0.750)	(1.421)	(0.042)	(0.051)	
Switzerland	0.290	-0.015	0.996	0.011	-0.006	-0.037	-0.059	0	0
	(0.004)	(0.004)	(0.034)	(0.007)	(600.0)	(0.016)	(0.006)		
		0.048	-0.968	0.992	0.319	-0.547	0	-0.717	0
		(0.002)	(0.883)	(0.123)	(0.210)	(0.341)		(0.196)	
		0.051	0.909	-0.020	0.699	-0.230	-0.075	-0.215	0
		(0.001)	(0.769)	(0.104)	(0.184)	(0.051)	(0.019)	(0.036)	
UK	0.707	0.067	0.434	-0.101	0.044	0.802	-0.099	0	0
	(0.013)	(0.008)	(0.591)	(0.204)	(0.047)	(1.074)	(0.008)		
		0.018	1.558	1.245	-0.066	-2.613	0	-0.341	0
		(0.006)	(1.382)	(0.469)	(0.111)	(2.669)		(0.024)	
		0.047	0.113	0.050	0.942	-0.536	-0.055	-0.118	0
		(0.002)	(0.666)	(0.152)	(0.039)	(0.704)	(0.001)	(0.004)	
SU	0.480	0.010		0.857	-0.019	0.040		-0.137	0
	(0.007)	(0.004)		(0.031)	(0.007)	(0.033)		(0.011)	
		0.039		0.205	1.017	-0.146		-0.081	0
		(0.001)		(0.035)	(0.00)	(0.010)		(0.013)	
The table pre on bonds wit state variable The paramet of the momen sample count is restricted $\varepsilon$ Point estimat twelve lags as with 100. Th	esents estimat h maturities es $(CP_{c,t}, G($ ers in $\delta_0, \delta_1,$ nt conditions erparts of the as in equation ces and stand s in Newey a e sample peri	The table presents estimation results of affine models with local on bonds with maturities of one month, and one to five years. C state variables $(CP_{c,t}, GCP_t)$ , and $Level_{c,t}$ , whereas the US 1 The parameters in $\delta_0$ , $\delta_1$ , $\rho^*$ , and $\lambda_0$ are estimated in a first s of the moment conditions in (25). The parameters in $\lambda_1$ are es- sample counterparts of the moment conditions in (28). The $\lambda_1$ r is restricted as in equation (26), and for the US it is restricted. Point estimates and standard errors, accounting for conditional twelve lags as in Newey and West (1987), are reported. Estima with 100. The sample period is January 1976 to December 2007.	affine mod , and one to , and one to $rel_{c,t}$ , when are estimate parameters ditions in ( counting fo 7), are repo	(els with le o five year reas the U ed in a fir s in $\lambda_1$ ar (28). The is restrict is restrict r conditio orted. Est ecember 2 ecember 2	ocal and glass. German, S. German, JS has two St step by e estimated $\lambda_1$ matrix f and netros in ec nal heteros inmates of I 007.	The table presents estimation results of affine models with local and global factors (see Section 4.1) for yields on bonds with maturities of one month, and one to five years. Germany, Switzerland, and the UK have three state variables $(CP_{c,t}, GCP_t, \text{ and } Level_{c,t})$ , whereas the US has two state variables $(GCP_t \text{ and } Level_{c,t})$ . The parameters in $\delta_0$ , $\delta_1$ , $\rho^*$ , and $\lambda_0$ are estimated in a first step by GMM, using the sample counterparts of the moment conditions in (25). The parameters in $\lambda_1$ are estimated in a second step by GMM, using the sample counterparts of the moment conditions in (28). The $\lambda_1$ matrix for Germany, Switzerland, and the UK is restricted as in equation (26), and for the US it is restricted as in equation (27). See Section 4 for details. Point estimates and standard errors, accounting for conditional heteroscedasticity and serial correlation up to twelve lags as in Newey and West (1987), are reported. Estimates of parameters in $\delta_0$ and $\delta_1$ are multiplied with 100. The sample period is January 1976 to December 2007.	see Section - l, and the U les $(GCP_t)$ the sample step by GM Switzerland See Section d serial cor $\delta_0$ and $\delta_1$	4.1) for $y_{i}^{(1)}$ for $y_{i}^{(1)}$ have that and <i>Leve</i> counterposed for the form $1$ , and the 4 for detain under the relation under the pare multiple of the form $1$ .	elds hree $l_{c,t}$ ). arts UK ails. p to blied

	1 month	1 year	2 years	3 years	4 years	5 years	RMSE	MAD
Germany	-0.028 $\{0.654\}$	0.007 $\{0.224\}$	0.007 $\{0.064\}$	-0.003 $\{0.080\}$	-0.007 $\{0.100\}$	0.006 $\{0.153\}$	0.295	0.155
Switzerland	-0.011 $\{0.481\}$	-0.008 $\{0.080\}$	-0.020 $\{0.128\}$	$0.001$ {0.033}	$0.014$ {0.112}	0.006 $\{0.167\}$	0.222	0.120
UK	0.002 {1.050}	-0.005 $\{0.468\}$	0.006 $\{0.135\}$	$0.000$ $\{0.095\}$	-0.003 $\{0.230\}$	0.002 $\{0.361\}$	0.504	0.310
US	-0.017 $\{0.732\}$	0.055 $\{0.323\}$	-0.026 $\{0.155\}$	-0.034 $\{0.080\}$	0.005 $\{0.167\}$	0.017 $\{0.293\}$	0.362	0.227

Table 8: Yield Diagnostics of the Estimated Affine Models

The table presents diagnostics of the estimated affine models with local and global factors (see Section 4.1) for yields on bonds with maturities of one month, and one to five years. Averages and standard deviations (in curly brackets) of yield errors, are reported in the first six columns. The last two columns report a root mean squared error (RMSE) and a mean absolute deviation (MAD) of yield errors. All statistics are expressed in % per year.

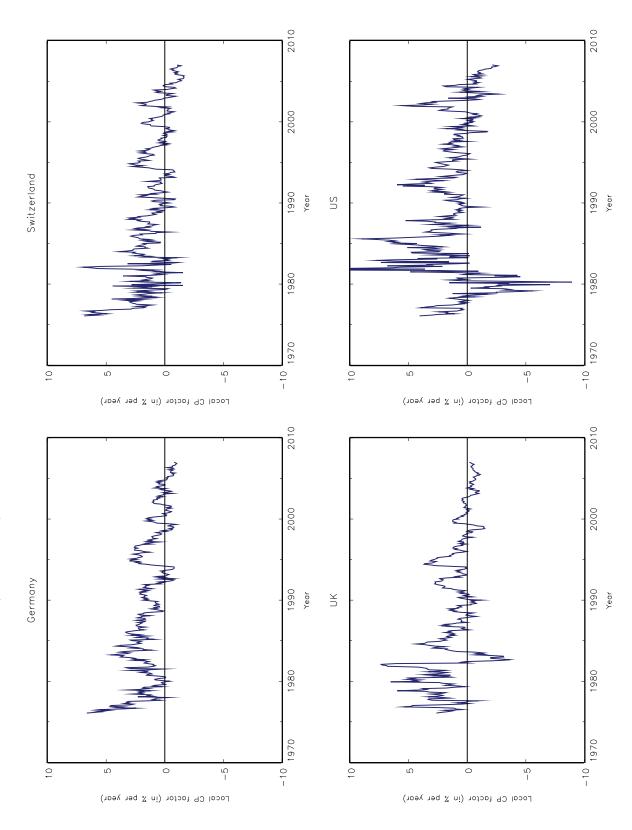
	Variable	Horizon	1 month	5 year
Germany	Local CP	1	0.39	0.37
		120	0.12	0.38
	Global CP	1	0.00	0.02
		120	0.06	0.03
	Level	1	0.61	0.62
		120	0.82	0.59
Switzerland	Local CP	1	0.30	0.49
		120	0.49	0.68
	Global CP	1	0.00	0.00
		120	0.02	0.00
	Level	1	0.70	0.51
		120	0.49	0.32
UK	Local CP	1	0.00	0.31
		120	0.13	0.13
	Global CP	1	0.09	0.02
		120	0.15	0.09
	Level	1	0.91	0.67
		120	0.72	0.78
US	Global CP	1	0.50	0.02
		120	0.12	0.09
	Level	1	0.50	0.98
		120	0.88	0.91

 Table 9: Variance Decompositions

The table presents variance decompositions of yield forecast errors, attributed to each state variable at horizons of one month and 120 months for yields on a one-month bond and a five-year bond.

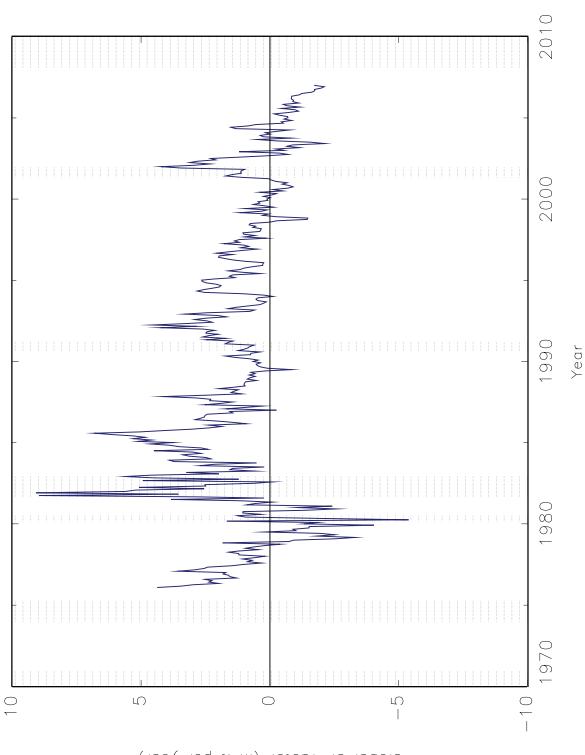


The figure shows the local CP factors (in % per year) for Germany, Switzerland, the UK, and the US.



## Figure 2: Global CP Factor

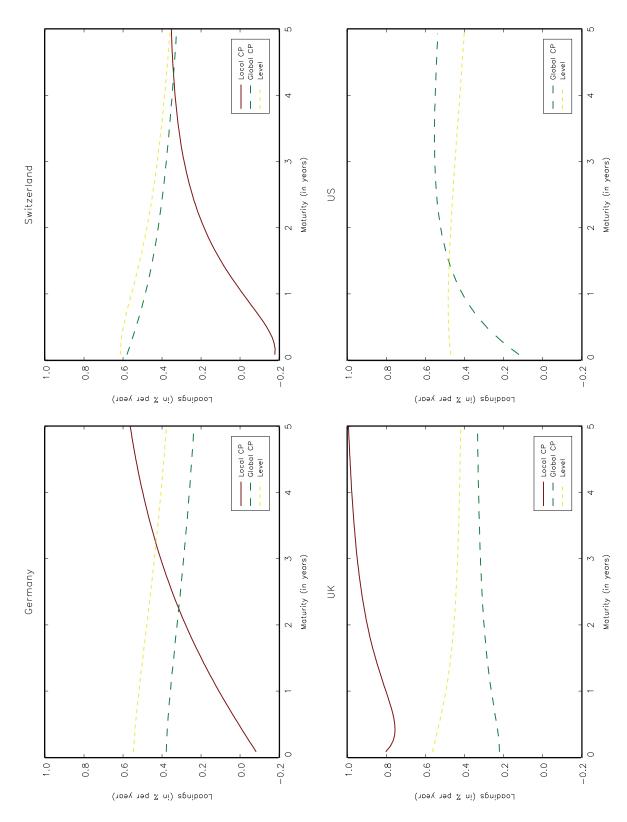
The figure shows the global CP factor (in % per year), which is a GDP-weighted average of the local CP factors (for Germany, Switzerland, the UK, and the US). The shaded areas mark US contractions (peaks to troughs) as dated by the NBER.



Global CP factor (in % per year)

### Figure 3: Yield Loadings

The figure shows the yield loadings (in % per year) for Germany, Switzerland, the UK, and the US. The loadings are  $-\frac{B'_n}{n}$ , presented in Equation(12). The solid line refers to the local CP factor, the dashed line to the global CP factor, and the short-dashed line to the local level factor.



# Figure 4: Impulse Response Functions

The figure shows the impulse response functions (in % per year) for the yield on a one-month bond and the yield on a five-year bond given a one standard deviation shock to each state variable. The solid line refers to impulses from the local CP factor, the dashed line to the global CP factor, and the short-dashed line to the local level factor.

