Asset Allocation in SVCJ Models:
How much does model choice matter?

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Abstract

This paper analyzes the optimal portfolio decision of a CRRA-investor in models with stochastic volatility and stochastic jumps. The investor has access to one additional derivative and is restricted to a buy-and-hold strategy. We find that the fine structure of the risk premia has a significant impact on the optimal portfolio decision and the utility gain due to having access to derivatives. This structure is of equal importance as the impact of model choice. The model and the risk premia also have an impact on whether the investor prefers to trade OTM or ATM options. The dependence of the optimal portfolio on the specific model and on the specific assumptions on the risk premia leads to significant utility losses in case of model mis-specification, in particular when OTM puts are the seemingly optimal choice.

Keywords: stochastic volatility, jumps, market prices of risk, asset allocation, buy-and-hold strategy, model mis-specification

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1 Introduction and Motivation

State-of-the-art option pricing models include stochastic volatility, stochastic jumps in stock returns, and in some cases also stochastic jumps in volatility. Broadie, Chernov, and Johannes (2008) study put option returns in these models. They show that the generated returns can well explain empirical option returns observed at the market if one includes a realistic risk premium for jumps in the stock price. Branger, Hansis, and Schlag (2008) additionally analyze call returns and returns over different holding periods. They show that the exact structure of the risk premia and in particular the structure of the jump risk premium has a significant impact on the returns of OTM options.

In this paper, we analyze the impact of the fine structure of the risk premia on the optimal asset allocation decision in state-of-the art option pricing models and compare it to the impact of the model choice, i.e. the decision on which risk factors to include. We consider a CRRA-investor who follows a buy-and-hold strategy and has access to the stock, the money-market account, and one additional option on the stock.

We show that the model choice has a significant impact on the optimal portfolio and on the utility gain of the buy-and-hold investor. It additionally has an impact on the optimal moneyness level for which the investor profits most from trading options. Secondly, we show that assumptions on the fine structure of the risk premia can have an impact that is as large as the impact of adding or omitting jumps in the stock price and in the volatility. We focus on the risk premium for variance diffusion risk and on the fine structure of the risk premia for jump risk, i.e. the market prices of risk for jump intensity risk, jump size risk, jump variance risk, and jumps in volatility, which are hard to identify empirically.[1] Finally, we analyze the utility losses of an investor due to model mis-specification. While they can be devastating if an incorrect model is used, they are smaller, but still highly economically significant, if incorrect risk premia are used.

The portfolio planning problem we consider is the most simple one in which this

[1]For a detailed discussion and an overview over the (contradictory) empirical findings, see Broadie, Chernov, and Johannes (2007).

analysis can be done. A buy-and-hold investor who has access to one derivative besides the stock and the money market account is just able to take some position in volatility risk and to deviate somehow from the fixed relation between the risk factors offered by the stock. However, he is still far away from the (ideal) situation of continuous trading in infinitely many derivatives, where he could fine-tune his portfolios to take all subtleties of the model and the risk premia into account. His limited ability to adjust his portfolio to all aspects of the model will most probably lead to a lower impact of the actual structure of the model. We thus expect our results to provide a conservative estimate for the impact of the model choice and the fine structure of the risk premia on the portfolio planning problem.

Our paper is related to the literature on portfolio planning with derivatives. Liu and Pan (2003) solve the portfolio planning problem of a CRRA-investor in a model with stochastic volatility and jumps in stock returns. They assume that the investor can trade continuously in infinitely many derivatives. Branger, Schlag, and Schneider (2008) extend this analysis to the case where volatility can also jump. Branger and Hansis (2009) study the impact of the structure of the risk premia on optimal portfolio decisions and on the utility in a complete market. In contrast to these papers, we consider a buy-and-hold investor who has access to one derivative only and compare the impact of assumptions on the risk premia and assumptions on the model structure. Driessen and Maenhout (2007) study optimal portfolios of stocks, bonds and one derivative contract, where they also consider portfolios of OTM puts and ATM straddles. Their analysis is done for empirical time series of returns observed in the market, and they compare various utility functions. They show that the investor wants to take a short position in put options in nearly all cases, which raises the question who actually buys these options. In contrast to their paper, we assume that stock and option returns are generated by some option pricing model and look at the implications the choice of the model as well as the fine structure of the risk premia have.

We consider the model of Black-Scholes, the stochastic-volatility model of Heston (1993), an SVJ-model with jumps in the stock price, and an SVCJ-model which also includes simultaneous jumps in volatility. The parameters for the different models are taken from Broadie, Chernov, and Johannes (2007). All models have been calibrated to exactly the same option prices and are thus as similar as possible. The portfolio planning problem is solved via a Monte-Carlo simulation, since there is no closed form solution for a of buy-and-hold investor. For each option pricing model and each moneyness level, we simulate 25,000 returns of the stock and the derivative and then solve for the optimal portfolio weights using a numerical optimization routine.

The choice of the model has a significant impact on the optimal portfolio decision. While the buy-and-hold investor takes a long position in put options in the Heston model, he wants to go short in models including jumps. The fine structure of the risk premia turns out to be of similar importance. Restricting the premium for variance risk in stock price jumps to zero, e.g., can change the optimal position in puts by a factor of two.

The utility gain from having access to one additional option is economically highly significant across all models besides the Black-Scholes model. Like the optimal portfolio, it depends significantly on the model and on the assumptions on the risk premia. In models with jumps, the utility improvement from trading derivatives is significantly larger than in a model with stochastic volatility only. Furthermore, the utility improvement increases in general when the risk premium for volatility diffusion risk and/or for the variance in stock price jumps is restricted to zero. The utility losses from earning zero risk premia on the restricted risk factors are thus more than offset by the larger risk premia for the risk factors that are still priced.

Our results furthermore show that the choice of the moneyness level has a big impact on the utility improvement for the buy-and-hold investor. By choosing the optimal moneyness, he can increase his certainty equivalent return by up to 5% a year. While the investor prefers ATM options in the Heston model, he profits most from OTM and ITM options in models with jumps. If we restrict the risk premium on volatility diffusion risk as well as the risk premium on the variance in stock price jumps to be equal to zero, the optimal strike moves towards ATM again. Again, the fine structure of the risk premia turns out to have an impact that is similar to the impact of the model choice.

Given the dependence of the optimal portfolio on the choice of the model and the fine structure of the risk premia, a natural question is how much an investor is going to lose when he relies on an incorrect model. We study the impact of model mis-specification for the buy-and-hold investor when he incorrectly omits all jumps or jumps in volatility from the model. Furthermore we look at the impact of incorrectly restricting some risk premia to be equal to zero. When the investor omits all jumps from his model, he loses up to 8% of the yearly certainty equivalent return. If he only omits jumps in volatility and follows the seemingly optimal strategy of selling OTM puts, he can even end up with a negative wealth, which is prohibitively bad for a CRRA-investor. Incorrect assumptions about the risk premia can also induce significant losses in the certainty equivalent return. With up to 4%, they are, however, not as devastating as losses due to incorrect assumptions about the structure of the model.

The remainder of the paper is organized as follows. In Section 2 we present the model setup and the portfolio planning problem of the investor. Section 3 gives the results for the impact of the model choice, while Section 4 gives the results for the fine structure of the risk premia. Model mis-specification is analysed in Section 5. Section 6 concludes.

2 Model Setup

2.1 Model

We consider an SVCJ model with stochastic volatility and jumps both in the stock price and in its volatility, which is e.g. discussed in Duffie, Pan, and Singleton (2000) or Broadie, Chernov, and Johannes (2007). The dynamics of the stock price (or index level) S and its variance V under the physical measure $\mathbb{P}$ are

$$dS_t = (b + 0.5V_t + \bar{\mu}^{\mathbb{P}}\lambda^{\mathbb{P}})S_t dt + \sqrt{V_t} S_t dW_t^{S,\mathbb{P}} + \left(e^{\xi} - 1\right) S_{t-} dN_t - \bar{\mu}^{\mathbb{P}}\lambda^{\mathbb{P}} S_t dt \quad (1)$$

$$dV_t = \kappa^{\mathbb{P}}(\theta^{\mathbb{P}} - V_t)dt + \sqrt{V_t}\sigma_V \left(\rho dW_t^{S,\mathbb{P}} + \sqrt{1-\rho^2} dW_t^{V,\mathbb{P}}\right) + \Psi dN_t, \quad (2)$$

where $W_t^{S,\mathbb{P}}$ and $W_t^{V,\mathbb{P}}$ are independent Wiener processes. N_t is a Poisson process with constant intensity $\lambda^{\mathbb{P}}$, and we assume that the stock price and the variance jump simultaneously. The jump size Ψ in the variance is exponentially distributed with expectation $\mu_V^{\mathbb{P}}$, i.e. $\Psi \sim \exp\left\{\mu_V^{\mathbb{P}}\right\}$. Conditional on the realized variance jump, the jump size ξ in the stock return follows a normal distribution: $\xi \sim \mathcal{N}\left(\mu_S^{\mathbb{P}} + \rho_J^{\mathbb{P}}\Psi, (\sigma_S^{\mathbb{P}})^2\right)$. The mean jump size in the stock price is thus

$$\bar{\mu}^{\mathbb{P}} = \frac{\exp\left\{\mu_S^{\mathbb{P}} + \frac{(\sigma_S^{\mathbb{P}})^2}{2}\right\}}{1 - \rho_J^{\mathbb{P}}\mu_V^{\mathbb{P}}} - 1.$$

where we assume $\rho_J^{\mathbb{P}}\mu_V^{\mathbb{P}} < 1$.[2] We assume that there are no dividend payments. The expected return on the stock is $b + 0.5V_t + \bar{\mu}^{\mathbb{P}}\lambda^{\mathbb{P}}$ for some constant b, where we follow the specification of Eraker, Johannes, and Polson (2003).

The SVCJ model nests several option pricing models. Setting $\lambda^{\mathbb{P}} = \sigma_V = 0$ and $V_t = \theta^{\mathbb{P}}$ gives the Black-Scholes model. For the Heston (1993) (SV) model, we set $\lambda^{\mathbb{P}} = 0$, and for the SVJ model of Bakshi, Cao, and Chen (1997) and Bates (1996), we set $\Psi = 0$.

The dynamics under the risk-neutral measure $\mathbb{Q}$ are

$$\begin{aligned} dS_t &= rS_t dt + \sqrt{V_t} S_t dW_t^{S,\mathbb{Q}} + \left(e^{\xi} - 1\right) S_{t-} dN_t - \bar{\mu}^{\mathbb{Q}}\lambda_S^{\mathbb{Q}} S_t dt \\ dV_t &= \kappa^{\mathbb{Q}}(\theta^{\mathbb{Q}} - V_t)dt + \sqrt{V_t}\sigma_V \left(\rho dW_t^{S,\mathbb{Q}} + \sqrt{1-\rho^2} dW_t^{V,\mathbb{Q}}\right) + \Psi dN_t. \end{aligned}$$

The mean-reversion speed and the mean-reversion level of the variance are given by

$$\begin{aligned} \kappa^{\mathbb{Q}} &= \kappa^{\mathbb{P}} + \eta_V \\ \kappa^{\mathbb{Q}}\theta^{\mathbb{Q}} &= \kappa^{\mathbb{P}}\theta^{\mathbb{P}}, \end{aligned}$$

where $\eta_V V_t$ is the premium for (total) volatility diffusion risk, i.e. for the diffusion term $\sqrt{V_t}\sigma_V \left(\rho dW_t^{S,\mathbb{P}} + \sqrt{1-\rho^2} dW_t^{V,\mathbb{P}}\right)$.

The intensity of the jump process under $\mathbb{Q}$ is $\lambda^{\mathbb{Q}}$. For the jump sizes, we assume that they still follow an exponential and a conditional normal distribution, respectively, but that all parameters of these distributions may change. It then holds that $\Psi \sim \exp\left\{\mu_V^{\mathbb{Q}}\right\}$ and $\xi \sim \mathcal{N}\left(\mu_S^{\mathbb{Q}} + \rho_J^{\mathbb{Q}}\Psi, (\sigma_S^{\mathbb{Q}})^2\right)$. In the following, we set $\rho_J^{\mathbb{Q}} \equiv 0$, again following Broadie, Chernov, and Johannes (2007).

[2] An average jump in volatility can thus not lead to an upward jump in the stock price of more than 100%, which is not a binding restriction in reality.

The expected excess return on the stock is $b + 0.5V_t + \bar{\mu}^{\mathbb{P}}\lambda^{\mathbb{P}} - r$. The compensation for jump risk is $\bar{\mu}^{\mathbb{P}}\lambda^{\mathbb{P}} - \bar{\mu}^{\mathbb{Q}}\lambda^{\mathbb{Q}}$. The premium per unit of stock diffusion risk, i.e. for $\sqrt{V_t}dW_t^{S,\mathbb{P}}$, is given by $b + 0.5V_t + \bar{\mu}^{\mathbb{Q}}\lambda^{\mathbb{Q}} - r$. Note that – different from the setup of Liu and Pan (2003) e.g. – the premium is affine, but not linear in V. For the analysis, it turns out to be useful to denote this premium by $\eta_S(V_t) \cdot V_t$, and it holds that

$$b + 0.5V_t + \lambda^{\mathbb{Q}}\bar{\mu}^{\mathbb{Q}} - r = \eta_S(V_t)V_t. \tag{3}$$

Analogously, we can define the premium $\eta_V^{pure}(V_t) \cdot V_t$ for pure volatility diffusion risk $\sqrt{V_t}dW_t^{V,\mathbb{P}}$, which follows from

$$\eta_V = \sigma_V \left(\eta_S(V_t)\rho + \eta_V^{pure}(V_t)\sqrt{1-\rho^2} \right).$$

For our analysis, we rely on the estimates from Eraker, Johannes, and Polson (2003) and Broadie, Chernov, and Johannes (2007). Table 1 gives the parameters under the $\mathbb{P}$-measure as estimated by Eraker, Johannes, and Polson (2003) from the time series of index returns. Broadie, Chernov, and Johannes (2007) calibrate the corresponding risk-neutral $\mathbb{Q}$-parameters from the cross-section of option prices. The resulting estimates are given in Table 2. Similar to Broadie, Chernov, and Johannes (2007), we also consider several restricted models, where some risk premia are set equal to zero.

2.2 Portfolio Planning Problem: Buy-and-Hold Investor

We consider a CRRA-investor with planning horizon T who derives utility from terminal wealth only. He can invest into a stock (or index), the risk-free asset, and into one derivative. Furthermore, we assume that he can only trade at the initial point in time, i.e. that he follows a buy-and-hold strategy.

The portfolio planning problem of this investor is

$$\max_{\alpha_E, \alpha_D} E\left[U(W_T)\right], \tag{4}$$

where α_E and α_D are the fractions of wealth invested in the stock and the derivative respectively. The terminal wealth W_T is given by

$$W_T = \left[R_f + \alpha_E(R_E - R_f) + \alpha_D(R_D - R_f)\right] W_0, \tag{5}$$

where R_i is the gross return of the equity index ($i = E$) and the derivative ($i = D$) from 0 to T. The first order conditions for the optimal weights are ($i = D, E$)

$$E\left[U'\left(\left[R_f + \alpha_E(R_E - R_f) + \alpha_D(R_D - R_f)\right] W_0\right)(R_i - R_f)\right] = 0. \tag{6}$$

The expectation (6) cannot be calculated in closed form, and we need to solve for the optimal portfolio weights numerically. We use a Monte-Carlo simulation with 25,000 runs. The dynamics of the stock price and its variance are discretized using an Euler-discretization with 10 time steps per day. The prices of the options today and at the end of the planning horizon are calculated by Fourier inversion, see e.g. Duffie, Pan, and Singleton (2000). For each model and each set of assumptions on the risk premia, we thus get 25,000 sets of returns on the stock and each of the options. For each moneyness, we can then calculate the left-hand side of Equation (6) for different values of α_D and α_E. The optimal portfolio weights are found by a numerical optimization.[3]

2.3 Analysis of Optimal Buy-and-Hold Portfolio

The solution of the portfolio planning problem is given by the optimal portfolio weights α_E and α_D. For the interpretation, however, portfolio weights turn out to be misleading, so that we focus on the absolute number of assets in the portfolio instead. First, note that the investor uses options to achieve a non-linear payoff. The amount of 'non-linearity' in the payoff profile depends on the absolute number of options held as compared to the absolute number of stocks, not on the relation of the portfolio weights. Second, there are huge price differences between options with different moneyness levels, so that differences in portfolio weights across moneyness levels are barely meaningful. Finally, due to put-call parity exactly the same payoff can be achieved by adding calls, puts or straddles, each with the same moneyness, to the portfolio. Replacing OTM puts (or calls) by ITM calls (or puts) would not change the terminal payoff, whereas the optimal weight in options would increase significantly.

[3]To control for the accuracy of the optimization, we run the optimization routine several times with different starting values. We only report those results where all solutions coincide up to rounding errors.

The investor follows a buy-and-hold strategy with one additional option, so that the terminal payoff of the optimal portfolio at the maturity date of the option will have exactly one kink. The main question is then whether the investor chooses a convex or a concave payoff profile, i.e. whether he takes a long or a short position in options. In addition to the payoff profile at maturity of the options (two months in our example), we also consider the density of the portfolio return at the end of the planning horizon (one month in our example). As a benchmark, we calculate the density for the case that the investor can only trade the stock and the money market account.

In addition to the hitherto global characteristics of the portfolio, we also look at the local exposures to the risk factors. The dynamics of the optimal wealth are given by

$$\begin{aligned}\frac{dW_t}{W_t} &= r\,dt + \theta_t^S \left(dW_t^{S,\mathbb{P}} + \eta_S(V_t)\sqrt{V_t}dt \right) + \theta_t^V \left(dW_t^{V,\mathbb{P}} + \eta_V^{\text{pure}}(V_t)\sqrt{V_t}dt \right) \\ &\quad + \theta_t^N(X_t, \Psi_t)dN_t + E^{\mathbb{P}} \left[\theta_t^N(X_t, \Psi_t) \right] \lambda^{\mathbb{P}} dt - E^{\mathbb{Q}} \left[\theta_t^N(X_t, \Psi_t) \right] \lambda^{\mathbb{Q}} dt, \end{aligned} \tag{7}$$

where θ_t^S, θ_t^V, and $\theta_t^N(X_t, \Psi_t)$ denote the exposure with respect to stock diffusion risk, pure volatility diffusion risk, and jump risk, respectively. These exposures follow from the dynamics of the asset prices and the number of assets held in the portfolio, as shown in Liu and Pan (2003).

To measure the investor's benefit from trading the stock and a derivative, we calculate the certainty equivalent return

$$CER_T = \frac{\ln \frac{CE_T}{W_0}}{T},$$

where the certainty equivalent CE_T is defined by

$$U(CE_T) = E\left[U(W_T^*)\right].$$

The certainty equivalent return CER_T is the deterministic return for which the investor is indifferent between investing at this deterministic return and investing into the optimal risky portfolio. When the investor can not only trade the stock and the money market account, but also has access to one derivative, the CER_T increases. The size of this increase is an economic measure for the utility improvement. A comparison across different moneyness levels allows us to find the optimal moneyness for the traded option.

3 Results: Comparison Across Models

We consider a CRRA-investor with a planning horizon of one month and a relative risk aversion of $\gamma = 5$.[4] The buy-and-hold investor has access to the stock, the money market account, and one put option on the stock with a time to maturity of two months and a given moneyness, defined as the strike price divided by the stock price. We consider moneyness levels between 86% and 110%.[5] Due to put-call parity, the set of payoffs available to the investor would not change if we replaced the put by a call option or a straddle, etc.

The physical and risk-neutral parameters of the models are given in Tables 1 and 2, respectively. Note that all models are calibrated to the same time-series of index returns and the same cross-section of option prices. The models are thus as similar as possible. Differences between portfolios can therefore be attributed completely to the different structures of the models.

3.1 Benchmark: No Derivatives

As a benchmark, we first look at the optimal portfolio when the investor can only invest in the money market account and the stock. Figure 1 shows the optimal weights of the stock index as a function of risk aversion. In line with intuition, the weight of the stock is decreasing in risk aversion for all models. Depending on the model, it is equal to one (so that the investor holds the stock only) for a risk aversion of $\gamma = 2.0$ (SVJ model), $\gamma = 3.1$ (SVCJ model), $\gamma = 3.3$ (SV model) or $\gamma = 4.9$ (Black-Scholes model). For $\gamma = 5$, as assumed in the subsequent analysis, the investor is thus in all models more risk averse than the market.

[4]We have redone the analysis also for a relative risk aversion of $\gamma = 10$. With the higher risk aversion, the results are basically kind of 'compressed'. The general patterns, however, do not change significantly, and the relations between the optimal portfolios across different moneyness levels and different models do not differ much from the case $\gamma = 5$.

[5]OTM-puts are more liquid than OTM-calls, which is why our moneyness interval not symmetric around 100%. See also Broadie, Chernov, and Johannes (2008).

The optimal weight of the stock differs between the four models we consider. This is due to differences in the expected excess returns and in the return variances. It is also due to the different risk factors the stock is exposed to, and due to the different relative compensations for diffusion risk and the various elements of jump risk. Put differently, the results show that the investor does not only care about the absolute risk and the absolute risk premium, but also about their decomposition.

3.2 Buy-and-Hold Strategies With Derivatives

3.2.1 Black-Scholes model

In case of continuous trading, the investor puts a constant fraction of his wealth into the risky asset. For $\gamma = 5$, this fraction is smaller than one, resulting in an optimal terminal payoff which is a concave function of the stock price. The investor is thus willing to forego some payoffs for very high and very low stock prices in order to get a higher payoff in the case of only moderate (and more probable) stock price changes.

In our setup, the investor is restricted to a buy-and-hold strategy and has access to one additional option. He can then still achieve a concave terminal payoff by taking a short position in puts. Figure 2 shows that he is indeed short in puts for very low and in particular for very high moneyness levels. For intermediate moneyness levels, the position in puts is approximately equal to zero. For a high moneyness level, the position in the stock decreases to offset the implicit positive exposure from the put position. As can be seen from Figure 3, the (local) exposure to stock diffusion risk is rather constant across different moneyness levels.

The upper left graph in Figure 4 characterizes the optimal portfolios by their payoff profiles, and the upper left graph in Figure 5 gives the density of the payoff at the end of the planning horizon. They both confirm that the optimal portfolio is rather similar to the one which consists of the stock and the money market account only. The only exception is given by the put with the largest moneyness, where the investor gives up some upside potential and accepts slightly more downside risk to increase the probability of moderately positive returns.

In line with these findings, the excess certainty equivalent returns due to having access to one option shown in Figure 6 are barely larger than zero. To get the intuition, note that one reason to add derivatives to the portfolio is to circumvent the problems caused by discrete instead of continuous trading. In a Black-Scholes setup and for moderate risk premia, however, even a buy-and-hold portfolio in the stock and the money market account is rather close to the overall optimal portfolio, as shown in Rogers (2001) and Branger, Breuer, and Schlag (2008). This also holds true in the other models we consider. The need for derivatives induced by discrete trading is thus rather small. In the other models (but not in the model of BS), there is a second reason to include derivatives. They allow the investor to earn the risk premia on further risk factors like stochastic volatility and to deviate from the relation of diffusion and jump risk offered by the stock. Our results in the following sections show that this second arguments indeed provides a strong motive for trading derivatives.

3.2.2 Heston model

In the Heston model, the investor takes a long position in puts. The number of puts is on average decreasing in moneyness (as shown in Figure 2) and is thus largest for OTM puts. To offset the implicit negative exposure to stock price risk, the long position in the stock is larger than under the Black-Scholes model. Figure 3 shows that the resulting exposure to stock diffusion risk is again rather constant across different moneyness levels. However, it is lower than in the Black-Scholes model, since there is a second priced risk factor. This induces the investor to substitute some stock diffusion risk by volatility diffusion risk, using kind of a diversification effect.

The exposure to volatility risk is – with a long position in puts – positive, and the investor earns the positive volatility risk premium. The exposure is largest for ATM puts, which have the largest vega. ITM and OTM options have a smaller vega, and the investor would need to hold a larger number of these options to get the same exposure to volatility risk. The resulting high leverage, however, raises some issues about the stability of the exposure over time, when the stock price and the variance change while the investor can not adjust his portfolio. He thus accepts a lower volatility risk exposure in exchange for a lower leverage of the portfolio (even if he still buys ten puts for the lowest moneyness).

Figure 4 shows the payoff as a function of the stock price. The kink in the payoff profiles is rather extreme for all moneyness levels. This is also reflected in the return densities shown in Figure 5. The densities of the portfolios which include puts differ significantly from the density in the benchmark case, where only the stock and the money market account are available. Basically, the investor uses the options to buy a portfolio which is more right-skewed.

The increase in the certainty equivalent returns in Figure 6 confirms that the investor profits significantly from having access to derivatives. He gains up to 1.2% per year. Since the utility gains differ across the moneyness levels, the choice of moneyness actually matters. The highest utility gains are achieved by ATM options, that is by those options which are best suited for trading volatility risk.

3.2.3 SVJ model

In the SVJ model, the investor takes a short position in puts for all moneyness levels and thus chooses a concave payoff profile. As in the model of Black-Scholes, the resulting positive exposure to stock price risk is offset by an additional short position in the stock. This effect is now so strong that the optimal position in the stock becomes negative for all but the lowest moneyness level. The resulting overall exposure to stock diffusion risk, however, is still positive.

A further analysis of the exposures in Figure 3 shows that the investor has a negative exposure to volatility risk – although the volatility risk premium is positive – and a negative exposure to jump risk.[6] He thus earns a positive risk premium on downward jumps, but foregoes the positive risk premium on volatility diffusion risk. The reason is that there

[6]The exposures of the investor to jump risk are a function of the jump size in the stock and in the variance. Figure 3 shows these exposures for the mean of the jump in the stock and in the volatility. As

is only one option available in which he can trade both to achieve a certain volatility risk exposure as well as to change the relation between the exposures to stock diffusion risk and jump risk offered by the stock. Given our parameterization, the premium for jump risk turns out to be more attractive than the premium for volatility risk. Consequently, the investor takes a short position in the put and thus increases his jump risk exposure beyond the one offered by the stock.

The payoff profile of the optimal portfolio, shown in Figure 4, is concave. The payoff is largest if the terminal stock price is equal to the strike price and is thus bounded from above. Figure 5 shows the corresponding densities of the terminal payoffs. Compared to the benchmark, the distribution is more left-skewed when options are included.

When the investor has the opportunity to invest in a derivative, his certainty equivalent return increases as compared to the case where he can only trade the money market account and the stock. Figure 6 shows that the utility gain depends on the moneyness of the option the investor uses. The difference between the lowest and highest gain in certainty equivalent returns is around 4-5%. A large fraction of the utility gain from trading derivatives thus comes from the choice of the optimal strike price. In this model, options with an extreme moneyness are best, i.e. the investor profits most from trading deep OTM puts and deep OTM calls. In line with intuition, deep OTM puts are those contracts which allow the investor to trade the risk of large downward jumps in the stock price.

3.2.4 SVCJ model

The SVCJ investor is short in puts for nearly all moneyness levels, as can be seen from the optimal positions shown in Figure 2. The overall pattern of the position is rather similar to the case of an SVJ investor, whereas the absolute size of the position in puts is

a robustness check, we have also calculated the expected exposures, where the expectation is taken over the joint distribution of the jumps in the stock price and in the variance. The pattern of the jump risk exposure across models and over the moneyness levels stays the same. To save space, we do not show the resulting graphs here.

slightly lower. Nevertheless, the exposure to jump risk is much larger in the SVCJ model than in the SVJ model, as can be seen from Figure 3. To get the intuition, note that the exposure of a put to jump risk is larger in the SVCJ model than in the SVJ model. If there is a downward jump in the stock price, the put price increases due to the change in the underlying and also due to the simultaneous upward jump in volatility. Consequently, a smaller position in puts can indeed lead to a larger jump risk exposure. On the other hand, the exposure of the puts to volatility jump risk implies a lower exposure to volatility diffusion risk. The different exposure dispartment together with the smaller position in puts leads to a lower volatility risk exposure for the SVCJ investor in absolute terms. Since the investor is willing to take a certain amount of risk and now faces a lower volatility risk exposure, he can take on more stock diffusion risk (and also more jump risk). Therefore, his exposure to stock diffusion risk also increases as compared to the SVJ investor.

The terminal payoff profiles and the return densities of the SVCJ model in Figures 4 and 5 are rather similar to the SVJ case. This can be attributed to the fact that the optimal portfolio positions and exposures of the SVCJ investor follow the same pattern as those of the SVJ investor. For the same reason, the increase in certainty equivalent returns due to trading derivatives is rather comparable for the SVJ and the SVCJ model. Again, the investor profits most from trading OTM options.

Put together, both the optimal portfolio and the utility gain from trading derivatives depend on the model choice. While the investor profits most from a long position in ATM options in an SV model, he is best off by selling OTM puts in models with jumps in the stock price and also in models with jumps both in the stock price and in volatility. The choice of the optimal strike price can change the increase in the certainty equivalent by less than 1.0% in the SV model and by more than 5% in the SVCJ model and is thus more important in the more sophisticated models.

4 Restrictions on Risk Premia

We now look at restricted versions of the models where different risk premia are assumed to be equal to zero. Since the physical measure does not change, these restrictions have no impact on our results when the investment opportunity set consists of the stock index and the money market account only. They significantly change, however, the expected returns on options as well as the exposures offered by the options. This in turn has a significant impact on the optimal portfolios in an incomplete market.

4.1 SVJ model, jump variance risk premium is set equal to zero

In the unrestricted case, it holds that $\sigma_S^{\mathbb{Q}} > \sigma_S^{\mathbb{P}}$. Restricting the jump variance risk premium to be equal to zero thus implies a lower jump variance under the $\mathbb{Q}$-measure, which reduces both the left and – in particular – the right tail of the risk-neutral distribution. Furthermore, the estimated mean jump size under the risk-neutral measure increases in absolute terms, which increases the left tail of the distribution and again reduces the right tail. Altogether, OTM calls become cheaper, while the price of OTM puts is approximately the same as in the unrestricted version, as also shown in more detail in Branger, Hansis, and Schlag (2008). Selling OTM calls (or selling ITM puts, respectively) is thus less attractive for the SVJ investor, and Figure 7 shows that he indeed takes a less extreme position in options for high moneyness levels. For low and intermediate moneyness levels, on the other hand, the optimal position in options does not change much.

The exposure to jump risk decreases in absolute terms for all moneyness levels (see Figure 8). To get the intuition, note that a CRRA investor wants to earn a premium for jump size risk and jump intensity risk, but not for jump variance risk (see also Naik and Lee (1990)). In the restricted version of the model, jump variance risk is no longer priced, while the premium for jump size risk increases. Thus, the investor earns a risk premium exactly on a risk factor which he actually cares about. He no longer has to 'waste' some risk premium on jump variance risk, which he does not want to be priced in the first place. Overall, the investor thus has to take less total jump exposure to get the 'right' amount of exposure to jump size risk. The smaller position in the put also leads to an exposure to volatility diffusion risk which is less negative than in the unrestricted case. Since the investor would actually prefer a positive volatility risk exposure (due to the positive volatility risk premium), he profits from this decrease.

Figure 11 shows that the utility gain of the investor from trading derivatives is larger than in the unrestricted version of the model for moneyness categories around 100%. For the extreme moneyness categories, on the other hand, the increase in the certainty equivalent return is significantly lower.

4.2 SVJ model, volatility risk premium is set equal to zero

Setting the volatility risk premium equal to zero has rarely any effects on the risk premia for jump risk, as can be seen from Table 2. Nevertheless, it can be shown that OTM calls are more expensive than under the unrestricted version, whereas the prices of OTM puts stay approximately the same. Selling OTM calls is thus more attractive for the investor, and his position in the corresponding ITM puts becomes more aggressive. This can be seen by the optimal positions shown in Figure 7 as well as by the terminal payoff profiles in Figure 9.

The more extreme short position in puts results in an exposure to diffusive volatility risk that is significantly more negative than in the unrestricted version of the model (see Figure 8). However, this does not imply a worse situation for the investor, who will actually care about the deviation from the optimal exposure in a complete market. This optimal exposure is positive with no restrictions and close to zero in the restricted case, since an η_V equal to zero implies a premium for pure volatility diffusion risk which is very close to zero. The restriction thus implies that the investor is willing to accept a more negative exposure to volatility diffusion risk than before.

Figure 10 shows the density of the payoff from the optimal portfolio at the end of the planing horizon. Due to the more aggressive position in puts, in particular for moderate moneyness levels, the overall variability of the terminal payoff from the optimal portfolio is larger. In exchange, there is a slightly higher probability of earning high returns, even if the upside potential is still limited.

The increase in the certainty equivalent return is larger than in the unrestricted version of the model, as be seen in Figure 11. Since the certainty equivalent return of the benchmark case (only the money market account and the stock are traded) does not change due to the restriction, this implies that the investor has a higher utility in the restricted model than in its unrestricted version. The difference between the two models is largest for calls which are slightly out of the money. Irrespective of these improvements for intermediate moneyness levels, however, the investor is still best of if he uses deep OTM or ITM puts.

4.3 SVJ model, both jump variance premium and volatility risk premium set equal to zero

In a next step, we restrict both the jump variance premium and the volatility risk premium to be equal to zero. Figure 7 shows that the effect on the optimal positions is basically some mixture of the effects we observe in case of one restriction only. However, it also suggests that restricting the premium for volatility diffusion risk has a larger influence than restricting the premium on jump variance risk. In Figure 8, we see that the exposure to diffusive volatility risk is more negative than in the unrestricted case. Again, it is much closer to the exposure in the case when only volatility diffusion risk is restricted to zero.

The certainty equivalent return is now larger than in the unrestricted case for nearly all moneyness levels, and it is larger than in the case with only one restriction for intermediate moneyness levels. In our setup, the investor is thus better off if neither volatility diffusion risk nor jump variance risk are priced. Furthermore, he profits much more if the premium for diffusive volatility is set to zero than if the jump variance risk premium is set to zero. The optimal options to trade are now ATM options.

4.4 SVCJ model

For the SVCJ model, we consider the same set of restrictions as for the SVJ model. The effect of restricting the jump variance premium to zero is smaller than in the SVJ model. To get the intuition, note that the uncertainty if a jump has happened is due to the stochastic jump size in the stock and – in case of the SVCJ model – also the stochastic jump size in the volatility. Restricting the jump variance risk premium to its lower value under the physical measure, however, reduces the uncertainty about the jump size in the stock price only. As a result, that restriction is less important in the SVCJ than in the SVJ model. This is also confirmed by the results for expected excess call returns in Branger, Hansis, and Schlag (2008).

5 Model Mis-Specification

The optimal portfolio depends on the model structure and on the parameters of the model, including the risk premia for the various risk factors. Our analysis in the last section has shown that the assumptions on the risk premia and on the risk factors to include are of equal importance.

In reality, the investor knows neither of these components with certainty, but has to estimate both the model and its parameters. He is thus exposed to model risk, i.e. to the risk that the model he uses deviates from the true data-generating process. Following our analysis above, we look at two kinds of scenarios. First, we consider the case where the investor omits risk factors and ignores either all jumps or just jumps in the volatility. Second, we consider the case where the investor relies on the correct model, but restricts some risk premia to be equal to zero.

The investor relies on his model to determine the optimal portfolio which he anticipates to give him certain exposures to the risk factors. We compare these 'seemingly optimal' exposures to two benchmarks, calculated under the true data-generating process. The first one is given by the 'truly optimal' exposures. The second one is given by the 'realized exposures', i.e. by the exposures of the portfolio bought by the investor in the true model. A comparison of the exposures shows how much the investor deviates from the position he wants to have, as well as from the portfolio he actually should hold.

For the certainty equivalent return (CER), we also determine the 'seemingly optimal' CER, the 'truly optimal' one, and the 'realized' one. A comparison of the realized CER with the truly optimal CER shows how much the investor looses due to model misspecification. Note that we show here the certainty equivalent return of the investment and no longer the *increase* in CER due to the inclusion of derivatives.

5.1 Omission of Risk Factors

We look first at the case where the investor relies on a model with stochastic volatility only (Heston model), whereas the true data-generating process incorporates jumps in the stock price process (SVJ model). Figure 12 shows that the realized exposures of the investor deviate enormously from the seemingly optimal as well as from the truly optimal ones. The realized exposure to stock diffusion risk is higher than the seemingly optimal one and exceeds the truly optimal one for slightly OTM puts by a factor of more than eight. The realized exposure to volatility diffusion risk is significantly smaller than the seemingly optimal one across all moneyness levels. While it is positive, the truly optimal exposure is negative. The differences are also large for jumps. The investor thinks that his exposure to jump risk is equal to zero, since he did not include jumps in his model setup. The realized exposure to jumps, however, is significantly negative and up to five times larger in absolute terms than the optimal one. Overall, the investor faces a way higher overall exposure than he would choose under the true data-generating process.

These large deviations from the optimal portfolio are reflected in the certainty equivalent returns, which are shown in Figure 13. The investor anticipates a CER around 8%. His realized CER, however, is dramatically lower - between 0.3 and 4%. Compared to the truly optimal portfolio he looses between 4 and 8% of CER per year. Furthermore, utility losses are largest for ATM options, which the investor incorrectly thinks to be the optimal choice. The omission of jumps in the stock price is therefore very expensive for the investor. If the true data-generating process incorporates jumps in the stock price as well as jumps in volatility (SVCJ model), while the investor still relies on a model with stochastic volatility only, the results are quite similar to the previous case.

Overall, the results show that the investor suffers huge utility losses if he incorrectly omits jumps from the model. Whether the true model includes jumps in volatility or not, however, is of second order importance and does not have a big impact on the utility losses.

We now turn to the case where the investor uses an SVJ model, while the true data-generating process is an SVCJ model. Figure 14 shows that the deviations of the realized exposures from the truly optimal exposures are smaller than in the cases just considered. For stock diffusion risk, the differences are rather small. The realized exposure to volatility diffusion as well as to jump risk is larger in absolute terms than the truly optimal one, especially for puts that are slightly in the money. The investor thus holds a portfolio that is too risky. The respective CER is shown in Figure 15. Ignoring jumps in volatility has dramatic consequences especially for an investment in OTM puts. There are scenarios where he looses nearly 90% of his wealth within one month, or where his wealth even becomes negative. The CER is then very small (and negative) or even equal to minus infinity[7]. The seemingly optimal strategy of investing in OTM puts is therefore a very risky strategy for the investor and has devastating consequences if the true data-generating process allows for jumps in volatility. If the investor chooses ITM puts, he foregoes some utility gains if he is right about the model, but also faces only small losses due to model risk.

5.2 Mis-Estimation of Risk Premia

Even if the investor makes the correct assumptions on the risk factors in the true data-generating process, he might still have difficulties to determine the correct structure of

[7] Since the negative values are too large, they are not shown in the figure.

the risk premia. In the literature there is general disagreement regarding the size and the sign of the volatility risk premium as well as the jump risk premium.[8] The structure of the jump risk premia is even more an open question. Often the premium for jump variance risk is restricted to be equal to zero ($\sigma_S^{\mathbb{Q}} = \sigma_S^{\mathbb{P}}$).[9] The investor might thus rely on a model where volatility risk as well as jump variance risk are not priced, while the true data-generating process follows the SVCJ model where both of these risk factors are priced.

Figure 16 compares the exposures to the risk factors. For medium to high moneyness levels, the deviation between the realized and the truly optimal exposure to volatility diffusion risk is larger than in the case where the investor ignores jumps in volatility completely (see Figure 14). The same holds true for the exposure to jump risk, with a realized exposure to jump risk that is up to twice as large as the truly optimal one. The resulting certainty equivalent returns therefore show a decline of up to 4% solely due to a mis-estimation of the risk premia, and the largest losses occur for ATM options. The decline as compared to the anticipated (seemingly optimal) investment strategy is even larger. As compared to his anticipation, the investor has to put up with a CER that is up to 9% lower. For the seemingly optimal choice of slightly OTM-puts, the realized CER is still between 5 and 8% lower than anticipated. The investor is thus clearly too optimistic on the profits he gets from having access to derivatives and by far overestimates his certainty equivalent return.

6 Conclusion

We analyze the impact of the model choice as well as of the fine structure of the risk premia on the optimal asset allocation decision of a CRRA-investor. The investor follows a buy-and-hold strategy. Besides the stock and the money market account, he can additionally

[8]See i.e. Eraker (2004), Pan (2002), Broadie, Chernov, and Johannes (2007)

[9]To the best of our knowledge, Broadie, Chernov, and Johannes (2007) are the first and only paper to drop this assumption.

trade one option. We look at the utility gains due to derivatives as well as at the losses due to the omission of risk factors as well as due to a mis-estimation of the risk premia.

First, we find that the choice of the model has a significant impact on the optimal portfolio decision. Even if all models are calibrated to exactly the same option prices, they imply very different optimal positions in the stock and the option. Furthermore, the utility improvements from trading derivatives are largest in a model which includes jumps.

Second, we find that the fine structure of the risk premia can be as important as the choice of the model. Restricting some elements of the jump risk premia to be equal to zero can change the optimal number of options and stocks by more than 100% in the buy-and-hold strategy and leads to large changes in the certainty equivalent returns.

Third, we find that the choice of the moneyness makes a big difference for the buy-and-hold investor. In particular in models including jumps, the investor can gain up to 5% by choosing the optimal strike price. In a model with stochastic volatility, he is best off if he trades ATM options, while he usually profits much more from trading deep OTM puts and, to a slightly smaller degree, deep OTM calls in case of jumps. Again, a change in the fine structure of the risk premia changes the optimal strike price.

Finally, we analyze the utility losses the investors suffers from if he uses an incorrect model or incorrect assumptions on the fine structure of the risk premia. If he incorrectly omits jumps from the model, the CER decreases by up to 10% for some moneyness levels. If he incorrectly omits only jumps in volatility, the seemingly optimal strategy of selling OTM puts can even lead to a CER of minus infinity. Finally, a mis-estimation of the fine structure of the jump risk premia has less devastating effects, but can still lead to a loss in CER of around 5%.

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Param.	BS	SV	SVJ	SVCJ
b	0.1119	0.1119	0.1250	0.1396
$\kappa^{\mathbb{P}}$		5.8212	3.2256	6.5520
$\theta^{\mathbb{P}}$	0.0228	0.0228	0.0205	0.0135
σ_V		0.3614	0.2404	0.1991
ρ		-0.3974	-0.4668	-0.4838
$\mu_V^{\mathbb{P}}$				0.0374
$\mu_S^{\mathbb{P}}$			-0.0259	-0.0175
$\rho_J^{\mathbb{P}}$				-0.2384
$\sigma_S^{\mathbb{P}}$			0.0407	0.0289
$\lambda^{\mathbb{P}}$			1.5120	1.6632
equity risk premium	0.0783	0.0783	0.0528	0.0732
average local variance	0.0228	0.0228	0.0240	0.0154
ERP/variance	3.4342	3.4342	2.2000	4.7532

Table 1: Parameters under the $\mathbb{P}$-measure

The table gives the parameters under the objective measure as estimated by Eraker, Johannes, and Polson (2003) (EJP) for the Black-Scholes model (BS), the Heston-model (SV), the model with stochastic volatility and jumps in the stock price (SVJ), and the model with stochastic volatility and jumps both in the stock price and in volatility (SVCJ). All parameters are given as annual decimals.

Parameters	restriction	η_V	$\eta_V^{pure}(\theta^{\mathbb{P}})$	$\mu_S^{\mathbb{Q}}$	$\sigma_S^{\mathbb{Q}}$	$\mu_V^{\mathbb{Q}}$	$\lambda^{\mathbb{Q}}$
SV		1.26	5.2866				
SVJ		1.512	7.7539	-0.0491	0.0994		$\lambda^{\mathbb{P}}$
SVJ	$\eta_V = 0$	$0^{(*)}$	0.6707	-0.0482	0.0981		$\lambda^{\mathbb{P}}$
SVJ	$\sigma_S^Q = \sigma_S^P$	2.52	10.5119	-0.0997	$\sigma_S^{\mathbb{P}(*)}$		$\lambda^{\mathbb{P}}$
SVJ	$\eta_V = 0$, $\sigma_S^Q = \sigma_S^P$	$0^{(*)}$	-1.2424	-0.0969	$\sigma_S^{\mathbb{P}(*)}$		$\lambda^{\mathbb{P}}$
SVCJ		7.812	45.5169	-0.0539	0.0578	0.2213	$\lambda^{\mathbb{P}}$
SVCJ	$\eta_V = 0$	$0^{(*)}$	1.0050	-0.0501	0.0751	0.0935	$\lambda^{\mathbb{P}}$
SVCJ	$\sigma_S^Q = \sigma_S^P$	7.56	43.2261	-0.0658	$\sigma_S^{\mathbb{P}(*)}$	0.2724	$\lambda^{\mathbb{P}}$
SVCJ	$\eta_V = 0$, $\sigma_S^Q = \sigma_S^P$	$0^{(*)}$	-0.5863	-0.0725	$\sigma_S^{\mathbb{P}(*)}$	0.1333	$\lambda^{\mathbb{P}}$

Table 2: Parameters under the $\mathbb{Q}$-measure

The table gives the parameters under the risk-neutral measure $\mathbb{Q}$ as estimated by Broadie, Chernov, and Johannes (2007). All parameters are given as annual decimals. The market of risk η_V^{pure} for pure variance diffusion risk is defined in Section 2.1 and is given for the variance equal to its mean-reversion level $\theta^{\mathbb{P}}$ under the physical measure $\mathbb{P}$.
The symbol '(*)' indicates that this parameter has been restricted either to its value under the true measure $\mathbb{P}$ or to zero.

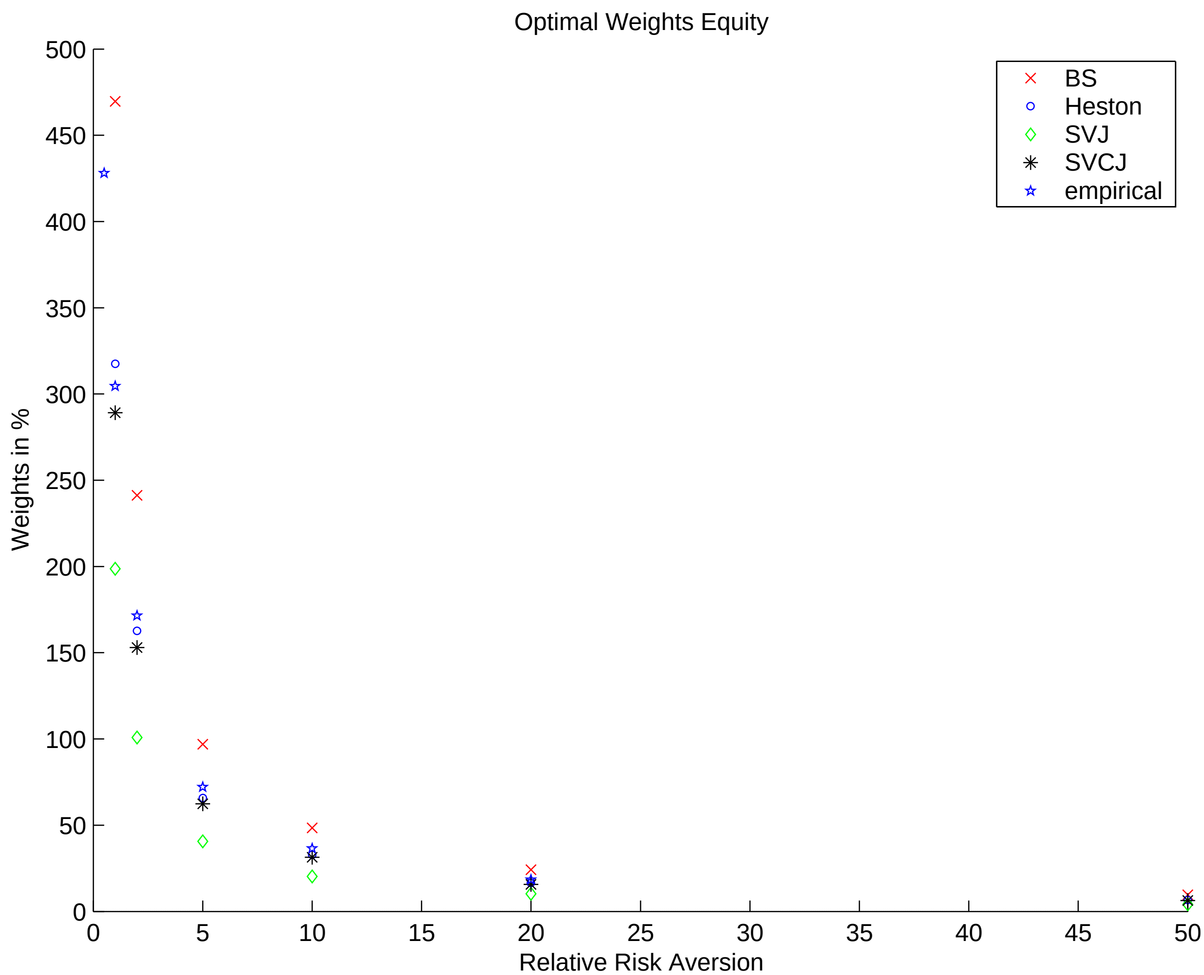

Figure 1: Optimal Asset Allocation without Derivatives
The graphs show the optimal weights for an asset allocation without derivatives as a function of relative risk aversion. The relative risk aversion is γ=5, the planning horizon is one month. The parameters are taken from Eraker, Johannes, and Polson (2003) (for the $\mathbb{P}$-measure) and Broadie, Chernov, and Johannes (2007) (for the $\mathbb{Q}$-measure). The empirical values are taken from Driessen and Maenhout (2007).

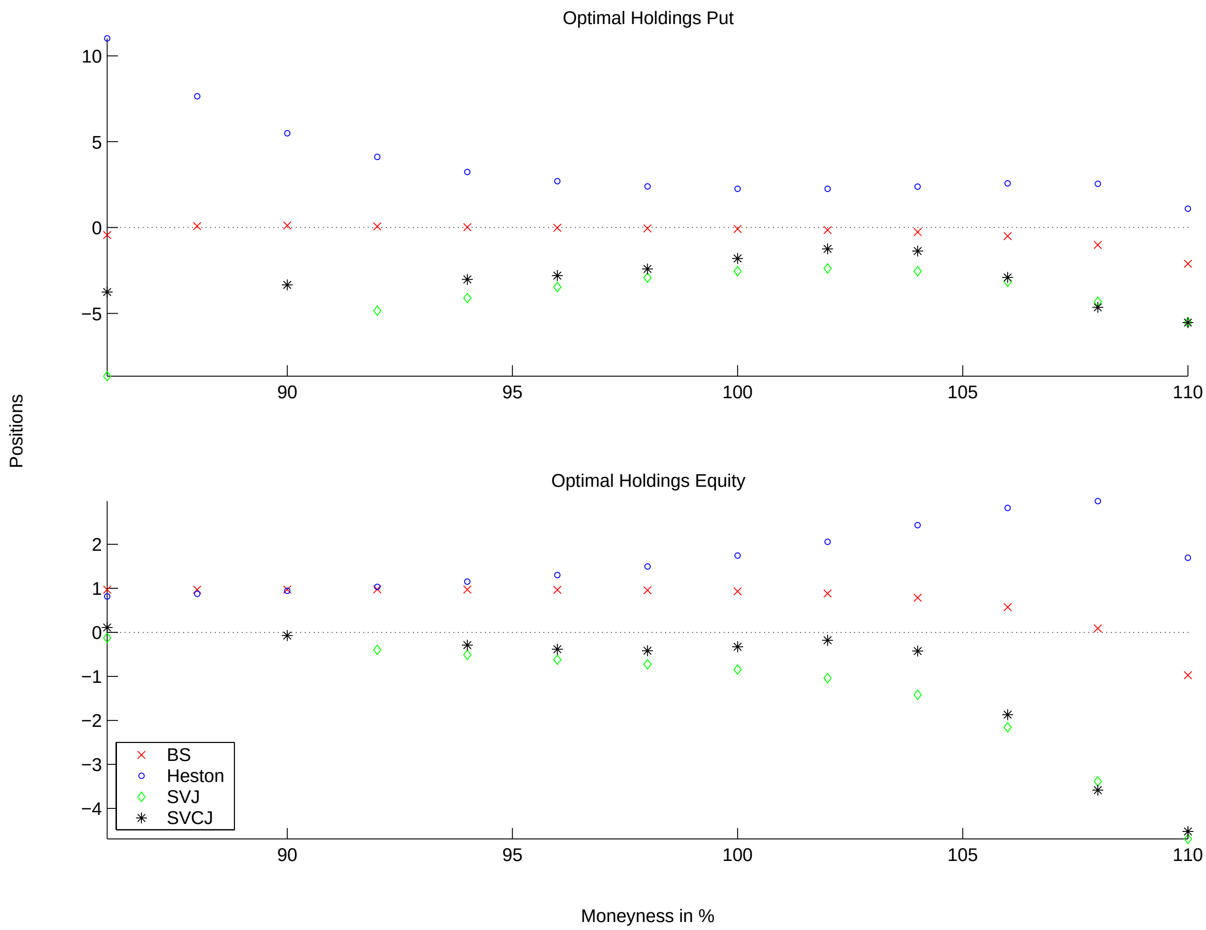

Figure 2: Optimal Asset Allocation with Puts, γ=5

The graphs show the optimal **positions** for an asset allocation with puts as a function of moneyness. The relative risk aversion is γ=5, the planning horizon is one month. The parameters are taken from Eraker, Johannes, and Polson (2003) (for the $\mathbb{P}$-measure) and Broadie, Chernov, and Johannes (2007) (for the $\mathbb{Q}$-measure).

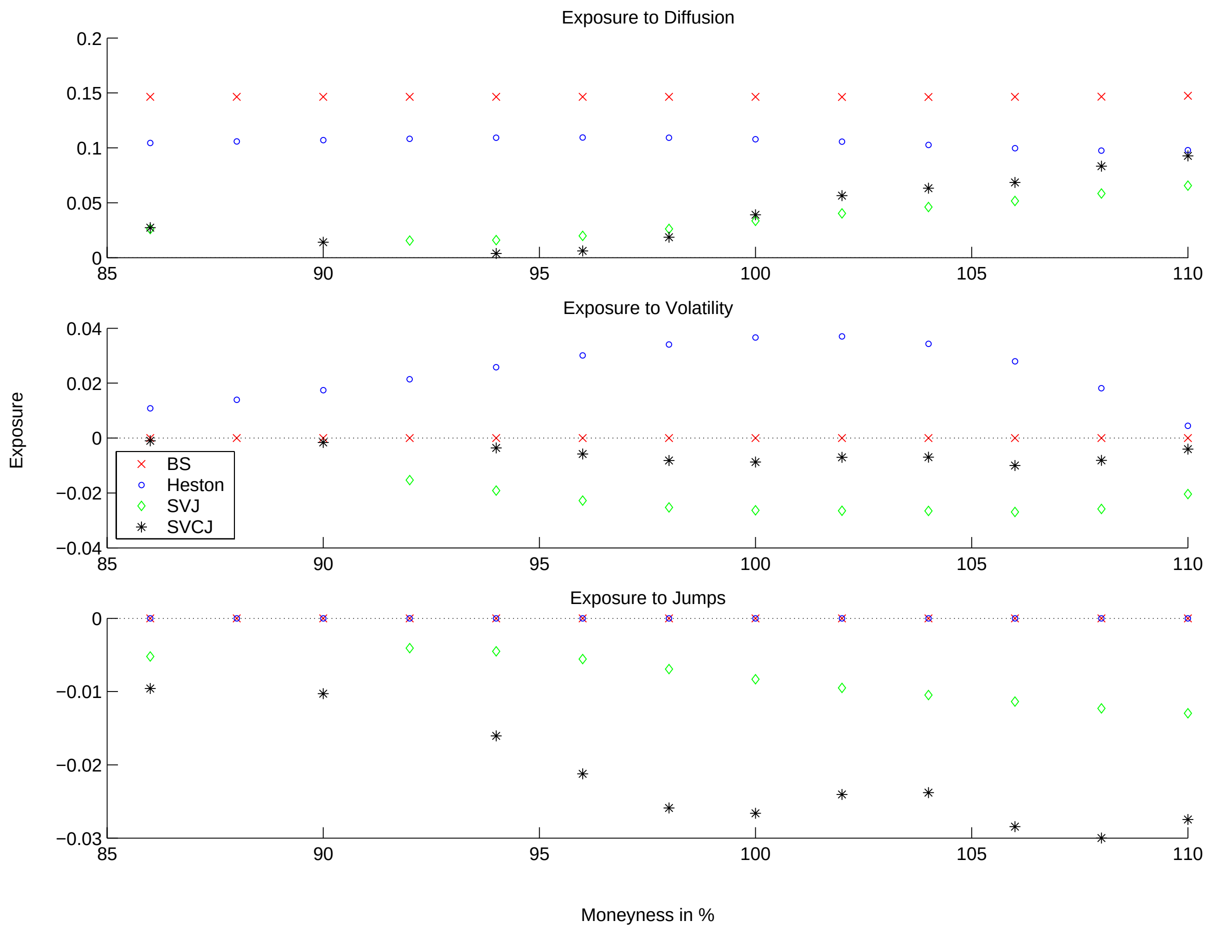

Figure 3: Optimal Asset Allocation with Puts, γ=5

The graphs show the optimal **exposures** for an asset allocation with puts as a function of moneyness. The upper graph shows the optimal exposure to diffusion risk θ^S, and the second graph shows the optimal exposure to diffusive volatility risk θ^V. The last graph shows the optimal exposure to jump risk $\theta^N(\bar{\mu}^{\mathbb{P}}, \mu_V^{\mathbb{P}})$ for the **mean** jump size in the stock price and in volatility. The relative risk aversion is γ=5, the planning horizon is one month. The parameters are taken from Eraker, Johannes, and Polson (2003) (for the $\mathbb{P}$-measure) and Broadie, Chernov, and Johannes (2007) (for the $\mathbb{Q}$-measure).

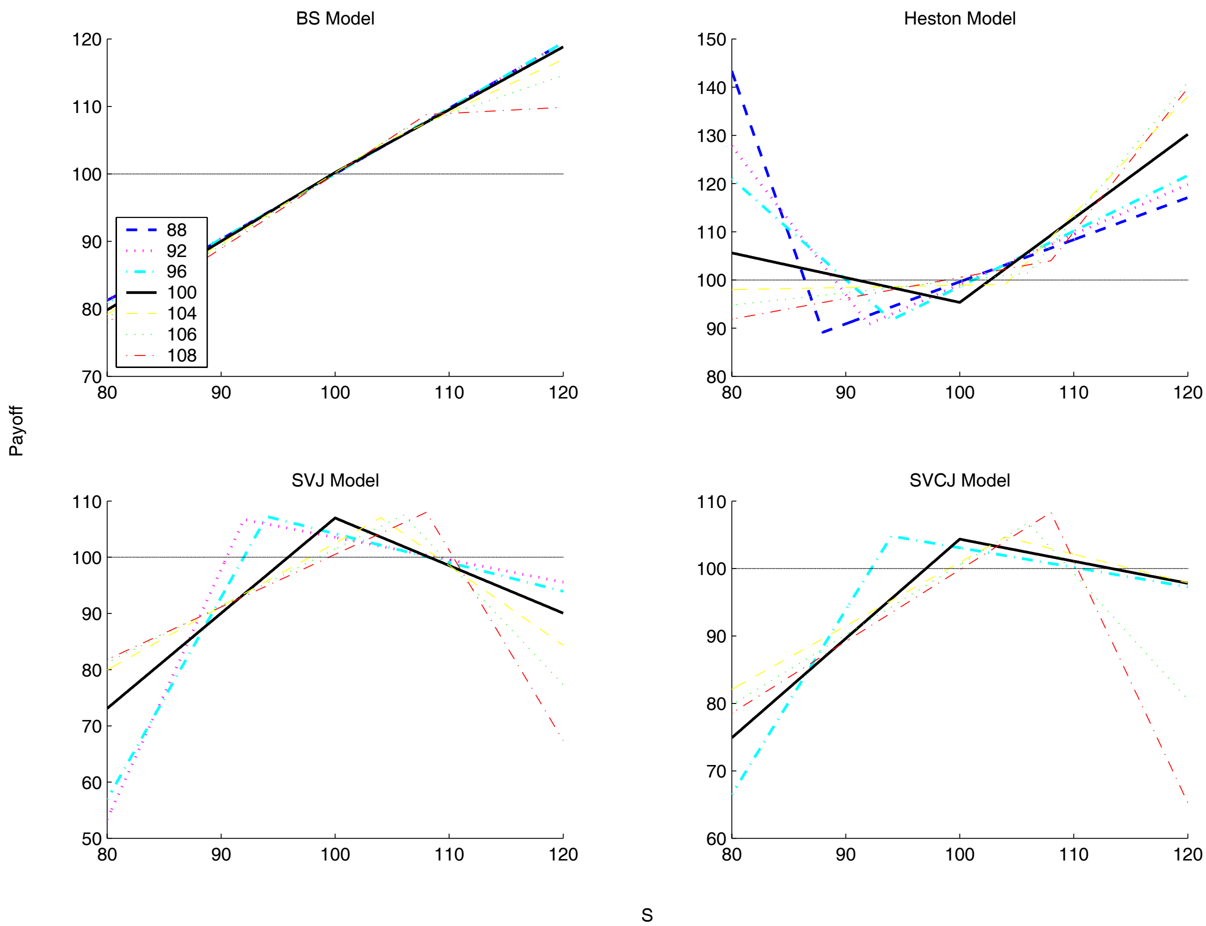

Figure 4: Optimal Asset Allocation with Puts, γ=5

The graphs show the optimal **payoff profils** for an asset allocation with puts as a function of moneyess. The relative risk aversion is γ=5, the planning horizon is one month. The parameters are taken from Eraker, Johannes, and Polson (2003) (for the $\mathbb{P}$-measure) and Broadie, Chernov, and Johannes (2007) (for the $\mathbb{Q}$-measure).

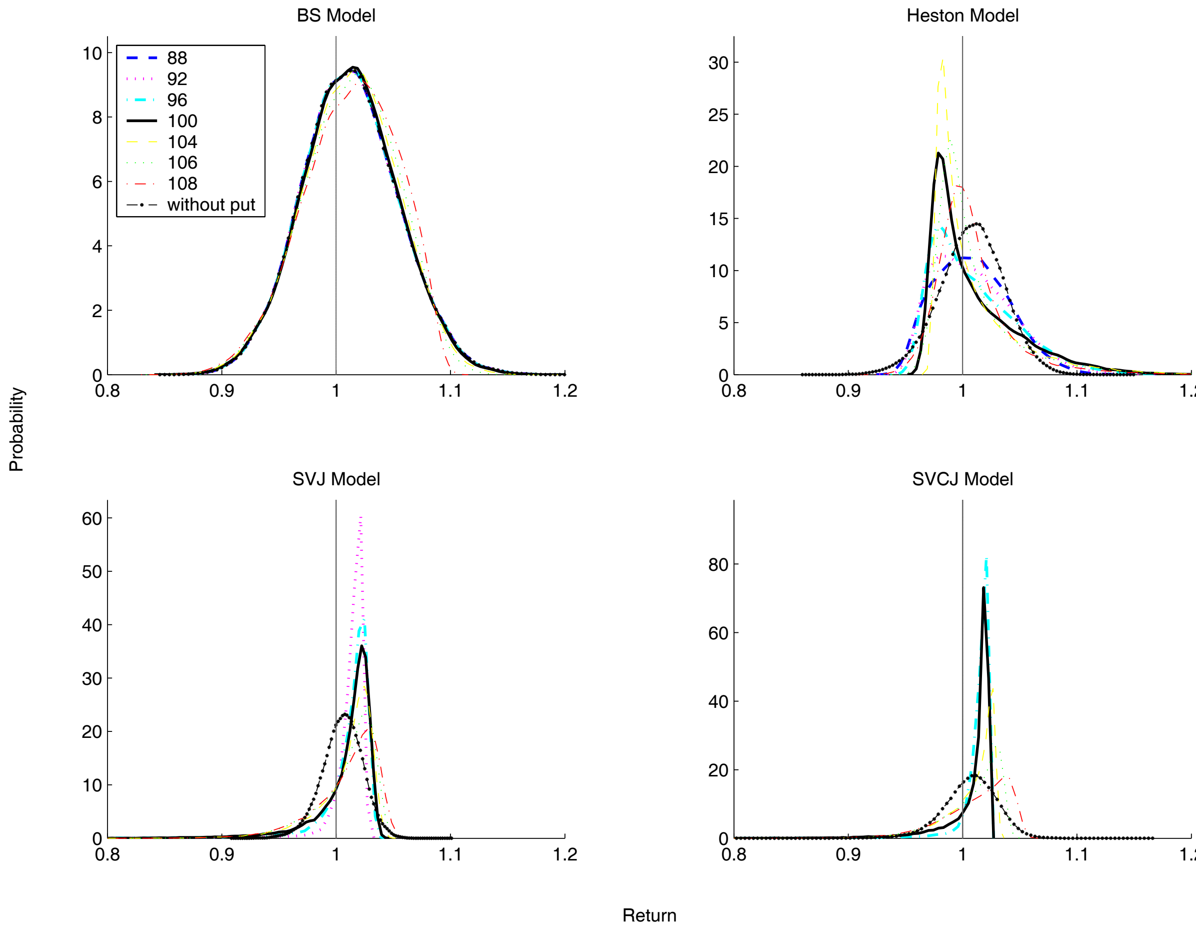

Figure 5: Density of Optimal Asset Allocation with Puts, γ=5
The graphs show the densities of the optimal terminal wealth when the investor can invest also in puts (considering puts with different moneyness). As a benchmark, we also give the density of the optimal terminal wealth when the investor has only access to the stock and the money market account. The relative risk aversion is γ=5, the planning horizon is one month. The parameters are taken from Eraker, Johannes, and Polson (2003) (for the $\mathbb{P}$-measure) and Broadie, Chernov, and Johannes (2007) (for the $\mathbb{Q}$-measure).

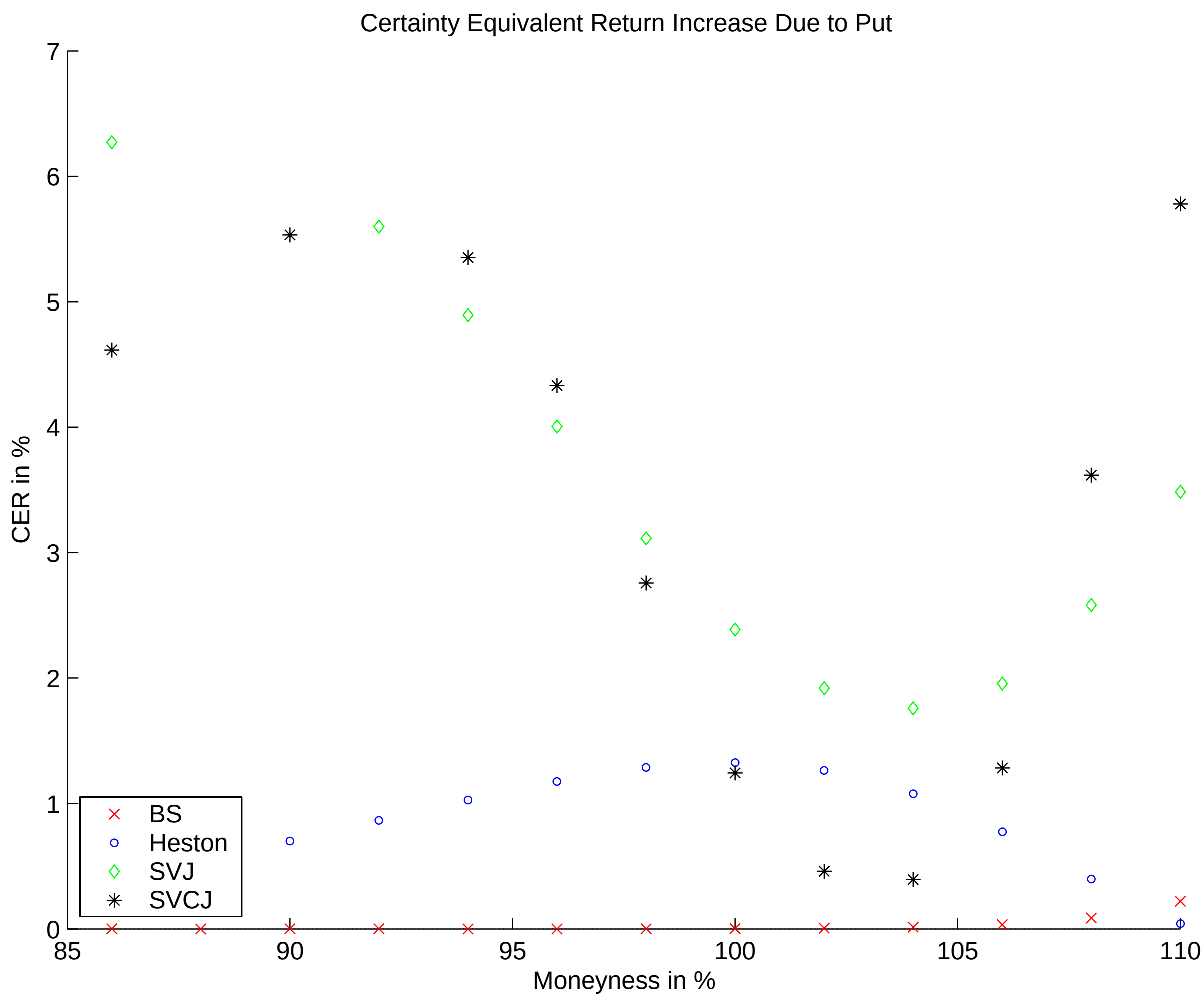

Figure 6: Increase of certainty equivalent return due to puts, γ=5
The graph shows the increase of the certainty equivalent return due to puts as a function of moneyness. The relative risk aversion is γ=5, the planning horizon is one month. The parameters are taken from Eraker, Johannes, and Polson (2003) (for the $\mathbb{P}$-measure) and Broadie, Chernov, and Johannes (2007) (for the $\mathbb{Q}$-measure).

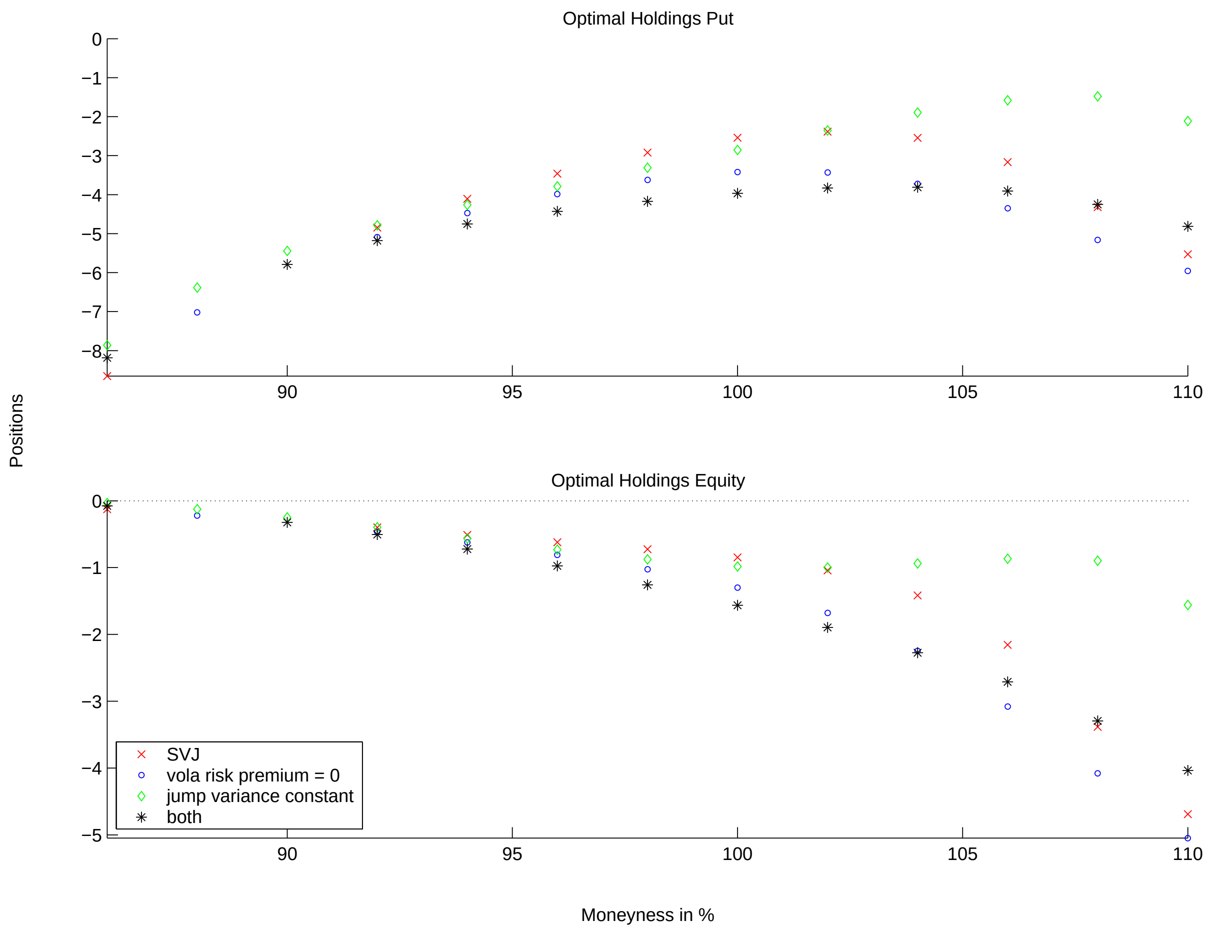

Figure 7: Optimal Asset Allocation with Puts, γ=5

The graphs show the optimal **positions** for an asset allocation with puts as a function of moneyness for different versions of the SVJ model. The relative risk aversion is γ=5, the planning horizon is one month. The parameters are taken from Eraker, Johannes, and Polson (2003) (for the $\mathbb{P}$-measure) and Broadie, Chernov, and Johannes (2007) (for the $\mathbb{Q}$-measure).

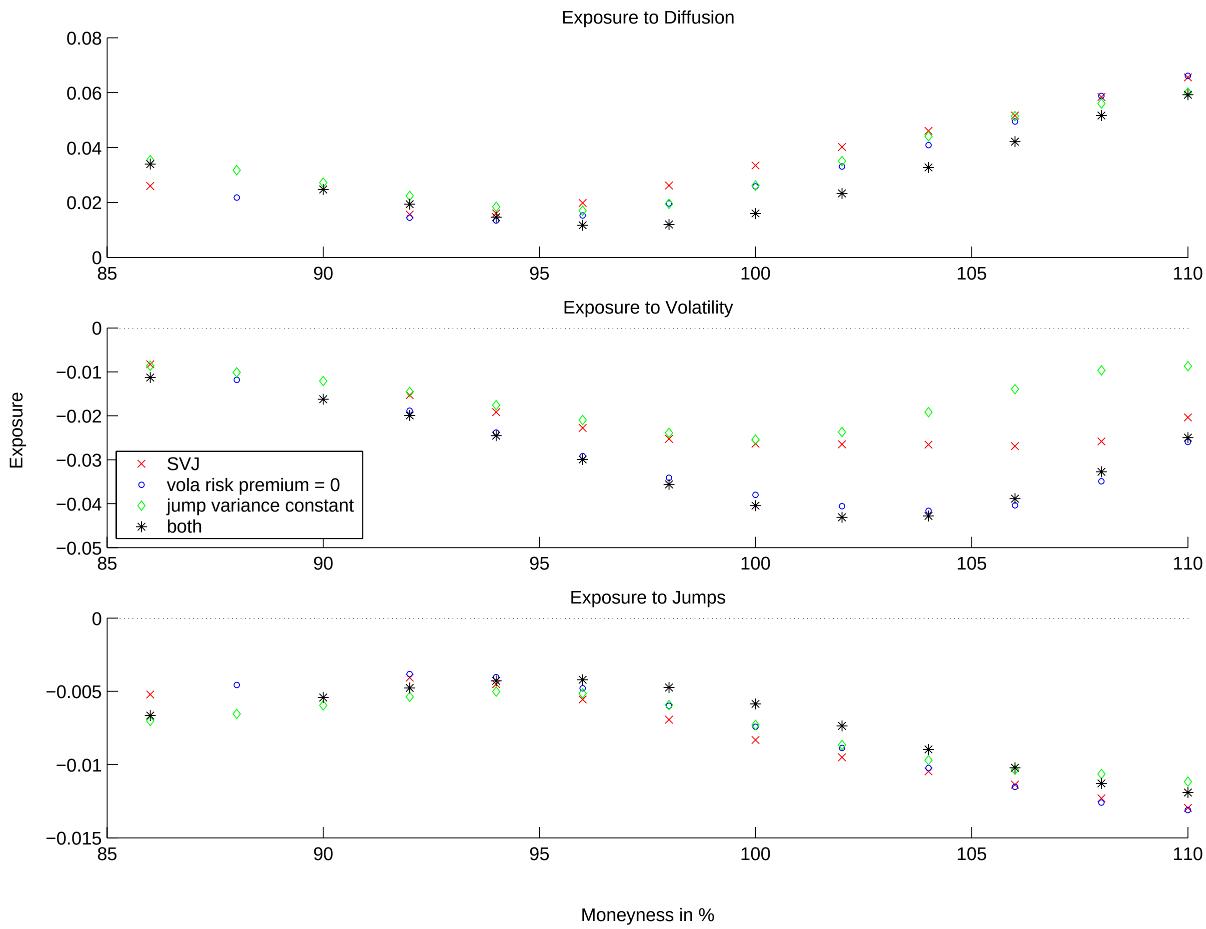

Figure 8: Optimal Asset Allocation with Puts, γ=5
The graphs show the optimal **exposures** for an asset allocation with puts as a function of moneyness for different versions of the SVJ model. The first graph shows the optimal exposure to diffusion risk θ^S, the second graph shows the optimal exposure to diffusive volatility risk θ^V. The last graph shows the optimal exposure to jump risk $\theta^N(\bar{\mu}^{\mathbb{P}}, \mu_V^{\mathbb{P}})$ for the **mean** jump size in the stock price and in volatility. The relative risk aversion is γ=5, the planning horizon is one month. The parameters are taken from Eraker, Johannes, and Polson (2003) (for the $\mathbb{P}$-measure) and Broadie, Chernov, and Johannes (2007) (for the $\mathbb{Q}$-measure).

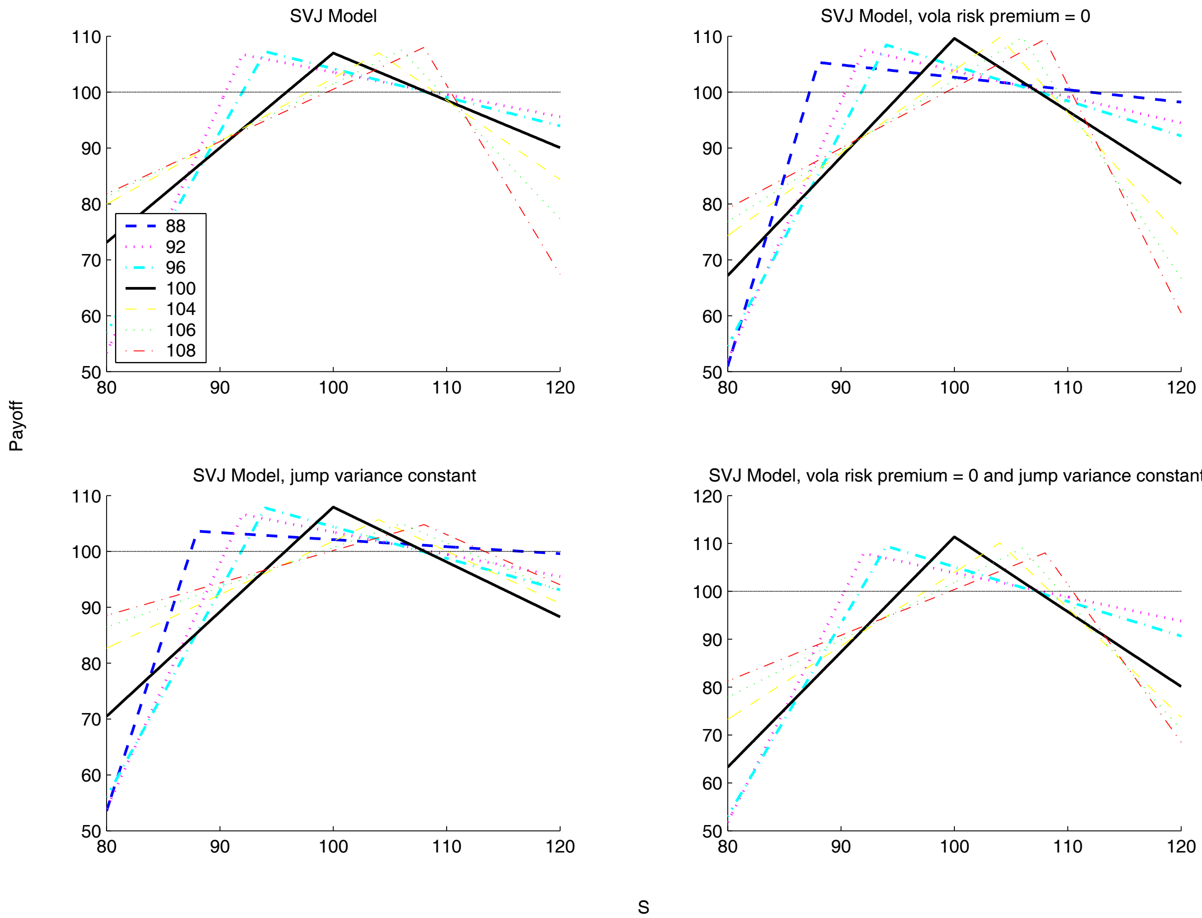

Figure 9: Optimal Asset Allocation with Puts, γ=5

The graphs show the optimal **payoff profils** for an asset allocation with puts as a function of moneyness for different versions of the SVJ model. The relative risk aversion is γ=5, the planning horizon is one month. The parameters are taken from Eraker, Johannes, and Polson (2003) (for the $\mathbb{P}$-measure) and Broadie, Chernov, and Johannes (2007) (for the $\mathbb{Q}$-measure).

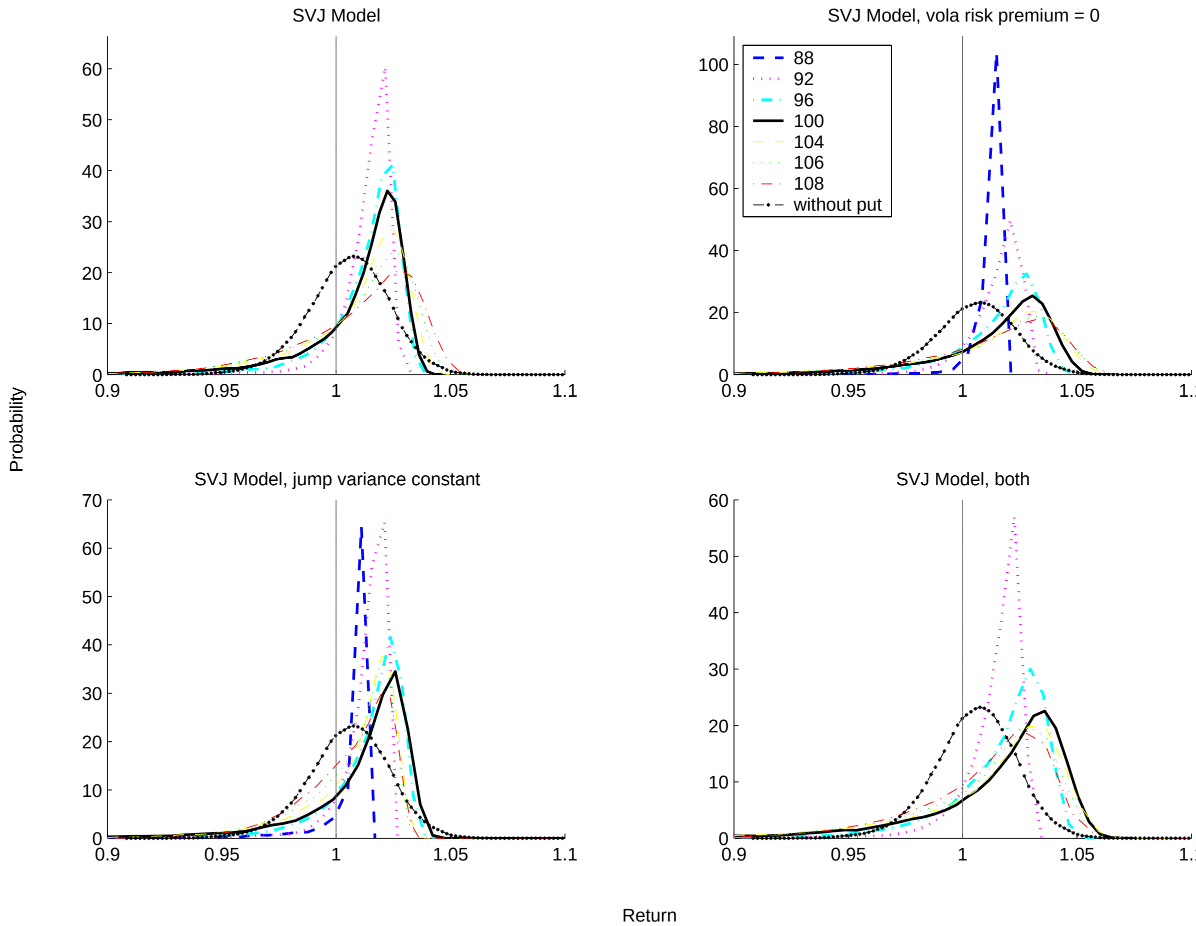

Figure 10: Density of Optimal Asset Allocation with Puts, γ=5

The graphs show the densities of the optimal asset allocations when the investor can invest also in puts (considering puts with different moneyness). As a benchmark, we also give the density of the optimal terminal wealth when the investor has only access to the stock and the money market account. The relative risk aversion is γ=5, the planning horizon is one month. The parameters are taken from Eraker, Johannes, and Polson (2003) (for the $\mathbb{P}$-measure) and Broadie, Chernov, and Johannes (2007) (for the $\mathbb{Q}$-measure).

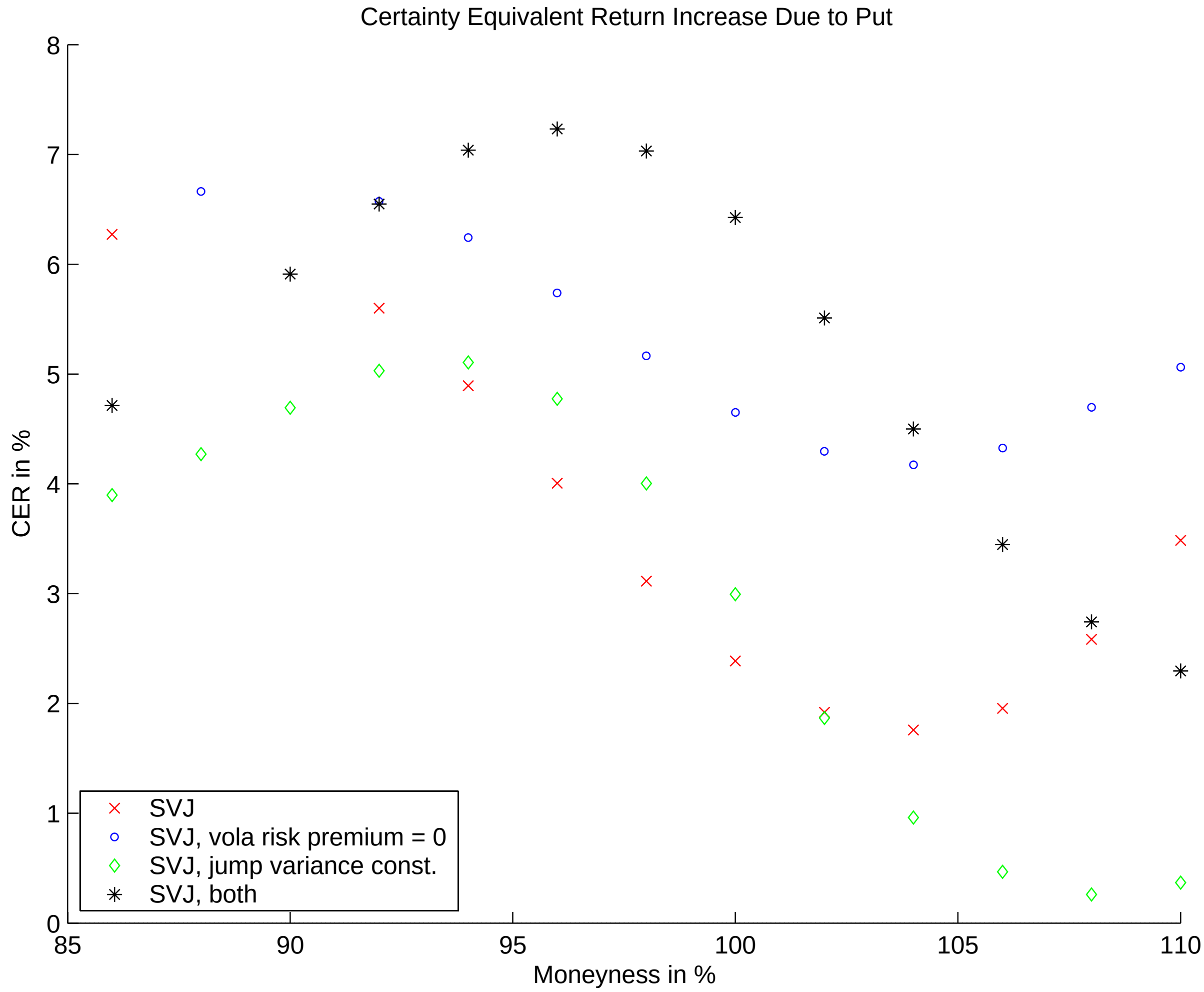

Figure 11: Increase of certainty equivalent return due to puts, γ=5

The graph shows the increase of the certainty equivalent return due to puts as a function of moneyness. We consider different restricted versions of the SVJ model. The relative risk aversion is γ=5, the planning horizon is one month. The parameters are taken from Eraker, Johannes, and Polson (2003) (for the $\mathbb{P}$-measure) and Broadie, Chernov, and Johannes (2007) (for the $\mathbb{Q}$-measure).

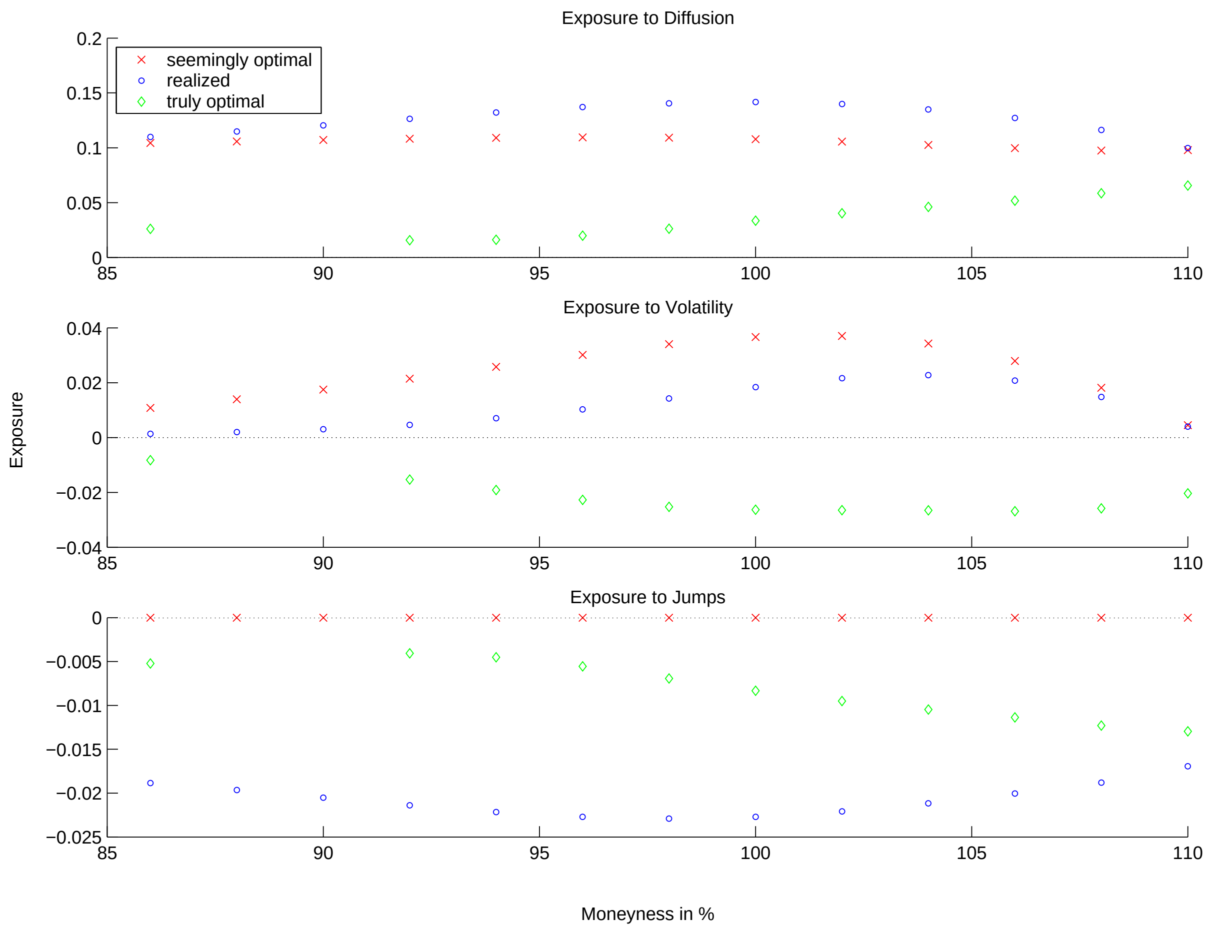

Figure 12: Exposures when investor omits jumps, γ=5

The graphs show the optimal **exposures** for an asset allocation with puts as a function of moneyness under model risk. The investor relies on a model with stochastic volatility only and omits jumps. His anticipation is to get the *seemingly optimal* exposures. Instead he faces the *realized* exposures. The *truly optimal* are those exposures that are optimal under the true data-generating process.

The first graph shows the optimal exposure to diffusion risk θ^S, the second graph shows the optimal exposure to diffusive volatility risk θ^V. The last graph shows the optimal exposure to jump risk $\theta^N(\bar{\mu}^{\mathbb{P}}, \mu_V^{\mathbb{P}})$ for the **mean** jump size in the stock price and in volatility. The relative risk aversion is γ=5, the planning horizon is one month. The parameters are taken from Eraker, Johannes, and Polson (2003) (for the $\mathbb{P}$-measure) and Broadie, Chernov, and Johannes (2007) (for the $\mathbb{Q}$-measure).

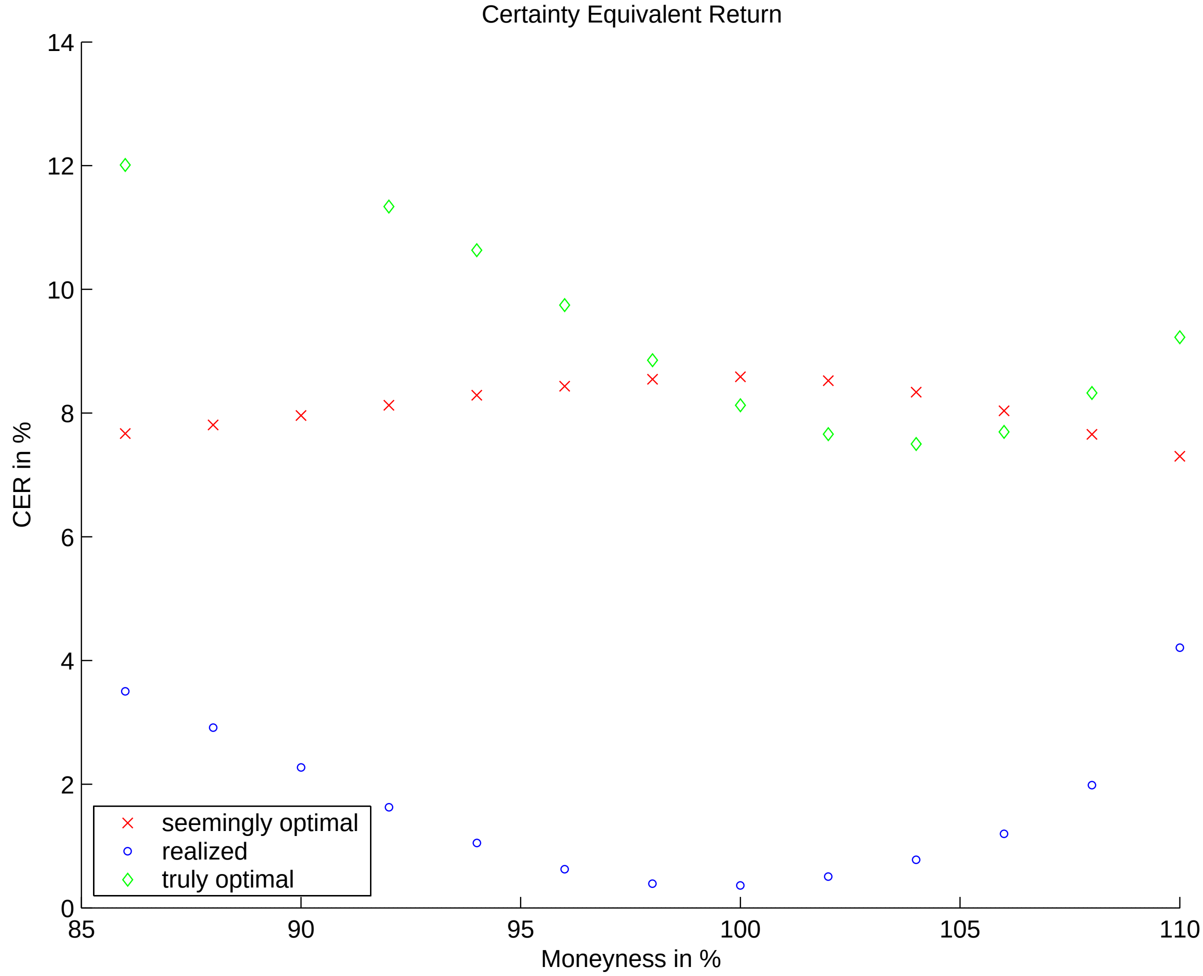

Figure 13: Certainty equivalent return when investor omits jumps, γ=5

The graphs show the **certainty equivalent return** (CER) for an asset allocation with puts as a function of moneyness under model risk. The investor relies on a model with stochastic volatility only and omits jumps. His anticipation is to get the *seemingly optimal* CER. Instead he faces the *realized* CER. The *truly optimal* CER is the CER that is optimal under the true data-generating process.

The relative risk aversion is γ=5, the planning horizon is one month. The parameters are taken from Eraker, Johannes, and Polson (2003) (for the $\mathbb{P}$-measure) and Broadie, Chernov, and Johannes (2007) (for the $\mathbb{Q}$-measure).

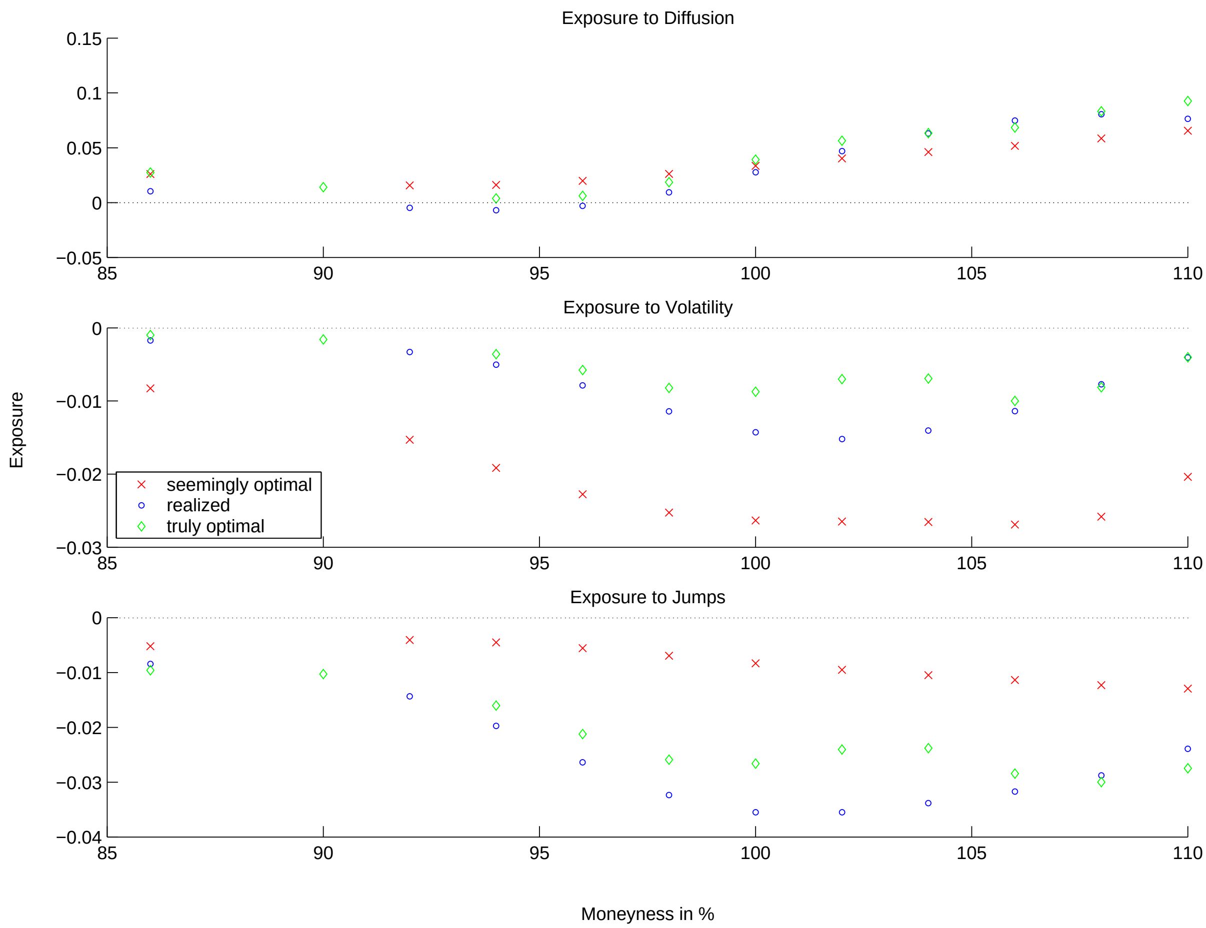

Figure 14: Exposures when investor omits jumps in volatility, γ=5

The graphs show the optimal **exposures** for an asset allocation with puts as a function of moneyness under model risk. The investor relies on a model with stochastic volatility and jumps in the stock price only. He omits jumps in volatility. His anticipation is to get the *seemingly optimal* exposures. Instead he faces the *realized* exposures. The *truly optimal* are the exposures that are optimal under the true data-generating process.

The first graph shows the optimal exposure to diffusion risk θ^S, the second graph shows the optimal exposure to diffusive volatility risk θ^V. The last graph shows the optimal exposure to jump risk $\theta^N(\bar{\mu}^{\mathbb{P}}, \mu_V^{\mathbb{P}})$ for the **mean** jump size in the stock price and in volatility. The relative risk aversion is γ=5, the planning horizon is one month. The parameters are taken from Eraker, Johannes, and Polson (2003) (for the $\mathbb{P}$-measure) and Broadie, Chernov, and Johannes (2007) (for the $\mathbb{Q}$-measure).

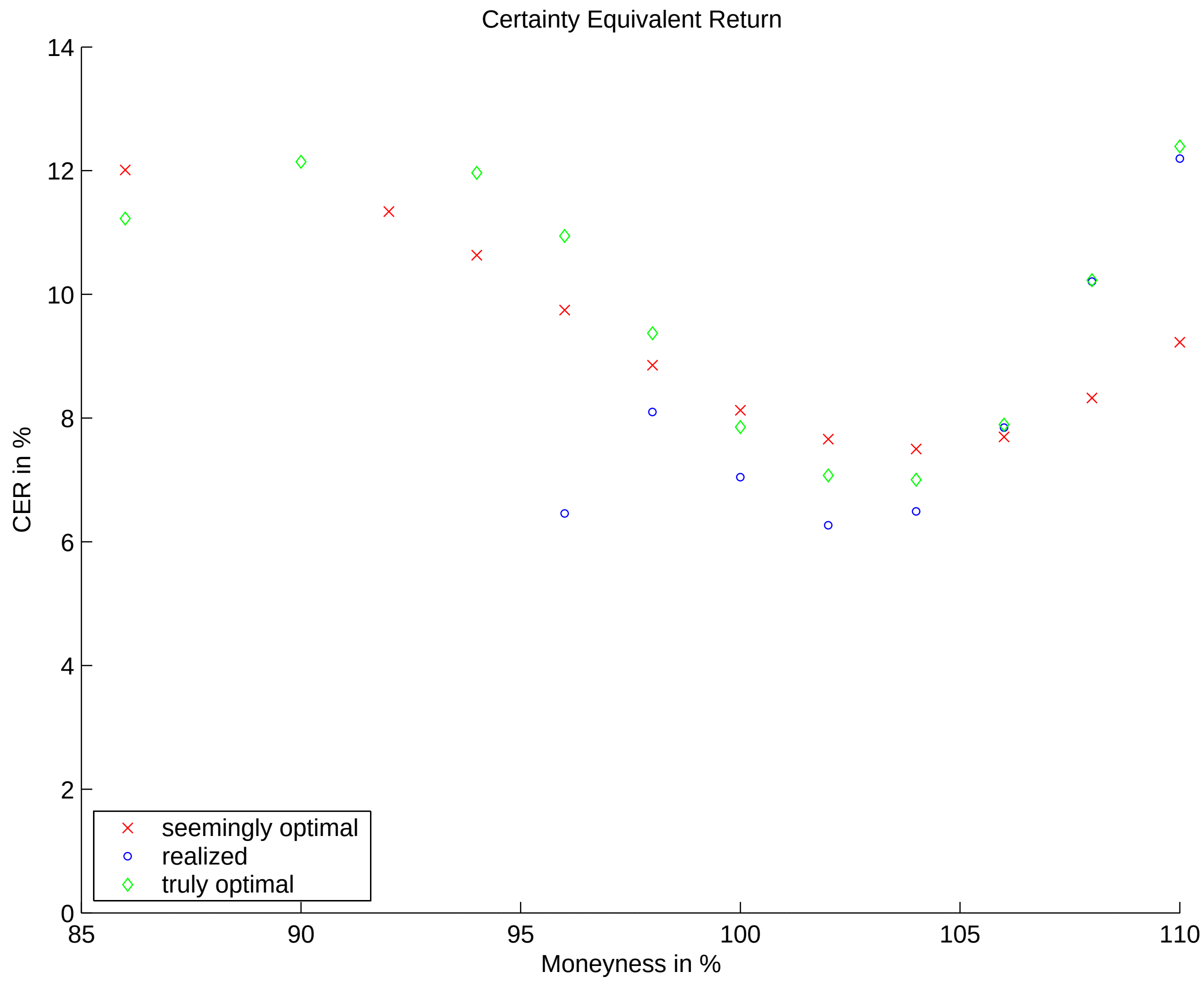

Figure 15: Certainty equivalent return when investor omits jumps in volatility, γ=5
The graphs show the **certainty equivalent return** (CER) for an asset allocation with puts as a function of moneyness under model risk. The investor relies on a model with stochastic volatility and jumps in the stock price only. He omits jumps in volatility. His anticipation is to get the *seemingly optimal* CER. Instead he faces the *realized* CER. The *truly optimal* CER is the CER that is optimal under the true data-generating process. The relative risk aversion is γ=5, the planning horizon is one month. The parameters are taken from Eraker, Johannes, and Polson (2003) (for the $\mathbb{P}$-measure) and Broadie, Chernov, and Johannes (2007) (for the $\mathbb{Q}$-measure).

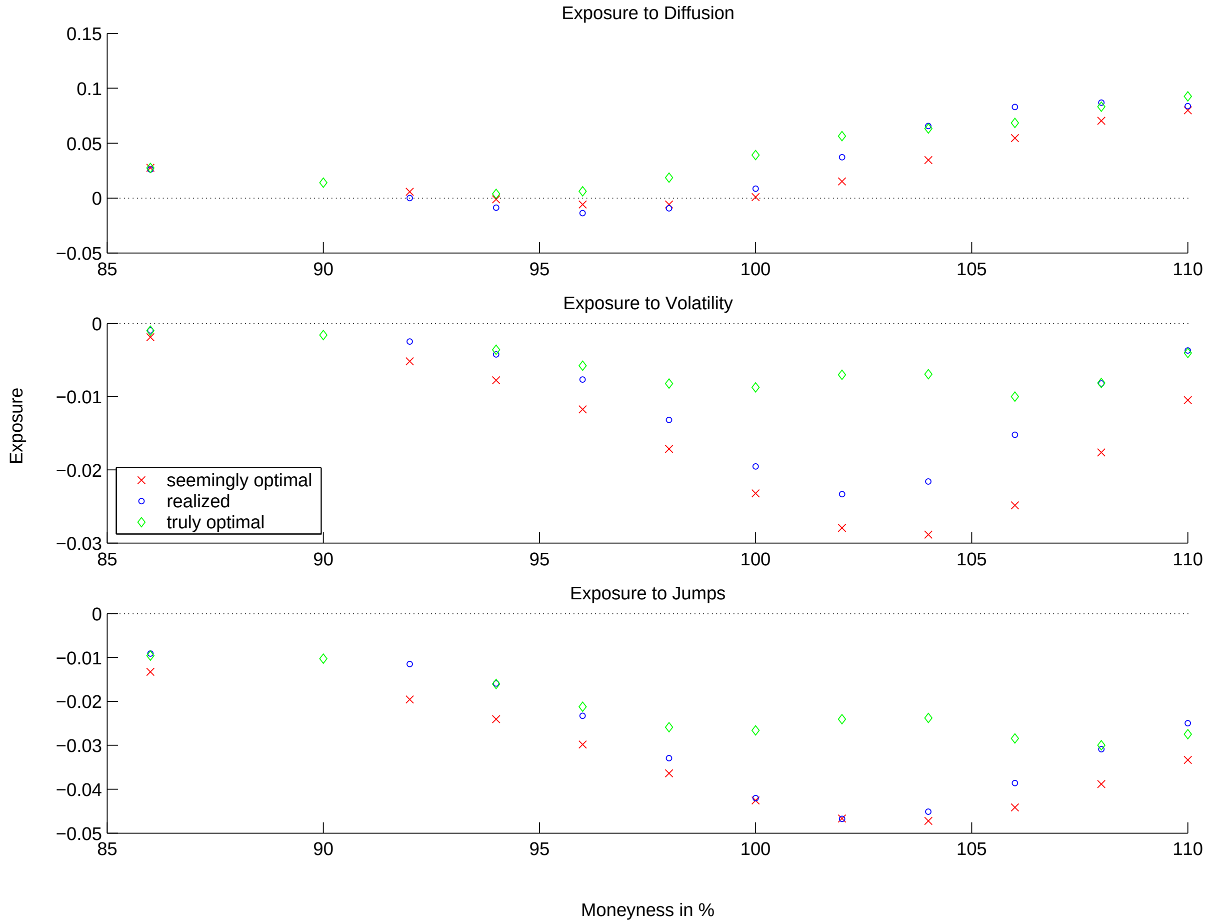

Figure 16: Exposures when investor mis-estimates the risk premia, γ=5

The graphs show the optimal **exposures** for an asset allocation with puts as a function of moneyness under model risk. The investor relies on a model where volatility risk and jump variance risk are not priced. In the real model both of these risk factors are priced. His anticipation is to get the *seemingly optimal* exposures. Instead he faces the *realized* exposures. The *truly optimal* exposures are the exposures that are optimal under the true data-generating process.

The first graph shows the optimal exposure to diffusion risk θ^S, the second graph shows the optimal exposure to diffusive volatility risk θ^V. The last graph shows the optimal exposure to jump risk $\theta^N(\bar{\mu}^{\mathbb{P}}, \mu_V^{\mathbb{P}})$ for the **mean** jump size in the stock price and in volatility. The relative risk aversion is γ=5, the planning horizon is one month. The parameters are taken from Eraker, Johannes, and Polson (2003) (for the $\mathbb{P}$-measure) and Broadie, Chernov, and Johannes (2007) (for the $\mathbb{Q}$-measure).

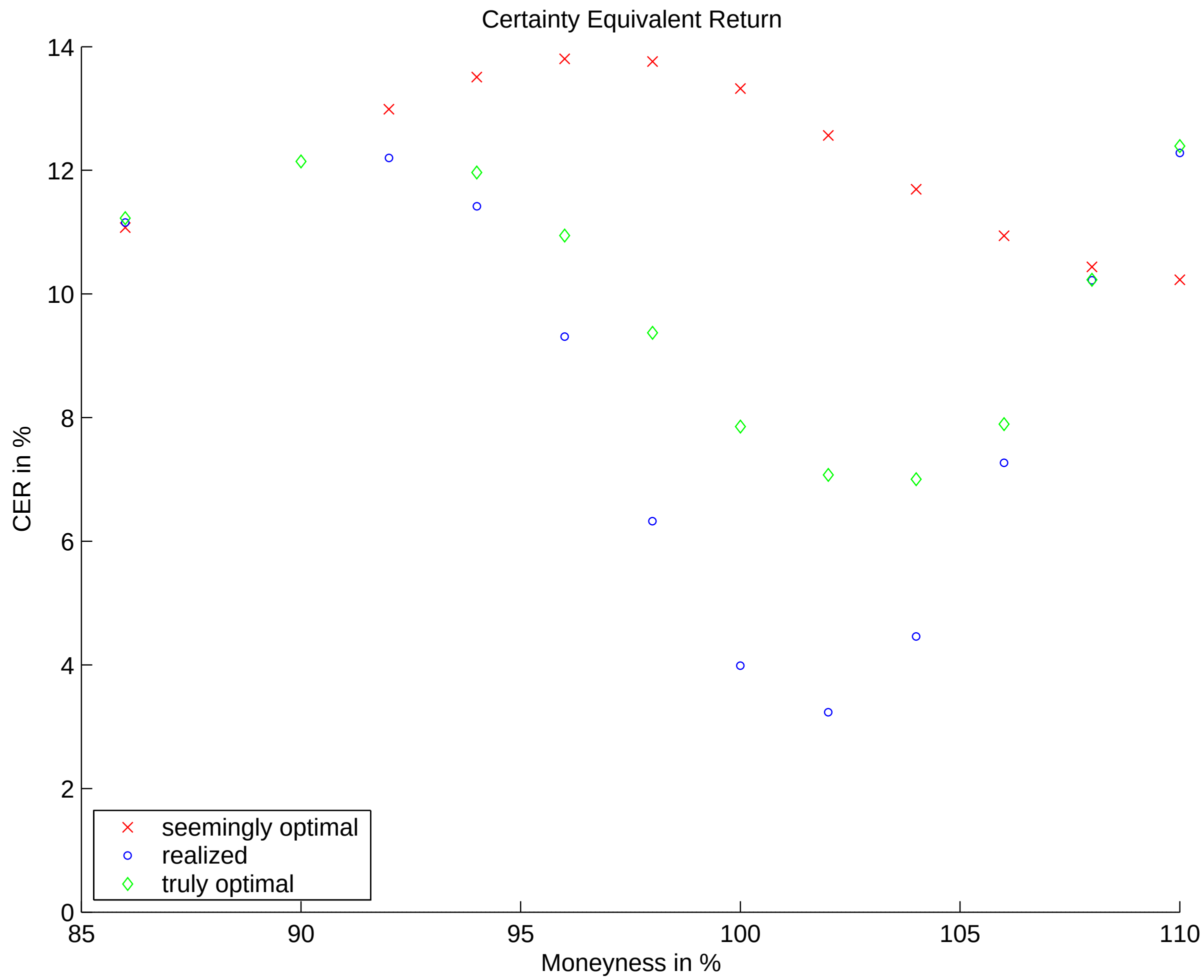

Figure 17: Certainty equivalent return when investor mis-estimates the risk premia, γ=5
The graphs show the **certainty equivalent return** (CER) for an asset allocation with puts as a function of moneyness under model risk. The investor relies on a model where volatility risk and jump variance risk are not priced. In the real model both of these risk factors are priced. His anticipation is to get the *seemingly optimal* CER. Instead he faces the *realized* CER. The *truly optimal* CER is the CER that is optimal under the true data-generating process.
The relative risk aversion is γ=5, the planning horizon is one month. The parameters are taken from Eraker, Johannes, and Polson (2003) (for the $\mathbb{P}$-measure) and Broadie, Chernov, and Johannes (2007) (for the $\mathbb{Q}$-measure).