Habit persistence, consumption based asset pricing, and time-varying expected returns

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Summary

Within the consumption based asset pricing framework, the habit persistence model of Campbell and Cochrane (1999) model has become one of the leading models in explaining asset pricing behavior. Campbell and Cochrane show that their model explains a number of stylized facts on the US stock market, including pro-cyclical stock prices, time-varying counter-cyclical expected returns on stocks, and it has the ability to explain the equity premium puzzle without facing a risk-free rate puzzle. Campbell and Cochrane and subsequent applications of their model only rely on calibration and simulation exercises and do not engage in formal econometric estimation and testing of the model. Given the fact that the Campbell-Cochrane model seems to work so well in several dimensions, it is also of great relevance to estimate and test the model econometrically.

In the first chapter "*An iterated GMM procedure for estimating the Campbell-Cochrane habit formation model, with an application to Danish stock and bond returns*" (joint work with Tom Engsted), we perform formal econometric estimation and testing of the model using Danish stock and bond returns. To our knowledge, there have been no formal econometric studies of the Campbell-Cochrane model on data from other countries than the US. Our paper is the first attempt to fill this gap. Denmark is interesting because historically over a long period of time the average return on Danish stocks has not been nearly as high as in the US and most other countries, and at the same time the return on Danish bonds has been somewhat higher than in other countries, see e.g. Engsted and Tanggaard (1999), Engsted (2002), and Dimson et al. (2002). Thus, the Danish equity premium is not nearly as high as in most other countries, and might not even be regarded a puzzle. The results we obtain using our GMM procedure on Danish asset market returns do not in general support the conclusions from the US studies. Although there is some evidence of time-varying counter-cyclical risk aversion in recent years, the Campbell-Cochrane model does not produce lower pricing errors or more plausible parameter values than the benchmark CRRA model.[1]

The second chapter "*Habit persistence: Explaining cross-sectional variation in returns and time-varying expected returns*" estimates and tests the Campbell-Cochrane model along both cross-sectional and time-series dimensions of the US stock market. The model is estimated in a cross-sectional setting using the 25 Fama and French value and size portfolios, which has not been tried previously, cf. Cochrane (2007). The cross-sectional estimation documents that the model is able to explain the size premium, but fails to explain the value premium. Besides cross-sectional variation in returns, I examine whether the model is able to account for variation in expected returns over time. Consistently with the model, I find that low surplus consumption ratios in recession times predict high future stock returns. Thus, the model captures time-varying counter-cyclical expected returns on stocks.[2]

In the third chapter "*Habit formation, surplus consumption and return predictability: International evidence*" (joint work with Tom Engsted and Stuart Hyde), we present

[1] The paper is forthcoming in the *International Journal of Finance and Economics.*

[2] The paper has been invited for third resubmission to the *Journal of Empirical Finance.*

further international evidence on the relative performance of the Campbell-Cochrane model and the benchmark CRRA model. There seems to be quite large cross-country differences in the ability of the Campbell-Cochrane model to explain stock and bond return movements over time, but for the majority of the countries in our sample, the model gets empirical support in a variety of different dimensions. The model generates counter-cyclical time-varying relative risk aversion, and in contrast to the benchmark CRRA model, the Campbell-Cochrane model has the important ability to escape the risk-free rate puzzle. Moreover, we find that the surplus consumption ratio is a strong predictive variable of future stock and bond returns. Since a common limitation to existing predictive variables is that they only contain information about *either* future stock returns *or* future bond returns, the ability of the surplus consumption ratio to capture predictive patterns in both stock and bond markets is particularly interesting.

The fourth chapter "*Consumption growth and time-varying expected returns*" examines the ability of the consumption growth rate to predict future stock returns. Previous studies show that the consumption growth rate has no predictive power for future stock returns. However, I find that the consumption growth rate based upon fourth quarter data is a strong predictive variable of future stock returns. The fourth quarter consumption growth rate explains a substantial amount of the variation in 1-year ahead stock returns and is a better predictive variable than traditional benchmark predictive variables such as the price-dividend ratio [Campbell and Shiller (1988) and Fama and French (1988, 1989)] and performs marginally better than new predictive variables such as the consumption-wealth ratio [Lettau and Ludvigson (2001)] in predicting future stock returns. Interestingly, when the consumption growth rate is measured based upon other quarters, the predictive power breaks down. This striking evidence is consistent with the insight of Jagannathan and Wang (2007) that investors tend to review their consumption and investments plans during the end of each calendar year, and at possibly random times in between. Importantly, the fourth quarter consumption growth rate is an almost i.i.d. process, which eliminates potential concerns about finding spurious evidence of return predictability, cf. Stambaugh (1999).[3]

[3] The paper is published in *Finance Research Letters*, 2008 volume 5, pages 129-136.

Dansk resumé (Danish summary)

Indenfor det forbrugsbaserede asset pricing framework er Campbell og Cochrane's (1999) habit persistence model blevet en af de førende modeller i at forklare prisfastsættelsen på aktiver. Campbell og Cochrane viser, at deres model forklarer en række stiliserede facts på det amerikanske aktiemarked inklusiv pro-cykliske aktiekurser, tidsvarierende kontra-cykliske forventede afkast på aktier, og modellen har evnen til at forklare equity premium puzzlet uden at blive konfronteret med et risk-free rate puzzle. Campbell og Cochrane og efterfølgende applikationer af deres model beror udelukkende på kalibrering og simulering og anvender ikke formel økonometrisk estimering og testning af modellen. Givet det faktum, at Campbell-Cochrane modellen synes at fungere så godt i flere dimensioner, er det også af væsentlig relevans at estimere og teste modellen økonometrisk.

I det første kapitel "*An iterated GMM procedure for estimating the Campbell-Cochrane habit formation model, with an application to Danish stock and bond returns*" (fælles arbejde med Tom Engsted) foretager vi formel økonometrisk estimering og testning af modellen ved brug af danske aktie- og obligationsafkast. Ud fra vores kendskab har der ikke været formelle økonometriske studier af Campbell-Cochrane modellen på data fra andre lande end USA. Vores artikel er det første forsøg på at udfylde dette hul i litteraturen. Danmark er interessant, fordi historisk set over en lang tidsperiode har det gennemsnitlige afkast på danske aktier ikke været nært så højt som i USA og de fleste andre lande, og samtidigt har afkastet på danske obligationer været noget højere end i andre lande, se eksempelvis Engsted and Tanggaard (1999), Engsted (2002), og Dimson mf. (2002). Dermed er den danske risikopræmie ikke nær så høj som i de fleste andre lande og anses muligvis ikke engang for værende et puzzle. De resultater vi opnår med vores GMM procedure anvendt på danske aktie- og obligationsafkast støtter generelt ikke konklusionerne fra de amerikanske studier. Selvom der i nogen grad er beviser på tidsvarierende kontracyklisk risikoaversion i de seneste år, producerer Campbell-Cochrane modellen ikke lavere prisfejl eller mere plausible parameterværdier end benchmark CRRA modellen.[4]

Det andet kapital "*Habit persistence: Explaining cross-sectional variation in returns and time-varying expected returns*" estimerer og tester Campbell-Cochrane modellen på både tværsnits- og tidsseriedimensioner af det amerikanske aktiemarked. Modellen estimeres i et tværsnit setup ved at bruge de 25 Fama og French value og size porteføljer, hvilket ikke har været forsøgt tidligere, jævnfør Cochrane (2007). Tværsnitsestimeringen dokumenterer, at modellen er i stand til at forklare the size premium, men kan ikke forklare the value premium. Foruden tværsnitsvariation i aktieafkast undersøger jeg, hvorvidt modellen er i stand til at forklare variation i forventede afkast over tid. I overensstemmelse med modellen finder jeg, at lave overskudsforbrugsratioer i recessionstider forudsiger høje fremtidige aktieafkast. Dermed opfanger modellen tidsvarierende kontra-cykliske afkast på aktier.[5]

I det tredje kapitel "*Habit formation, surplus consumption and return predictabil-*

[4] Artiklen udkommer i *International Journal of Finance and Economics*.

[5] Artiklen er blevet inviteret til tredje genindsendelse til *Journal of Empirical Finance*.

ity: International evidence" (fælles arbejde med Tom Engsted og Stuart Hyde) præsenterer vi yderligere international dokumentation af den relative performance af Campbell-Cochrane modellen og benchmark CRRA modellen. På tværs af lande synes der at være ganske store forskelle i Campbell-Cochrane modellens evne til at forklare bevægelser i aktie- og obligationsafkast over tid. For hovedparten af landene i vores stikprøve opnår modellen dog empirisk støtte i en lang række forskellige dimensioner. Modellen genererer kontra-cyklisk tidsvarierende relativ risikoaversion, og i modsætning til benchmark CRRA modellen, har Campbell-Cochrane modellen den vigtige evne til at slippe fri af risk-free rate puzzlet. Ydermere finder vi, at overskudsforbrugsratioen er en stærk forecastvariabel af fremtidige aktie- og obligationsafkast. Da en fælles begrænsning for eksisterende forecastvariable er, at de udelukkende indeholder information om *enten* fremtidige aktieafkast *eller* fremtidige obligationsafkast, er overskudsforbrugsratioens evne til at opfange forudsigelige mønstre i både aktie- og obligationsmarkedet særligt interessant.

Det fjerde kapitel "*Consumption growth and time-varying expected returns*" undersøger forbrugsvækstens evne til at forudsige fremtidige aktieafkast. Tidligere undersøgelser viser, at forbrugsvæksten ikke har forecaststyrke for fremtidige aktieafkast. Imidlertid finder jeg, at forbrugsvæksten baseret på fjerde kvartalsdata er en stærk forecastvariabel af fremtidige aktieafkast. Forbrugsvæksten i fjerde kvartal forklarer en væsentlig del af variationen i aktieafkast 1 år frem i tiden og er en bedre forecastvariabel end traditionelle benchmark forecastvariable såsom pris-dividende ratioen [Campbell og Shiller (1988) og Fama og French (1988, 1989)] og klarer sig marginalt bedre end nye forecastvariable såsom forbrug-formue ratioen [Lettau og Ludvigson (2001)] i at forudsige fremtidige aktieafkast. Når forbrugsvæksten måles baseret på andre kvartaler, bryder forecaststyrken sammen. Dette iøjnefaldende resultat er konsistent med Jagannathan og Wang's (2007) indsigt, at investorerne har en tendens til at revurdere deres forbrug- og investeringsplaner ved afslutningen af hvert kalenderår og på mulige tilfældige tidspunkter ind imellem. Hvad der er nok så vigtigt, er forbrugsvæksten i fjerde kvartal tæt på at være en i.i.d. proces, hvilket eliminerer potentielle bekymringer om at finde falske beviser på afkastforudsigelighed, jf. Stambaugh (1999).[6]

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Chapter 1

An iterated GMM procedure for estimating the Campbell-Cochrane habit formation model, with an application to Danish stock and bond returns

An iterated GMM procedure for estimating the Campbell-Cochrane habit formation model, with an application to Danish stock and bond returns*

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Forthcoming in the *International Journal of Finance and Economics*

Abstract

We suggest an iterated GMM approach to estimate and test the consumption based habit persistence model of Campbell and Cochrane (1999), and we apply the approach on annual and quarterly Danish stock and bond returns. For comparative purposes we also estimate and test the standard CRRA model. In addition, we compare the pricing errors of the different models using Hansen and Jagannathan's (1997) specification error measure. The main result is that for Denmark the Campbell-Cochrane model does not seem to perform markedly better than the CRRA model. For the long annual sample period covering more than 80 years there is absolutely no evidence of superior performance of the Campbell-Cochrane model. For the shorter and more recent quarterly data over a 20-30 year period, there is some evidence of counter-cyclical time-variation in the degree of risk-aversion, in accordance with the Campbell-Cochrane model, but the model does not produce lower pricing errors or more plausible parameter estimates than the CRRA model.

Keywords: Consumption-based model, habit persistence, GMM, pricing error.

JEL codes: C32, G12

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1 Introduction

Since Mehra and Prescott's (1985) seminal study, explaining the observed high equity premium within the consumption based asset pricing framework has occupied a large number of researchers in finance and macroeconomics. Despite an intense research effort, still no consensus has emerged as to why stocks have given such a high average return compared to bonds. At first sight the natural response to the equity premium puzzle is to dismiss the consumption based framework altogether. However, as emphasized by Cochrane (2005), within the rational equilibrium paradigm of finance, there is really no alternative to the consumption based model, since other models are not alternatives to − but special cases of − the consumption based model. Thus, despite its poor empirical performance, the consumption based framework continues to dominate studies of the equity premium on the aggregate stock market.

In a recent paper Chen and Ludvigson (2006) argue that within the equilibrium consumption based framework, habit formation models are the most promising and successful in describing aggregate stock market behaviour. The most prominent habit model is the one developed by Campbell and Cochrane (1999). In this model people slowly develop habits for a high or low consumption level, such that risk-aversion becomes time-varying and counter-cyclical. The model is able to explain the high US equity premium and a number of other stylized facts for the US stock market. A special feature of the model is that the average risk-aversion over time is quite high, but the risk-free rate is low and stable. Thus, the model solves the equity premium puzzle by high risk-aversion, but without facing a risk-free rate puzzle.

Campbell and Cochrane (1999) themselves, and most subsequent applications of their model, do not estimate and test the model econometrically. Instead they calibrate the model parameters to match the historical risk-free rate and Sharpe ratio, and then simulate a chosen set of moments which are informally compared to those based on actual historical data. Only a few papers engage in formal econometric estimation and testing of the model. Tallarini and Zhang (2005) use an Efficient Method of Moments technique to estimate and test the model on US data. They statistically reject the model and find that it has strongly counterfactual implications for the risk-free interest rate, although they also find that the model performs well in other dimensions. Fillat and Garduno (2005) and Garcia et al. (2005) use an iterated Generalized Method of Moments approach to estimate and test the model on US data. Fillat and Garduno strongly reject the model by Hansen's (1982) J-test. On the other hand Garcia et al. do not reject the model at conventional significance levels. However, Garcia et al. face the problem that their iterated GMM approach does not lead to convergence with positive values of the risk-aversion parameter. Finally, Møller (2008) estimates the model by GMM in a cross-sectional setting using the Fama-French 25 value and size portfolios. He finds support for the model although it has difficulties in explaining the value premium.

To our knowledge, there have been no formal econometric studies of the Campbell-Cochrane model on data from other countries than the US. Our paper is the first attempt to fill this gap.[1] We examine the Campbell-Cochrane model's ability to explain Danish stock and bond returns. Denmark is interesting because historically over a long period of time the average return on Danish stocks has not been nearly as high as in the US and most other countries, and at the same time the return on Danish bonds has been somewhat higher than in other countries, see e.g. Engsted and Tanggaard (1999), Engsted (2002), and Dimson et al. (2002). Thus, the Danish equity premium is not nearly as high as in most other countries, and might not even be regarded a puzzle.

On annual Danish data for the period 1922-2004 and quarterly data for the period 1977-2006 we estimate and test both the standard model based on constant relative risk-aversion (CRRA) and the Campbell-Cochrane model based on habit formation. We basically follow the iterated GMM approach set out in Garcia et al. (2005). However, in contrast to Garcia et al., − who estimate the model parameters in two successive steps − we do a joint GMM estimation of all parameters, thereby properly taking into account sampling error on all parameter estimates. We also compute Hansen and Jagannathan's (1997) specification error measure based on the second moment matrix of returns as weighting matrix. This measure has an intuitively appealing percentage pricing error interpretation, and it allows for direct comparison of the magnitude of pricing errors across models.

Our main findings are as follows. First, neither the CRRA model nor the Campbell-Cochrane model are statistically rejected by Hansen's J-test, and pricing errors are of the same magnitude for both models. Second, both models imply high risk-aversion and a low and plausible value for the real risk-free rate. Third, in most cases the CRRA model produces plausible values for the time discount factor while the Campbell-Cochrane model delivers implausibly low values for this parameter. These results are quite robust across different data sets and instrument sets. However, when it comes to the variation over time in the degree of relative risk-aversion in the Campbell-Cochrane model, there is some difference between the long annual data set and the shorter quarterly data sets. In the annual data there is no visible counter-cyclical movement in risk-aversion, while in the quarterly data there is some evidence of counter-cyclical variation over time in accordance with the Campbell-Cochrane model.

The rest of the paper is organized as follows. The next section briefly presents the consumption-based models. Section 3 explains the iterated GMM approach used to estimate the models. Section 4 presents the empirical results based on Danish data. Finally, section 5 offers some concluding remarks.

[1] Hyde and Sherif (2005), Hyde et al. (2005), and Li and Zhong (2005) examine the Campbell-Cochrane model using international data, but with the calibrated parameter values from the original US study by Campbell and Cochrane. In Engsted et al. (2008) we apply the iterated GMM approach from the present paper to estimate and test the Campbell-Cochrane model using an international post World War II annual dataset.

2 The consumption based models

In this section we start by describing the standard CRRA utility version of the consumption based model. Since this version of the model is well-known and familiar to most readers, the description will be very brief. Then we give a more detailed description of the Campbell-Cochrane habit based model.

2.1 The CRRA utility model

Standard asset pricing theory implies that the price of an asset at time t, P_t, is determined by the expected future asset payoff, Y_{t+1}, multiplied by the stochastic discount factor, M_{t+1}: $P_t = E_t(M_{t+1}Y_{t+1})$. The payoff is given as prices plus dividends, $Y_{t+1} = P_{t+1} + D_{t+1}$, and the stochastic discount factor depends on the underlying asset pricing model. In consumption based models M_{t+1} is the intertemporal marginal rate of substitution in consumption. With power utility (constant relative risk-aversion), $U(C_t) = \frac{C_t^{1-\gamma}-1}{1-\gamma}$, where $\gamma \geq 0$ is the degree of relative risk-aversion, the stochastic discount factor becomes $M_{t+1} = \delta \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}$, where $\delta = (1+t_p)^{-1}$ and t_p is the rate of time-preference. Defining the gross return as $R_{t+1} = \frac{P_{t+1}+D_{t+1}}{P_t}$, the asset pricing relationship can be stated as:

$$0 = E_t\left[\delta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} R_{t+1} - 1\right]. \tag{1}$$

Equation (1) captures the basic idea that risk-adjusted equilibrium returns are unpredictable. In the consumption based model, risk-adjustment takes place by multiplying the raw return with the intertemporal marginal rate of substitution in consumption. Risk-averse consumers want to smooth consumption over time, and for that purpose they use (dis)investments in the asset, thereby making a direct connection between consumption growth and the asset return. The correlation between consumption growth and returns then becomes crucial for the equilibrium expected return. From (1) expected returns are given as:

$$E_t\left[R_{t+1}\right] = \frac{1 - \mathrm{Cov}_t\left[R_{t+1}, \delta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\right]}{E_t\left[\delta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\right]}. \tag{2}$$

The higher the correlation between consumption growth and returns (the lower the correlation between the stochastic discount factor and returns), the higher will be expected equilibrium return (*ceteris paribus*), because the higher the correlation, the less able the asset will be in helping to smooth consumption over time, which means that the asset will be considered riskier and thereby demand a higher return.

Equation (1) lends itself directly to empirical estimation and testing within the GMM framework, c.f. section 3. Empirically the consumption based power utility model has run into trouble because consumption growth and stock returns are not sufficiently positively correlated to explain the historically observed high return on common stocks, unless the degree of risk-aversion γ is extremely high. The basic problem is that unless γ is very high, the variability of the intertemporal marginal rate of substitution cannot match the variability of stock returns. Perhaps people *are* highly risk-averse, but then the power utility model faces another problem, namely that with a high γ, the risk-free rate implied by the model becomes implausibly high. For the risk-free rate the covariance with the stochastic discount factor is zero, thus from (2):

$$R_{f,t+1} = \frac{1}{E_t\left[\delta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\right]}. \tag{3}$$

Thus, within the standard CRRA utility framework, the equity premium puzzle cannot be solved without running into a risk-free rate puzzle. This has led to the development of alternative utility models with a higher volatility of the stochastic discount factor, and with plausible implications for the risk-free rate. The habit persistence model described in the next subsection is one such model.

2.2 The Campbell-Cochrane model

Habit formation models differ from the standard power utility model by letting the utility function be time-nonseparable in the sense that the utility at time t depends not only on consumption at time t, but also on previous periods consumption. The basic idea is that people get used to a certain standard of living and thereby the utility of some consumption level at time t will be higher (lower) if previous periods consumption was low (high) than if previous periods consumption was high (low).

Habit formation can be modelled in a number of different ways. In the Campbell-Cochrane model utility is specified as

$$U(C_t, X_t) = \frac{(C_t - X_t)^{1-\gamma} - 1}{1-\gamma}, \quad C_t > X_t \tag{4}$$

where X_t is an external habit level that depends on previous periods consumption. Define the *surplus consumption ratio* as $S_t = \frac{C_t - X_t}{C_t}$. Then the stochastic discount factor can be stated as $M_{t+1} = \delta\left(\frac{S_{t+1}}{S_t}\frac{C_{t+1}}{C_t}\right)^{-\gamma}$ and the pricing equation becomes

$$0 = E_t\left[\delta\left(\frac{S_{t+1}}{S_t}\frac{C_{t+1}}{C_t}\right)^{-\gamma} R_{t+1} - 1\right]. \tag{5}$$

Compared to the standard power utility model in (1), the Campbell-Cochrane model implies a stochastic discount factor that not only depends on consumption growth but also on growth in the surplus consumption ratio. In this model relative risk-aversion is no longer measured by γ but as $\frac{\gamma}{S_t}$. This shows that relative risk-aversion is time-varying and counter-cyclical: when consumption is high relative to habit, relative risk-aversion is low and expected returns are low. By contrast, when consumption is low and close to habit, relative risk-aversion is high leading to high expected returns. Basically the model explains time-varying and counter-cyclical ex ante returns (which implies pro-cyclical stock prices) as a result of time-varying and counter-cyclical risk-aversion of people. From (5) expected returns are given as:

$$E_t\left[R_{t+1}\right] = \frac{1 - \mathrm{Cov}_t\left[R_{t+1}, \delta\left(\frac{S_{t+1}}{S_t}\frac{C_{t+1}}{C_t}\right)^{-\gamma}\right]}{E_t\left[\delta\left(\frac{S_{t+1}}{S_t}\frac{C_{t+1}}{C_t}\right)^{-\gamma}\right]}. \tag{6}$$

A crucial aspect in operationalizing the model is the modelling of the risk-free rate. Campbell and Cochrane specify the model in such a way that the risk-free rate is constant and low by construction. First, assume that consumption is lognormally distributed such that consumption growth is normally distributed and *iid*:

$$\Delta c_{t+1} = g + v_{t+1}, \quad v_{t+1} \sim niid(0, \sigma_v^2) \tag{7}$$

where $c_t \equiv \log(C_t)$. g is the mean consumption growth rate. Next, specify the log surplus consumption ratio $s_t = \log(S_t)$ as a stationary first-order autoregressive process

$$s_{t+1} = (1-\phi)\overline{s} + \phi s_t + \lambda(s_t)v_{t+1}, \tag{8}$$

where $0 < \phi < 1$, $\overline{s}$ is the steady state level of s_t, and $\lambda(s_t)$ is the sensitivity function to be specified below. Note that shocks to consumption growth are modelled to have a direct impact on the surplus consumption level, and for ϕ close to one, habit responds slowly to these shocks.

The sensitivity function $\lambda(s_t)$ is specified as follows:

$$\lambda(s_t) = \left\{ \begin{array}{cc} \frac{1}{\overline{S}}\sqrt{1-2(s_t-\overline{s})}-1 & \text{if } s_t \leq s_{\max} \\ 0 & \text{else} \end{array} \right\} \tag{9}$$

where

$$\overline{S} = \sqrt{\frac{\sigma_v^2\gamma}{1-\phi}}, \qquad s_{\max} \equiv \overline{s} + \frac{1}{2}(1-\overline{S}^2), \qquad \overline{s} = \log(\overline{S}).$$

Specifying $\lambda(s_t)$ in this way implies the following equation for the log risk-free rate:

$$r_{f,t+1} = -\log(\delta) + \gamma g - \frac{\gamma^2\sigma_v^2}{2}\left(\frac{1}{\overline{S}}\right)^2. \tag{10}$$

As seen, no time-dependent variables appear in (10), thus the risk-free rate is constant over time. Economically this property of the model is obtained by letting the effects of intertemporal substitution and precautionary saving – which have opposite effects on the risk-free rate – cancel each other out, see Campbell and Cochrane (1999) for details.

Campbell and Cochrane calibrate their model with parameters chosen to match post war US data: mean real consumption growth rate (g), mean real risk-free rate (r_f), volatility (σ_v), etc. Then, based on the calibrated model, simulated time-series for returns, price-dividend ratios, etc., are generated and their properties are compared to the properties of the actually observed post war data. In the present paper we instead estimate the model parameters in a GMM framework. The next section describes how.

3 GMM estimation of the models

The GMM technique developed by Hansen (1982) estimates the model parameters based on the orthogonality conditions implied by the model. Let the asset pricing equation be $0 = E_t\left[M_{t+1}(\theta)R_{t+1} - 1\right]$, where M_{t+1} is the stochastic discount factor, R_{t+1} is a vector of asset returns, and the vector θ contains the model parameters. In the present context this equation corresponds to either (1) or (5) with $\theta = (\delta \;\; \gamma)'$. Define a vector of instrumental variables, Z_t, observable at time t. Then the asset pricing equation implies the following orthogonality conditions $E\left[(M_{t+1}(\theta)R_{t+1} - 1) \otimes Z_t\right] = 0$. GMM estimates θ by making the sample counterpart to these orthogonality conditions as close to zero as possible, by minimizing a quadratic form of the sample orthogonality conditions based on a chosen weighting matrix. Define $g_T(\theta) = \frac{1}{T}\sum_{t=1}^{T}(M_{t+1}(\theta)R_{t+1} - 1) \otimes Z_t$ as the sample orthogonality conditions based on T observations. Then the parameter vector θ is estimated by minimizing

$$g_T(\theta)'W g_T(\theta), \tag{11}$$

where W is the weighting matrix. The statistically optimal (most efficient) weighting matrix is obtained as the inverse of the covariance matrix of the sample orthogonality conditions. Other weighting matrices can be chosen, however, and often a fixed and model-independent weighting matrix (the identity matrix, for example) is used in order to make it possible to compare the magnitude of estimated pricing errors across different models. Such a comparison cannot be done if the statistically optimal weighting matrix is used because this matrix is model-dependent.

GMM estimation of the standard CRRA utility model (1) is straightforward. However, estimation of the Campbell-Cochrane model, equation (5), is complicated by the fact that the surplus consumption ratio, S_t, is not observable in the same way as returns, R_t, and consumption, C_t, are directly observable. Garcia et al. (2005) suggest to generate a process for s_t by initially estimating the parameters ϕ, g and σ_v^2, and setting γ to some initial value, which then gives $\overline{s}$, from which s_t can be constructed using (8) and a starting value for s_t at $t = 0$. Garcia et al. set $s_0 = \overline{s}$. Having obtained a series for the surplus consumption ratio, GMM can be applied directly. Since the surplus consumption ratio depends on γ, however, the resulting GMM estimate of γ may not correspond to the value initially imposed in generating s_t. Therefore, Garcia et al. iterate over γ by estimating the model in each iteration using GMM with the statistically optimal weighting matrix. Unfortunately, this procedure does not lead to convergence with a positive value of γ in their application. Instead they do a grid search that implies an estimated value of γ close but not identical to the initially picked value.

Our procedure differs from Garcia et al.'s in the following way: They estimate ϕ, g and σ_v^2 separately in an initial step. Then, given these parameter estimates, they estimate δ and γ using GMM. Instead we do a joint GMM estimation of all parameters and, hence, take into account sampling error on all parameters. We report results for different instrument sets. However, in order to economize on the number of orthogonality conditions, we fix the instrument set to contain just a constant for the estimation of g and σ_v^2, and a constant and lagged log price-dividend ratio for the estimation of ϕ. Moreover, following Cochrane's (2005) suggestion, we use the identity matrix as weighting matrix across all GMM estimations. Thereby we attach equal weight to each asset in the estimation.[2] The use of the identity matrix has the further advantage that in our application it leads to convergence with positive values of the risk-aversion parameter, in contrast to the case where we use the statistically optimal weighting matrix (Garcia et al. (2005) also face convergence problems, which might be due to their exclusive use of the statistically optimal weighting matrix). Thus, we restrict attention to the case with $W = I$.

The details of our estimation procedure are as follows. The moment conditions used in our estimation procedure are:

$$0 = E\left[\left(\delta\left(\frac{S_{t+1}}{S_t}\frac{C_{t+1}}{C_t}\right)^{-\gamma} R_{t+1} - 1\right) \otimes Z_t\right], \tag{12}$$

$$0 = E\left[\Delta c_{t+1} - g\right], \tag{13}$$

$$0 = E\left[\left(\Delta c_{t+1} - g\right)^2 - \sigma_v^2\right], \tag{14}$$

$$0 = E\left[\left(pd_t - \alpha - \phi pd_{t-1}\right)\left(1 \quad pd_{t-1}\right)'\right]. \tag{15}$$

Based on the asset equation in (5), we form the moment conditions in (12). In order to estimate the parameters ϕ, g and σ_v^2, the GMM estimation of the Campbell-Cochrane model requires additional moment conditions. Given the random walk model of consumption in (7), we estimate g and σ_v^2 based on the moment conditions in (13) and (14). Following Campbell and Cochrane (1999) and Garcia et al. (2005), we estimate ϕ as the first-order autocorrelation parameter for the log price-dividend ratio using the moment conditions in (15). This is feasible since in the Campbell-Cochrane model the surplus consumption ratio is the only state variable, whereby the log price-dividend ratio, pd_t, will inherit its dynamic properties from the log surplus consumption ratio, s_t.

As starting values in the GMM iterations we use OLS estimates of g, σ_v^2, and ϕ, and we choose an initial value of $\gamma = 1$ to obtain $\overline{S} = \sqrt{\frac{\sigma_v^2\gamma}{1-\phi}}$ and set $s_t = \overline{s}$ at $t = 0$. From the chosen parameter values, we obtain the s_t process recursively. Given s_t, S_t is obtained as $\exp(s_t)$. Using this S_t process and with the identity matrix as weighting matrix, we jointly estimate the moment conditions (12) to (15), which gives GMM estimates of all model parameters δ, γ, ϕ, g, and σ_v^2. The parameter estimates are used to generate a new S_t process and we repeat this procedure until convergence of all estimated parameters.

[2] We use a GMM programme written in MatLab. The programme is available upon request.

Since the chosen weighting matrix is not the efficient Hansen (1982) matrix but the identity matrix I, the formula for the covariance matrix of the parameter vector is (c.f. Cochrane (2005), chpt. 11):

$$Var(\widehat{\theta}) = \frac{1}{T}(d'Id)^{-1}d'ISId(d'Id)^{-1}, \tag{16}$$

where $d' = \partial g_T(\theta)/\partial\theta$, and the spectral density matrix $S = \sum_{j=-\infty}^{\infty} E[\ g_T(\theta)g_{T-j}(\theta)']$ is computed with the usual Newey and West (1987) estimator with a lag truncation. Similarly, the J-test of overidentifying restrictions is computed based on the general formula (c.f. Cochrane (2005) chpt. 11):

$$J_T = Tg_T(\widehat{\theta})' \left[(I - d(d'Id)^{-1}d'I)S(I - Id(d'Id)^{-1}d')\right]^{-1} g_T(\widehat{\theta}). \tag{17}$$

J_T has an asymptotic χ^2 distribution with degrees of freedom equal to the number of overidentifying restrictions. (17) involves the covariance matrix $Var(g_T(\widehat{\theta})) = \frac{1}{T}(I - d(d'Id)^{-1}d'I)S(I - Id(d'Id)^{-1}d')$, which is singular, so it is inverted using the Moore-Penrose pseudo-inversion.

In addition to formally testing the model using the J-test, we also compute the Hansen and Jagannathan (1997) misspecification measure, HJ, as

$$HJ = \left[E(M_{t+1}(\theta)R_{t+1} - 1)'(E(R_{t+1}R'_{t+1}))^{-1}E(M_{t+1}(\theta)R_{t+1} - 1)\right]^{\frac{1}{2}}. \tag{18}$$

HJ measures the minimum distance between the candidate stochastic discount factor M_{t+1} and the set of admissible stochastic discount factors. HJ can be interpreted as the maximum pricing error per unit payoff norm. Thus, it has an intuitively appealing percentage pricing error interpretation. It is a measure of the magnitude of pricing errors that gives a useful *economic* measure of fit, in contrast to the statistical measure of fit given by Hansen's J-test. In addition, since the HJ measure is based on a model-independent weighting matrix, it can be used to compare pricing errors across models. The HJ measure is computed at the GMM estimates of δ and γ. We compute the asymptotic standard error of $\widehat{HJ}$ using the Hansen et al. (1995) procedure.[3]

4 Empirical results

We estimate the models on annual data from 1922 to 2004 and quarterly data from 1977:1 to 2006:3. For the quarterly data we measure consumption as per capita, seasonally adjusted, expenditures on non-durables and services from IMF International Financial Statistics.[4] We adopt the Campbell (2003) beginning of period timing assumption

[3]The asymptotic distribution of $\widehat{HJ}$ is degenerate when $HJ = 0$. Thus, the asymptotic standard error of $\widehat{HJ}$ cannot be used to test whether $HJ = 0$. Instead, the standard error gives a measure of the precision of the estimate of HJ.

[4]The use of seasonally adjusted consumption data is standard practice in this field. As an exception, Ferson and Harvey (1992) examine consumption based asset pricing using seasonally unadjusted con-

that consumption during period t takes place at the beginning of period t. We use the dividend-adjusted stock market return from Morgan Stanley Capital International and derive the price-dividend ratio from return indices with and without dividend reinvestment. As is standard practice, the dividend series used in the price-dividend ratio is the sum over the last four quarters. This accounts for any seasonality in dividends. We use long-term (10 years) and short-term (3 month) government bond returns from Datastream and Global Financial Data. Nominal returns and nominal consumption are converted to real units using the consumption deflator from IMF International Financial Statistics. Our annual data set is an updated version of the data set in Engsted (2002). As instruments in the GMM estimations, we use lags of stock returns, bond returns, consumption growth, and the price-dividend ratio.

Table 1 reports summary statistics for the real gross stock and bond returns and the instruments. As seen in Table 1, the average annual arithmetic real stock return, R_S, over the 1922-2004 period is 6.72%, while the long-term, R_{LB}, and short-term, R_{SB}, real bond returns are 4.44% and 2.40%, respectively. The corresponding standard deviations are 20.94%, 12.03%, and 5.23%. Thus, stocks give higher average returns than bonds, but are also more volatile. The average ex post yearly equity premium, i.e. the yearly stock return in excess of the 3-month government bond return, is 4.33%, with a standard deviation of 20.91%. Thus, the Danish equity premium is lower than in most other countries, and in the US in particular, but it is just as volatile as in other countries (in fact, the Danish equity premium is not statistically significant: the standard error of the average premium is 2.31%). This is similar to what Engsted and Tanggaard (1999), Engsted (2002), and Dimson et al. (2002) have found using long-term annual Danish data.

Table 1 also reports summary statistics for quarterly data from 1977:1 to 2006:3 and from 1984:4 to 2006:3 (quarterly observations on long-term government bonds start in 1984:4). As seen, over these shorter quarterly sample periods, the average yearly equity premium is $4 \times (2.66\% - 1.42\%) = 4.99\%$ and $4 \times (2.86\% - 1.22\%) = 6.54\%$, respectively, which is somewhat higher than the average of 4.33% for the annual sample. Table 1 also shows that quarterly real stock returns are slightly positively autocorrelated, whereas real bond returns show strong positive autocorrelation.

In a qualitative sense, the consumption based model implies that the stochastic discount factor should be negatively correlated with stock returns in order to generate a positive equity-premium. Table 2 reports correlations between M_{t+1} and real stock returns $R_{S,t+1}$, where M_{t+1} is either equal to $\delta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma}$ (i.e. the standard power utility model, CRRA), or $\delta \left(\frac{S_{t+1}}{S_t} \frac{C_{t+1}}{C_t} \right)^{-\gamma}$ (i.e. the Campbell-Cochrane model), and where S_{t+1} has been constructed as described in section 3 from OLS estimates of ϕ, g and σ_v^2 and with values of γ ranging from 1 to 20 in the CRRA utility case, and from 0.5 to 2.0 in the

sumption data. They consider a model with seasonal habit persistence, which implies that habit depends on previous consumption in the same season. To stay consistent with the Campbell-Cochrane model, we do not incorporate seasonal effects in the level of habit and, hence, work with seasonally adjusted consumption data.

Campbell-Cochrane case corresponding to values of relative risk-aversion γ/S_t ranging from 10 to 40, which is consistent with the GMM estimates reported below. For both models − and across the different values for risk-aversion − stock returns are negatively correlated with the stochastic discount factor in both the annual and quarterly data sets. However, all correlations are close to zero, so although in a qualitative sense this is consistent with the basic consumption-based framework, the evidence does not strongly support it and certainly does not allow us to discriminate between the standard CRRA utility model and the Campbell-Cochrane model.

Now we turn to formal estimation of the parameters and statistical tests of the models. Table 3 reports the iterated GMM estimates and associated test statistics for the long annual data set, while Tables 4 and 5 report the results for the shorter quarterly data. We report results using six different instrument sets for the return moment conditions, see the notes to Table 3. For the annual data, the vector of returns includes real returns on stocks, long-term bonds, and short-term bonds. For the standard CRRA utility model, Panel A in Table 3 shows that the annual subjective discount factor δ is precisely estimated at slightly below unity. The estimated risk-aversion parameter γ is around 8-9 and statistically significant. The J-test does not in any case reject the model at conventional significance levels, and the HJ measure indicates pricing errors of around 11%. The annual real risk-free rate, r_f, implied by these estimates is around 6%, which is high but not completely unreasonable.

The estimates in Panel B of Table 3 do not indicate that the Campbell-Cochrane model performs better than the simple CRRA model. The model is not statistically rejected and pricing errors and average risk-aversion are of the same magnitude as for the CRRA model. However, the estimates of δ of around 0.90 (implying an annual rate of time-preference of 10%) is somewhat low. On the other hand, the implied risk-free rate of around 2.7% is more reasonable than the 6% implied by the CRRA model. The estimated average geometric per capita consumption growth rate, g, is 1.56% p.a., with a standard deviation, σ_v, of around 5% ($\sigma_v^2 = 0.0024$), and the estimated persistence parameter of $\phi = 0.88$ implies that the price-dividend ratio and, hence, the surplus consumption ratio are stationary but highly persistent. Figure 1 shows the movement over time in the implied degree of relative risk-aversion, γ/S, computed from column 2 in Table 3, Panel B.[5] There is no systematic strong counter-cyclical time-variation in relative risk-aversion; the most interesting aspect of the figure is the dramatic increase in risk-aversion associated with the decline in real consumption at the outbreak of World War II. Overall, based on these annual results, it is impossible to discriminate between the CRRA and Campbell-Cochrane models.

Turning to the quarterly data, Table 4 reports results for stocks and short-term bonds over the period 1977:1-2006:3. As for the annual data, neither the CRRA model nor the Campbell-Cochrane model are statistically rejected by the J-test, and HJ pricing errors are quite low (below 10%) for both models. The estimated quarterly time discount factor δ is reasonable for the CRRA model, but implausibly low for the Campbell-Cochrane

[5] The time-series movement in γ/S_t is essentially similar to the one in Figure 1 if parameter values from the other columns in Table 3 are used. This also holds for Figures 2 and 3 below.

model. The real quarterly risk-free rate is around 1% in both models. In the CRRA model, the degree of risk-aversion is very high − ranging from 13 to 22, depending on the instrument set − but imprecisely estimated. In the Campbell-Cochrane model the estimated values of γ imply an average degree of risk-aversion from 15 to 24, similar to the estimated values for the CRRA model. However, Figure 2 shows that − in contrast to the annual data − the Campbell-Cochrane model now produces visible counter-cyclical time-variation in the degree of risk-aversion: High risk-aversion during the cyclical downturns in the late 1970s, beginning of the 1980s, late 1980's, and start of the new millennium. And low risk-aversion during the booming years of the mid 1980s, mid to late 1990s and the final years of the sample, 2005-2006. (Figure 2 uses the parameter values from column 2 in Table 4, Panel B).

In Table 5 and Figure 3 we include in the return vector long-term bonds in addition to stocks and short-term bonds, and we look at the shorter quarterly sample period, 1984:4-2006:3, since there are no quarterly return data for long-term bonds before 1984:4. The main differences to the quarterly results in Table 4 and Figure 2 are that now δ exceeds one in the CRRA model, r_f is slightly negative in the Campbell-Cochrane model, and HJ pricing errors increase to around 25% for both models even though the J-test still does not reject the models statistically. This is an illustration of the fact emphasized by Hansen and Jagannathan (1997), Cochrane (2005), and others, that a statistical non-rejection by the J-test does not necessarily imply low pricing errors. Figure 3 resembles Figure 2 in showing counter-cyclical time-variation in the degree of risk-aversion, in accordance with the predictions of the Campbell-Cochrane model.

The main conclusion we draw from the empirical analysis is that for Denmark the Campbell-Cochrane habit formation model does not seem to perform markedly better than the standard time-separable power utility model in explaining stock and bond returns. For the long annual sample period covering more than 80 years there is absolutely no evidence of superior performance of the Campbell-Cochrane model. For the shorter and more recent quarterly data over a 20-30 year period, there is some evidence of counter-cyclical time-variation in the degree of risk-aversion, in accordance with the Campbell-Cochrane model, but the model does not produce lower pricing errors than the time-separable model with constant risk-aversion. Further, the Campbell-Cochrane model resembles the standard time-separable power utility model in the sense that it has to rely on very high values of risk-aversion to explain the Danish asset returns.[6]

5 Concluding remarks

The habit persistence model developed by Campbell and Cochrane (1999) has become one of the most prominent consumption based asset pricing models, in particular with

[6] The risk-aversion estimates reported in this study may be perceived as implausibly high. However, they are not higher than in other studies; see e.g. Campbell (2003) for a comprehensive international study. In fact, our yearly estimates are much lower than in other studies and do not exceed 10 considered plausible by Mehra and Prescott (1985).

respect to aggregate stock market returns. It explains pro-cyclical stock prices, time-varying and counter-cyclical expected returns, and high and time-varying equity premia as a result of high but time-varying and counter-cyclical risk-aversion, and it does this while keeping the risk-free rate low and stable.

When the Campbell-Cochrane model is calibrated to actual historical data from the US, the model is found to match a number of key aspects of the data. However, only a few attempts have been made to formally estimate and test the model, and almost exclusively on US data. These formal estimations and tests generally have led to statistical rejection of the model. Thus, while there is evidence that the Campbell-Cochrane model has empirical content on US data, and that it clearly outperforms the standard CRRA utility model, it is also clear that the model does involve significant pricing errors.[7]

In this paper we have performed a formal econometric estimation and testing of both the standard CRRA model and the Campbell-Cochrane model using Danish stock and bond market returns and aggregate consumption. We have used an iterated GMM procedure that for the Campbell-Cochrane model estimates all parameters in one comprehensive step while generating – within the iterations – a process for the unobservable surplus consumption ratio and, hence, the degree of relative risk-aversion.

The results we obtain using this procedure on Danish asset market returns do not in general support the conclusions from the US studies. Although there is some evidence of time-varying counter-cyclical risk-aversion in recent years, the Campbell-Cochrane model does not produce lower pricing errors or more plausible parameter values than the CRRA model. In Engsted et al. (2008) we present further international evidence on the relative performance of the two models. There seems to be quite large cross-country differences in the ability of the Campbell-Cochrane model to explain asset return movements over time. With no doubt, investigations of consumption-based models with habit persistence will continue in the future.

[7] As noted by Campbell and Cochrane (1999) themselves (p.236), the worst performance of the model occurs during the end of their sample period, i.e. the first half of the 1990s.

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	Mean (std.dev)	Autocorr. (std.err)
Annual, 1922-2004		
R_S	1.0672 (0.2094)	−0.0963 (0.1104)
R_{LB}	1.0444 (0.1203)	0.0468 (0.1104)
R_{SB}	1.0240 (0.0523)	0.5924 (0.1104)
C/C_{-1}	1.0162 (0.0484)	0.1149 (0.1104)
pd	3.3211 (0.4631)	0.8836 (0.1104)
Quarterly, 1977:1-2006:3		
R_S	1.0266 (0.0979)	0.2462 (0.0921)
R_{SB}	1.0142 (0.0112)	0.7593 (0.0921)
C/C_{-1}	1.0031 (0.0165)	−0.1449 (0.0921)
pd	3.8137 (0.4944)	0.9730 (0.0921)
Quarterly, 1984:4-2006:3		
R_S	1.0286 (0.0941)	0.1806 (0.1072)
R_{LB}	1.0193 (0.0319)	0.4086 (0.1072)
R_{SB}	1.0122 (0.0109)	0.8542 (0.1072)
C/C_{-1}	1.0037 (0.0134)	−0.0682 (0.1072)
pd	4.0434 (0.2407)	0.8769 (0.1072)

Notes: R_S, R_{LB}, and R_{SB} are real gross returns on stocks, long-term bonds, and short-term bonds. C/C_{-1} is the real per capita gross consumption growth rate. pd is the log price-dividend ratio.

Table 1: Summary statistics for asset returns and instruments.

	$\mathrm{Corr}(R_S,\, M^{CRRA})$		$\mathrm{Corr}(R_S,\, M^{CC})$
Annual, 1922-2004			
$\gamma = 1$	-0.1803	$\gamma = 0.5$	-0.1903
$\gamma = 5$	-0.1603	$\gamma = 1$	-0.1649
$\gamma = 10$	-0.1188	$\gamma = 1.5$	-0.1460
$\gamma = 20$	-0.0699	$\gamma = 2$	-0.1309
Quarterly, 1977:1-2006:3			
$\gamma = 1$	-0.1332	$\gamma = 0.5$	-0.1118
$\gamma = 5$	-0.1317	$\gamma = 1$	-0.1323
$\gamma = 10$	-0.1290	$\gamma = 1.5$	-0.1368
$\gamma = 20$	-0.1210	$\gamma = 2$	-0.1371
Quarterly, 1984:4-2006:3			
$\gamma = 1$	-0.1286	$\gamma = 0.5$	-0.0991
$\gamma = 5$	-0.1195	$\gamma = 1$	-0.0810
$\gamma = 10$	-0.1079	$\gamma = 1.5$	-0.0751
$\gamma = 20$	-0.0845	$\gamma = 2$	-0.0763

Notes: M^{CRRA} and M^{CC} are the stochastic discount factors in the CRRA utility model and Campbell-Cochrane model, respectively.

Table 2: Correlations between stock returns and the stochastic discount factor.

Instrument set	1	2	3	4	5	6
Panel A: CRRA model						
δ	0.9804	0.9876	0.9870	0.9943	0.9772	0.9910
	(0.0559)	(0.0545)	(0.0521)	(0.0479)	(0.0552)	(0.0514)
γ	9.4339	8.9449	8.9982	8.4589	9.5197	8.6616
	(3.7035)	(3.6191)	(3.6723)	(3.3385)	(3.6188)	(3.5970)
J-test	4.1415	7.6336	8.3655	10.3327	5.7790	10.6004
	(0.3872)	(0.3660)	(0.3015)	(0.1705)	(0.5658)	(0.3895)
HJ	0.1182	0.1138	0.1140	0.1135	0.1193	0.1132
	(0.0968)	(0.0803)	(0.0814)	(0.0770)	(0.0998)	(0.0770)
r_f	0.0610	0.0568	0.0571	0.0525	0.0637	0.0548
Panel B: Campbell-Cochrane model						
δ	0.9018	0.9098	0.9051	0.9099	0.8949	0.9157
	(0.0521)	(0.0508)	(0.0500)	(0.0488)	(0.0539)	(0.0488)
γ	1.8599	1.6150	1.7017	1.6166	1.9895	1.5033
	(0.7548)	(0.6723)	(0.7238)	(0.6837)	(0.7965)	(0.6489)
g	0.0156	0.0156	0.0156	0.0156	0.0156	0.0156
	(0.0050)	(0.0050)	(0.0050)	(0.0050)	(0.0050)	(0.0050)
σ_v^2	0.0024	0.0024	0.0024	0.0024	0.0024	0.0024
	(0.0011)	(0.0011)	(0.0011)	(0.0011)	(0.0011)	(0.0011)
ϕ	0.8854	0.8854	0.8854	0.8854	0.8854	0.8854
	(0.0502)	(0.0502)	(0.0502)	(0.0502)	(0.0502)	(0.0502)
J-test	4.1258	7.1558	7.7175	9.3613	5.5754	9.9166
	(0.3892)	(0.4128)	(0.3582)	(0.2277)	(0.5901)	(0.4478)
HJ	0.1143	0.1127	0.1118	0.1127	0.1188	0.1154
	(0.0857)	(0.0794)	(0.0782)	(0.0794)	(0.1002)	(0.0833)
r_f	0.0258	0.0271	0.0287	0.0270	0.0280	0.0254
γ/S	8.4206	7.9115	8.2066	7.9171	8.7130	7.4951

Notes: The table reports parameter estimates of the CRRA utility and Campbell-Cochrane models using the iterated GMM approach described in section 3, with asymptotic standard errors in parentheses. J-test is Hansen's test of overidentifying restrictions, computed as in (17), with asymptotic p-value in parenthesis. HJ is the Hansen-Jagannathan specification error measure, computed as in (18), with asymptotic standard error in parenthesis. r_f is the log real risk-free rate, computed from (3) and (10). S in γ/S is the average value of S over the sample. The instrument sets for the return moment conditions (12) are:

1: Constant, pd.

2: Constant, pd, R_S.

(*table continues next page*)

3: Constant, pd, R_{SB}.

4. Constant, pd, C/C_{-1}.

5. Constant, pd, and its lag.
6. Constant, pd, R_S, R_{SB}.

Table 3: GMM estimation of the CRRA utility and Campbell-Cochrane models using real annual returns on stocks, long-term bonds, and short-term bonds, 1922-2004.

Instrument set	1	2	3	4	5	6
Panel A: CRRA model						
δ	0.9930	0.9934	0.9972	0.9972	0.9809	0.9969
	(0.0310)	(0.0296)	(0.0194)	(0.0195)	(0.0547)	(0.0199)
γ	17.2626	16.9044	13.3399	13.4246	21.6905	13.5817
	(13.2284)	(13.2178)	(11.0521)	(11.0774)	(15.7843)	(11.2701)
J-test	1.1357	6.8080	1.3439	5.0895	4.6080	7.2043
	(0.5667)	(0.1464)	(0.8539)	(0.2782)	(0.3299)	(0.3024)
HJ	0.0845	0.0856	0.0967	0.0965	0.0691	0.0960
	(0.1163)	(0.1164)	(0.1162)	(0.1162)	(0.1155)	(0.1163)
r_f	0.0136	0.0138	0.0147	0.0146	0.0150	0.0148
Panel B: Campbell-Cochrane model						
δ	0.9559	0.9572	0.9671	0.9672	0.9376	0.9666
	(0.0471)	(0.0446)	(0.0306)	(0.0305)	(0.0734)	(0.0310)
γ	2.3238	2.2216	1.5362	1.5264	3.4249	1.5626
	(1.6252)	(1.5730)	(1.1300)	(1.1263)	(2.3380)	(1.1538)
g	0.0026	0.0026	0.0026	0.0026	0.0026	0.0026
	(0.0014)	(0.0014)	(0.0014)	(0.0014)	(0.0014)	(0.0014)
σ_v^2	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003
	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)
ϕ	0.9628	0.9628	0.9628	0.9628	0.9628	0.9628
	(0.0293)	(0.0293)	(0.0293)	(0.0293)	(0.0293)	(0.0293)
J-test	1.0280	6.9322	1.1747	4.3896	4.6146	7.2783
	(0.5981)	(0.1395)	(0.8823)	(0.3566)	(0.3296)	(0.2959)
HJ	0.0768	0.0785	0.0906	0.0908	0.0605	0.0901
	(0.1176)	(0.1177)	(0.1177)	(0.1176)	(0.1161)	(0.1177)
r_f	0.0080	0.0082	0.0089	0.0089	0.0097	0.0090
γ/S	19.7728	19.3104	15.7690	15.7114	24.0862	15.237

See the notes to Table 3.

Table 4: GMM estimation of the CRRA utility and Campbell-Cochrane models using real quarterly returns on stocks and short-term bonds, 1977:1-2006:3.

Instrument set	1	2	3	4	5	6
Panel A: CRRA model						
δ	1.0107	1.0179	1.0142	1.0135	1.0165	1.0184
	(0.0839)	(0.0370)	(0.0560)	(0.0614)	(0.0371)	(0.0323)
γ	29.6037	23.7190	27.1305	27.7042	23.2281	22.4344
	(44.5128)	(29.9511)	(36.4666)	(38.1363)	(31.7599)	(27.3983)
J-test	2.8620	5.1253	5.5748	5.6734	4.7891	7.4711
	(0.5812)	(0.6447)	(0.5902)	(0.5784)	(0.6857)	(0.6803)
HJ	0.2677	0.2660	0.2669	0.2671	0.2659	0.2657
	(0.1636)	(0.1590)	(0.1615)	(0.1620)	(0.1586)	(0.1581)
r_f	0.0175	0.0172	0.0177	0.0176	0.0188	0.0174
Panel B: Campbell-Cochrane model						
δ	0.8827	0.9485	0.9056	0.9006	0.9448	0.9489
	(0.1807)	(0.0796)	(0.1330)	(0.1494)	(0.0956)	(0.0765)
γ	2.5159	0.8882	1.9772	2.1094	0.9293	0.8777
	(2.4446)	(1.0266)	(1.8623)	(2.0764)	(1.2098)	(0.9961)
g	0.0036	0.0036	0.0036	0.0036	0.0036	0.0036
	(0.0014)	(0.0014)	(0.0014)	(0.0014)	(0.0014)	(0.0014)
σ_v^2	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
ϕ	0.8729	0.8729	0.8729	0.8729	0.8729	0.8729
	(0.0584)	(0.0584)	(0.0584)	(0.0584)	(0.0584)	(0.0584)
J-test	2.7031	5.4612	5.6297	6.9086	4.9142	6.9216
	(0.6087)	(0.6039)	(0.5836)	(0.4385)	(0.6704)	(0.7328)
HJ	0.2528	0.2670	0.2547	0.2532	0.2668	0.2671
	(0.1579)	(0.1530)	(0.1546)	(0.1552)	(0.1514)	(0.1512)
r_f	−0.0261	−0.0004	−0.0193	−0.0218	0.0011	−0.0002
γ/S	36.8241	19.7193	32.2148	33.3273	20.3868	19.5562

See the notes to Table 3.

Table 5: GMM estimation of the CRRA utility and Campbell-Cochrane models using real quarterly returns on stocks, long-term bonds, and short-term bonds, 1984:4-2006:3.

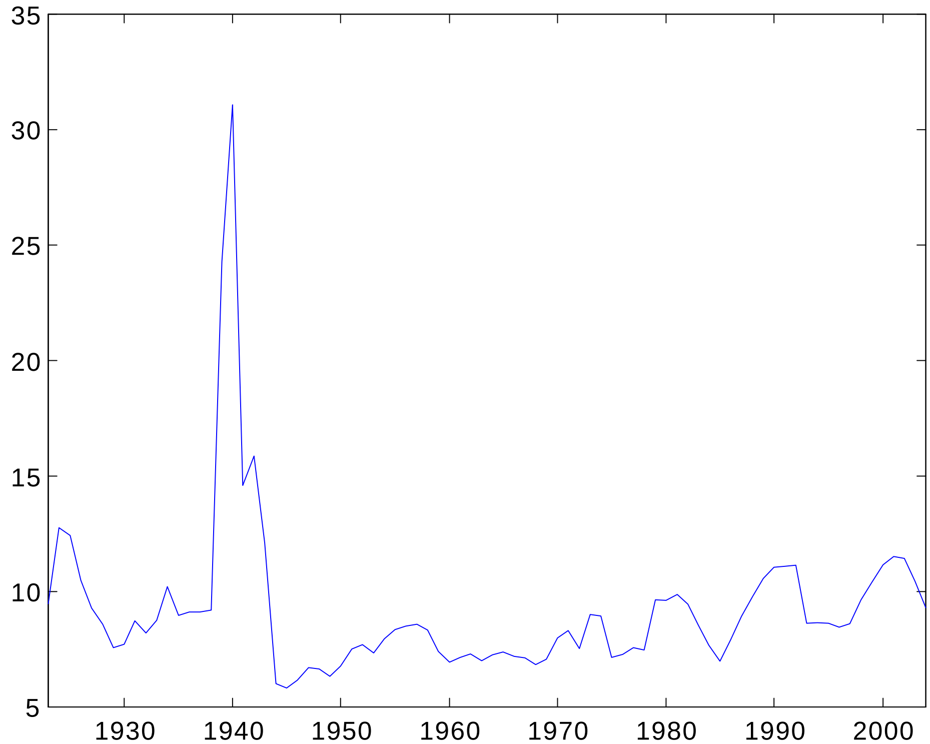

Figure 1: Relative risk aversion, γ/S_t, in the Campbell-Cochrane model, Denmark 1922-2004.

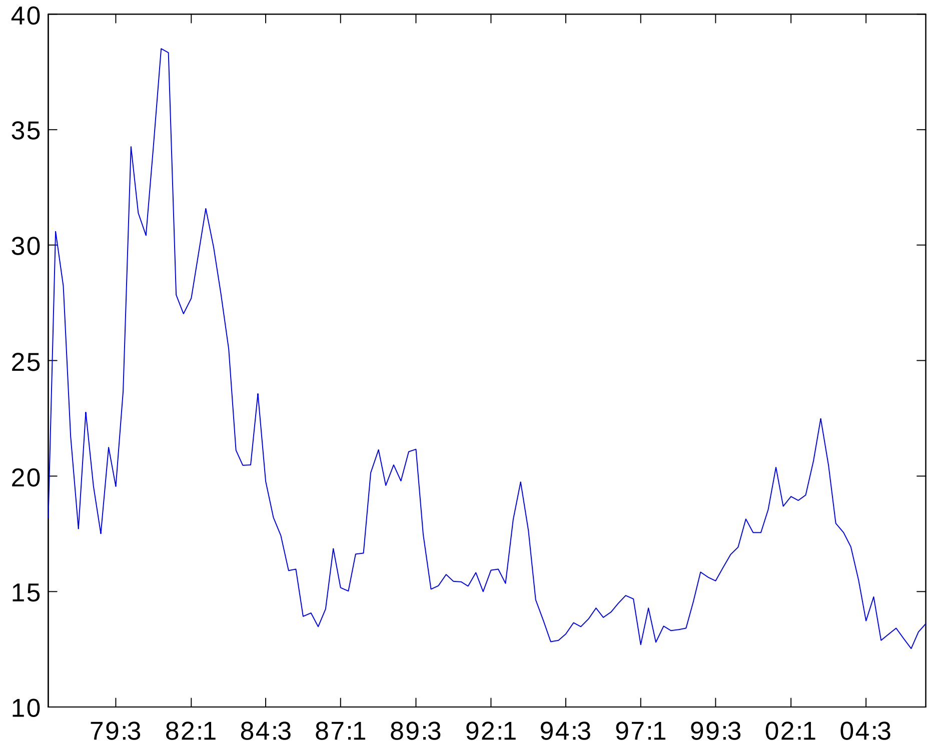

Figure 2: Relative risk aversion, γ/S_t, in the Campbell-Cochrane model, Denmark 1977:1-2006:3.

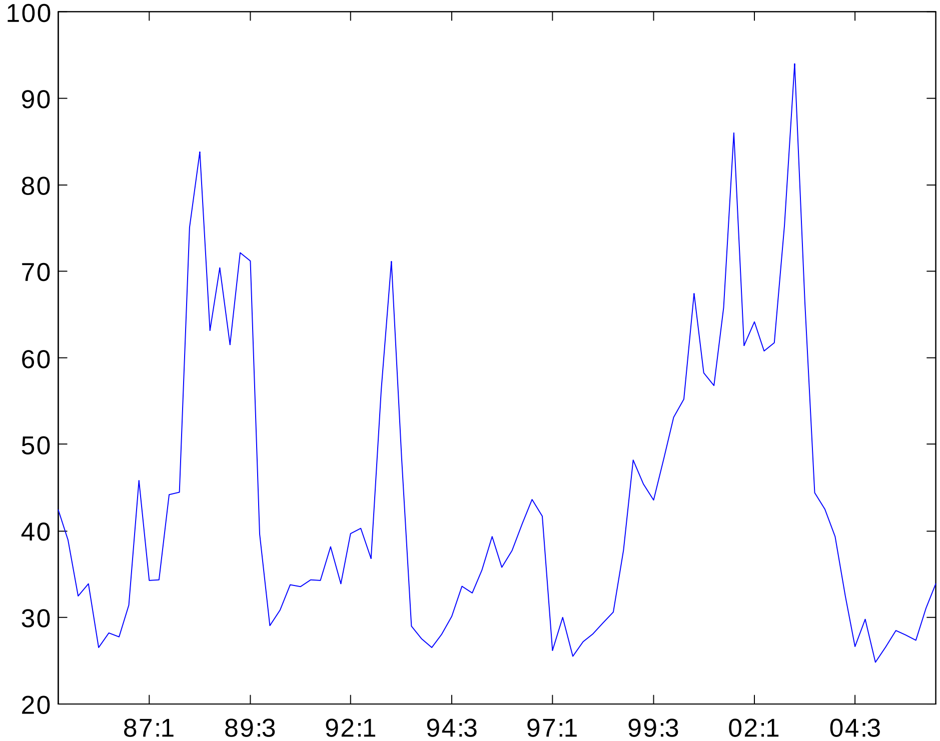

Figure 3: Relative risk aversion, γ/S_t, in the Campbell-Cochrane model, Denmark 1984:4-2006:3.

Chapter 2

Habit persistence: Explaining cross-sectional variation in returns and time-varying expected returns

Habit persistence: Explaining cross-sectional variation in returns and time-varying expected returns*

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Invited for third resubmission to the *Journal of Empirical Finance*.

Abstract

This paper uses an iterated GMM approach to estimate and test the consumption based habit persistence model of Campbell and Cochrane (1999) on the US stock market. The empirical evidence shows that the model is able to explain the size premium, but fails to explain the value premium. Further, the state variable of the model − the surplus consumption ratio − explains counter-cyclical time-varying expected returns on stocks. The model also produces plausible low real risk-free rates despite high relative risk aversion.

Keywords: Campbell-Cochrane model, 25 Fama-French portfolios, GMM, return predictability by surplus consumption ratio.
JEL codes: C32, G12.

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1 Introduction

Within the consumption based asset pricing framework, the habit persistence model of Campbell and Cochrane (1999) model has become one of the leading models in explaining asset pricing behavior. The Campbell-Cochrane model explains a number of stylized facts on the US stock market, including pro-cyclical stock prices, time-varying counter-cyclical expected returns, and it has the ability to explain the equity premium puzzle without facing a risk-free rate puzzle. Campbell and Cochrane and most subsequent applications of their model rely on calibration and simulation exercises and do not engage in formal econometric estimation and testing of the model. They calibrate the structural parameters of the model to match historical means of the risk-free rate and the Sharpe ratio, and then simulate a chosen set of moments which are informally compared to those based on the actual historical data.

Instead of calibrating and simulating the Campbell-Cochrane model, this paper uses an iterated GMM approach to estimate and test the model on the US stock market over the period 1947-2005. The model is estimated in a cross-sectional setting using the 25 Fama and French value and size portfolios, which has not been tried previously, cf. Cochrane (2007). Following the suggestion of Lewellen et al. (2008), the portfolio set is expanded beyond the value and size dimensions by including 10 industry portfolios. The estimation of the model reveals that it has difficulties in explaining the value premium, but provides a great fit of the size premium. The inability of the model to explain the value premium is consistent with recent work by Lettau and Wachter (2007) and Santos and Veronesi (2008). They argue that due to the negative correlation between changes in consumption and the price of risk, the Campbell-Cochrane model is likely to generate a growth premium instead of a value premium.

Besides cross-sectional variation in stock returns, the paper examines whether the model captures time-variation in expected stock returns. The Campbell-Cochrane model has the intuitively appealing implication that expected stock returns vary counter-cyclically over the business cycle. As a result, investors require a higher expected stock return in recession times when consumption is close to habit. The empirical evidence shows that the surplus consumption ratio is significantly negatively related to future excess stock returns, implying that low surplus consumption − when consumption gets close to habit in recession times − predicts high future excess stock returns. These findings are consistent with Li (2001, 2005) who uses Campbell and Cochrane's calibrated parameter values to examine the predictive power of the surplus consumption ratio.

Following the suggested extension in Wachter (2006), the paper allows for a time-varying real risk-free rate in order to generate cyclical variation in interest rates and a nontrivial term structure. Despite high relative risk aversion, the Campbell-Cochrane model implies plausible low values for the real risk-free rate, i.e. the model explains the equity premium puzzle without facing a risk-free rate puzzle. However, the estimated structural parameters of the model imply counterfactual implications for the slope of the yield curve.

Only a few papers engage in formal econometric estimation of the Campbell-Cochrane model. Fillat and Garduño (2005), Garcia et al. (2005), and Tallarini and Zhang (2005) estimate the model on US data.[1] However, they all consider the baseline version of the model with a constant real risk-free rate and only use a small cross section of equities. This paper differs by allowing for time-variation in the real risk-free rate and by testing whether the model accounts for the cross-sectional variation in returns on value, size and industry portfolios, as well as variation of expected returns over time.[2]

The paper relates to Bekaert et al. (2005), Buraschi and Jiltsov (2007), and Wachter (2006) who explore extensions of the Campbell-Cochrane model to explain the full term structure of interest rates. Moreover, Verdelhan (2008) extends the Campbell-Cochrane model to explain the foreign exchange risk premium. Bekaert et al. (2007) consider time-varying counter-cyclical risk aversion as well as economic uncertainty as sources of risk and find that both are important in explaining many asset pricing phenomena.[3]

The paper also relates to the growing body of literature documenting time-varying expected stock returns. Financial variables such as the price-dividend ratio, the term spread on bonds, and the relative interest rate have been documented as forecasters of stock returns, cf. Campbell and Shiller (1988), Fama and French (1989), Campbell (1991), and Hodrick (1992). Fama and French (1989) link the financial forecasting variables to the business cycle and suggest that investors require a higher expected return at a business cycle trough than they do at a business cycle peak. As an extension to these financial forecasting variables, Lettau and Ludvigson (2001) introduce the consumption-wealth ratio, which is a macroeconomic variable that forecasts stock returns. Similarly, the surplus consumption ratio in the Campbell-Cochrane model is a macroeconomic variable that provides a direct linkage between the business cycle and expected stock returns.

The paper is organized as follows. Section 2 introduces the Campbell-Cochrane model, Section 3 describes the empirical methodology, Section 4 describes the data, Section 5 reports the empirical results, and Section 6 concludes.

2 The Campbell-Cochrane model

The utility function of the representative investor is:

$$E_t \sum_{j=0}^{\infty} \delta^j \frac{(C_{t+j} - X_{t+j})^{1-\gamma} - 1}{1-\gamma}. \tag{1}$$

C_t is real consumption, X_t is the external habit level, δ is the impatience parameter, and γ is the utility curvature parameter. Campbell and Cochrane capture the relation

[1] Engsted and Møller (2008) estimate the model outside the US and find that it does not perform better than the simple CRRA model in explaining Danish asset returns.

[2] Chen and Ludvigson (2008) also estimate a habit-based model on the 25 Fama and French portfolios, but they treat the functional form of the habit as unknown.

[3] Bansal and Yaron (2004) develop a long-run risk model and stress the importance of economic uncertainty.

between consumption and habit through the surplus consumption ratio:

$$S_t \equiv \frac{C_t - X_t}{C_t}, \tag{2}$$

and specify the logarithm of the surplus consumption ratio $s_t = \log(S_t)$ as a stationary first-order autoregressive process:

$$s_{t+1} = (1-\phi)\,\bar{s} + \phi s_t + \lambda(s_t)\, v_{t+1}, \tag{3}$$

where $0 < \phi < 1$ is the habit persistence parameter, $\bar{s}$ is the steady state level of s_t, and $\lambda(s_t)$ is the sensitivity function that determines how innovations in consumption growth v_{t+1} influence s_{t+1}. The consumption growth process is given by:

$$\triangle c_{t+1} = g + v_{t+1}, \qquad v_{t+1} \sim niid\left(0, \sigma_c^2\right), \tag{4}$$

where $c_t = \log(C_t)$, and g is the mean consumption growth rate. The sensitivity function $\lambda(s_t)$ is specified as follows:

$$\lambda(s_t) = \left\{ \begin{array}{ll} \frac{1}{\overline{S}}\sqrt{1 - 2(s_t - \bar{s})} - 1, & s_t \leq s_{\max} \\ & \\ 0 & s_t \geq s_{\max} \end{array} \right\}, \tag{5}$$

where

$$\overline{S} = \sigma_c \sqrt{\frac{\gamma}{1 - \phi - B/\gamma}}, \qquad s_{\max} = \bar{s} + \frac{1}{2}(1 - \overline{S}^2), \qquad \bar{s} = \log(\overline{S}).$$

Specifying $\lambda(s_t)$ in this way implies that the real risk-free rate is a linear function of s_t. From the Euler equation,

$$1 = E_t\left[R_{i,t+1} M_{t+1}\right], \tag{6}$$

where $R_{i,t+1}$ is the real gross return on any asset i, and M_{t+1} is the stochastic discount factor:

$$M_{t+1} = \delta \left(\frac{S_{t+1}}{S_t}\frac{C_{t+1}}{C_t}\right)^{-\gamma} = \delta e^{-\gamma\{g + (\phi-1)(s_t - \bar{s}) + [1 + \lambda(s_t)]v_{t+1}\}}, \tag{7}$$

the log real risk-free rate is:

$$r_{f,t+1} = \log\left[\frac{1}{E_t\left[M_{t+1}\right]}\right] \tag{8}$$

$$= -\log(\delta) + \gamma g - \frac{\gamma(1-\phi) - B}{2} - B(s_t - \bar{s}). \tag{9}$$

B governs the cyclicality of the real risk-free rate and the slope of the yield curve. $B > 0$ implies a counter-cyclical real risk-free rate and an upward-sloping yield curve. $B < 0$ implies a pro-cyclical real risk-free rate and a downward-sloping yield curve. $B = 0$ corresponds to the baseline version of the Campbell-Cochrane model with a constant real risk-free rate.

From the Euler equation (6), the expected excess stock return can be stated as:

$$E_t\left(r^e_{i,t+1}\right)+\frac{1}{2}\sigma^2_{i,t}=\gamma\left[1+\lambda\left(s_t\right)\right]\sigma_{ic,t}, \tag{10}$$

where $\frac{1}{2}\sigma^2_{i,t}$ is a Jensen's inequality term. (10) shows that the expected excess stock return is given by the state-dependent price of risk, $\gamma\left[1+\lambda\left(s_t\right)\right]$, times the amount of risk, $\sigma_{ic,t}$ (the conditional covariance between the return on asset i and the consumption growth). Li (2001) finds that $\sigma_{ic,t}$ is close to being constant through time. This lack of time-variation in the amount of risk suggests that time-varying expected excess stock returns are generated entirely by time-variation in the price of risk. Since $\lambda\left(s_t\right)$ is decreasing in s_t, it follows that expected excess stock returns vary counter-cyclically with s_t. Thus, investors require a higher expected excess stock return in recession times when consumption is close to habit.

3 Empirical methodology

The Campbell-Cochrane model is estimated using Hansen's (1982) GMM based on the following moment conditions:

$$\begin{aligned}
0_{N\times 1} &= E\left[R_{t+1}\delta e^{-\gamma\{g+(\phi-1)(s_t-\bar{s})+[1+\lambda(s_t)]v_{t+1}\}}-1\right], && (11)\\
0_{2\times 1} &= E\left[\left(r^e_{t+1}-\alpha-\beta s_{t-1}\right)\left(1\quad s_{t-1}\right)'\right], && (12)\\
0 &= E\left[r_{f,t+1}+\log\left(\delta\right)-\gamma g+\frac{\gamma\left(1-\phi\right)-B}{2}+B\left(s_t-\bar{s}\right)\right], && (13)\\
0 &= E\left[\left(y_{2,t}-r_{f,t+1}\right)+\frac{1}{2}\left[\begin{array}{c}-0.5(\gamma\left(1-\phi\right)-B)\\ +(\gamma\left(1-\phi\right)+B(\phi-2))(s_t-\bar{s})\\ +0.5\sigma^2_c\left[B\lambda\left(s_t\right)-\gamma-\gamma\lambda\left(s_t\right)\right]^2\end{array}\right]\right], && (14)\\
0 &= E\left[\triangle c_{t+1}-g\right], && (15)\\
0 &= E\left[\left(\triangle c_{t+1}-g\right)^2-\sigma^2_c\right]. && (16)
\end{aligned}$$

The moment conditions are chosen in order to examine whether the Campbell-Cochrane model simultaneously explains the cross-sectional variation in returns on stocks, time-varying expected returns on stocks, and the mean values of the real risk-free rate and the real yield spread.

First, using the Euler equation (6) and the stochastic discount factor in (7), I form the moment conditions in (11), where R_{t+1} contains real gross returns on a vector of N assets. The purpose is to examine whether the model is able to explain the cross-sectional variation in mean stock returns on portfolios sorted on size, book-to-market and industry. The returns are not scaled with instruments since this would result in an unmanageable large number of moment conditions relative to the number of sample observations.

Second, in order to examine whether the model captures time-variation in expected stock returns, I form the moment conditions in (12). By approximating equation (10), I examine the linear relationship between the surplus consumption ratio and the future excess stock return.[4] Li (2005) also examines the linear predictive power of the surplus consumption ratio, but uses the calibrated parameter values of Campbell and Cochrane (1999) to generate the surplus consumption ratio. Since the surplus consumption ratio is estimated using Campbell's (2003) beginning of period consumption timing convention, it is lagged twice in (12).

Third, I examine whether the model is able to explain the mean values of the real risk-free rate and the real yield spread. In this way GMM estimates the model parameters based on both stock and bond market data. Using the specification of the real risk-free rate in (8), I obtain the moment condition in (13), and using the analytical solution of the 2-year yield spread shown in the appendix, I obtain the moment condition in (14). I restrict the attention to the 2-year yield spread because it is not possible to find analytical solutions for a higher maturity than 2 years.

Finally, given the random walk model of consumption in (4), I estimate the mean of the consumption growth rate and its volatility based on moment conditions (15) and (16) .

The estimation of the Campbell-Cochrane model is complicated by the fact that the surplus consumption ratio is not observable in the same way as returns and consumption are directly observable. To observe the s_t process, I use $s_0 = \overline{s}$ as starting value of s_t at $t = 0$. Then I choose initial values of the model parameters from which the s_t process can be constructed using (3). By iterating over the model parameters, the GMM procedure simultaneously generates the s_t process and estimates the model parameters.

Defining $g_T(\theta)$ as the sample moment conditions based on T observations, the parameter vector $\theta = (\delta \quad \gamma \quad \phi \quad B \quad g \quad \sigma_c \quad \alpha \quad \beta)'$ is estimated by minimizing the quadratic form:

$$g_T(\theta)' W g_T(\theta), \tag{17}$$

where W is a positive definite weighting matrix. The identity matrix, I, is used to give equal weight to all moment conditions.

Since the chosen weighting matrix is not the efficient Hansen (1982) matrix but the identity matrix, the formula for the covariance matrix of the parameter vector is (cf. Cochrane (2005), chpt. 11):

$$Var(\widehat{\theta}) = \frac{1}{T}(d'Id)^{-1}d'ISId(d'Id)^{-1}, \tag{18}$$

where $d' = \partial g_T(\theta)/\partial\theta$, and the spectral density matrix $S = \sum_{j=-\infty}^{\infty} E[\ g_T(\theta)g_{T-j}(\theta)']$ is computed with the usual Newey and West (1987) estimator with a lag truncation. Similarly, the J-test of overidentifying restrictions is computed based on the general

[4] The GMM moment conditions in (12) correspond to the normal OLS equations used in the return predictability literature.

formula (cf. Cochrane (2005) chpt. 11):

$$J_T = T g_T(\widehat{\theta})' \left[(I - d(d'Id)^{-1}d'I)S(I - Id(d'Id)^{-1}d')\right]^{-1} g_T(\widehat{\theta}). \tag{19}$$

J_T has an asymptotic χ^2 distribution with degrees of freedom equal to the number of overidentifying restrictions. (19) involves the covariance matrix $Var(g_T(\widehat{\theta})) = \frac{1}{T}(I - d(d'Id)^{-1}d'I)S(I - Id(d'Id)^{-1}d')$, which is singular, so it is inverted using the Moore-Penrose pseudo-inversion.

As a measure of the cross-sectional fit, the root mean squared error across the N assets is used:

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\bar{R}_i - E\left[R_i\right]\right)^2}, \tag{20}$$

where $\bar{R}_i$ is the average return on asset i, and $E\left[R_i\right]$ is the model predicted average return on asset i.

As a measure of the predictive power of the surplus consumption ratio, the R^2-statistic is used:

$$R^2 = \frac{Var\left(\beta s_{t-1}\right)}{Var\left(r^e_{t+1}\right)}. \tag{21}$$

4 Data

The Campbell-Cochrane model is estimated on annual post-war data for the period 1947 to 2005. Consumption is measured as expenditures on non-durables and services obtained from the National Income and Product Accounts (NIPA) table 2.3.5. Following the beginning of period timing convention of Campbell (2003), consumption during year t is assumed to take place at the beginning of year t. Nominal consumption is converted to real units using the consumption deflator from NIPA table 2.3.4. Real per capita consumption is obtained using the population numbers in NIPA table 2.1.

The log excess stock return is calculated as the log real return on the value weighted CRSP index including NYSE, AMEX, and NASDAQ firms minus the log real risk-free rate. The log real risk-free rate (defined as the log real 1-year yield) and the log real 2-year yield are based on the McCulloch-Kwon dataset available at J. Huston McCulloch's website and the Fama-Bliss dataset available at the CRSP. The McCulloch-Kwon dataset contains yield data from 1947 to 1990, and this dataset is extended up to 2005 using the Fama-Bliss dataset.

The model is estimated on the 25 Fama and French portfolios sorted on book-to-market and size. Following the suggestion of Lewellen et al. (2008), the portfolio set is expanded beyond the value and size dimensions by including 10 industry portfolios. To check the robustness, the model is estimated on other portfolios as well. Due to potential small sample problems, the model is estimated on a small cross section of the 6 Fama and French portfolios underlying the Fama and French (1993) factors SMB and HML together with 5 industry portfolios. To evaluate the pricing abilities along the value dimension, the model is estimated on 10 decile portfolios sorted on book-to-market together with 5 industry portfolios. To evaluate the pricing abilities along the size dimension, the model is estimated on 10 decile portfolios sorted on size together with 5 industry portfolios. All data are taken from Kenneth French's website where details on the construction of the portfolios are also available. Real returns are obtained using the consumption deflator.

Table 1 shows average real gross returns on the 25 Fama and French portfolios with standard deviations in parentheses. Value firms with high book-to-market ratios have higher returns than growth firms with low book-to-market ratios, and small firms have higher average returns than large firms. In the following, I examine whether the Campbell-Cochrane model can explain these value and size premiums.

5 Empirical results

The Campbell-Cochrane model is estimated based on the GMM procedure described in Section 3. The following subsections present the parameter estimates, the cross-sectional fit, the time-series fit, and some robustness checks.

5.1 Parameter estimates

As seen in Table 2, the structural parameter estimates are economically plausible and quite robust across the different portfolio sets. The estimates of the utility curvature parameter, γ, are statistically significant and range from 3.406 to 4.568 across the different portfolios sets. Based on the estimated parameters for the 25 Fama and French portfolios joint with the 10 industry portfolios, Fig. 1 plots the relative risk aversion defined as γ/S_t. The shaded areas represent the NBER recession dates. Consistently with the findings of Campbell and Cochrane (1999), the steady state relative risk aversion has a high value of 66, and in recession times with low surplus consumption ratios, the relative risk aversion is even higher. The figure shows visible counter-cyclical time-variation in the degree of relative risk aversion. In particular, the relative risk aversion tends to increase during recessions and reaches its highest values near troughs.

The estimates of the impatience parameter, δ, are statistically significant and range from 0.791 to 0.895 across the different portfolios sets. Importantly, δ takes on a value less than 1, implying that the Campbell-Cochrane model has the ability to explain the equity premium puzzle without facing a risk-free rate puzzle. Depending on the portfolio set, the mean value of the model implied real risk-free rate ranges from 2.06% to 2.69%, which is close to the empirical counterpart of 1.86%. Hence, despite the high relative risk aversion, the model provides a reasonable fit of the mean value of the real risk-free rate, and it does this while keeping the impatience parameter below 1.

The interest rate parameter, B, is estimated to be negative, implying a pro-cyclical real risk-free rate and a downward-sloping yield curve. However, the estimates of B are quite imprecise and statistically insignificant. The negative sign on B has counterfactual implications for the slope of the yield curve. In fact, the mean value of the model implied 2-year yield spread ranges from -0.82% to -0.26%, while the empirical counterpart has a positive value of 0.42%. In addition, the negative sign on B implies a pro-cyclical real risk-free rate such that high (low) levels of surplus consumption correspond with high (low) levels of the real risk-free rate. A pro-cyclical real risk-free rate is consistent with the findings of Ang et al. (2008) and Verdelhan (2008), but inconsistent with the findings of Wachter (2006). Given the mixed evidence on the cyclicality of the real risk-free rate, it is perhaps not surprising that the estimates of B are statistically insignificant.

The estimates of the persistence parameter, ϕ, are statistically significant and range from 0.852 to 0.963. The high degree of persistence implies that the habit moves slowly to changes in consumption. Ljungqvist and Uhlig (2003) show that the habit may decrease after an increase in consumption in the Campbell-Cochrane model. Importantly, this problem is not present since the habit moves nonnegatively with consumption throughout the entire sample.

Finally, the return predictability parameters α and β are in most cases statistically significant, implying that the surplus consumption ratio captures time-variation in expected excess stock returns. This will be discussed further below in Section 5.3.

5.2 Cross-sectional variation in expected returns

Now I turn to the cross-sectional fit of the Campbell-Cochrane model. As seen in Table 2, the $RMSE$ of the 25 Fama and French portfolios joint with the 10 industry portfolios has a rather high value of 2.16%. To illustrate the cross-sectional fit further, Fig. 2 plots realized average returns against model predicted average returns. A perfect fit requires that all portfolios lie along the 45 degree line. The figure illustrates that the model captures some of the cross-sectional variation in the returns, but the pricing errors seem economically large. Not surprisingly, the J-test implies that the pricing errors are statistically significantly different from zero. Looking at the results for the small cross section of the 6 Fama and French portfolios joint with the 5 industry portfolios, the $RMSE$ still takes on a high value of 1.84%, and the J-test still rejects the model.

To examine whether the mispricing is along the size dimension or the value dimension, the model is respectively estimated on 10 size portfolios joint with 5 industry portfolios and 10 book-to-market portfolios joint with 5 industry portfolios. The $RMSE$ across the 10 size portfolios and the 5 industry portfolios is 1.03%, and the J-test does not reject the model. Fig. 3 illustrates that the Campbell-Cochrane misprices one of the industry portfolios, but provides a great fit of the size premium. The model intuition is that small stocks earn higher returns than large stocks since they pay of badly in recession times when consumption is close to habit. The $RMSE$ across the 10 book-to-market portfolios and the 5 industry portfolios is 1.39%, and the J-test rejects the model. Fig. 4 illustrates that the Campbell-Cochrane model has great difficulties in explaining the value premium. The model predicted average returns are too high for the low book-to-market portfolios and too low for the high book-to-market portfolios. Consequently, it seems that the Campbell-Cochrane model does well on the size portfolios, but misprices the value portfolios. This evidence relates to the recent work of Lettau and Wachter (2007) and Santos and Veronesi (2008). They show that due to the negative correlation between changes in consumption and the price of risk, the Campbell-Cochrane model implies a higher risk premium on long-duration assets than short-duration assets. If growth stocks are considered to be long-duration assets and value stocks short-duration assets, then the Campbell-Cochrane model would induce a growth premium instead of a value premium. This implies that the Campbell-Cochrane model would do a poor job in explaining the cross-sectional variation in returns on book-to-market portfolios. The empirical evidence presented here is consistent with the findings of Lettau and Wachter (2007) and Santos and Veronesi (2008).

5.3 Time-variation in expected returns

The Campbell-Cochrane model has the intuitively appealing implication that expected stock returns vary counter-cyclically over the business cycle such that investors require a higher expected stock return in recession times when consumption is close to habit. This feature of the model is examined by using the surplus consumption ratio as a predictive variable of the future excess stock return. Table 2 shows that the R^2-statistic varies across the different portfolio sets from 5.84% to 10.70%. The predictive power of the surplus consumption ratio seems to depend on the degree of persistence. When the surplus consumption ratio gets too smooth to capture the variability of future excess stock returns, the R^2-statistic falls. The slope estimate, β, is significantly negative such that low surplus consumption ratios predict high future excess stock returns, i.e. expected excess stock returns are high at business cycle troughs when consumption is close to habit and low at business cycle peaks when consumption is well above habit. These findings are consistent with Li (2001, 2005) who uses Campbell and Cochrane's calibrated parameter values to document the predictive power of the surplus consumption ratio.

Stambaugh (1999) demonstrates that the use of a highly persistent predictive variable with innovations correlated with the innovations in returns may lead to spurious evidence of return predictability in a small sample. The surplus consumption ratio is a stationary but highly persistent predictive variable. However, since the surplus consumption ratio is a pure macroeconomic variable, its innovations have very low correlation with the innovations in returns, which basically eliminates the small sample bias.

5.4 Robustness: restrictions on ϕ

Campbell and Cochrane (1999) and Wachter (2006) fix the persistence parameter ϕ to match the first-order autocorrelation coefficient of the log price-dividend ratio. This is feasible since in the Campbell-Cochrane model the surplus consumption ratio is the only state variable, whereby the log price-dividend ratio inherits its dynamic properties from the log surplus consumption ratio. It is not possible to find an analytical solution of the price-dividend ratio in the Campbell-Cochrane model. Instead, Campbell and Cochrane (1999) and Wachter (2006) solve the price-dividend ratio numerically based on a grid search. Fig. 5 shows the solution of the price-dividend ratio based on the estimated parameter values from column 2 in Table 2.[5] Similarly to the findings of Campbell and Cochrane (1999) and Wachter (2006), the figure illustrates that price-dividend ratio is a nearly linear function of the surplus consumption ratio.

The strict version of the Campbell-Cochrane model predicts a perfect relationship between the surplus consumption ratio and the price-dividend ratio. In real data, however, the two series do not move one for one, but they are closely related. Fig. 6 shows that the two series move together quite closely, but to some extent become disconnected in the first part of the 1990s. Campbell and Cochrane (1999) themselves also find that during the first part of the 1990s – the end of their sample period – the calibrated price-dividend ratio moves in the opposite direction of the actual price-dividend ratio.

Following Campbell and Cochrane (1999) and Wachter (2006), I now fix the persistence parameter ϕ to match the first-order autocorrelation coefficient of the log price-dividend ratio and reestimate the model. The persistence parameter ϕ is set equal to 0.956. The results of this robustness check are shown in Table 3. The table shows that the restriction on ϕ does not influence the parameter estimates much. The table also shows that the restriction on ϕ has very little impact on the cross-sectional fit of the model. The $RMSE$ is basically unchanged across the different portfolio sets. The only change in the results is that the predictive power of the surplus consumption ratio for future excess stock returns is slightly lower.

6 Conclusions

Campbell and Cochrane (1999) show by calibration and simulation that their model explains a number of stylized facts on the US stock market. Instead of calibrating and simulating the model, this paper uses an iterated GMM approach to estimate and test the model. The GMM estimation of the model shows that it is able to explain the size premium, but has great difficulties in explaining the value premium. This is consistent with Lettau and Wachter (2007) and Santos and Veronesi (2008) who show that the Campbell-Cochrane model implies a growth premium due to the negative correlation between changes in consumption and the price of risk. While the model has difficulties in explaining cross-sectional variation in expected stock returns, the model has the ability to explain time-variation in expected stock returns. The model implies that relative risk aversion and expected stock returns are time-varying and counter-cyclical. When

[5] I use a fixed-point solution method (written in MatLab) similar to the one used by Campbell and Cochrane (1999). I use a fine grid with 100 linearly spaced points between 0 and $S_{\max}$, and I also include $S_{\max}$ in the grid. Wachter (2005) shows that the solution of the price-dividend function converges as the grid becomes increasingly fine.

consumption is close to habit in recession times, the relative risk aversion is high and investors require a higher expected risk premium to invest in stocks. Thus, the model provides a direct linkage between time-varying expected stock returns and the business cycle.

7 Appendix

The Euler equation for bonds is:

$$P_{n,t} = E_t \left[P_{n-1,t+1} M_{t+1} \right],$$

where $P_{n,t}$ is the time t real price of a bond with maturity n, and the log real yield is given by $y_{n,t} = -\frac{1}{n} \log P_{n,t}$. It is possible to find closed form solutions for $n = 1$ and $n = 2$, but not for higher values of n. For $n = 1$ the real price is:

$$\begin{aligned} P_{1,t} &= E_t \left[P_{0,t+1} M_{t+1} \right] \\ &= E_t \left[M_{t+1} \right] \\ &= e^{-r_{f,t+1}}. \end{aligned}$$

where $r_{f,t+1}$ is known at time t. For $n = 2$ the real price is:

$$\begin{aligned} P_{2,t} &= E_t \left[P_{1,t+1} M_{t+1} \right] \\ &= E_t \left[e^{-\bar{r}+B(s_{t+1}-\bar{s})} \delta e^{-\gamma\{g+(\phi-1)(s_t-\bar{s})+[1+\lambda(s_t)]v_{t+1}\}} \right] \\ &= \delta e^{-\bar{r}} e^{-\gamma g} e^{(\gamma(1-\phi)+B\phi)(s_t-\bar{s})} e^{0.5\sigma_c^2[B\lambda(s_t)-\gamma-\gamma\lambda(s_t)]^2}, \end{aligned}$$

where $\bar{r} = -\log(\delta) + \gamma g - \frac{\gamma(1-\phi)-B}{2}$. It follows that the yield spread is:

$$\begin{aligned} y_{2,t} - y_{1,t} &= y_{2,t} - r_{f,t+1} \\ &= -\frac{1}{2} \left[\begin{array}{c} -0.5(\gamma(1-\phi) - B) \\ +(\gamma(1-\phi) + B(\phi-2))(s_t - \bar{s}) \\ +0.5\sigma_c^2 \left[B\lambda(s_t) - \gamma - \gamma\lambda(s_t) \right]^2 \end{array} \right]. \end{aligned}$$

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Table 1. The 25 Fama and French portfolios.

	BM1	BM2	BM3	BM4	BM5
S1	1.065 (33.6)	1.125 (30.4)	1.126 (25.0)	1.154 (24.8)	1.175 (27.1)
S2	1.072 (26.8)	1.108 (21.8)	1.135 (22.5)	1.143 (22.3)	1.160 (24.1)
S3	1.082 (22.6)	1.113 (19.8)	1.117 (19.0)	1.136 (22.0)	1.151 (24.0)
S4	1.091 (20.6)	1.092 (17.7)	1.127 (19.2)	1.128 (20.6)	1.140 (23.4)
S5	1.086 (18.7)	1.086 (16.1)	1.101 (16.1)	1.104 (18.5)	1.112 (21.4)

This table reports average real gross returns on the 25 Fama and French portfolios formed on size and book-to-market. Standard deviations in percent are in parentheses.

Table 2. GMM estimation of the Campbell-Cochrane model.

Parameter estimates				
	FF25I10	FF6I5	S10I5	BM10I5
δ	0.791	0.827	0.895	0.855
	(0.120)	(0.153)	(0.130)	(0.093)
γ	3.406	4.295	4.568	3.950
	(1.378)	(2.565)	(2.684)	(1.740)
ϕ	0.852	0.886	0.963	0.919
	(0.174)	(0.244)	(0.262)	(0.238)
B	−0.069	−0.042	−0.211	−0.121
	(0.632)	(1.036)	(1.202)	(0.985)
g	0.023	0.023	0.023	0.023
	(0.002)	(0.002)	(0.002)	(0.002)
σ_c	0.011	0.011	0.011	0.011
	(0.001)	(0.001)	(0.001)	(0.001)
α	−0.328	−0.349	−0.205	−0.275
	(0.155)	(0.168)	(0.142)	(0.153)
β	−0.128	−0.150	−0.108	−0.123
	(0.051)	(0.062)	(0.058)	(0.057)
Model fit				
	FF25I10	FF6I5	S10I5	BM10I5
R^2	10.70	10.15	5.84	8.46
$RMSE$	2.16	1.84	1.03	1.39
J_T	56.28	21.19	10.73	27.14
	(0.007)	(0.012)	(0.634)	(0.012)

This table reports results of estimating the Campbell-Cochrane model on different portfolio sets. FF25I10 is the 25 Fama and French portfolios sorted on book-to-market and size joint with 10 industry portfolios. FF6I5 is the 6 Fama and French portfolios underlying the Fama and French (1993) factors SMB and HML joint with 5 industry portfolios. S10I5 is 10 decile portfolios sorted on size joint with 5 industry portfolios. BM10I5 is 10 decile portfolios sorted on book-to-market joint with 5 industry portfolios. The upper panel of the table reports parameter estimates with standard errors in parentheses. The lower panel of the table reports measures of model fit. J_T is Hansen's (1982) test of overidentifying restrictions calculated as in (19). P-values are in parentheses. $RMSE$ is the root mean squared error calculated as in (20). R^2 is the explanatory power calculated as in (21).

Table 3. GMM estimation of the Campbell-Cochrane model with restrictions on ϕ.

Parameter estimates				
	FF25I10	FF6I5	S10I5	BM10I5
δ	0.864	0.881	0.892	0.880
	(0.111)	(0.148)	(0.145)	(0.128)
γ	3.613	4.727	4.613	3.991
	(1.114)	(1.772)	(1.973)	(1.260)
B	-0.251	-0.219	-0.188	-0.218
	(0.264)	(0.338)	(0.327)	(0.292)
g	0.023	0.023	0.023	0.023
	(0.002)	(0.002)	(0.002)	(0.002)
σ_c	0.011	0.011	0.011	0.011
	(0.001)	(0.001)	(0.001)	(0.001)
α	-0.158	-0.221	-0.227	-0.186
	(0.117)	(0.147)	(0.149)	(0.130)
β	-0.079	-0.113	-0.116	-0.094
	(0.043)	(0.059)	(0.061)	(0.050)
Model fit				
	FF25I10	FF6I5	S10I5	BM10I5
R^2	5.60	6.33	6.39	5.94
$RMSE$	2.19	1.82	1.04	1.38
J_T	56.42	21.60	12.07	26.44
	(0.009)	(0.017)	(0.601)	(0.023)

The persistence parameter is restricted to match the first-order autoregressive coefficient of the log price-dividend ratio. Otherwise see the notes to Table 2.

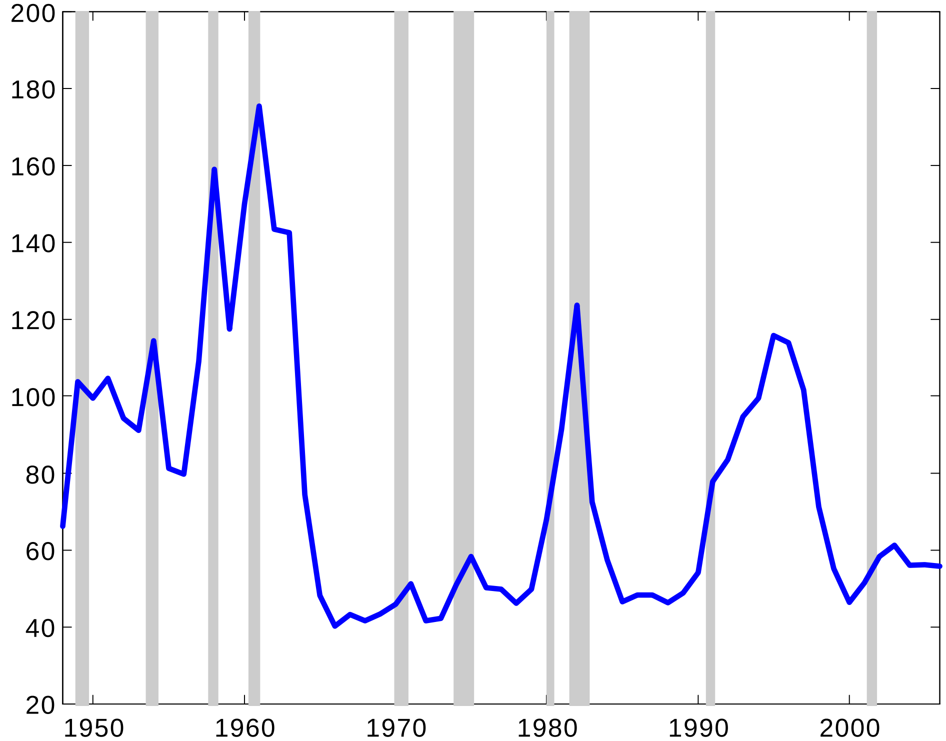

Fig. 1. The relative risk aversion.

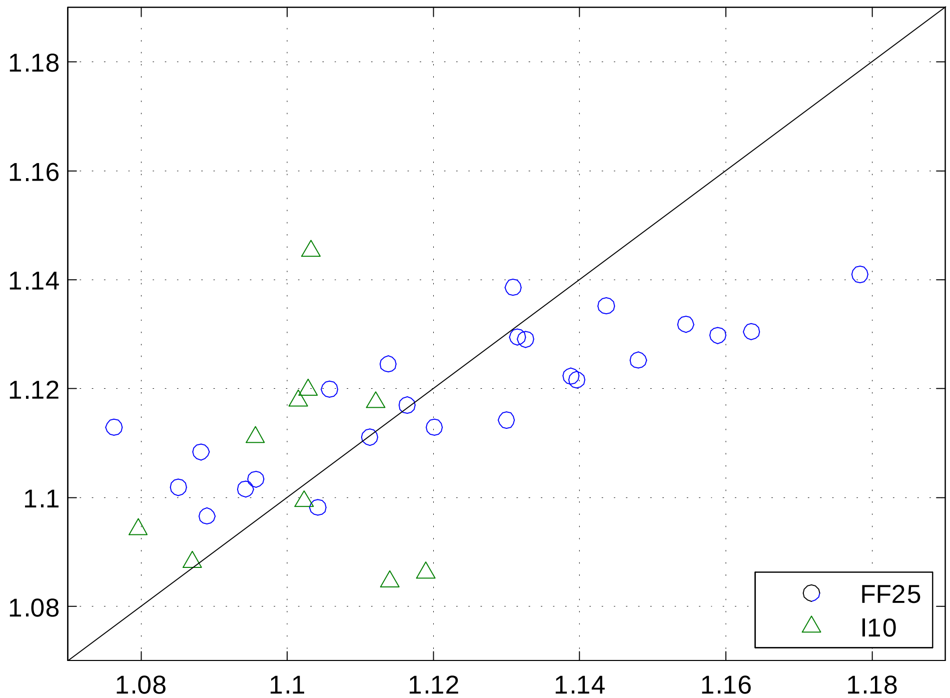

Fig. 2. Realized average returns against model predicted average returns: 25 Fama and French portfolios joint with 10 industry portfolios.

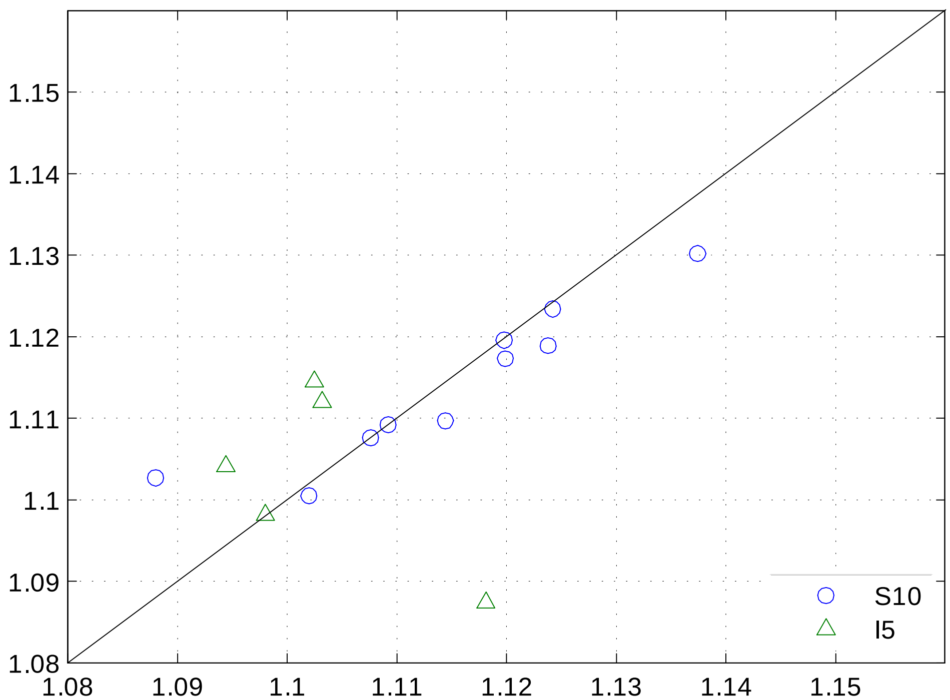

Fig. 3. Realized average returns against model predicted average returns: 10 size portfolios joint with 5 industry portfolios.

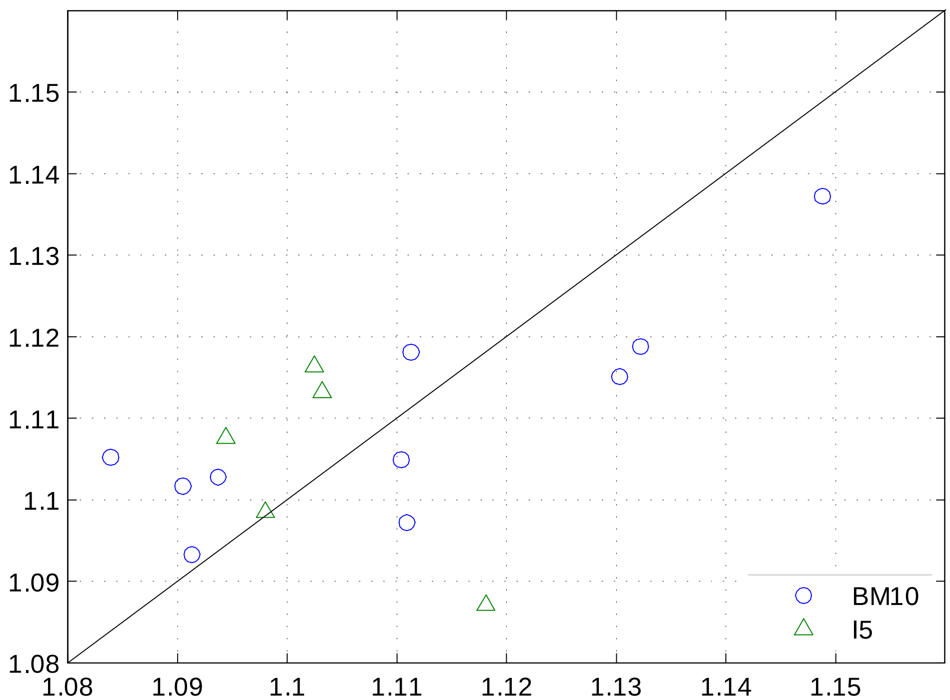

Fig. 4. Realized average returns against model predicted average returns: 10 book-to-market portfolios joint with 5 industry portfolios.

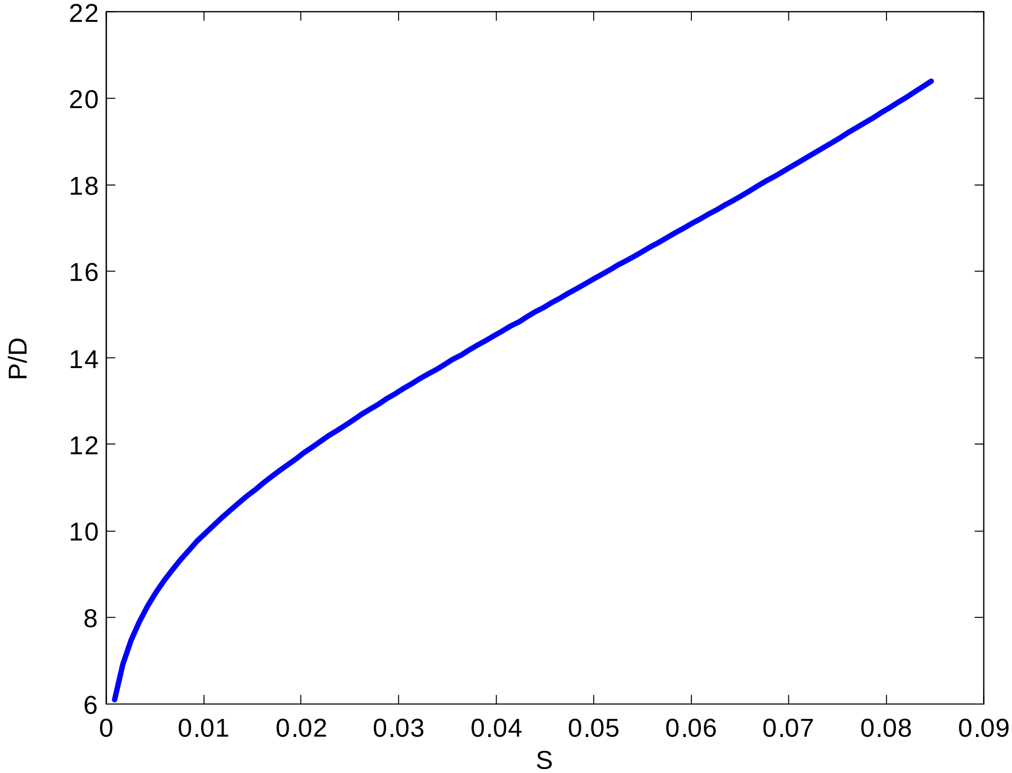

Fig. 5. The price-dividend ratio as a function of the surplus consumption ratio.

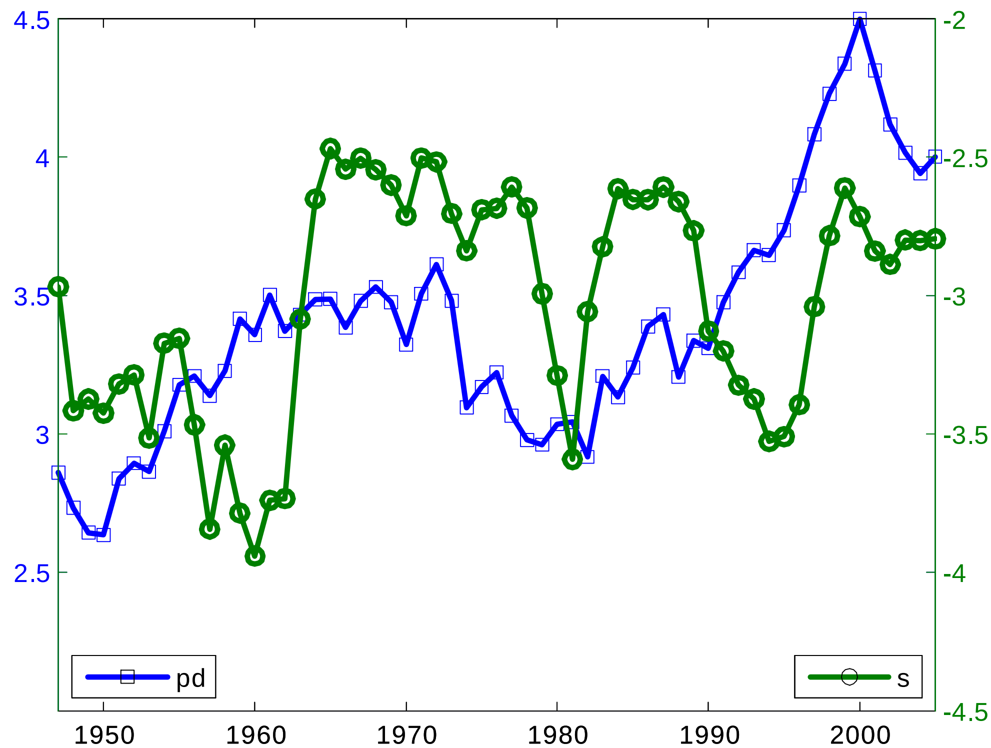

Fig. 6. The price-dividend ratio and the estimated surplus consumption ratio.

Chapter 3

Habit formation, surplus consumption and return predictability: International evidence

Habit formation, surplus consumption and return predictability: International evidence*

Tom Engsted[†] Stuart Hyde[‡] Stig Vinther Møller[§]

Abstract

On an international post World War II dataset, we use an iterated GMM procedure to estimate and test the Campbell-Cochrane (1999) habit formation model with a time-varying risk-free rate. In addition, we analyze the predictive power of the surplus consumption ratio for future stock and bond returns. We find that, although there are important cross-country differences and economically significant pricing errors, for the majority of countries in our sample the model gets empirical support in a variety of different dimensions, including reasonable estimates of risk-free rates. Further, for the majority of countries the surplus consumption ratio captures time-variation in expected returns. Together with the price-dividend ratio, the surplus consumption ratio contains significant information about future stock returns, also during the 1990s. In addition, in most countries the surplus consumption ratio is also a powerful predictor of future bond returns. Thus, the surplus consumption ratio captures time-varying expected returns in both stock and bond markets.

JEL Classification: E21, G12, G15
Keywords: Habit formation, Campbell-Cochrane model, surplus consumption ratio, GMM estimation, pricing errors, return predictability.

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1 Introduction

Rumors of the death of the consumption based capital asset pricing model (C-CAPM) have been widely exaggerated. Throughout the 1980s asset pricing tests of the time separable power utility model provided direct evidence against the ability of the model to capture the behavior of asset returns, although initially Hansen and Singleton (1982) reported results favorable to the model. Indeed, Mehra and Prescott (1985) demonstrated that the return on US equity in excess of the risk-free rate is greater than that which can be explained by the standard consumption based model with a reasonable degree of risk aversion. Lund and Engsted (1996) reported evidence against the standard C-CAPM for a number of European countries. However, the development of alternative approaches which either relax the assumption of separation between states (Epstein and Zin, 1989, 1991) or abandon the time-separability constraint allowing habit formation (Abel, 1990; Constantinides, 1990; Campbell and Cochrane, 1999) breathed new life into the consumption based framework. Subsequent empirical tests have proved to be more supportive, resuscitating the C-CAPM, and leading Chen and Ludvigson (2006) to argue that within the equilibrium consumption based framework, habit formation models are the most promising and successful in describing aggregate stock market behavior.

In this paper we provide fresh international evidence on the pricing and predictability of asset returns. We investigate the performance of the Campbell and Cochrane (1999) habit formation specification compared to the benchmark time-separable power utility model. By adding the surplus consumption ratio to the standard C-CAPM with power utility, Campbell and Cochrane show by calibration that their habit formation model accounts for a number of stylized facts on the US stock market, including time-varying expected returns. The model implies that individuals slowly develop habits for high or low consumption such that the price of risk (risk aversion) becomes time-varying and counter-cyclical: when consumption is well above habit in cyclical upswings, the price of risk is low leading to low expected returns and high asset prices. In contrast, when consumption is close to habit, the price of risk is high leading to high expected returns and low asset prices. However, there is scant evidence using non-US data on the performance of the Campbell-Cochrane model.[1] Indeed, even with US data many studies employ the calibrated values from the original study rather than re-estimating and testing the model empirically.[2] We address this lack of international evidence by estimating the Campbell-Cochrane model using a post World War II sample of eight countries.[3] Following the suggested extension in Wachter (2006), we allow for cyclical variation in expected returns on bonds as well as stocks. We use an iterated GMM approach where, in each iteration,

[1]Engsted and Møller (2008) estimate the Campbell-Cochrane model on Danish data. However, other international evidence (Hyde and Sherif, 2005; Hyde, Cuthbertson and Nitzsche, 2005; Li and Zhong, 2005) employs US calibrated values when applying the model to non-US samples.

[2]For example, Li (2001, 2005) employs the calibrated values rather than estimating the parameters of the model. Tallarini and Zhang (2005), Fillat and Garduño (2005), Garcia, Renault and Semenov (2005) estimate the model with limited success while Møller (2008) estimates and reports supportive evidence for the specification.

[3]We assume that the national economies are closed, whereas Li and Zhong (2005) examine habit formation models in the context of world market integration.

a new time series for the surplus consumption ratio is generated, which is then used to obtain the moment conditions of the model. We find that, although there are important cross-country differences and economically significant pricing errors, for the majority of countries in our sample the Campbell-Cochrane model is not rejected statistically and produces economically plausible parameter values, including reasonable values for the risk-free rate.

Next, using the same international sample, we provide evidence on the power of the surplus consumption ratio as a predictor for returns. It is a well established fact of empirical finance that stock returns are predictable. Evidence that aggregate valuation ratios such as the price-dividend ratio and price-earnings ratio or financial/monetary variables such as the term spread or relative interest rate can account for the time-variation in expected returns is provided by Fama and French (1988, 1989), Campbell and Shiller (1988a, 1988b), Campbell (1991) and Hodrick (1992). However, the documented inability of the price-dividend ratio to capture time-variation in expected returns during the 1990s has resulted in the emergence of a number of new predictor variables. For instance, Boudoukh et al. (2007) argue that the net payout ratio is more appropriate than the price-dividend ratio since it captures more accurately the extent of distributing cash to shareholders and show that it has greater ability to predict future returns.

Alternatively, many of these new predictors are linked to macroeconomic factors such as consumption, labor income, and output demonstrating the strong links between the financial and real sectors of the economy. From the representative consumer's budget constraint, Lettau and Ludvigson (2001a, 2001b) show that the consumption-wealth ratio, *cay*, contains information about future returns.[4] Further, Santos and Veronesi (2006) introduce a labor income-consumption ratio which predicts US returns well both independently and in addition to the price-dividend ratio. Julliard (2007) argues that the consumption-wealth ratio should be combined with expected future labor income growth to predict future US returns, demonstrating not only that expected changes in labor income have high predictive power for future returns but that together the consumption-wealth ratio and expected changes in labor income explain much of the variation in the cross-section of returns. Using the calibrated parameter values from Campbell and Cochrane (1999), Li (2005) examines the forecasting power of the surplus consumption ratio in addition to the consumption-wealth ratio for US stock returns, documenting that the surplus consumption ratio contains incremental information not incorporated in the consumption-wealth ratio. Additionally, Møller (2008) provides evidence that the estimated surplus consumption ratio has strong predictive abilities. Focusing on output rather than consumption, Rangvid (2006) advocates the adoption of a price-output variable and provides evidence that it explains more of the time-variation in expected returns than either the price-earnings or price-dividend ratios and performs as well as *cay* for US returns. Rangvid (2006) also provides evidence on an international sample, showing that the ability of the price-output ratio to predict returns is robust outside the US.

[4] The consumption-wealth ratio is measured as the residuals of the cointegrating relationship between log consumption, log asset wealth and log labor income.

Time-varying expected returns have also been documented on the bond market. Fama and Bliss (1987), Fama and French (1989), Campbell and Shiller (1991), and Cochrane and Piazzesi (2005) find that excess bond returns are forecastable by financial variables such as forward spreads and term spreads. New work by Ludvigson and Ng (2007) link time-varying expected bond returns together with macroeconomics. They form macro factors based on principal components of a large number of macroeconomic series and find that these macro factors predict counter-cyclical movements in excess bond returns. Buraschi and Jiltsov (2007) also develop a model which links bond returns with underlying macro factors, allowing for time-variation in interest rates. They highlight the ability of habit formation to describe important characteristics of the US nominal term structure.

Here we examine bond and stock return predictability using annual data over the post World War II period up to 2004. Given the debate regarding disappearing predictability in the 1990s we also report results for a shorter sample that ends in 1990. We demonstrate that the surplus consumption ratio significantly predicts future stock returns in the majority of countries. To check the robustness of this result we examine bivariate regressions with alternative predictors. We consider two of the traditional return predictors; the price-dividend ratio and the term spread. We show that the ability of the surplus consumption ratio to predict future returns is not diminished by including these additional predictors. In particular, the surplus consumption ratio together with the price-dividend ratio contains significant information about future stock returns in most countries and, interestingly, the predictive power remains statistically significant during the 1990s. Furthermore, the surplus consumption ratio is shown to be a powerful predictor of future bond returns in most countries. This is also robust to the inclusion of alternative predictor variables. Our findings therefore support the extended version of the Campbell-Cochrane model in which expected returns on stocks and bonds move counter-cyclically with the surplus consumption ratio. In general, the countries in which the surplus consumption ratio is a useful return predictor are the same that get most empirical support in the GMM estimations. Thus, our analysis implies that, although there are clear cross-country differences, for many countries several of the implications of the Campbell-Cochrane model are supported empirically.

The remainder of the paper is set out as follows. Section 2 describes the consumption based asset pricing models. Section 3 provides details on the GMM estimation of the models while section 4 gives details of the data and section 5 reports the empirical results. Section 6 concludes.

2 The models

In the consumption based asset pricing framework the representative agent makes consumption and investment decisions in order to maximize expected lifetime utility. This maximization problem implies the following first order condition that all correctly priced assets must satisfy:

$$E_t\left[R_{i,t+1}M_{t+1}\right] = 1. \tag{1}$$

$R_{i,t+1}$ is the real gross return of investing in asset i at time t and selling it at time $t+1$, and M_{t+1} is the stochastic discount factor:

$$M_{t+1} = \delta\frac{U'\left(C_{t+1}\right)}{U'\left(C_t\right)}, \tag{2}$$

where δ is the subjective time discount factor, C_t is real consumption, and $U'(\cdot)$ is marginal utility. To observe the stochastic discount factor the representative agent's utility function has to be specified. With standard CRRA utility,

$$U\left(C_t\right) = \frac{C_t^{1-\gamma}-1}{1-\gamma}, \tag{3}$$

where γ is the coefficient of relative risk aversion, the stochastic discount factor equals:

$$M_{t+1} = \delta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}. \tag{4}$$

Inserting (4) in (1), the log expected excess return can be stated approximately as:

$$E_t\left[r_{i,t+1}\right] - r_{f,t+1} + \frac{1}{2}\sigma_{i,t}^2 = \gamma Cov_t\left[r_{i,t+1}, \log\left(\frac{C_{t+1}}{C_t}\right)\right], \tag{5}$$

where $r_{i,t+1} \equiv \log\left(R_{i,t+1}\right)$, $\sigma_{i,t}^2$ is the variance of $r_{i,t}$, and $r_{f,t+1}$ is the log risk-free rate.

Empirically the consumption based CRRA model has run into trouble because the covariance between stock returns and consumption growth is too low to explain the historically high excess return on stocks, unless the degree of risk aversion γ is extremely high. In addition, even if an extremely high value of γ is accepted, it would imply an implausibly high real risk-free rate. Assuming that returns and consumption growth are *iid* and jointly lognormally distributed, the log real risk-free rate is:

$$r_{f,t+1} = -\log(\delta) + \gamma g - \frac{1}{2}\gamma^2\sigma_c^2, \tag{6}$$

where g and σ_c^2 are mean and variance of consumption growth. Thus, the real risk-free rate is very sensitive to the mean consumption growth rate for high values of γ. Consequently, the standard specification cannot solve the equity premium puzzle without running into a risk-free rate puzzle. Furthermore, the CRRA model also faces a return predictability puzzle since the consumption covariance with returns is too smooth to account for time-variation in expected returns, cf. Ferson and Harvey (1993), Li (2001) among others. New consumption based models have been developed to solve these puzzles. In what follows we present the habit persistence model of Campbell and Cochrane (1999) with the extension suggested by Wachter (2006) to allow for a time-varying risk-free rate in order to generate a nontrivial term structure.[5]

[5] The working paper version of Campbell and Cochrane (1999) [Campbell and Cochrane (1995)] also contains an analysis with a time-varying risk-free rate.

The utility function in the Campbell-Cochrane model is specified as follows:

$$U(C_t, X_t) = \frac{(C_t - X_t)^{1-\gamma} - 1}{1-\gamma}, \quad C_t > X_t, \tag{7}$$

where X_t is an external habit level of consumption. With this specification the surplus consumption ratio, $S_t = \frac{C_t - X_t}{C_t}$, becomes a business cycle variable that is high in cyclical upswings and low in cyclical downturns such that relative risk aversion, γ/S_t, moves counter-cyclically. Rather than specifying a process for the habit, X_t, Campbell and Cochrane specify a process for the log surplus consumption ratio, $s_t = \log(S_t)$, to ensure that consumption is above habit at all times. The log surplus consumption ratio is modeled as a stationary first-order autoregressive process:

$$s_{t+1} = (1-\phi)\,\bar{s} + \phi s_t + \lambda(s_t)\,v_{t+1}, \tag{8}$$

where $0 < \phi < 1$ is the habit persistence parameter, $\bar{s}$ is the steady state level of s_t, and $\lambda(s_t)$ is the sensitivity function that determines how innovations in consumption growth v_{t+1} influence s_{t+1}. The consumption growth process is given by:

$$\triangle c_{t+1} = g + v_{t+1}, \qquad v_{t+1} \sim niid\left(0, \sigma_c^2\right), \tag{9}$$

where $c_t = \log(C_t)$, and g is the mean consumption growth rate, as in (6). The sensitivity function $\lambda(s_t)$ is specified as follows:

$$\lambda(s_t) = \left\{ \begin{array}{cc} \frac{1}{\overline{S}}\sqrt{1 - 2(s_t - \bar{s})} - 1 & \text{if} \quad s_t \leq s_{\max} \\ 0 & \text{otherwise} \end{array} \right\}, \tag{10}$$

where

$$\overline{S} = \sqrt{\frac{\sigma_c^2 \gamma}{1 - \phi - B/\gamma}}, \qquad s_{\max} = \bar{s} + \frac{1}{2}(1 - \overline{S}^2), \qquad \bar{s} = \log(\overline{S}).$$

Specifying $\lambda(s_t)$ in this way implies that the real risk-free rate is a linear function of s_t. Using the stochastic discount factor,

$$M_{t+1} = \delta\left(\frac{S_{t+1}}{S_t}\frac{C_{t+1}}{C_t}\right)^{-\gamma}, \tag{11}$$

the log real risk-free rate can be derived as:

$$r_{f,t+1} = \bar{r} - B s_t^e, \tag{12}$$

where $s_t^e = (s_t - \bar{s})$ and $\bar{r} = -\log(\delta) + \gamma g - \frac{\gamma(1-\phi) - B}{2}$ is the average real risk-free rate. B governs the cyclicality of real risk-free rates; if $B > 0$, the real risk-free rate moves counter-cyclically; if $B < 0$, the real risk-free rate moves pro-cyclically; and if $B = 0$, we obtain the baseline Campbell-Cochrane model with a constant real risk-free rate.

The log expected excess return is given by the following:

$$E_t\left[r_{i,t+1}\right] - r_{f,t+1} + \frac{1}{2}\sigma_{i,t}^2 = \gamma\left[1 + \lambda(s_t)\right] Cov_t\left[r_{i,t+1}, \log\left(\frac{C_{t+1}}{C_t}\right)\right], \tag{13}$$

which states that expected excess returns move counter-cyclically with s_t since $\lambda(s_t)$ is decreasing in s_t. When surplus consumption falls in cyclical downturns, investors require a higher expected return and vice versa. Thus, in contrast to the CRRA model, the Campbell-Cochrane model accounts for counter-cyclical time-variation in expected returns despite smooth consumption covariance with returns.

Campbell and Cochrane (1999) calibrate their model with a constant risk-free rate (i.e. $B = 0$) by choosing parameter values to match certain moments of postwar US data. They find that the model explains a number of stylized facts for the US stock market, including stock return predictability. Wachter (2006) also calibrates the model by matching certain moments of postwar US data, but now with a time-varying risk-free rate (i.e. $B \neq 0$). Her focus is on explaining nominal bond yields, and she finds that the model accounts for many of the observed features of the term structure of interest rates. Instead of doing such calibration exercises, we use an iterated GMM approach to estimate all the parameters of the model. In the next section we describe the econometric approach.

3 GMM estimation of the models

We estimate the CRRA model and the Campbell-Cochrane model using the GMM technique of Hansen (1982). From the first order condition (1), we estimate the CRRA model based on the following moment conditions:

$$0 = E\left[\left(\left(R_{S,t+1} - R_{f,t+1}\right)\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\right) Z_t\right], \tag{14}$$

$$0 = E\left[\left(\left(R_{LB,t+1} - R_{f,t+1}\right)\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\right) Z_t\right], \tag{15}$$

$$0 = E\left[\left(\left(R_{f,t+1}\right)\delta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} - 1\right) Z_t\right]. \tag{16}$$

$R_{S,t+1}$ is the real gross returns on stocks, $R_{LB,t+1}$ is the real gross return on long-term bonds, and $R_{f,t+1}$ is the real risk-free rate proxied by the real gross return on 3-month T-bills, i.e. the test assets in (14) to (16) are the excess stock return, the excess bond return, and the ex post real T-bill rate. Following Kocherlakota (1996), among others, we include a moment condition for the real T-bill rate as well as the excess stock return in order to focus on both the equity premium puzzle and the risk-free rate puzzle. Since the expectation in (1) is conditional on information available at time t, we use a vector of instruments Z_t observable at time t to capture the relevant conditioning information. In addition to a constant, the instrument vector Z_t contains the log price-dividend ratio.[6]

[6] Campbell and Shiller (1988b) and Fama and French (1988, 1989) document that the price-dividend ratio is a strong return predictor, suggesting that the price-dividend ratio is a useful instrument in the GMM estimations. We have also used an expanded set of instruments, but in general different combinations of instruments do not change the main results.

Defining $g_T(\theta_{CRRA})$ as the sample moment conditions of (14) to (16) based on T observations, we estimate the parameter vector $\theta_{CRRA} = (\delta, \gamma)'$ by minimizing the quadratic form, $g_T(\theta_{CRRA})' W g_T(\theta_{CRRA})$, where W is a positive definite weighting matrix. As weighting matrix, W, we use the identity matrix, I, to give equal weight to all moment conditions, following Cochrane's (2005) suggestion.

The Campbell-Cochrane model is estimated based on the following moment conditions:

$$0 = E\left[\left(\left(R_{S,t+1} - R_{f,t+1}\right)\left(\frac{S_{t+1}}{S_t}\frac{C_{t+1}}{C_t}\right)^{-\gamma}\right) Z_t\right], \tag{17}$$

$$0 = E\left[\left(\left(R_{LB,t+1} - R_{f,t+1}\right)\left(\frac{S_{t+1}}{S_t}\frac{C_{t+1}}{C_t}\right)^{-\gamma}\right) Z_t\right], \tag{18}$$

$$0 = E\left[\left(r_{f,t+1} - \left(-\log(\delta) + \gamma g - \frac{\gamma(1-\phi) - B}{2}\right) + B s_t^e\right)\left(1 \quad s_t^e\right)'\right], \tag{19}$$

$$0 = E\left[\triangle c_{t+1} - g\right], \tag{20}$$

$$0 = E\left[\left(\triangle c_{t+1} - g\right)^2 - \sigma_c^2\right], \tag{21}$$

$$0 = E\left[\left(pd_{t+1} - \alpha - \phi pd_t\right)\left(1 \quad pd_t\right)'\right]. \tag{22}$$

As for the CRRA model, we estimate the Campbell-Cochrane model using the excess stock return, the excess bond return, and the real T-bill rate as test assets, which leads to the moment conditions in (17), (18), and (19). Following the specification in (12), the moment conditions in (19) impose the restriction that the average log real risk-free rate is given by $\overline{r} = -\log(\delta) + \gamma g - \frac{\gamma(1-\phi)-B}{2}$. From the random walk model of consumption in (9), we obtain the moment conditions in (20) and (21). Since consumption follows a random walk, the natural choice of instruments is a constant only. Following Campbell and Cochrane (1999) and Wachter (2006), we estimate the persistence parameter, ϕ, as the first-order autoregressive coefficient of the log price-dividend ratio, pd_t, which leads to the moment conditions in (22). This is feasible since in the model the surplus consumption ratio is the only state variable, whereby the price-dividend ratio inherits the dynamic properties from the surplus consumption ratio.

The estimation of the Campbell-Cochrane model is complicated by the fact that the surplus consumption ratio is not observable in the same way as returns and consumption are directly observable. To observe the s_t process, we use $s_0 = \overline{s}$ as starting value of s_t at $t = 0$. Then we choose starting values of the model parameters and obtain the s_t process recursively using (8).[7] We use an iterated GMM approach where, in each iteration, a new time series for the surplus consumption ratio is generated, which is then used to obtain the moment conditions of the model. The iterations are continued until convergence of all model parameters.

Defining $g_T(\theta_{CC})$ as the sample moment conditions based on T observations, the parameter vector $\theta_{CC} = (\delta, \gamma, g, \sigma_c^2, \alpha, \phi, B)'$ is estimated by minimizing the quadratic

[7] We use the calibrated parameter values in Campbell and Cochrane (1999) as starting values.

form, $g_T(\theta_{CC})' W g_T(\theta_{CC})$. As for the CRRA model, we use the identity matrix as weighting matrix to give equal weight to all moment conditions.

The use of a fixed and model-independent weighting matrix such as the identity matrix allows us to compare the magnitude of the estimated pricing errors across different models. Since the chosen weighting matrix is not the efficient Hansen (1982) matrix but the identity matrix, I, the formula for the covariance matrix of the parameter vector is (cf. Cochrane (2005), chpt. 11):

$$Var(\widehat{\theta}) = \frac{1}{T}(d'Id)^{-1}d'ISId(d'Id)^{-1}, \tag{23}$$

where $d = \partial g_T(\theta)/\partial\theta'$, and the spectral density matrix $S = \sum_{j=-\infty}^{\infty} E[\, g_T(\theta)g_{T-j}(\theta)']$ is estimated using the Newey and West (1987) estimator with a Bartlett kernel. To evaluate the model fit we use Hansen's J-test of overidentifying restrictions:

$$J_T = T g_T(\widehat{\theta})' \left[Var(g_T(\widehat{\theta}))\right]^{-1} g_T(\widehat{\theta}), \tag{24}$$

where $Var(g_T(\widehat{\theta})) = \frac{1}{T}(I - d(d'Id)^{-1}d'I)S(I - d(d'Id)^{-1}d'I)'$ is singular and hence inverted using the Moore-Penrose pseudo-inversion. J_T has an asymptotic χ^2 distribution with degrees of freedom equal to the number of moment conditions minus the number of parameters.

The J-test provides a statistical test whether the moment conditions for a given model are significantly different from zero. As a supplement to the J-test we use Hansen and Jagannathan's (1997) distance measure that provides a useful *economic* measure of the model fit. The Hansen-Jagannathan distance is given by:

$$HJ = \left[E(M_{t+1}(\theta)R_{t+1} - 1)'(E(R_{t+1}R'_{t+1}))^{-1}E(M_{t+1}(\theta)R_{t+1} - 1)\right]^{\frac{1}{2}}. \tag{25}$$

HJ gives the minimum distance from the stochastic discount factor of a given model to the set of true stochastic discount factors that price assets correctly. It is a measure of the maximum percentage pricing error associated with a given model and hence gives a comparable measure of model misspecification.

4 Data and summary statistics

We study the following eight countries: Belgium, Canada, France, Germany, Italy, Sweden, the UK, and the US. We select these countries on the basis of data availability. Our samples of annual observations begin between 1949 and 1953, depending on the country, and end in 2004. We measure consumption as private total consumption from IMF International Financial Statistics and adopt the Campbell (2003) beginning of period timing assumption that consumption during year t takes place at the beginning of year t. Nominal consumption is converted to real units using the consumer prices indices from IMF International Financial Statistics. Real per capita consumption is constructed using the population numbers from Global Financial Data. Returns on stocks, long-term (10-year) bonds, and 3-month T-bills are obtained from Global Financial Data. All return series are deflated using consumer price indices from IMF International Financial Statistics. We compare the ability of the surplus consumption ratio to predict stock and bond returns with alternative return predictors. As alternative return predictors, we use the price-dividend ratio and the term spread between long-term bonds and 3-month T-bills. Both predictors are obtained from Global Financial Data.

Table 1 provides summary statistics for the real gross return on equity, long-term bonds and 3-month T-bills in each of the eight countries. The reported statistics are consistent with the stylized facts for international equity and bond markets over the past half century. Mean real stock returns range from 7.5% (Canada) to 11.2% (Sweden) with the return on 3-month T-bills between 1.2% (US) and 3.1% (Belgium) implying that the average equity premium ranges from 4.3% in Belgium to 9.7% in Sweden. The average long-term real bond returns are between 2.1% (Sweden) and 4.6% (Germany).

5 Empirical results

In the following sub-sections we report parameter estimates of the CRRA and Campbell-Cochrane models, statistical tests of the models' implied overidentifying restrictions, estimates of Hansen-Jagannathan pricing errors and, finally, predictive return regressions based on the surplus consumption ratio.

5.1 GMM estimates and tests

Table 2 reports GMM results for the CRRA and Campbell-Cochrane models for each of the eight countries in our sample. The CRRA model is estimated on moment conditions (14) to (16), and the Campbell-Cochrane model is estimated on moment conditions (17) to (22).

For the CRRA model, the estimates of the constant relative risk aversion, γ, have the correct sign, but the estimates tend to be quite imprecise. Only two of them are statistically significant at a 5% level. Consistent with other studies, the γ estimates are extremely high. Furthermore, the estimates of the subjective time discount factor, δ, are greater than one, which shows that the time-separable power utility model is unable to solve the equity premium puzzle without facing a risk-free rate puzzle. Although the estimates of δ and γ seem economically implausible, the J-test of overidentifying restrictions does not statistically reject the model at conventional significance levels. However, this may be due to low power of the test.[8]

[8] The identity matrix is used as weighting matrix in the estimations. If instead the statistically optimal weighting matrix is used (the inverse of the covariance matrix of the sample orthogonality conditions), the results are qualitatively similar to the results for the CRRA model in Table 2, except that the parameters are estimated more precisely, as expected. For the Campbell-Cochrane model,

The Campbell-Cochrane model resembles the CRRA model in terms of an overall high level of relative risk aversion. The steady state relative risk aversion, $\gamma/\bar{S}$, varies across countries from 25 in Belgium to 75 in France. Unlike the CRRA model, though, the Campbell-Cochrane model has the important ability to escape the risk-free rate puzzle. In fact, the estimates of the time discount factor, δ, are less than 1 in all countries. For some countries, however, the δ estimates seem too low to be economically reasonable; this is in particular the case for France. Despite the cross-country differences, the J-test does not reject the Campbell-Cochrane model in any country. Again, this may be due to low power of the test. The estimates of the persistence parameter, ϕ, range from 0.74 in Germany to 0.95 in the US, implying highly persistent but stationary log price-dividend ratio's and, hence, surplus consumption ratio's. The mean consumption growth rate ranges from around 2% in most of the countries to around 3% in two of the countries.

Table 2 also reports the implied risk-free rate (r_f) for the CRRA model, and the estimated mean risk-free rate ($\overline{r}$) and cyclicality parameter (B) in the Campbell-Cochrane model. For the CRRA model the implied r_f varies considerably across countries, ranging from -52.72% in Germany to 10.39% in Sweden. For the Campbell-Cochrane model, reasonable $\overline{r}$ values are obtained for all countries, ranging from 1.24% in the US to 3.00% in Belgium. Except for the UK, all B estimates are positive, implying a counter-cyclical risk-free rate, an upward-sloping yield curve, and positive bond risk-premia. Note, however, that some of the B estimates are statistically insignificant, or only marginally significant.

Finally, Table 2 reports estimates of the Hansen and Jagannathan (1997) distance for the CRRA and Campbell-Cochrane models using the preference parameter estimates from the GMM estimation. The Hansen-Jagannathan distance provides the maximum percentage pricing error associated with a given model and is suitable for direct model comparisons. The distances are estimated using the excess stock return, the excess bond return and the real T-bill rate. Although the J-test does not statistically reject the models, the magnitude of the distances are economically significant for both models. Moreover, the CRRA model has lower distances than the Campbell-Cochrane model. Since we use $W = I$ and not $W = E(R_{t+1}R'_{t+1})^{-1}$ as weighting matrix, we do not estimate the parameters of the models to minimize the Hansen-Jagannathan distance, but still it is surprising that the Campbell-Cochrane model performs worse than the CRRA model.

The evidence in this subsection gives mixed results regarding the consumption based framework's ability to explain international asset returns. The CRRA and Campbell-Cochrane models are not rejected statistically at conventional significance levels, but the Hansen-Jagannathan measure does indicate economically large pricing errors also for the Campbell-Cochrane model. In the remaining part of the paper we investigate the Campbell-Cochrane model in another dimension, by analyzing whether the surplus consumption ratio contains useful information about future returns.

using the statistically optimal weighting matrix does not lead to convergence with positive values of γ in the GMM iterations, a problem also faced by Garcia, Renault and Semenov (2005). Thus, we restrict attention to the case with $W = I$.

5.2 Time-varying expected returns

The Campbell-Cochrane model implies that the surplus consumption ratio captures time-varying expected returns. When consumption is well above habit in cyclical upswings, relative risk aversion and expected returns on risky assets are low. In contrast, when consumption is close to habit in cyclical downturns, relative risk aversion and expected returns on risky assets are high. To test this feature of the Campbell-Cochrane model, we run predictability regressions of excess returns on stocks and bonds with the surplus consumption ratio as predictor.[9] Moreover, since the Campbell-Cochrane model implies that the surplus consumption ratio is the only state variable in the economy, it should capture all relevant information about time-varying expected returns. We test this implication of the model using bivariate predictability regressions with the surplus consumption ratio and alternative return predictors. We use two traditional benchmark predictors of stock and bond returns; the price-dividend ratio, pd_t (Campbell and Shiller, 1988; Fama and French, 1989; Hodrick, 1992), and the term spread, $TERM_t$ (Fama and French, 1989; Campbell and Shiller, 1991).

Figure 1 plots the surplus consumption ratio, s_t (based on the estimates from Table 2), and the price-dividend ratio, pd_t. Both series are in logs and standardized to have mean 0 and variance 1. The price-dividend ratio is used as proxy variable for the surplus consumption ratio to estimate the persistence parameter ϕ and the Campbell-Cochrane model therefore implies a one for one relationship between these two variables. The figure shows that the two series tend to move together, but in many countries they become less connected from the beginning of the 1990s and onwards. In fact, up to 1990 s_t and pd_t are positively correlated, but the correlations are reduced by including data up to 2004. For example, for the US the correlation between s_t and pd_t up to 1990 is 0.55 which is reduced to 0.01 for the whole sample. This indicates a deteriorating performance of the Campbell-Cochrane model in recent years (as already anticipated by Campbell and Cochrane (1999) themselves who note a poor fit for their model at the end of their sample period). Thus, it will be interesting to see whether the predictive ability of the surplus consumption ratio deteriorates by including data after 1990.[10]

5.2.1 Stock return predictability

To examine whether the surplus consumption ratio is able to track time-varying expected stock returns, we run predictability regressions of 1-year ahead log excess stock returns, $r_{S,t+1} - r_{f,t+1}$, with the log surplus consumption ratio, s_{t-1}, as predictor: $r_{S,t+1} - r_{f,t+1} = \alpha + \beta_S s_{t-1} + e_{t+1}$. The log surplus consumption ratio is lagged twice relative to returns because we use Campbell's (2003) beginning of period consumption timing convention.

[9] Following Li (2005), we examine the linear relationship between the surplus consumption ratio and future excess returns. This allows direct comparison with alternative return predictors.

[10] As a further check on the time-series movements of the surplus consumption ratio, we have correlated it with Hodrick-Prescott filtered GDP. In each of the eight countries the correlation is positive, and strongest for Belgium, Canada, Sweden, and the US (details are available upon request).

Table 3 reports standard Newey and West (1987) corrected t-statistics and adjusted R^2-statistics.[11] The upper panel reports the full sample results up to 2004, and the lower panel reports the sub sample results up to 1990. Consistently with the Campbell-Cochrane model, the table shows that the surplus consumption ratio is negatively related to future excess stock returns, such that low surplus consumption ratio's in cyclical downturns predict high future excess stock returns. However, the predictive power varies strongly across countries. Based on the t-statistics and adjusted R^2-statistics, there is statistical evidence of stock return predictability in Belgium, Sweden, the UK, and the US (and also France and Italy if the sample is restricted to end in 1990), but the surplus consumption ratio has low predictive power in Canada and Germany.

Next, to compare the surplus consumption ratio's predictive power for future stock returns with alternative predictors, we use in turn the price-dividend ratio and the term spread in bivariate predictability regressions together with the surplus consumption ratio. Throughout the predicted variable is the 1-year ahead log excess stock return.

Table 4 shows that the surplus consumption ratio does not drive out the price-dividend ratio in bivariate predictability regressions of future excess stock returns. In fact, the price-dividend ratio is a significant predictor in all countries except Germany. The surplus consumption ratio remains a significant predictor in Belgium, France, Italy, Sweden, and the US, but is driven out as a significant predictor in the UK. The overall conclusion is that both predictors have significant forecasting ability for future excess stock returns in the majority of countries and, interestingly, they remain significant when including data from the 1990s, which is surprising in light of the findings in other recent studies, e.g. Ang and Bekaert (2007).

Turning to bivariate predictability regressions of future stock returns with the surplus consumption ratio and the term spread as predictors, Table 5 shows that the term spread does not bring much additional information about future stock returns relative to the surplus consumption ratio. The exception is Canada, but otherwise the term spread is not significant in bivariate predictability regressions with the surplus consumption ratio.

The overall impression from the results so far is that the surplus consumption ratio significantly captures time-varying expected stock returns to a greater or lesser extent in Belgium, France, Italy, Sweden, the UK, and the US, but not in Canada and Germany. Furthermore, the surplus consumption ratio appears to be a stronger stock return predictor than the term spread. However, the surplus consumption ratio does not consistently drive out the price-dividend ratio in bivariate predictability regressions, which suggests that the surplus consumption ratio does not capture all relevant information about future stock returns.

[11] Valkanov (2003) shows that the use of overlapping data in a small sample may lead to spurious evidence of return predictability. By using only non-overlapping data we are less exposed to such concerns.

5.2.2 Bond return predictability

The basic version of the Campbell-Cochrane model does not generate time-varying expected returns on bonds since bond returns at all maturities are equal to the constant risk-free rate. However, our version of the model incorporates the extension suggested by Wachter (2006) such that the surplus consumption ratio captures counter-cyclical time-variation in both stock and bond returns. We analyze this extended version of the model by running predictability regressions of 1-year ahead log excess bond returns with the log surplus consumption ratio as predictor. Table 6 shows that, indeed, high surplus consumption ratio's predict low future excess bond returns. As with excess stock returns, the ratio significantly predicts excess bond returns in Belgium, France, Italy, Sweden, the US, and to some extent the UK. Furthermore, the surplus consumption ratio also predicts excess bond returns in Canada. Once again, though, the surplus consumption ratio does not have predictive power in Germany.

Table 7 shows that the price-dividend ratio is generally not a significant predictor of excess bond returns. It appears that the price-dividend ratio only predicts excess stock returns, but not excess bond returns, whereas the surplus consumption ratio captures a common source of predictability in both excess stock and bond returns. The most often used predictor for excess bond returns is the term spread. In Table 8 we include this variable together with the surplus consumption ratio in bivariate predictability regressions. The table shows that the surplus consumption ratio is a better bond return predictor than the term spread for the majority of countries. The main exception is the US, where the term spread seems to drive out the surplus consumption ratio as a significant predictor.

There is growing body of literature about return predictability on both stocks and bonds. However, a common limitation to existing return predictors is that they only contain information about either future stock returns or future bond returns. Interestingly, we find that the surplus consumption ratio captures predictive patterns in both stock and bond markets. Our findings therefore support the extended version of the Campbell-Cochrane model in which expected returns on stocks and bonds move counter-cyclically with the surplus consumption ratio.

5.2.3 Small sample bias

As a final robustness check, we examine to what extent our results suffer from small sample bias. Stambaugh (1999) shows that standard OLS inference is biased in finite samples when the predictive variable is highly persistent and its innovations are correlated with the innovations in returns. Amihud and Hurvich (2004) extend the work of Stambaugh (1999) by developing a straightforward estimation method to obtain bias-adjusted beta estimates and t-statistics. Tables 9 and 10 report the Amihud-Hurvich bias-adjusted results using the surplus consumption ratio to predict excess stock and bond returns, respectively.[12] The tables show that the small sample bias is very modest, which is due to low correlation between the surplus consumption innovations and the return innovations. Hence, even the bias-adjusted results imply that the surplus consumption ratio has the ability to predict stock and bond returns.

6 Concluding remarks

Consumption based models with habit persistence, and in particular the Campbell and Cochrane (1999) model, is at present one of the leading frameworks within the equilibrium based paradigm to explain financial market returns and how they vary over time and across assets. The Campbell-Cochrane model has the intuitively appealing implication that risk aversion moves counter-cyclically, and the model implies return predictability based on the surplus consumption ratio.

Most previous analyses using the Campbell-Cochrane model have been on US data, and in the few existing international studies using the model, the calibrated parameter values from the original US study are employed in the analyses. In the present paper we have analyzed the Campbell-Cochrane model on an international dataset in which, for each country, we have used an iterative GMM procedure to formally estimate and test the model. In addition, based on the parameter estimates, we have constructed time series for the surplus consumption ratio in each country, which we have used as a predictor variable in predictability regressions for stock and bond returns.

We find that there are large cross-country differences in the Campbell-Cochrane model's ability to explain financial market returns. Clearly the model does not give a perfect description of the data in any of the countries, which is of course not surprising given the highly stylized nature of the model. However, for the majority of countries (Belgium, Italy, Sweden, the UK and the US), the Campbell-Cochrane model gets empirical support in a variety of different dimensions: Economically plausible estimates of preference parameters and the risk-free rate, time-varying counter-cyclical risk-aversion, and statistically significant return predictability for both stocks and bonds based on the surplus consumption ratio (and in the 'right' direction, i.e. increasing (decreasing) consumption relative to habit during economic up(down)turns predicts lower (higher) future returns). For another group of countries (Canada and France), the results are mixed. For Germany, however, there is not much empirical support for the Campbell-Cochrane model.

Thus, there seems to be important cross-country differences in how habit persistence affects equilibrium pricing in the financial markets. We leave a deeper investigation into the nature of these cross-country differences for future research.

[12] See Amihud and Hurvich (2004) for details of their bias-correction method.

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	Sample	R_S	R_{LB}	R_f
BEL	1953-2004	1.078	1.043	1.031
		(0.188)	(0.081)	(0.023)
CAN	1949-2004	1.075	1.035	1.021
		(0.160)	(0.087)	(0.029)
FRA	1952-2004	1.096	1.038	1.020
		(0.255)	(0.091)	(0.030)
GER	1952-2004	1.100	1.046	1.019
		(0.257)	(0.077)	(0.019)
ITA	1951-2004	1.078	1.041	1.020
		(0.282)	(0.138)	(0.031)
SWE	1950-2004	1.112	1.021	1.014
		(0.249)	(0.069)	(0.035)
UK	1952-2004	1.099	1.030	1.018
		(0.240)	(0.070)	(0.032)
US	1949-2004	1.094	1.027	1.012
		(0.174)	(0.098)	(0.021)

Table 1. Summary statistics.

The table reports average real gross returns on stocks, R_S, long-term bonds, R_{LB}, and 3-month T-bills, R_f. Standard deviations are in parentheses.

	BEL	CAN	FRA	GER	ITA	SWE	UK	US
				Campbell-Cochrane				
δ	0.8756	0.9160	0.3588	0.5059	0.8337	0.6455	0.5947	0.9100
	(0.0752)	(0.0342)	(0.2910)	(0.2250)	(0.1078)	(0.7211)	(0.2158)	(0.0553)
γ	4.0547	4.3097	27.4462	6.6164	2.7821	6.8423	6.9516	17.3711
	(3.1021)	(2.0479)	(21.8943)	(4.4526)	(2.1442)	(17.1836)	(5.4311)	(6.6481)
ϕ	0.8999	0.9216	0.8779	0.7408	0.8101	0.8291	0.8215	0.9494
	(0.0684)	(0.0560)	(0.0708)	(0.1002)	(0.0541)	(0.0677)	(0.0825)	(0.0473)
g	0.0206	0.0197	0.0232	0.0283	0.0342	0.0203	0.0225	0.0199
	(0.0036)	(0.0033)	(0.0039)	(0.0045)	(0.0038)	(0.0029)	(0.0033)	(0.0030)
σ_c^2	0.0006	0.0005	0.0006	0.0007	0.0006	0.0004	0.0004	0.0004
	(0.0003)	(0.0001)	(0.0002)	(0.0002)	(0.0001)	(0.0001)	(0.0001)	(0.0001)
B	0.0333	0.0335	0.0689	0.0129	0.0108	0.0428	-0.0803	0.0226
	(0.0145)	(0.0115)	(0.0301)	(0.0068)	(0.0101)	(0.0148)	(0.0259)	(0.0471)
J_T	2.7194	3.8155	0.8588	2.5592	3.0389	2.6613	0.8075	3.1478
	(0.4369)	(0.2821)	(0.8354)	(0.4647)	(0.3857)	(0.4468)	(0.8477)	(0.3694)
HJ	0.2735	0.1597	0.4355	0.4885	0.0490	0.3466	0.3677	0.2811
$\overline{r}$	0.0300	0.0206	0.0203	0.0175	0.0183	0.0135	0.0155	0.0124
$\gamma/\bar{S}$	25.1482	25.8992	75.4072	48.6861	28.4500	56.6544	55.0689	46.0492
				CRRA				
δ	1.2339	1.3385	1.3383	2.3773	1.9001	1.3511	1.9462	1.5673
	(0.1789)	(0.2150)	(0.9572)	(0.8553)	(0.7574)	(1.6421)	(0.4606)	(0.2816)
γ	17.4557	26.2481	75.0523	64.0067	25.8590	90.1772	60.0946	43.6025
	(11.0357)	(12.3599)	(56.2377)	(62.4814)	(19.6749)	(104.6803)	(58.3921)	(17.0264)
J_T	4.7902	5.4265	5.2691	4.6562	2.1823	3.1197	5.2316	4.6984
	(0.3095)	(0.2463)	(0.2608)	(0.3244)	(0.7023)	(0.5380)	(0.2643)	(0.3197)
HJ	0.2387	0.0921	0.1573	0.1063	0.0228	0.1871	0.2505	0.1576
r_f	0.0596	0.0703	-0.1760	-0.5272	0.0294	0.1039	-0.1023	0.0353

Table 2. GMM estimation of the Campbell-Cochrane model and the CRRA model.

The table reports GMM estimates of the parameters of the Campbell-Cochrane and CRRA models. Standard errors are in parentheses. The CRRA model is estimated on moment conditions (14) to (16), and the Campbell-Cochrane model is estimated on moment conditions (17) to (22). J_T is Hansen's (1982) test of overidentifying restrictions with p-values in parentheses. HJ is the Hansen and Jagannathan (1997) distance. $\gamma/\bar{S}$ is the steady state relative risk aversion.

	BEL	CAN	FRA	GER	ITA	SWE	UK	US
				Full sample period, ends in 2004				
β_s	-0.21	-0.06	-0.25	-0.03	-0.12	-0.29	-0.28	-0.85
t_{NW}	-3.00	-1.05	-1.06	-0.32	-1.53	-3.20	-2.28	-2.72
$\bar{R}^2$	8.24	-0.76	0.05	-1.82	2.56	11.12	4.33	8.57
				Sub-sample period, ends in 1990				
β_s	-0.22	-0.12	-1.01	-0.15	-0.52	-0.32	-0.27	-0.87
t_{NW}	-3.35	-0.81	-2.58	-1.19	-3.11	-1.95	-1.87	-2.30
$\bar{R}^2$	9.62	-1.45	5.85	-0.10	10.92	8.28	2.84	9.35

Table 3. Predicting stock returns with the surplus consumption ratio.

The table reports results of predictability regressions, $r_{S,t+1} - r_{f,t+1} = \alpha + \beta_s s_{t-1} + e_{t+1}$, where $r_{S,t+1} - r_{f,t+1}$ is the log excess stock return, and s_{t-1} is the log surplus consumption ratio. t_{NW} is the Newey and West (1987) corrected t-statistic, and $\bar{R}^2$ denotes the adjusted R^2-statistic (in %).

	BEL	CAN	FRA	GER	ITA	SWE	UK	US
				Full sample period, ends in 2004				
β_s	-0.26	-0.19	-0.42	-0.08	-0.20	-0.39	-0.05	-0.89
t_{NW}	-4.56	-2.48	-1.87	-0.91	-2.14	-4.78	-0.52	-2.55
β_{pd}	-0.14	-0.17	-0.17	-0.21	-0.23	-0.23	-0.40	-0.12
t_{NW}	-2.34	-3.16	-2.42	-1.71	-3.58	-3.60	-2.90	-2.18
$\bar{R}^2$	14.51	8.38	6.12	1.77	10.98	24.64	20.27	15.45
				Sub-sample period, ends in 1990				
β_s	-0.22	-0.05	-1.00	-0.07	-0.52	-0.29	-0.06	-0.44
t_{NW}	-4.07	-0.28	-2.91	-0.47	-2.61	-2.62	-0.42	-1.34
β_{pd}	-0.13	-0.36	-0.13	-0.24	-0.25	-0.32	-0.74	-0.23
t_{NW}	-1.63	-4.26	-2.09	-1.07	-3.25	-2.76	-2.99	-3.36
$\bar{R}^2$	12.65	15.67	9.02	2.40	19.66	25.13	31.09	16.17

Table 4. Predicting stock returns with the surplus consumption ratio and the price-dividend ratio.

The table reports results of predictability regressions, $r_{S,t+1} - r_{f,t+1} = \alpha + \beta_s s_{t-1} + \beta_{pd} pd_t + e_{t+1}$, where $r_{S,t+1} - r_{f,t+1}$ is the log excess stock return, s_{t-1} is the log surplus consumption ratio, and pd_t is the log price-dividend ratio. t_{NW} is the Newey and West (1987) corrected t-statistic, and $\bar{R}^2$ denotes the adjusted R^2-statistic (in %).

	BEL	CAN	FRA	GER	ITA	SWE	UK	US
				Full sample period, ends in 2004				
β_s	-0.15	-0.01	-0.26	-0.08	-0.12	-0.29	-0.20	-0.79
t_{NW}	-2.18	-0.15	-1.09	-0.90	-1.47	-3.18	-1.36	-2.42
β_T	0.04	0.02	0.00	0.03	0.02	-0.00	0.02	0.01
t_{NW}	1.53	1.88	0.29	1.40	0.82	-0.15	0.73	0.74
$\bar{R}^2$	10.71	1.88	-1.94	-1.74	1.99	9.37	3.60	7.27
				Sub-sample period, ends in 1990				
β_s	-0.18	-0.03	-1.00	-0.24	-0.54	-0.31	-0.17	-0.76
t_{NW}	-2.07	-0.17	-2.56	-2.24	-3.02	-1.90	-0.96	-1.94
β_T	0.11	0.03	-0.01	0.04	0.03	-0.00	0.02	0.02
t_{NW}	1.28	2.20	-0.38	1.49	1.31	-0.18	0.75	1.64
$\bar{R}^2$	14.44	4.15	3.21	0.31	12.78	5.40	1.92	9.68

Table 5. Predicting stock returns with the surplus consumption ratio and the term spread.

The table shows results of predictability regressions, $r_{S,t+1} - r_{f,t+1} = \alpha + \beta_s s_{t-1} + \beta_T TERM_t + e_{t+1}$, where $r_{S,t+1} - r_{f,t+1}$ is the log excess stock return, s_{t-1} is the log surplus consumption ratio, and $TERM_t$ is the term spread on bonds. t_{NW} is the Newey and West (1987) corrected t-statistic, and $\bar{R}^2$ denotes the adjusted R^2-statistic (in %).

	BEL	CAN	FRA	GER	ITA	SWE	UK	US
				Full sample period, ends in 2004				
β_s	−0.10	−0.10	−0.21	−0.02	−0.08	−0.06	−0.08	−0.35
t_{NW}	−3.10	−3.41	−3.24	−1.06	−2.44	−2.33	−1.77	−2.51
$\bar{R}^2$	13.01	12.30	10.08	−0.73	6.54	6.80	3.69	4.61
				Sub-sample period, ends in 1990				
β_s	−0.09	−0.14	−0.42	−0.02	−0.26	−0.04	−0.10	−0.34
t_{NW}	−1.88	−2.45	−2.72	−0.43	−2.64	−2.27	−2.13	−2.14
$\bar{R}^2$	10.15	5.98	13.22	−2.36	15.44	1.55	7.08	4.29

Table 6. Predicting bond returns with the surplus consumption ratio.

The table reports results of predictability regressions, $r_{LB,t+1} - r_{f,t+1} = \alpha + \beta_s s_{t-1} + e_{t+1}$, where $r_{LB,t+1} - r_{f,t+1}$ is the log excess bond return, and s_{t-1} is the log surplus consumption ratio. t_{NW} is the Newey and West (1987) corrected t-statistic, and $\bar{R}^2$ denotes the adjusted R^2-statistic (in %).

	BEL	CAN	FRA	GER	ITA	SWE	UK	US
				Full sample period, ends in 2004				
β_s	-0.11	-0.12	-0.22	-0.03	-0.09	-0.05	-0.06	-0.34
t_{NW}	-3.89	-3.53	-3.41	-1.26	-2.45	-2.34	-1.40	-2.53
β_{pd}	-0.03	-0.02	-0.02	-0.03	-0.03	0.02	-0.03	0.02
t_{NW}	-1.07	-0.93	-0.54	-0.70	-0.67	0.95	-0.69	0.97
$\bar{R}^2$	12.70	11.04	8.77	-1.79	5.31	6.47	2.83	3.66
				Sub-sample period, ends in 1990				
β_s	-0.09	-0.14	-0.42	-0.01	-0.26	-0.04	-0.06	-0.42
t_{NW}	-2.16	-2.28	-2.73	-0.14	-2.57	-2.47	-1.32	-2.08
β_{pd}	-0.04	0.00	-0.00	-0.04	-0.02	0.01	-0.13	0.04
t_{NW}	-1.16	0.04	-0.01	-0.60	-0.45	0.38	-2.46	0.96
$\bar{R}^2$	11.39	3.44	10.67	-3.94	13.55	-0.78	17.64	2.99

Table 7. Predicting bond returns with the surplus consumption ratio and the price-dividend ratio.

The table reports results of predictability regressions, $r_{LB,t+1} - r_{f,t+1} = \alpha + \beta_s s_{t-1} + \beta_{pd} pd_t + e_{t+1}$, where $r_{LB,t+1} - r_{f,t+1}$ is the log excess bond return, s_{t-1} is the log surplus consumption ratio, and pd_t is the log price-dividend ratio. t_{NW} is the Newey and West (1987) corrected t-statistic, and $\bar{R}^2$ denotes the adjusted R^2-statistic (in %).

	BEL	CAN	FRA	GER	ITA	SWE	UK	US
				Full sample period, ends in 2004				
β_s	-0.08	-0.07	-0.21	-0.04	-0.08	-0.06	-0.08	-0.16
t_{NW}	-1.88	-2.30	-3.49	-1.68	-2.47	-2.41	-1.56	-1.12
β_T	0.02	0.02	0.00	0.01	0.01	0.00	-0.00	0.03
t_{NW}	1.98	1.99	0.27	1.12	1.18	0.31	-0.07	4.05
$\bar{R}^2$	18.08	18.78	8.39	0.37	6.32	4.58	1.70	20.45
				Sub-sample period, ends in 1990				
β_s	-0.08	-0.09	-0.42	-0.05	-0.27	-0.04	-0.10	-0.17
t_{NW}	-1.67	-1.52	-2.77	-1.21	-2.74	-2.11	-1.85	-1.07
β_T	0.02	0.02	-0.00	0.02	0.01	0.00	-0.00	0.03
t_{NW}	0.96	3.09	-1.03	1.34	1.59	0.15	-0.08	3.67
$\bar{R}^2$	8.86	20.02	11.80	1.11	18.63	-2.02	4.36	24.81

Table 8. Predicting bond returns with the surplus consumption ratio and the term spread.

The table reports results of predictability regressions, $r_{LB,t+1} - r_{f,t+1} = \alpha + \beta_s s_{t-1} + \beta_T TERM_t + e_{t+1}$, where $r_{LB,t+1} - r_{f,t+1}$ is the log excess bond return, s_{t-1} is the log surplus consumption ratio, and $TERM_t$ is the term spread on bonds. t_{NW} is the Newey and West (1987) corrected t-statistic, and $\bar{R}^2$ denotes the adjusted R^2-statistic (in %).

	BEL	CAN	FRA	GER	ITA	SWE	UK	US
			Full sample period, ends in 2004					
β_s^{AH}	-0.19	-0.06	-0.17	-0.02	-0.09	-0.28	-0.29	-0.86
t_{AH}	-2.10	-0.71	-0.63	-0.22	-1.16	-2.61	-1.84	-2.45
			Sub-sample period, ends in 1990					
β_s^{AH}	-0.19	-0.11	-0.97	-0.14	-0.51	-0.31	-0.28	-0.89
t_{AH}	-1.79	-0.61	-1.70	-0.92	-2.28	-2.02	-1.47	-2.23

Table 9. Predicting stock returns with the surplus consumption ratio: Bias-adjusted results.

The table reports the Amihud-Hurvich bias-adjusted beta estimates and t-statistics using the surplus consumption ratio to forecast the log excess stock return.

	BEL	CAN	FRA	GER	ITA	SWE	UK	US
			Full sample period, ends in 2004					
β_s^{AH}	-0.11	-0.12	-0.22	-0.03	-0.09	-0.06	-0.09	-0.36
t_{AH}	-3.04	-3.21	-2.65	-0.91	-2.36	-2.15	-1.93	-1.93
			Sub-sample period, ends in 1990					
β_s^{AH}	-0.10	-0.15	-0.45	-0.02	-0.27	-0.04	-0.12	-0.33
t_{AH}	-2.34	-2.00	-2.63	-0.43	-2.91	-1.10	-2.22	-1.60

Table 10. Predicting bond returns with the surplus consumption ratio: Bias-adjusted results.

The table reports the Amihud-Hurvich bias-adjusted beta estimates and t-statistics using the surplus consumption ratio to forecast the log excess bond return.

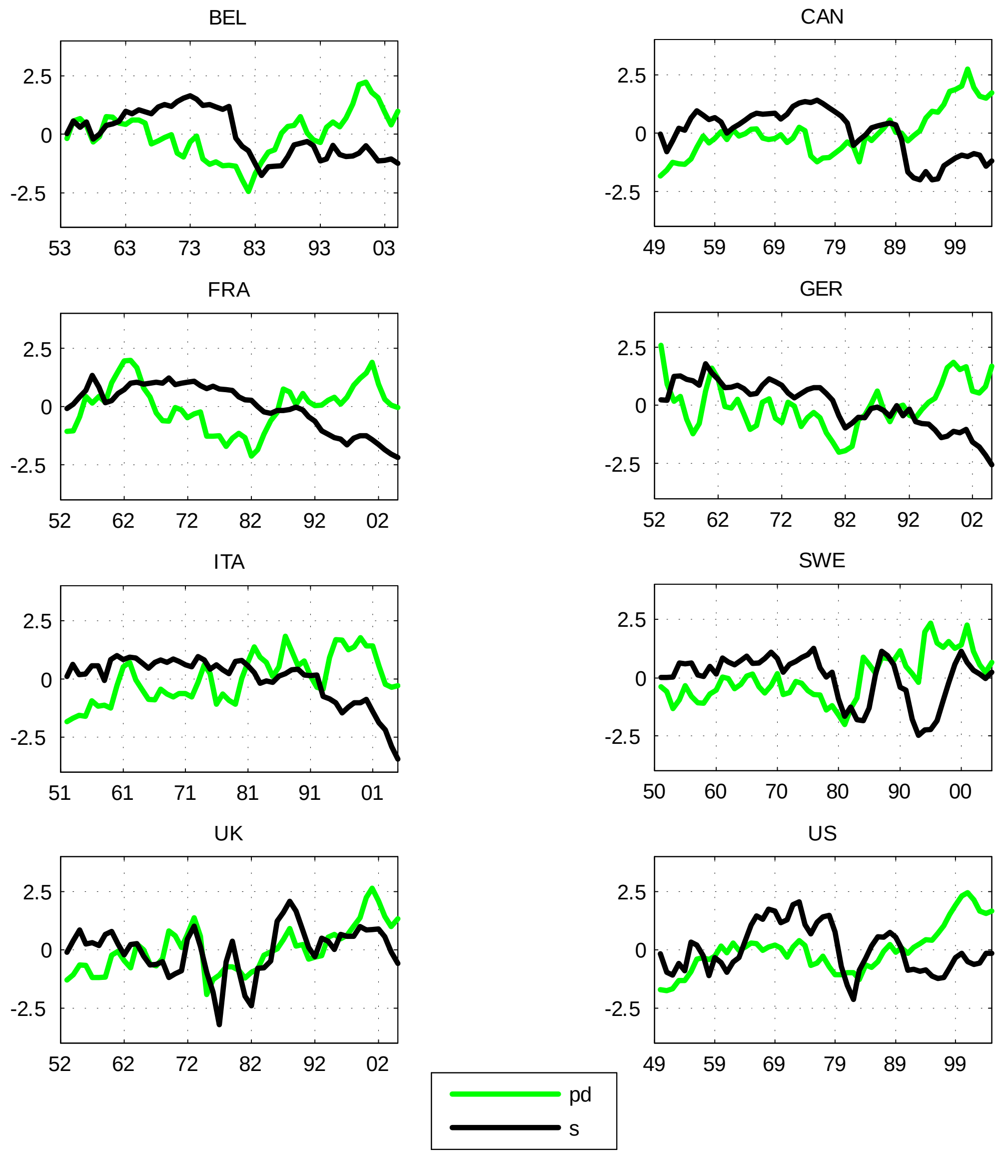

Figure 1. The surplus consumption ratio and the price-dividend ratio. Both series are standardized to have mean 0 and variance 1.

Chapter 4

Consumption growth and time-varying expected stock returns

Consumption growth and time-varying expected stock returns*

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Abstract

When the consumption growth rate is measured based upon fourth quarter data, it tracks predictable variation in future excess stock returns. Low fourth quarter consumption growth rates predict high future excess stock returns such that expected returns are high at business cycle troughs and low at business cycle peaks. The consumption growth rate loses predictive power when it is measured based upon other quarters. This is consistent with the insight of Jagannathan and Wang (2007) that investors tend to review their consumption and investment plans during the end of each calendar year, and at possibly random times in between. The consumption growth rate measured based upon fourth quarter data is a much stronger predictive variable than benchmark predictive variables such as the dividend-price ratio, the term spread, and the default spread.

Keywords: Return predictability; Consumption growth
JEL codes: C12; E21; E44; G12

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1 Introduction

An extensive empirical literature in finance has demonstrated that expected stock returns vary over time. Campbell and Shiller (1988) and Fama and French (1988, 1989) among others use financial predictive variables based on stock and bond market data such as the dividend-price ratio, the term spread, and the default spread to document that stock returns display predictable variation over time. Fama and French (1989) link the financial predictive variables to the business cycle and suggest that investors require a higher expected return at a business cycle trough than they do at a business cycle peak. More recently, macro predictive variables such as the consumption-wealth ratio (Lettau and Ludvigson 2001) have been shown to predict stock returns providing a direct linkage between time-varying expected returns and the business cycle.[1]

This paper examines the ability of the consumption growth rate to capture predictable variation in stock returns over the business cycle. The consumption growth rate has a clear business cycle pattern and is closely related to the business cycles as measured by the National Bureau of Economic Research (NBER). To mitigate the effect of measurement error in consumption data as well as the effect of infrequent adjustment of consumption plans that may disrupt the linkage between the consumption growth rate and expected returns, I follow Jagannathan and Wang (2007) and measure the consumption growth rate based upon fourth quarter data. I examine the predictive power of the fourth quarter consumption growth rate by running regressions of future excess stock returns on the lagged fourth quarter consumption growth rate. I find strong support for the ability of the fourth quarter consumption growth rate to predict future excess stock returns using US post-war data from 1947 to 2005. The $\bar{R}^2$-statistic is as high as 19% at the 1-year horizon, and the slope estimate is strongly significantly negative such that low consumption growth rates predict high future excess stock returns. Hence, expected returns are high at business cycle troughs and low at business cycle peaks, which is consistent with the findings of Fama and French (1989). The fourth quarter consumption growth rate − a pure macroeconomic variable − is a much stronger predictive variable than the traditional financial predictive variables such as the dividend-price ratio, the term spread, and the default spread. In fact, the fourth quarter consumption growth rate drives out the financial predictive variables in multiple regressions. Moreover, the fourth quarter consumption growth rate also provides substantial additional information about future excess stock returns beyond that contained in the consumption-wealth ratio.

The consumption growth rate loses predictive power when it is measured based upon other quarters. This is consistent with the insight of Jagannathan and Wang (2007) that investors tend to review their consumption and investment plans during the end of each calendar year, and at possibly random times in between. Possible explanations include more leisure time during the Christmas holiday season, the resolution of uncertainty about end of year bonuses, and end of year tax consequences of portfolio choices; see Jagannathan and Wang (2007) and the references therein.

[1] See Cochrane (2007) for a comprehensive survey on the return predictability literature.

The fourth quarter consumption growth rate has a number of distinct properties as a predictive variable. First of all, the fourth quarter consumption growth rate is a pure macroeconomic variable that provides a direct linkage between time-varying expected returns and the business cycle. In addition, the fourth quarter consumption growth rate is easily constructed from the National Income and Product Accounts (NIPA) and does not rely on estimating a cointegration relationship (such as for example the consumption-wealth ratio). Finally, the fourth quarter consumption growth rate is an almost i.i.d. process and is much less persistent than alternative predictive variables. Given the controversy about return predictability using highly persistent predictive variables, it is noteworthy that the fourth quarter consumption growth rate – an almost i.i.d. process – can predict future stock returns.[2]

2 Data

The empirical analysis is based on US post-war data for the period 1947 to 2005. Consumption is measured as seasonally adjusted real per capita expenditures on non-durables and services. The consumption data is obtained from the National Income and Product Accounts (NIPA) and is available on quarterly frequency starting from 1947. The annual log excess stock return is calculated as the log return on the value weighted CRSP index including NYSE, AMEX, and NASDAQ firms minus the log return on a 3-month Treasury bill rate. As benchmark predictive variables, I use the log dividend-price ratio (dp_t), the term spread between long-term government bond yields and Treasury bill yields ($TERM_t$), the default spread between BAA and AAA corporate bond yields (DEF_t), and the consumption-wealth ratio ($\widehat{cay}_t$). dp_t is derived from CRSP value weighted returns with and without dividend capitalization. $TERM_t$, DEF_t, and $\widehat{cay}_t$ are obtained from Amit Goyal's website.

Table 1 provides summary statistics of the consumption growth rate. In the upper panel, the consumption growth rate is measured annually as year to year growth rates in quarterly consumption, i.e. 4Q-4Q is the consumption growth rate calculated from the fourth quarter in year $t-1$ to the fourth quarter in year t. In the lower panel, the consumption growth rate is measured quarterly, i.e. 3Q-4Q is the fourth quarter consumption growth rate calculated from the third quarter in year t to the fourth quarter in year t. The means and standard deviations of the year to year growth rates in quarterly consumption are similar across quarters, but the range is largest for the fourth quarter. This replicates the findings of Jagannathan and Wang (2007). Moreover, the quarterly consumption growth rate has a higher standard deviation and range in the fourth quarter compared to the first, second and third quarters.

Figure 1 plots the annual 4Q-4Q consumption growth rate and the quarterly 3Q-4Q consumption growth rate. The shaded areas represent the NBER recession dates. The annual 4Q-4Q consumption growth rate and the quarterly 3Q-4Q consumption growth

[2]Stambaugh (1999) demonstrates that the use of highly persistent predictive variables may lead to spurious evidence of return predictability.

rate have similar patterns, but the latter is more volatile and takes on more extreme values at peaks and troughs than the former. The correlation coefficient between the two series is 0.63. Furthermore, the figure illustrates two distinct properties of the consumption growth rate as a predictive variable. First, the consumption growth rate has a clear business cycle pattern; it rises during business cycle expansions and reaches its highest values near peaks and falls during business contractions and reaches its lowest values near troughs. For instance, the consumption growth rate drops substantially just after the recession years of the oil shock of 1973-1975. Second, the consumption growth rate has a very low degree of persistence, implying that the consumption growth rate does not suffer from the statistical problems that arise using a highly persistent predictive variable, cf. Stambaugh (1999).

3 Predicting stock returns

Now I turn to testing the ability of consumption growth rates to predict future excess stock returns. This is done by 1-year ahead predictive regressions:

$$r^e_{t+1} = \alpha + \beta G^c_t + e_{t+1}, \tag{1}$$

where r^e_{t+1} is the 1-year ahead log excess stock return and G^c_t is the consumption growth rate. Table 2 reports OLS estimates, Newey and West (1987) corrected t-statistics, and $\bar{R}^2$-statistics. Significant estimates at the five percent level are in bold. The upper panel of Table 2 reports the results for annual consumption growth rates measured as year to year growth rates in quarterly consumption. When the annual consumption growth rate is based upon fourth quarter data, it tracks a substantial amount of the variation in future excess stock returns. The $\bar{R}^2$-statistic is 12.01%, and the slope estimate is significantly negative such that low consumption growth rates predict high future excess stock returns, i.e. expected returns are high at business cycle troughs and low at business cycle peaks. When the annual consumption growth rate is measured based upon other quarters, it loses predictive power; both the t-statistic and the $\bar{R}^2$-statistic fall. The lower panel of Table 2 reports the results for quarterly consumption growth rates. Here the evidence is even more striking. The fourth quarter consumption growth rate produces an $\bar{R}^2$-statistic of 18.89%, and the slope estimate is strongly significant (t-statistic of -5.75). When the quarterly consumption growth rate is measured based upon the first, second, and third quarters, the slope estimates are borderline significant or insignificant, and the $\bar{R}^2$-statistics are negligible. These dramatic results relate to the findings of Jagannathan and Wang (2007). They emphasize that the use of fourth quarter data mitigates the effect of measurement error in consumption data as well as the effect of infrequent adjustment of consumption plans that may disrupt the linkage between the consumption growth rate and expected returns.

The above evidence implies that both the 4Q-4Q and 3Q-4Q consumption growth rates have predictive power for future excess stock returns. The 4Q-4Q (annual) consumption growth rate is the sum of the 4Q-3Q (first three quarters) and the 3Q-4Q (fourth quarter) consumption growth rates. To examine whether the 4Q-3Q consumption growth rate also predicts future excess stock returns, I regress the 1-year ahead calendar year excess stock return on the 4Q-3Q consumption growth rate. The slope estimate is -3.24, the Newey-West corrected t-statistic is -1.66, and the $\bar{R}^2$-statistic is 3.31. Hence, the consumption growth rate of the first three quarters does not contain much predictive power for future excess stock returns, implying that the predictive power of the consumption growth rate is related to the fourth quarter.

3.1 Controlling for benchmark predictive variables

To control for benchmark predictive variables, I run predictive regressions of the form:

$$r^e_{t+1} = \alpha + \beta G^c_t + \Phi' Z_t + e_{t+1}, \tag{2}$$

where Z_t is a vector of benchmark predictive variables and G^c_t is the fourth quarter consumption growth rate. I compare the performance of G^c_t with traditional financial predictive variables (dp_t, $TERM_t$, and DEF_t) and the most prominent macro predictive variable ($\widehat{cay}_t$). The benchmark predictive variables are measured on an annual frequency. Table 3 shows that G^c_t contains substantial additional information about future excess stock returns relative to the traditional financial predictive variables. dp_t has a significant slope estimate and explains 6.86% of the variation in 1-year ahead excess stock returns, whereas $TERM_t$ and DEF_t are not able to predict excess stock returns in the post-war period from 1947 to 2005; their slope estimates are insignificant, and the $\bar{R}^2$-statistics are close to zero or negative. When G^c_t is included in the predictive regression with dp_t, the $\bar{R}^2$-statistic increases to 22.34%, and the slope estimate turns out to be insignificant for dp_t. Table 3 shows that G^c_t is also robust to the inclusion of $\widehat{cay}_t$. $\widehat{cay}_t$ has a significant slope estimate, and it produces an $\bar{R}^2$-statistic of 17.48% as a sole predictive variable. By including G^c_t along with $\widehat{cay}_t$, the $\bar{R}^2$-statistic increases to 29.57% and both predictive variables remain significant. To confirm the robustness, I run a predictive regression that includes all the predictive variables; G^c_t, dp_t, $TERM_t$, DEF_t and $\widehat{cay}_t$. The financial predictive variables (dp_t, $TERM_t$ and DEF_t) are all insignificant, and the $\bar{R}^2$-statistic does not increase once these variables are included. Hence, the relevant information about future excess stock returns is contained in G^c_t and $\widehat{cay}_t$; macro predictive variables that provide a direct linkage between time-varying expected returns and the business cycle.

As a further robustness check, I examine the predictive power of the benchmark predictive variables measured based upon fourth quarter data. Table 4 shows that the predictive power of dp_t and $\widehat{cay}_t$ does not change much when they are measured based upon fourth quarter data instead of annual data.[3] dp_t produces an $\bar{R}^2$-statistic of 7.89% on fourth quarter data compared to 6.86% on annual data, whereas $\widehat{cay}_t$ produces an $\bar{R}^2$-statistic of 13.75% on fourth quarter data compared to 17.48% on annual data.[4] Both

[3] I only report results for dp_t and $\widehat{cay}_t$ since both $TERM_t$ and DEF_t produce the same results with fourth quarter data as they do with annual data.

[4] For $\widehat{cay}_t$ the fourth quarter data starts in 1951, while the annual data starts in 1948.

dp_t and $\widehat{cay}_t$ continue to be significant as sole predictive variables, but turn insignificant when G_t^c is included to the predictive regression. Overall, the evidence suggests that the fourth quarter effect is a pure consumption effect.

3.2 Small sample bias

This section deals with small sample bias in the predictive regression (1). Specifying the predictive variable (x_t) as a stationary first-order autoregressive process, Stambaugh (1999) sets up the following model:

$$r_{t+1}^e = \alpha + \beta x_t + e_{t+1}, \quad e_{t+1} \sim niid\left(0, \sigma_e^2\right) \tag{3}$$

$$x_{t+1} = \delta + \rho x_t + u_{t+1}, \quad u_{t+1} \sim niid\left(0, \sigma_u^2\right) \tag{4}$$

and derives the small sample bias in β as a function of the degree of persistence in x_t and the correlation between the innovations in (3) and (4). Stambaugh (1999) shows that the small sample bias is particularly severe for financial predictive variables such as the dividend-price ratio since it is highly persistent, and its innovations are highly correlated with the innovations in returns. The small sample bias is less relevant with the fourth quarter consumption growth rate as predictive variable for two reasons. First, the fourth quarter consumption growth rate is not highly persistent. It has an AR(1) coefficient of -0.05, whereas the dividend-price ratio has an AR(1) coefficient of 0.95. Second, since the fourth quarter consumption growth rate is a pure macroeconomic variable, its innovations have relatively low correlation with the innovations in returns. The correlation between the innovations in the fourth quarter consumption growth rate and the innovations in the excess stock return is 0.27, whereas the correlation between the innovations in the dividend-price ratio and the innovations in the excess stock return is -0.61. To confirm that small sample bias is not an issue, I apply the following bootstrap procedure:

First, I estimate the following model, where G_t^c is the fourth quarter consumption growth rate:

$$r_{t+1}^e = \alpha + e_{t+1}, \tag{5}$$

$$G_{t+1}^c = \delta + \rho G_t^c + u_{t+1}. \tag{6}$$

Following the common practice (Nelson and Kim 1993, Goetzmann and Jorion 1993, and Kothari and Shanken 1997), I bootstrap under the null of no predictability by imposing the constraint that $\beta = 0$ and assume that the predictive variable follows an AR(1) model.[5] Second, I construct $100,000$ bootstrap samples of length $T + 1,000$ by randomly selecting residual pairs from (5) and (6). I use the OLS estimates of α, δ, and ρ, and set the initial values of r_t^e and G_t^c equal to their sample averages. The first $1,000$ observations are thrown away to avoid any effects from using the sample averages as starting values. Third, I estimate β from each bootstrap sample using equation (1) and then calculate a

[5] I have also used an AR(2) model as the data generating process for G_t^c. The AR(2) coefficient is -0.32. Using an AR(2) model produces nearly identical results as the AR(1) model.

95% bootstrap confidence interval for β using the lower 2.5th percentile and the upper 97.5th percentile of the 100,000 bootstrap samples.

The average value of the 100,000 artificial slope coefficients simulated under the null of no predictability is -0.03, and the 95% bootstrap confidence interval is $[-1.94; 1.90]$. Since the confidence interval does not include the OLS estimate $\hat{\beta} = -3.19$ (reported in Table 2), the bootstrap analysis confirms the conclusion that the fourth quarter consumption growth ratio predicts future excess stock returns.

4 Conclusion

This paper shows that the consumption growth rate based upon fourth quarter data tracks predictable variation in future excess stock returns. When the consumption growth rate is measured based upon other quarters, it loses predictive power. This is consistent with Jagannathan and Wang (2007) who emphasize that the use of fourth quarter data mitigates the effect of measurement error in consumption data as well as the effect of infrequent adjustment of consumption plans that may disrupt the linkage between the consumption growth rate and expected returns.

The fourth quarter consumption growth rate is a pure macroeconomic variable and provides a direct linkage between time-varying expected returns and the business cycles; it predicts high excess stock returns at business cycle troughs and low excess stock returns at business cycle peaks. The fourth quarter consumption growth rate outperforms financial predictive variables such as the dividend-price ratio, the term spread, and the default spread. The fourth quarter consumption growth rate also provides statistically significant additional information about future excess stock returns beyond that contained in the consumption-wealth ratio. Importantly, the fourth quarter consumption growth rate is an almost i.i.d. process, which eliminates potential concerns about finding spurious evidence of return predictability, cf. Stambaugh (1999).

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Table 1. Consumption growth: summary statistics (in %)

	Annual consumption growth			
	1Q-1Q	2Q-2Q	3Q-3Q	4Q-4Q
Mean	2.29	2.28	2.29	2.33
SD	1.42	1.34	1.37	1.42
Min	−0.36	−0.31	−1.08	−0.78
Max	5.72	5.40	4.83	5.70
Range	6.08	5.71	5.91	6.48
	Quarterly consumption growth			
	4Q-1Q	1Q-2Q	2Q-3Q	3Q-4Q
Mean	2.22	2.43	2.24	2.33
SD	2.10	1.99	2.10	2.32
Min	−4.17	−4.62	−3.88	−4.93
Max	6.76	6.79	5.65	9.52
Range	10.92	11.40	9.52	14.45

Notes. The table reports summary statistics of consumption growth rates. In the upper panel, the consumption growth rate is measured annually as year to year growth rates in quarterly consumption, i.e. 4Q-4Q is the consumption growth rate calculated from the fourth quarter in year $t-1$ to the fourth quarter in year t. In the lower panel, the consumption growth rate is measured quarterly, i.e. 3Q-4Q is the fourth quarter consumption growth rate calculated from the third quarter in year t to the fourth quarter in year t. The quarterly consumption growth rates are scaled by 4 such that the unit of measurement is percentage points per year.

Table 2. Predicting excess stock returns with G_t^c.

	Annual consumption growth					
	Constant	1Q-1Q	2Q-2Q	3Q-3Q	4Q-4Q	$\bar{R}^2(\%)$
Estimate	**0.12**	**−2.86**				5.30
t-value	4.27	−2.71				
Estimate	**0.14**		**−3.76**			8.16
t-value	4.17		−2.72			
Estimate	**0.13**			−2.99		3.20
t-value	3.32			−1.76		
Estimate	**0.16**				**−4.26**	12.01
t-value	4.18				−3.24	
	Quarterly consumption growth					
	Constant	4Q-1Q	1Q-2Q	2Q-3Q	3Q-4Q	$\bar{R}^2(\%)$
Estimate	**0.07**	−0.69				−0.91
t-value	2.96	−0.90				
Estimate	**0.09**		**−1.33**			0.93
t-value	3.47		−2.07			
Estimate	**0.08**			−1.13		−0.14
t-value	3.35			−1.61		
Estimate	**0.13**				**−3.19**	18.89
t-value	5.92				−5.75	

Notes. This table reports results of predictive regressions for the 1-year ahead log excess return (r_{t+1}^e) on the lagged consumption growth rate (G_t^c): $r_{t+1}^e = \alpha + \beta G_t^c + e_{t+1}$. For each regression, the table reports OLS estimates, Newey-West corrected t-statistics, and $\bar{R}^2$-statistics. Significant estimates at the five percent level are in bold. In the upper panel, the consumption growth rate is measured annually as year to year growth rates in quarterly consumption, i.e. 4Q-4Q is the consumption growth rate calculated from the fourth quarter in year $t-1$ to the fourth quarter in year t. In the lower panel, the consumption growth rate is measured quarterly, i.e. 3Q-4Q is the fourth quarter consumption growth rate calculated from the third quarter in year t to the fourth quarter in year t. For 1Q-1Q and 4Q-1Q consumption growth rates, the 1-year ahead excess stock return is measured from April to the next March. For 2Q-2Q and 1Q-2Q consumption growth rates, the 1-year ahead excess stock return is measured from July to the next June. For 3Q-3Q and 2Q-3Q consumption growth rates, the 1-year ahead excess stock return is measured from October to the next September. For 4Q-4Q and 3Q-4Q consumption growth rates, the 1-year ahead excess stock return is measured over the calendar year.

Table 3. Controlling for alternative predictive variables.

	Constant	G_t^c	dp_t	$TERM_t$	DEF_t	$\widehat{cay}_t$	$\bar{R}^2(\%)$
Estimate	**0.45**		**0.11**				6.86
t-value	2.71		2.23				
Estimate	**0.42**	**−2.92**	0.09				22.34
t-value	2.77	−4.90	1.80				
Estimate	0.04			1.71			0.65
t-value	1.33			1.57			
Estimate	**0.11**	**−3.18**		1.70			19.85
t-value	3.57	−5.74		1.50			
Estimate	0.04				1.97		−1.54
t-value	0.77				0.45		
Estimate	**0.13**	**−3.19**			−0.00		17.38
t-value	2.60	−5.51			−0.00		
Estimate	**−0.92**					**4.07**	17.48
t-value	−3.56					3.86	
Estimate	**−0.67**	**−2.63**				**3.29**	29.57
t-value	−2.36	−4.55				2.88	
Estimate	−0.20	**−2.60**	0.07	1.40	−2.66	**2.43**	28.75
t-value	−0.51	−4.24	1.43	1.23	−0.83	2.04	

Notes. This table reports results of predictive regressions for the 1-year ahead log excess return (r_{t+1}^e) on lagged predictive variables: $r_{t+1}^e = \alpha + \beta G_t^c + \Phi' Z_t + e_{t+1}$. G_t^c is the fourth quarter consumption growth rate and Z_t is a vector of benchmark predictive variables. For each regression, the table reports OLS estimates, Newey-West corrected t-statistics, and $\bar{R}^2$-statistics. Significant estimates at the five percent level are in bold.

Table 4. Controlling for alternative predictive variables based on fourth quarter data.

	Constant	G_t^c	dp_t	$\widehat{cay}_t$	$\bar{R}^2(\%)$
Estimate	**0.57**		**0.11**		7.89
t-value	2.84		2.42		
Estimate	**0.50**	$\mathbf{-2.86}$	0.08		22.45
t-value	2.62	-4.67	1.85		
Estimate	**0.05**			**5.37**	13.75
t-value	2.82			2.86	
Estimate	**0.12**	$\mathbf{-2.95}$		3.89	26.44
t-value	3.97	-3.94		1.94	
Estimate	0.39	$\mathbf{-2.90}$	0.06	3.42	26.96
t-value	1.67	-4.27	1.13	1.64	

Notes. This table reports results of predictive regressions for the 1-year ahead log excess return (r_{t+1}^e) on lagged predictive variables: $r_{t+1}^e = \alpha + \beta G_t^c + \Phi' Z_t + e_{t+1}$. G_t^c is the fourth quarter consumption growth rate and Z_t is a vector of benchmark predictive variables measured based upon fourth quarter data. For each regression, the table reports OLS estimates, Newey-West corrected t-statistics, and $\bar{R}^2$-statistics. Significant estimates at the five percent level are in bold.

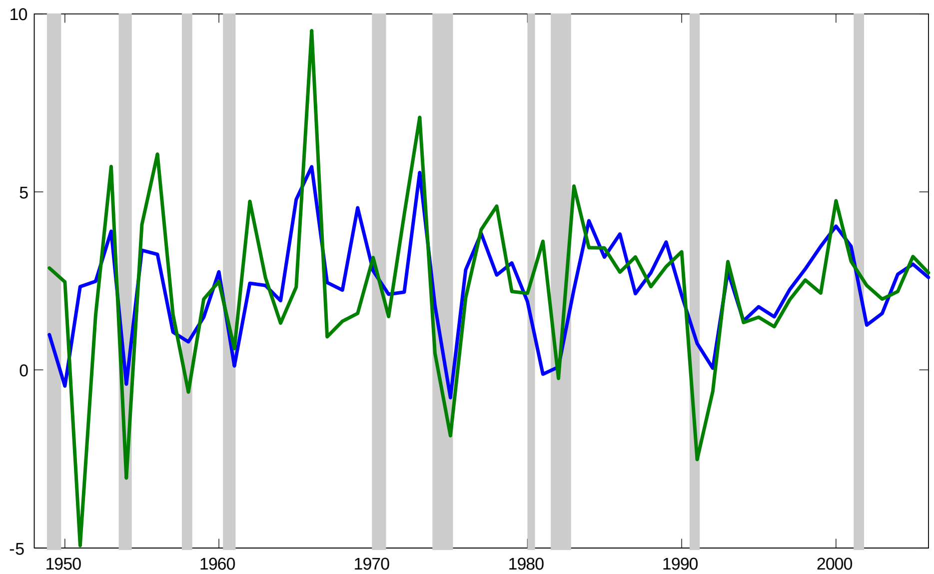

Fig. 1. The consumption growth rate.

The figure plots the annual 4Q-4Q consumption growth rate (blue line) and the quarterly 3Q-4Q consumption growth rate (green line). The quarterly 3Q-4Q consumption growth rate is scaled by 4 such that the unit of measurement is percentage points per year.