Money and Price Posting under Private Information*

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Abstract

We study price posting with undirected search in a search theoretic model with divisible money and divisible goods. *Ex ante* homogeneous buyers experience match specific preference shocks in bilateral trades. The shocks follow a continuous distribution and the realization of the shocks is private information. We show that generically there exists a unique price posting monetary equilibrium. In equilibrium, each seller posts a continuous pricing schedule that exhibits quantity discounts. Buyers spend only when they have high preferences, and as preferences are higher, they spend more. As inflation decreases the value of waiting for future trades, higher inflation induces more buyers to spend money and **those who spend before to spend their money faster**. The model naturally captures the hot potato effect of inflation along both the intensive margin and the extensive margin.

Keywords: Price posting, Undirected search, Private information, Hot potato effect, Inflation, Quantity discounts

JEL: D82, D83, E31

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1 Introduction

This paper builds a search theoretic monetary model based on the framework developed by Lagos and Wright (2005) and Rocheteau and Wright (2005), where sellers post prices and buyers engage in undirected search in the sense that buyers only see the posted prices after they are matched with sellers. Price posting with undirected search (hereafter price posting) is an interesting pricing mechanism to study because it captures the characteristics of many daily exchanges.[1] In many occasions, buyers randomly enter a store, read the prices and decide whether or not to make a purchase.

In the search theoretic monetary literature, very few papers have studied price posting because it is hard to generate equilibria with valued fiat money under this pricing mechanism. Generally, the existence of monetary equilibria hinges critically on the condition that the buyer extracts some surplus through trading with the seller. In a typical monetary model with price posting, sellers can extract all the trading surplus by proposing prices and quantities, and buyers decide whether to trade or not. In this case, sellers have all the bargaining power so that in equilibrium no buyers want to hold money when the nominal interest rate is positive. Monetary equilibrium thus unravels under price posting.

To generate monetary equilibria under price posting, we introduce match specific preference shocks which affect buyers' marginal willingness to consume and private information about the realization of the shocks. When the preference shocks follow a continuous distribution, we show that there exists a unique monetary equilibrium.

The model is then used to examine the seller's pricing schedule, the buyer's spending pattern and the effects of inflation. We find that in equilibrium, sellers post a non-linear pricing schedule and the unit price exhibits quantity discounts: the unit price is lower for purchases of larger sizes. Quantity discounts are frequently observed in practice and the traditional explanation is that the unit cost of producing a large quantity is lower. In our model, quantity discounts exist even when the unit cost is constant. It is used by sellers to induce buyers with higher preferences to consume more. Buyers spend money only when they have high enough marginal utilities and spend more when their marginal utilities are higher.

As for the effects of inflation, we show that buyers spend their money faster when inflation is higher; this is the "hot potato" effect of inflation. The intuition is that inflation reduces the benefit of retaining money for future trading opportunities and induces buyers to spend their money faster.

[1] We use price posting to refer to price posting with undirected search in the rest of the paper. When price posting is combined with directed search, it is generally labelled as competitive search in the literature.

More importantly, price posting equilibrium endogenously generates the hot potato effect along both the intensive (those who spend choose to spend more) and the extensive margins (those who do not spend start spending) without resorting to free entry or search intensity.

There have been a few papers that study price posting. As in our paper, the key to the existence of a monetary equilibrium under price posting is private information about match specific preference shocks. Jafarey and Masters (2003) and Curtis and Wright (2004) are built on the indivisible money model of Trejos and Wright (1995). In Curtis and Wright (2004), there are $K \geq 2$ realizations of the preference shock and in equilibrium, sellers post at most two prices. In Jafarey and Masters (2003), the preference shock follows a uniform distribution. Interestingly, there is only a single price posted in equilibrium. In general, there exists the law of two prices in models with indivisible money as emphasized by Curtis and Wright (2004). More recently, Ennis (2008) extends price posting to a divisible money framework as in Lagos and Wright (2005). He specifies a distribution of preference shocks that takes two values, and shows that sellers post a single price in equilibrium.

The above three papers generate monetary equilibria under price posting, however, they are not suitable to analyze sellers' pricing behavior and buyers' spending pattern and how they are affected by inflation. In equilibrium, at most two prices are posted, and buyers spend all their money during exchanges. In our model, money is divisible and preference shocks follow a continuous distribution. The model implies richer (and more realistic) pricing behavior and allows us to study the effect of inflation on buyers' spending pattern in a natural environment.

Several recent papers have tried to rationalize the hot potato effect of inflation using microfounded monetary theory. Lagos and Rocheteau (2005) endogenize search intensity, and inflation induces buyers to search more intensively. The explanation, however, is not robust and only applies to competitive search at low inflation. In Ennis (2009), sellers have more opportunities to rebalance money holdings than buyers, inflation makes buyers search harder to find sellers to off-load their cash. Liu *et al.* (forthcoming) offer another explanation. They assume that buyers pay the entry fee to enter the market. Inflation reduces buyers' trading surplus, so fewer buyers choose to enter, which increases the matching probability for those who do enter. Nosal (forthcoming) assumes that accepting a current trade reduces the probability of future trading. Inflation reduces the value of future trading and buyers are more likely to accept current trades.

The rest of the paper proceeds as follows. Section 2 describes the environment. In Section 3, we characterize the monetary equilibrium when the preference shock follows a uniform distribution. Section 4 examines the effect of inflation and rationalizes the hot potato effect of inflation. In Section 5, the model is extended to allow for a more general continuous distribution of the preference

shock. Finally, Section 6 concludes. The technical proofs of results in Section 5 are provided in the Appendix.

2 Environment

The model is based on Lagos and Wright (2005). Time is discrete and runs from 0 to ∞. A decentralized market (DM) and a centralized market (CM) open sequentially in each period. The discount factor between two periods is $0 < \beta < 1$. There are two permanent types of agents: buyers and sellers distinguished by their roles in the DM. There is one nonstorable good in each submarket: a CM good and a DM good.

The CM is a centrally located competitive spot market. In the CM, all agents can consume or produce the CM good x. The utility of consuming x units of the CM good is x. If $x < 0$, it means that an agent produces x units of the CM good and incurs disutility x. In the DM, agents are anonymous. Buyers and sellers are randomly matched so that one buyer meets one seller with probability 1. Buyers are those who want to consume but cannot produce. Sellers, on the other hand, can produce but do not want to consume. This generates a lack of double coincidence of wants problem. Together with anonymity, money becomes essential as the medium of exchange. For a seller, the disutility of producing q units of the DM good is $c(q)$ with $c(0) = 0$, $c' > 0$, and $c'' \geq 0$. By consuming q units of the DM good, a buyer's utility is $eu(q)$ where $e \geq 0$ is a preference parameter that determines the buyer's marginal utility of consumption. The utility function $u(q)$ satisfies $u(0) = 0$, $u' > 0 > u''$ and $u'(0) = \infty$.

All buyers are *ex ante* identical before being matched with sellers. *Ex post*, however, they are subject to match specific preference shocks and become heterogeneous during their matches with sellers. The realization of e follows a uniform distribution on the interval $[0, 1]$. Buyers hold private information about the realization of e.

The terms of trade in the DM are characterized by price posting with undirected search. Before a buyer and a seller are matched, the seller posts terms of trade which consist of a menu of price-quantity pairs, and the buyer does not observe the posting. Once they are matched, the buyer sees the posted terms of trade and decides which price-quantity pair to take from the menu. As in other papers that study price posting, buyers may choose not to trade at all after they are matched.[2] Figure 1 describes the timeline of events.

[2] Since agents are anonymous in the DM, they are perceived to lack commitment. Assuming that buyers can choose not to trade is consistent with the environment where there is a lack of commitment.

Fiat money is supplied by the monetary authority. Money supply grows at a constant rate $\gamma \geq \beta$ so that $M_t = \gamma M_{t-1}$. New money is used to finance a lump-sum transfer to buyers at the beginning of each CM. Let $T_t = (\gamma - 1)M_{t-1}$ be the amount of nominal transfer to each buyer.

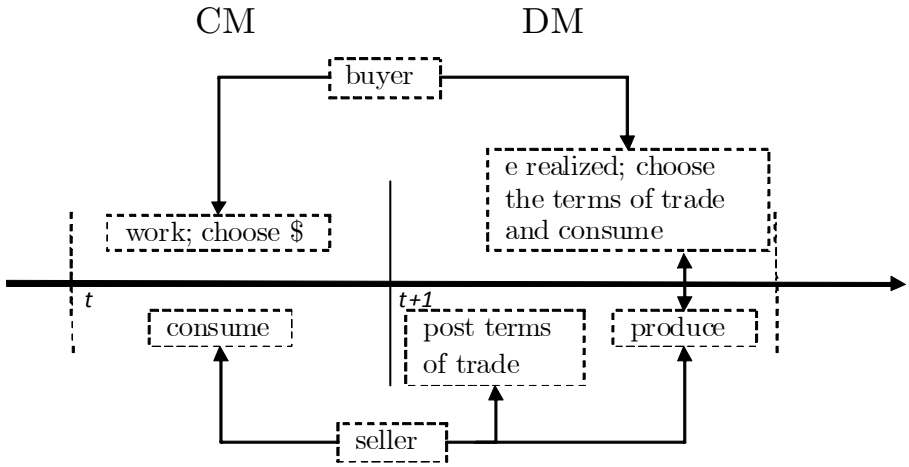

Figure 1: Timeline of Events

3 Price-Posting Equilibrium

Throughout this paper, we assume that money balances are observable. To solve the equilibrium, we begin with analyzing agents' choices in the CM and then move to consider the DM.

3.1 Decision Making in the CM

In the CM, agents rebalance their money holdings by trading money for the CM good x or vice versa. We first consider the buyer's problem. Let $W^b(m)$ denote a buyer's value function while entering the CM with m units of money. Let $\hat{m}_+$ be the money holding upon entering into the DM in the next period, and $V^b(\hat{m}_+)$ be the associated value function. The buyer's value function is

$$\begin{aligned} W^b(m) &= \max_{x,\hat{m}_+} \left[x + \beta V^b(\hat{m}_+)\right] \\ \text{s.t. } x &= \phi(m + T - \hat{m}_+). \end{aligned}$$

where ϕ is the value of money in the CM. Defining $z = \phi m$, $\tau = \phi T$ and $\hat{z}_+ = \phi \hat{m}_+$, we can rewrite the buyer's problem in real terms as

$$W^b(z) = \max_{\hat{z}_+} \left[z + \tau - \frac{\phi}{\phi_+} \hat{z}_+ + \beta V^b\left(\hat{z}_+\right) \right].$$

Note that due to quasilinearity, the choice of $\hat{z}_+$ is independent of z and $W^b(z)$ is linear in z, where $dW^b(z)/dz = 1$. The first order condition is

$$\beta \frac{dV^b\left(\hat{z}_+\right)}{d\hat{z}_+} = \frac{\phi}{\phi_+} \text{ if } \hat{z}_+ > 0.$$

For a seller with z units of real money balance upon entering the CM, let $W^s(z)$ be his value function. The seller simply spends all money that they accumulated in the previous DM and carry 0 money balance to the next DM. The seller's value function is $W^s(z) = z + \beta V^s(0)$ and $dW^s(z)/dz = 1$.

3.2 Decision Making in the DM

In the DM, the seller can potentially post a schedule of price-quantity pairs (q_e, z_e) for all $e \in [0, 1]$. We assume that the seller can observe $\hat{z}$, the buyer's choice of money balance to enter into the DM, so that the seller's posting may depend on $\hat{z}$.[3] The matching function is such that each buyer meets a seller. Upon matching, the buyer realizes the match specific preference shock and the shock is the buyer's private information. After seeing the seller's posting, the buyer decides whether to trade or not, and if trade, which (q_e, z_e) to take.

The seller's choice of pricing schedule is in essence a mechanism design problem, where the seller is the principal and the buyer is the agent with private information (about their preferences). We can apply the revelation principle to formulate the seller's problem as follows: taking the buyer's $\hat{z}$, and the distribution of the preference shocks as given, choose $(q_e, z_e)_{e \in [0,1]}$ to maximize his DM

[3] Refer to Ennis (2008) for a detailed discussion of this assumption.

value function. Formally,

$$
\begin{aligned}
V^s(0) \quad &= \quad \max_{\{q_e, z_e\}_{e\in[0,1]}} \int_0^1 [-c(q_e) + W^s(z_e)]de \\
&\text{s.t.} \begin{cases} q_e \geq 0, & \text{(NN)} \\ z_e \leq \hat{z}, & \text{(CC)} \\ eu(q_e) - z_e \geq eu(q_{e'}) - z_{e'} \text{ for all } e, e' \in [0,1], & \text{(IC)} \\ eu(q_e) - z_e \geq 0. & \text{(PC)} \end{cases}
\end{aligned}
$$

The four constraints are the non-negativity constraints, the cash constraints, the incentive constraints and the participation constraints, respectively. Notice that we use the property that $W^b(z)$ is linear in z to derive the IC. To solve the seller's optimization problem with private information, we use the result from Mas-Collell, Winston and Green (1995, Proposition 23.D.1, page 888) to find the necessary and sufficient conditions to guarantee that the incentive constraints are satisfied. Then we replace the incentive constraints with these conditions and convert the seller's problem into an optimal control problem.[4]

Let $v_e \equiv eu(q_e) - z_e$ denote the buyer's *ex post* trading surplus when the realization of the preference shock is e. The solution (q_e, z_e) is incentive compatible if and only if

$$
\begin{aligned}
& \partial q_e / \partial e \geq 0, \\
v_e \quad = \quad & \int_0^e \dot{v}_x dx = \int_0^e u(q_x)dx \text{ or } \dot{v}_e = u(q_e).
\end{aligned}
$$

We can ignore the PCs for all $e > 0$ because they are implied by the PC for $e = 0$ and the ICs. In addition, the PC binds for buyers with $e = 0$. The seller's problem can be restated as an optimal

[4] Faig and Jerez (2006) use a similar technique in a model where terms of trade are determined by competitive search and buyers have private information about the realization of preference shocks.

control problem with v_e as the state variable and q_e as the control variable,

$$\max_{\{q_e, v_e\}_{e\in[0,1]}} \int_0^1 [-c(q_e) + eu(q_e) - v_e]de$$

$$\text{s.t.} \begin{cases} q_e \geq 0, & \text{(NN)} \\ eu(q_e) - v_e \leq \hat{z}, & \text{(CC)} \\ \partial q_e/\partial e \geq 0, & \text{(IC1)} \\ \dot{v}_e = u(q_e), & \text{(IC2)} \\ v_0 = 0. & \text{(PC)} \end{cases}$$

To proceed, we first disregard the constraint $\partial q_e/\partial e \geq 0$ (and we will impose it later). The Hamiltonian of the optimal control problem is

$$H = [-c(q_e) + eu(q_e) - v_e] + \lambda_e u(q_e),$$

where λ_e is the co-state variable. The Lagrangian is

$$\begin{aligned} \mathcal{L} = &-c(q_e) + eu(q_e) - v_e + \lambda_e u(q_e) \\ &+ \mu_e[\hat{z} - eu(q_e) + v_e] + \theta_e q_e, \end{aligned}$$

where μ_e and θ_e are the Lagrangian multipliers associated with the cash constraint and the non-negativity constraint, respectively. The first-order conditions are

$$q_e \quad : \quad (e + \lambda_e - \mu_e e)u'(q_e) = c'(q_e) - \theta_e, \tag{1}$$

$$v_e \quad : \quad -1 + \mu_e = -\dot{\lambda}_e, \tag{2}$$

$$\lambda_e \quad : \quad u(q_e) = \dot{v}_e, \tag{3}$$

$$\mu_e \quad : \quad \hat{z} - eu(q_e) + v_e \geq 0, \text{ if } > 0, \text{ then } \mu_e = 0, \tag{4}$$

$$\theta_e \quad : \quad q_e \geq 0, \text{ if } > 0, \text{ then } \theta_e = 0. \tag{5}$$

The transversality condition is $\lambda_1 v_1 = 0$. For monetary equilibrium to exist, $v_1 > 0$ and $\lambda_1 = 0$.

Integrating $\dot{\lambda}_e$ over the interval $[e, 1]$, we have

$$\int_e^1 \dot{\lambda}_x dx = \int_e^1 (1-\mu_x)dx \to \lambda_1 - \lambda_e = 1 - e - \int_e^1 \mu_x dx \to \lambda_e = (e-1) + \Sigma_e \text{ where } \Sigma_e \equiv \int_e^1 \mu_x dx.$$

where the last step uses the transversality condition. It follows that (1) becomes

$$[(2e-1) + \Sigma_e - \mu_e e]u'(q_e) = c'(q_e) - \theta_e \tag{6}$$

Now we impose the constraint $\partial q_e/\partial e \geq 0$ (and as a result $\partial z_e/\partial e \geq 0$). Given this, we can consider the seller's problem in three regions of e divided by $0 \leq e_0 \leq \hat{e} \leq 1$.

Case (a). When e is small ($e \leq e_0$), no exchange occurs or $q_e = z_e = v_e = 0$. The NN binds and the CC is loose. In this case, q is constant at 0 so the constraint $\partial q_e/\partial e \geq 0$ is satisfied.

Case (b). For intermediate values of e ($e_0 \leq e \leq \hat{e}$), neither NN nor CC binds, i.e., $\mu_e = \theta_e = 0$. Hence, (6) reduces to[5]

$$[(2e-1) + \Sigma_{\hat{e}}]u'(q_e) = c'(q_e), \tag{7}$$

which can be used to solve q_e. Since $u'' < 0 \leq c''$, we can conclude that $\partial q_e/\partial e > 0$. As a result, the constraint $\partial q_e/\partial e \geq 0$ is automatically satisfied.

Case (c). When e is high ($e \geq \hat{e}$), the buyer is charged all his money holdings, or $z_e = \hat{z}$ and $q_e = \hat{q}$. The constraint $\partial q_e/\partial e \geq 0$ is again satisfied and we can solve $\hat{q}$ from

$$[(2\hat{e}-1) + \Sigma_{\hat{e}}]u'(\hat{q}) = c'(\hat{q}). \tag{8}$$

In the next step, we will find the term $\Sigma_{\hat{e}}$ in (7) and (8) as a function of $\hat{e}$. Since $q_e = \hat{q}$ for all $e > \hat{e}$, $\hat{q}$ also solves

$$[2e - 1 + \Sigma_e - \mu_e e]\, u'(\hat{q}) = c'(\hat{q}). \tag{9}$$

Combining (8) and (9), and using $\mu_e = -\dot{\Sigma}_e$, we have a differential equation of Σ_e,

$$2\hat{e} + \Sigma_{\hat{e}} = 2e + \Sigma_e + e\dot{\Sigma}_e. \tag{10}$$

[5] Notice that since $\mu_e = 0$ for $e \in [0, \hat{e}]$, we have $\Sigma_e = \Sigma_{\hat{e}}$ for $e \in [0, \hat{e}]$.

Together with the end-point conditions $\mu_{\hat{e}} = 0$ and $\Sigma_1 = 0$, (10) gives rises to the following result[6]

$$\begin{aligned} \Sigma_e &= \left[(1-e) + \hat{e}^2\left(1 - \frac{1}{e}\right)\right], \\ \Sigma_{\hat{e}} &= (1-\hat{e})^2, \\ \mu_e &= \left[1 - (\frac{\hat{e}}{e})^2\right]. \end{aligned} \tag{11}$$

The final step to complete the solution to the seller's problem is to determine e_0 and $\hat{e}$. First, note that e_0 can be expressed as a function of $\hat{e}$ and is determined by

$$[(2e_0 - 1) + \Sigma_{\hat{e}}]u'(0) = c'(0) \text{ or } e_0 = \hat{e} - \frac{\hat{e}^2}{2}.$$

It then suffices to find $\hat{e}$ as a function of $\hat{z}$. From IC2 and the definition of v_e, $\hat{e}$ is solved from[7]

$$\hat{z} = (\hat{e} - \frac{\hat{e}^2}{2})u(\hat{q}) + \frac{1}{2}c(\hat{q}).$$

In general, it is possible that $\hat{e}$ takes the corner solution $\hat{e} = 1$ (so that no buyer is cash constrained) when $\hat{z}$ is large enough. This situation occurs when $\hat{z} \geq \bar{z}$ with $\bar{z}$ given by

$$\bar{z} = \frac{1}{2}\left[u(q_1) + c(q_1)\right] \text{ and } u'(q_1) = c'(q_1).$$

In this situation, buyers with highest preferences ($e = 1$) buy q_1 amount of goods with $\bar{z}$ units of money.

To summarize, the seller's pricing schedule (q_e, z_e) and v_e are functions of $\hat{z}$. Lemma 1 states that the pricing schedule posted by the seller is unique and is a function of $\hat{z}$.

Lemma 1 *For any given $\hat{z} \leq \bar{z}$, the optimal solution (q_e, z_e, v_e) for all $e \in [0,1]$ to the seller's price posting problem is unique and is characterized by:*

(i) For $e \in [0, e_0]$, $q_e = z_e = v_e = 0$;

[6] We solve (10) by a guess-and-verify method. We guess that $\mu_e = 1 + \frac{k}{e^s}$ and find that $k = -\hat{e}^2$.

[7] Integrating both sides of IC2 with respect to q_e over $[0, \hat{e}]$ gives $\int_0^{\hat{e}}[2e - 1 + \Sigma_{\hat{e}}]u'(q_e)dq_e = \int_0^{\hat{e}} c'(q_e)dq_e$ or $\int_0^{\hat{e}} 2eu'(q_e)dq_e - (1 - \Sigma_{\hat{e}})u(\hat{q}) = c(\hat{q})$. Implementing integrating by parts on the first terms gives $2\left[\hat{e}u(\hat{q}) - \int_0^{\hat{e}} u(q_e)de\right] - (1 - \Sigma_{\hat{e}})u(\hat{q}) = c(\hat{q})$. Combine this with $v_e = \int_0^{\hat{e}} u(q_e)de = \hat{e}u(\hat{q}) - \hat{z}$ and we have the equation below.

(ii) For $e \in [e_0, \hat{e}]$,

$$
\begin{aligned}
q_e &: (2e - 2\hat{e} + \hat{e}^2)u'(q_e) = c'(q_e), && (12)\\
v_e &= \left(e - \hat{e} + \frac{\hat{e}^2}{2}\right) u(q_e) - \frac{1}{2}c(q_e), \\
z_e &= \left(\hat{e} - \frac{\hat{e}^2}{2}\right) u(q_e) + \frac{1}{2}c(q_e); && (13)
\end{aligned}
$$

(iii) For $e \in [\hat{e}, 1]$,

$$
\begin{aligned}
q_e &= \hat{q} : \hat{e}^2 u'(\hat{q}) = c'(\hat{q}), && (14)\\
z_e &= \hat{z}, && (15)\\
v_e &= eu(\hat{q}) - \hat{z};
\end{aligned}
$$

(iv) e_0 and $\hat{e}$ are given by

$$
\begin{aligned}
\hat{z} &= (\hat{e} - \frac{\hat{e}^2}{2})u(\hat{q}) + \frac{1}{2}c(\hat{q}) && (16)\\
e_0 &= \hat{e} - \frac{\hat{e}^2}{2} && (17)
\end{aligned}
$$

For $\hat{z} > \bar{z}$, (16) is replaced by $\hat{e} = 1$.

3.3 Monetary Equilibrium

Given the seller's pricing schedule in the DM, we are now ready to derive the buyer's demand for money in the CM. As the buyer knows how (q_e, z_e, v_e) depends on $\hat{z}$, the buyer's value function in the DM with $\hat{z}$ is given by

$$
\begin{aligned}
V^b(\hat{z}) &= \int_0^1 [eu(q_e(\hat{z})) + \hat{z} - z_e(\hat{z})]de + W^b(0) \\
&= S(\hat{z}) + \hat{z} + W^b(0),
\end{aligned}
$$

where $S(\hat{z}) \equiv \int_0^1 v_e(\hat{z})de$ is the expected trading surplus in the DM. In the CM, the buyer choice of $\hat{z}_+$ satisfies

$$
\frac{\phi}{\phi_+} = \beta \frac{\partial V^b(\hat{z}_+)}{\partial \hat{z}_+} = \beta \left[S'(\hat{z}_+) + 1\right].
$$

We will focus on the steady-state equilibrium where $\hat{z}$ is constant and $\phi/\phi_+ = \gamma$. The equilibrium $\hat{z}$ is given by

$$\frac{\gamma}{\beta} - 1 = S'(\hat{z}). \tag{18}$$

The nominal interest rate follows the Fisher equation and is given by $i = \gamma/\beta - 1$. As long as $\gamma > \beta$ or the nominal interest rate is positive, the cash constraint will bind for buyers with high enough preferences, so the optimal $\hat{z}$ cannot be greater than $\bar{z}$. The buyer's choice of $\hat{z}$ from (18), together with the seller's optimal pricing schedule, completes the description of the price posting monetary equilibrium.

Definition 1 *In the steady state, a price-posting monetary equilibrium is a list of (q_e, z_e) for all $e \in [0,1]$ and $\hat{z}$ such that [1] (q_e, z_e) maximize $V^s(0)$ given $\hat{z}$ (characterized by (12),(13),(14),(15), (16) and (17)) ; and [2] $\hat{z}$ maximizes $W^b(z)$ for any z (characterized by (18)).*

Proposition 1 *There exists a unique monetary equilibrium for generic values of γ.*

Proof. We first show that knowing how (q_e, z_e) depends on $\hat{z}$, buyers choose a unique $\hat{z}$ in the CM. Note that

$$S'(\hat{z}) = \frac{1}{2}(1-\hat{e}^2)u'(\hat{q})\frac{d\hat{q}}{d\hat{z}} - \frac{1}{2}(1-\hat{e})\left[\hat{e}^2 u(\hat{q}) - c(\hat{q})\right]\frac{d\hat{e}}{d\hat{z}} - (1-\hat{e}),$$

where $d\hat{q}/d\hat{z}$ and $d\hat{e}/d\hat{z}$ are solved by differentiating (14) and (16) with respect to $\hat{z}$. We can see that $S'(\hat{z})$ does not depend on i directly. Following the proof in Wright (2008), $\hat{z}$ is unique for generic values of γ. It follows from proposition 1 that a unique $\hat{z}$ leads to a unique pricing schedule. In addition, one can show that $S'(0) = \infty$. Hence, monetary equilibrium exists and it is unique for generic values of γ.■

In price posting equilibrium, all buyers choose the same $\hat{z}$ in the CM. As long as $\gamma < \infty$, we have $\hat{z} > 0$ and $\hat{e}, e_0, (\hat{e} - e_0) > 0$. A continuum of prices is observed when preference shocks follow a uniform distribution. The number of prices observed in equilibrium is measured by $\hat{e} - e_0$. This is in contrast to Ennis (2008), where only a single price is observed with a two-point distribution of preference shocks. This is also in contrast to Jafarey and Masters (2003), and Curtis and Wright (2004), where there can be at most two prices in equilibrium because money is indivisible. Therefore, the law of two prices cannot be generalized to models with divisible money. Compared with the previous papers on price posting, our model generates a richer pricing schedule (on the seller's side) and spending pattern (on the buyer's side).

When buyers hold private information about match specific preference shocks that follow a uniform distribution, sellers post prices such that the pricing schedule can differentiate buyers of

different ex post preferences. Buyers with $e \in [0, e_0]$ choose not to spend money and consume nothing. Buyers in this group do not value consumption much. As a result, it is better for them to hold on to their money balances and wait for future consumption opportunities. For buyers with $e \in [\hat{e}, 1]$, the cash constraint binds and they exhaust their money holdings during exchanges. Buyers with $e \in [e_0, \hat{e}]$ spend part of their money holdings and as e increases, q_e and z_e both increase. In equilibrium, a positive measure $(\hat{e} - e_0)$ of prices is observed. The unit price is given by

$$\frac{z_e}{q_e} = \left(\hat{e} - \frac{\hat{e}^2}{2}\right)\frac{u(q_e)}{q_e} + \frac{1}{2}\frac{c(q_e)}{q_e} \text{ for } e \in [e_0, \hat{e}]. \tag{19}$$

In general, z_e/q_e depends on q_e, which means that the pricing schedule is nonlinear. The next proposition establishes that larger quantities are associated with lower per unit prices when the cost function is linear.

Proposition 2 *Quantity discounts: when $c(q) = q$, $d(z_e/q_e)/dq_e < 0$ for $e \in [e_0, \hat{e}]$.*

The proof follows directly from (19). This result appears to be consistent with the pricing strategy in reality. One common practice used by sellers is to offer quantity discounts, i.e., lower unit prices for larger purchases. The traditional explanation for quantity discounts is that the unit cost for sellers to produce larger quantities is smaller. Here, we find that even if the unit cost is constant, sellers may still choose to offer quantity discounts. The model provides an alternative explanation for quantity discounts. When buyers' preferences are private information, sellers offer quantity discounts to induce buyers with higher preferences to buy more.

One may wonder if quantity discounts still hold when the cost function is convex. In this case, there are two factors that affect the per unit price – convex cost and private information. The convex cost function implies that per unit price tends to increase when the quantity becomes larger. In Peterson and Shi (2004), buyers experience match specific preference shocks. In each bilateral trade, price is determined by the buyer's take-or-leave-it offer. The model generates a continuous distribution of prices. Under the assumption of a convex cost function (a standard assumption adopted in the monetary search literature), the model predicts that higher quantities are associated with higher per unit prices.[8] With a convex cost function, it is not obvious analytically whether the per unit price becomes higher or lower as quantity increases. From the numerical exercise, quantity discounts hold even when the cost function is very convex. It seems that the effect of private information on per unit price dominates the effect of the convex cost function.

[8] It can be shown that competitive pricing with bilateral trades generates a similar pricing pattern.

4 Hot Potato Effect of Inflation

In this economy, it is easy to verify that the Friedman rule cannot achieve the first-best allocation. Inflation affects the economy along both the extensive and intensive margins. The extensive margin is captured by e_0, the measure of buyers who choose not to trade ex post. The intensive margin is captured by q_e for $e \in [e_0, 1]$.

Proposition 3 *The effects of inflation on* $(\hat{z}, e_0, \hat{e}, \hat{e} - e_0)$*:* $d\hat{z}/d\gamma < 0$*,* $de_0/d\gamma < 0$*,* $d\hat{e}/d\gamma < 0$ *and* $d(\hat{e} - e_0)/d\gamma < 0$*.*

Proof. Consider $\hat{z} = \bar{z}$. As $\hat{e} = 1$, it is straightforward that $S'(\bar{z}) = 0$. The optimal $\hat{z}$ satisfies $S'(\hat{z}) + 1 - \frac{\gamma}{\beta} = 0$. Since we focus on $\hat{z} \leq \bar{z}$, it implies that $S'(\bar{z}) + 1 - \frac{\gamma}{\beta} < 0$ for $\gamma > \beta$ and $\gamma \to \beta$ from above. As $S'(0) = \infty$, we know that $S'(0) + 1 - \frac{\gamma}{\beta} = \infty$. For $\hat{z} \in [0, \bar{z}]$, a generically unique optimal $\hat{z}$ implies that $S''(\hat{z}) < 0$. From $S'(\hat{z}) + 1 - \frac{\gamma}{\beta} = 0$, $\frac{d\hat{z}}{d\gamma} = \frac{1}{\beta S''(\hat{z})} < 0$.

Using (14), one can find $\frac{d\hat{q}}{d\gamma}$ in terms of $\frac{d\hat{e}}{d\gamma}$, which is substituted into (16) to get $\frac{d\hat{e}}{d\gamma} < 0$. It follows from (17) that $\frac{de_0}{d\gamma} < 0$ and $\frac{d(\hat{e}-e_0)}{d\gamma} < 0$ as well.■

Proposition 4 *The effects of inflation on* $(q_e, z_e, z_e/\hat{z})$ *for* $e \in [e_0, \hat{e}]$*:* $dq_e/d\gamma > 0$*,* $dz_e/d\gamma > 0$*, and* $d(z_e/\hat{z})/d\gamma > 0$ *if* $u''' \geq 0$ *and* $c''' \leq 0$*.*

Proof. As $d\hat{e}/d\gamma < 0$, proving $dq_e/d\gamma > 0$ and $dz_e/d\gamma > 0$ is equivalent to proving $dq_e/d\hat{e} < 0$ and $dz_e/d\hat{e} < 0$. We do this by proving $d\left(\frac{dq_e}{de}\right)/d\hat{e} < 0$ and $d\left(\frac{dz_e}{de}\right)/d\hat{e} < 0$, or $q_e(e;\hat{e})$ and $z_e(e;\hat{e})$ are steeper for lower values of $\hat{e}$.

We first show $d\left(\frac{dq_e}{de}\right)/d\hat{e} < 0$. Use (12) that characterizes q_e for $e \in [e_0, \hat{e}]$ to calculate dq_e/de as (to simplify notations, we omit the arguments of the functions $u(\cdot)$ and $c(\cdot)$):

$$\frac{dq_e}{de} = \frac{2u'^2}{c''u' - c'u''}$$

The term $\frac{(u')^2}{c''u'-c'u''} > 0$ decreases in $\hat{e}$ if $c''' \leq 0$ and $u''' \geq 0$:

$$\begin{aligned}
\frac{d\left(\frac{u'^2}{c''u'-c'u''}\right)}{d\hat{e}} &= \frac{d\left(\frac{u'^2}{c''u'-c'u''}\right)}{dq_e}\frac{dq_e}{d\hat{e}} \\
&\propto \frac{2u'(c''u' - c'u'') - u'^2(c'''u' + c''u'' - c''u'' - c'u''')}{(c''u' - c'u'')^2}\frac{dq_e}{d\hat{e}} \\
&= \frac{2u'(c''u' - c'u'') - u'^3c''' + u'^2c'u'''}{(c''u' - c'u'')^2}\frac{dq_e}{d\hat{e}} \\
&< 0 \text{ if } c''' < 0 \text{ and } u''' > 0.
\end{aligned}$$

Now we show $d\left(\frac{dz_e}{de}\right)/d\hat{e} < 0$. Recall that $dz_e/de = eu'(q_e)dq_e/de$, so

$$\frac{dz_e}{de} = \frac{eu'^3}{c''u' - c'u''} > 0.$$

The term $\frac{eu'^3}{c''u'-c'u''}$ decreases in $\hat{e}$ if $c''' \leq 0$ and $u''' \geq 0$ because

$$\begin{aligned}
& \frac{d\left(\frac{eu'^3}{c''u'-c'u''}\right)}{dq_e} \\
= \quad & e\frac{3u'^2(c''u' - c'u'') - u'^3(c'''u' + c''u'' - c''u'' - c'u''')}{(c''u' - c'u'')^2} \\
= \quad & e\frac{3u'^2(c''u' - c'u'') - u'^4c''' + u'^3c'u'''}{(c''u' - c'u'')^2} \\
> \quad & 0 \text{ if } c''' < 0 \text{ and } u''' > 0.
\end{aligned}$$

As $dz_e/d\gamma > 0$ and $d\hat{z}/d\gamma < 0$, $d(z_e/\hat{z})/d\gamma > 0$.■

Here, we have the standard result that inflation decreases the demand for real money balances. Besides that, the model provides a natural explanation about the "hot potato" effect of inflation, which says that buyers choose to spend money faster when inflation is higher. In price posting equilibrium, buyers spend money only when their preferences are high enough, and those with higher preferences spend more. Inflation reduces the future purchasing power of money and the benefit of retaining money, so buyers spend money faster. The model can capture the hot potato effect along both the extensive and intensive margins. As shown in Proposition 3, inflation reduces e_0, implying that more buyers start spending money. Proposition 4 implies that for those who spend money, higher inflation induces them to spend more money, captured by higher $z_e/\hat{z}$. Refer to Figure 2 for a graphical illustration of the hot potato effect.

Several papers in monetary theory have tried to rationalize the hot potato effect. Lagos and Rocheteau (2005) endogenize search intensity, and they show that inflation may increase buyer's trading surplus and their search intensity. However, the result holds only for low inflation rates when the pricing mechanism is competitive search. Bargaining cannot deliver similar results because inflation monotonically reduces the buyer's surplus in a match and thus the search effort. Ennis (2009) assumes that sellers have more opportunities to rebalance money holdings than buyers (i.e., they have more frequent access to the centralized market than buyers), inflation makes buyers search harder to find sellers to off-load their money.

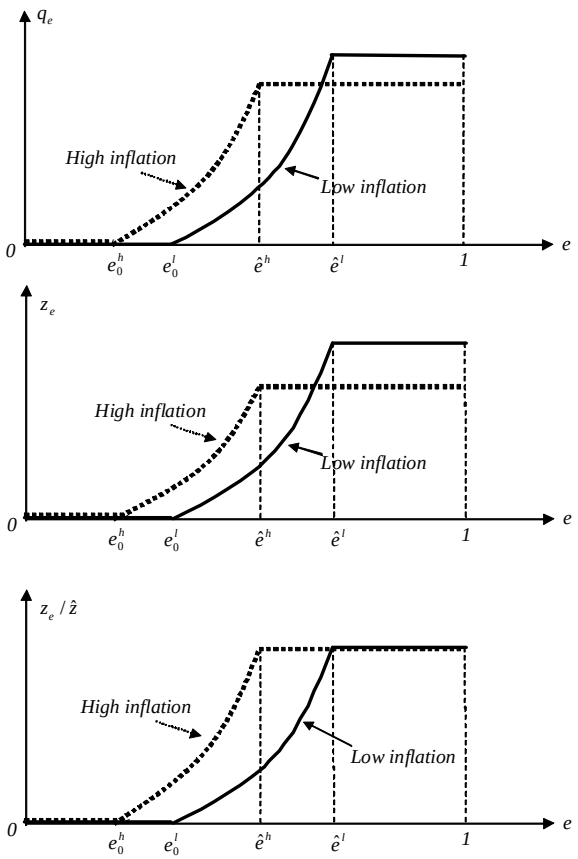

Figure 2: Hot Potato Effect of Inflation

More recently, Liu *et al.* (forthcoming) assume that buyers pay the entry fee to enter the market. Inflation reduces buyers' trading surplus, so fewer buyers choose to enter. For those who do enter, the matching probability is higher. Buyers are able to spend faster simply because they have more opportunities to trade, not because they actively try to get rid of their money. Furthermore, the result that the matching probability is higher for buyers when the inflation rate is higher depends on whether free entry is assumed by buyers or sellers. For example, Rocheteau and Wright (2005) allow free entry by sellers and inflation monotonically reduces buyers' trading probability. To explain the hot potato effect of inflation, it is not clear that one should assume free entry by buyers rather than by sellers.

In Nosal (forthcoming), it is assumed that accepting a current trade involves an exogenous cost that lowers the probability of future trading. Inflation reduces the value of future trading and people are more likely to accept a current trade. Our paper and Nosal (forthcoming) both emphasize that buyers spend faster in fear that money will lose purchasing power in the future due to inflation. We

provide a more integrated model and do not rely on assuming the exogenous reduction of future trading probability as a consequence of accepting a current trade.[9]

5 Extension to More General Distributions

The above analysis assumes that the distribution of preference shocks follows a uniform distribution. In this section, we extend our model by allowing preference shocks to follow a general continuous distribution. Let $f(e)$ and $F(e)$ denote the p.d.f. and the c.d.f. of the distribution, respectively. Without loss of generality, we still focus on $e \in [0,1]$. We assume that $ef(e)$ is increasing in e, i.e., $f(e) + ef'(e) > 0$.

5.1 Seller's Price Posting

The choice problems for buyers and sellers in each market remain the same as before. We first solve for the seller's optimal price posting problem, taking $\hat{z}$ as given. Similar to the case with a uniform distribution, $\bar{z}$ is defined as

$$\bar{z} = u(q_1) - \int_{e_0}^{1} u(q_x)dx$$

where q_1 remains the same as before and $(e_0, q_x)_{x\in[e_0,1]}$ solve

$$\begin{aligned} e_0 f(e_0) + F(e_0) &= 1, \\ \left[e - \frac{1-F(e)}{f(e)}\right] u'(q_x) &= c'(q_x). \end{aligned}$$

We characterize the solution in Lemma 2.

Lemma 2 *For any given $\hat{z} \leq \bar{z}$, the optimal solution (q_e, z_e, v_e) for all $e \in [0,1]$ to the seller's price posting problem is unique and is characterized by*

(i) For $e \in [0, e_0]$, $q_e = z_e = v_e = 0$;

(ii) For $e \in [e_0, \hat{e}]$,

$$\begin{aligned} q_e &: \left[e - \frac{1-F(e)-\Sigma_{\hat{e}}}{f(e)}\right] u'(q_e) = c'(q_e), \qquad (20) \\ \Sigma_{\hat{e}}(\hat{e}) &= \frac{[1-F(\hat{e})]^2}{\hat{e}f(\hat{e}) + 1 - F(\hat{e})}, \\ z_e &= eu(q_e) - v_e, \\ v_e &= \int_0^e u(q_e)dF(e); \end{aligned}$$

[9] In Nosal (forthcoming), the terms of trade in the DM is determined by buyers making take-it-or-leave-it offers.

(iii) For $e \in [\hat{e}, 1]$,

$$\begin{aligned} q_e &= \hat{q} : \left[\hat{e} - \frac{1 - F(\hat{e}) - \Sigma_{\hat{e}}}{f(\hat{e})}\right] u'(\hat{q}) = c'(\hat{q}), \\ z_e &= \hat{z}, \\ v_e &= eu(\hat{q}) - \hat{z}; \end{aligned} \tag{21}$$

(iv) e_0 and $\hat{e}$ are given by

$$\hat{e}u(\hat{q}) - \int_{e_0}^{\hat{e}} u(q_x)dx = \hat{z}, \tag{22}$$

$$e_0 f(e_0) + F(e_0) - 1 + \Sigma_{\hat{e}} = 0. \tag{23}$$

For $\hat{z} > \bar{z}$, (22) is replaced by $\hat{e} = 1$.

Proof. See the Appendix.■

With a general continuous distribution, the pricing schedule posted by sellers is again to extract buyers' private information about their preference shocks. Low preference buyers optimally choose to hold on to their money balances and wait for future consumption opportunities. The endogenous extensive margin effect still exists. Buyers with preferences higher than e_0 but below $\hat{e}$ spend part of their money balances and consume more as e increases. For buyers with $[\hat{e}, 1]$, they spend all their money. We also have the following comparative statics results.

Proposition 5 *The seller's optimal pricing schedule has the following properties: $d\hat{e}/d\hat{z} > 0$, $d\hat{q}/d\hat{z} > 0$, $de_0/d\hat{z} > 0$ and $dq_e/d\hat{z} < 0$ for $e \in [e_0, \hat{e}]$.*

Proof. See the appendix.■

Proposition 6 *Quantity discounts: when $c(q) = q$, $d(z_e/q_e)/dq_e < 0$ for $e \in [e_0, \hat{e}]$.*

Proof. See the appendix.■

5.2 Equilibrium

After deriving the seller's pricing schedule in the DM, we can solve the buyer's demand for money in the CM from (18). The definition of a price posting equilibrium remains the same as Definition 1. Since $S(\hat{z})$ and $S'(\hat{z})$ do not directly depend on i, a unique monetary equilibrium exists for generic values of γ. Recall that $ef(e)$ is increasing in e. From (23), one can show that $1 - \Sigma_{\hat{e}} < \hat{e}f(\hat{e}) + F(\hat{e})$ and hence $e_0 < \hat{e}$. There is a continuum of prices observed in equilibrium, which confirms that the

law of two prices in Curtis and Wright (2004) cannot be generalized to a divisible money framework with a general continuous distribution.

Proposition 7 *The effects of inflation on* $(\hat{z}, e_0, \hat{e})$*:* $d\hat{z}/d\gamma < 0$*,* $de_0/d\gamma < 0$ *and* $d\hat{e}/d\gamma < 0$*.*

Proof. See the appendix.■

Proposition 8 *The effects of inflation on* $(q_e, z_e, z_e/\hat{z})$ *for* $e \in [e_0, \hat{e}]$*:* $dq_e/d\gamma > 0$*,* $dz_e/d\gamma > 0$*, and* $d(z_e/\hat{z})/d\gamma > 0$ *if* $u''' \geq 0$ *and* $c''' \leq 0$ *and* $f'(e) \leq 0$*.*

Proof. See the appendix.■

Inflation reduces buyers' choice of the real money balance. Inflation also induces more buyers to spend their money. The model with a more general distribution can still capture the hot potato effect along the extensive margin. To show the hot potato effect along the intensive margin, we impose the assumption $f'(e) \leq 0$.

To summarize, most of our results from a uniform distribution of preference shocks remain valid when the distribution of preference shocks is extended to a more general continuous distribution. It shows that price posting with undirected search is an interesting pricing mechanism to examine because it captures several commonly observed pricing behaviors such as quantity discounts and the hot potato effect of inflation. It also verifies that the divisibility of money matters. The law of two prices found in the indivisible framework no longer holds when money becomes divisible.

6 Conclusion

This paper studies the monetary equilibrium when sellers post prices and search is undirected. It contributes to the literature by considering a continuous distribution of the match specific preference shocks in a divisible money framework. We show that price posting monetary equilibrium exists and it is unique. Unlike the predictions from the indivisible money framework or the divisible money framework with a two-point distribution of preference shocks, we find that there exists a continuum of price-quantity pairs and the law of two prices does not hold. The model can also be used to explain seller's pricing behavior and the buyer's spending pattern. The equilibrium pricing schedule features quantity discounts, a common pricing practice. The model offers a very natural explanation of the hot potato effect – buyers spend money faster when facing higher inflation. As inflation increases, the value of retaining money for future spending decreases. So more buyers start spending money and in addition, buyers spend a larger fraction of their money.

We think that it is useful to study price posting because it is more realistic than bargaining or price taking in many situations. One may argue that it is hard to categorize whether search is directed or undirected in the real world. In the future, we plan to study price posting with a mixture of directed search (informed buyers) and undirected search (uninformed buyers). It will be interesting to investigate how informed buyers interact with uninformed buyers and how sellers set prices in this context.

A Appendix

A.1 Proof of Proposition 6: Seller's Optimal Price Posting with a General Distribution

Proof. The seller's problem is

$$\max_{q_e, v_e} \int_0^1 [-c(q_e) + eu(q_e) - v_e] dF(e)$$

$$\text{s.t.} \begin{cases} q_e \geq 0; & \text{(NN)} \\ eu(q_e) - v_e \leq \hat{z}; & \text{(CC)} \\ \partial q_e / \partial e \geq 0; & \text{(IC1)} \\ \dot{v}_e = u(q_e); & \text{(IC2)} \\ v_0 = 0. & \text{(PC)} \end{cases}$$

for all $e \in [0, 1]$.

To proceed, we first ignore the constraint $\partial q_e / \partial e \geq 0$. We will impose the constraint later to find the solution. The Hamiltonian of the optimal control problem is $H = [-c(q_e) + eu(q_e) - v_e] f(e) + \lambda_e u(q_e)$ where λ_e is the co-state variable. The Lagrangian is

$$\begin{aligned} \mathcal{L} &= f(e)[eu(q_e) - c(q_e) - v_e] + \lambda_e u(q_e) \\ &\quad + \mu_e [\hat{z} - eu(q_e) + v_e] \\ &\quad + \theta_e q_e, \end{aligned}$$

where μ_e and θ_e are the Lagrangian multipliers associated with the cash constraint and the non-negativity constraint, respectively. The first-order conditions are

$$q_e \quad : \quad [ef(e) + \lambda_e - \mu_e e]\, u'(q_e) = f(e)c'(q_e) - \theta_e, \tag{24}$$

$$v_e \quad : \quad -f(e) + \mu_e = -\dot{\lambda}_e, \tag{25}$$

$$\lambda_e \quad : \quad u(q_e) = \dot{v}_e, \tag{26}$$

$$\mu_e \quad : \quad \hat{z} - eu(q_e) + v_e \geq 0, \text{ if } > 0, \text{ then } \mu_e = 0, \tag{27}$$

$$\theta_e \quad : \quad q_e \geq 0, \text{ if } > 0, \text{ then } \theta_e = 0. \tag{28}$$

The transversality condition is $\lambda_1 v_1 = 0$. For monetary equilibrium to exist, $v_1 > 0$ and $\lambda_1 = 0$. Integrating (25) over the interval $[e, 1]$, we have

$$\int_e^1 \dot{\lambda}_x dx = F(1) - F(e) - \int_e^1 \mu_x dx$$

and $\lambda_1 - \lambda_e = 1 - F(e) - \Sigma_e$,where $\Sigma_e = \int_e^1 \mu_x dx$. Since we focus on monetary equilibria, it follows from the transversality condition that

$$\lambda_e = F(e) - 1 + \Sigma_e. \tag{29}$$

Plugging (29) into (24), we get

$$[ef(e) + F(e) - 1 + \Sigma_e - \mu_e e]\, u'(q_e) = f(e)c'(q_e) - \theta_e. \tag{30}$$

Since $\partial q_e/\partial e \geq 0$ (and as a result $\partial z_e/\partial e \geq 0$), we can discuss the solution to the seller's problem by dividing e into three regions characterized by two threshold values of e, e_0 and $\hat{e}$. We will consider three cases in the following.

Case (a). For $e \leq e_0$, the NN binds and $q_e = z_e = v_e = 0$.

Case (b). For $e \in [e_0, \hat{e}]$, neither NN nor CC binds or $\mu_e = \theta_e = 0$. (30) reduces to[10]

$$[ef(e) + F(e) - 1 + \Sigma_{\hat{e}}]\, u'(q_e) = f(e)c'(q_e) \tag{31}$$

Case (c). For $e \in [\hat{e}, 1]$, CC binds. In this case, $z_e = \hat{z}$, $q_e = \hat{q}$ with $\hat{q}$ solving

$$[\hat{e}f(\hat{e}) + F(\hat{e}) - 1 + \Sigma_{\hat{e}}]\, u'(\hat{q}) = f(\hat{e})c'(\hat{q}) \tag{32}$$

In the next step, we solve for $\Sigma_{\hat{e}}$ as a function of $\hat{e}$. To do this, note that for $e \in (\hat{e}, 1)$, CC binds and $q_e = \hat{q}$ solves

$$[ef(e) + F(e) - 1 + \Sigma_e - \mu_e e]\, u'(\hat{q}) = f(e)c'(\hat{q}) \tag{33}$$

Combining (32) and (33), we reach

$$\begin{aligned} & \hat{e}f(\hat{e})f(e) + F(\hat{e})f(e) - f(e) + f(e)\Sigma_{\hat{e}} \\ = \; & ef(e)f(\hat{e}) + F(e)f(\hat{e}) - f(\hat{e}) + f(\hat{e})\Sigma_e + \dot{\Sigma}_e e f(\hat{e}). \end{aligned} \tag{34}$$

[10] Notice that $\Sigma_e = \Sigma_{\hat{e}}$ for $e \in [0, \hat{e}]$.

where we use $\mu_e = -\dot{\Sigma}_e$. Rewriting (34) as

$$\dot{\Sigma}_e + \Sigma_e \frac{1}{e} = \left[\hat{e} + \frac{F(\hat{e})}{f(\hat{e})} - \frac{1}{f(\hat{e})} + \frac{\Sigma_{\hat{e}}}{f(\hat{e})}\right] \frac{f(e)}{e} - f(e) - \frac{F(e)}{e} + \frac{1}{e} \equiv Q(e),$$

which is a first-order differential equation of Σ_e. Solving the first-order differential equation, we have

$$\Sigma_e = \frac{1}{e} \int eQ(e)de.$$

and

$$e\Sigma_e = \left[\hat{e} + \frac{F(\hat{e})}{f(\hat{e})} - \frac{1}{f(\hat{e})} + \frac{\Sigma_{\hat{e}}}{f(\hat{e})}\right] F(e) - eF(e) + e + C \tag{35}$$

where C is a constant determined by the boundary condition $\Sigma_1 = 0$:

$$C = \frac{1}{f(\hat{e})} - \hat{e} - \frac{F(\hat{e})}{f(\hat{e})} - \frac{\Sigma_{\hat{e}}}{f(\hat{e})}.$$

We can solve $\Sigma_{\hat{e}}$ as a function of $\hat{e}$ by using (35) for $e = \hat{e}$:

$$\Sigma_{\hat{e}}(\hat{e}) = \frac{[1 - F(\hat{e})]^2}{\hat{e}f(\hat{e}) + 1 - F(\hat{e})} = \frac{1 - F(\hat{e})}{\frac{\hat{e}f(\hat{e})}{1-F(\hat{e})} + 1} < 1 - F(\hat{e}). \tag{36}$$

Since $ef(e)$ increases in e, the numerator of $\Sigma_{\hat{e}}$ increases in $\hat{e}$. It follows that $\Sigma_{\hat{e}}$ decreases in $\hat{e}$.

Finally, we will prove that $\partial q_e/\partial e > 0$ for $e \in [e_0, \hat{e}]$ to justify that the solution satisfies the constraint $\partial q_e/\partial e \geq 0$. We rearrange the first-order condition for $q_{e\in[e_0,\hat{e}]}$ as

$$\frac{u'(q_e)}{c'(q_e)} = \frac{f(e)}{ef(e) + F(e) - 1 + \Sigma_{\hat{e}}} = \frac{\frac{1}{e}}{1 + \frac{F(e)-1}{ef(e)} + \frac{\Sigma_{\hat{e}}}{ef(e)}}.$$

As $\frac{1}{e}$ is decreasing in e, we only need to show that $\chi(e) = \frac{F(e)-1}{ef(e)} + \frac{\Sigma_{\hat{e}}}{ef(e)}$ is increasing in e. Since $\Sigma_{\hat{e}} < 1 - F(\hat{e})$ and $f(e) + ef'(e) > 0$,

$$\chi'(e) = \frac{e[f(e)]^2 + [1 - F(e) - \Sigma_{\hat{e}}][f(e) + ef'(e)]}{[ef(e)]^2} > 0.$$

Using the results above, the derivation of Proposition 6 is a straightforward exercise.■

A.2 Proof of Proposition 7: Properties of Seller's Optimal Pricing Schedule

Proof. Let $\varphi(\hat{e}) \equiv \hat{e} - \frac{1-F(\hat{e})-\Sigma_{\hat{e}}}{f(\hat{e})}$, (32) can be rewritten $\varphi(\hat{e}) = \frac{c'(\hat{q})}{u'(\hat{q})}$. Differentiate it with respect to $\hat{e}$ and we have

$$\frac{d\hat{q}}{d\hat{e}} = \frac{\varphi'(\hat{e})(u')^2}{c''u' - c'u''}$$

Use (36) to rearrange $\varphi(\hat{e})$ as

$$\varphi(\hat{e}) = \frac{\hat{e}}{1 + \frac{1-F(\hat{e})}{\hat{e}f(\hat{e})}}.$$

It is easy to show that $\varphi'(\hat{e}) > 0$. If then follows that $d\hat{q}/d\hat{e} > 0$.

Differentiate (23) with respect to $\hat{e}$ and we have

$$\frac{de_0}{d\hat{e}} = -\frac{\frac{d\Sigma_{\hat{e}}}{d\hat{e}}}{2f(e_0) + ef(e_0)} > 0.$$

Differentiating (20) with respect to $\hat{e}$ gives

$$\frac{dq_e}{d\hat{e}} = \frac{\frac{1}{f(e)}\frac{d\Sigma_{\hat{e}}}{d\hat{e}}}{\frac{c''u'-c'u''}{u'^2}} < 0.$$

Differentiate (22) with respect to $\hat{z}$,

$$\hat{e}u'(\hat{q})\frac{d\hat{q}}{d\hat{e}}\frac{d\hat{e}}{d\hat{z}} - \frac{d\hat{e}}{d\hat{z}}\int_{e_0}^{\hat{e}} u'(q_x)\frac{dq_x}{d\hat{e}}dx = 1,$$

from which we derive

$$\frac{d\hat{e}}{d\hat{z}} = \frac{1}{\hat{e}u'(\hat{q})\frac{d\hat{q}}{d\hat{e}} - \int_{e_0}^{\hat{e}} u'(q_x)\frac{dq_x}{d\hat{e}}dx}$$

Since $d\hat{q}/d\hat{e} > 0$, $dq_e/d\hat{e} < 0$ for $e \in [e_0, \hat{e}]$, we have $d\hat{e}/d\hat{z} > 0$. It then follows that $d\hat{q}/d\hat{z} > 0$, $de_0/d\hat{z} > 0$ and $dq_e/d\hat{z} < 0$ for $e \in [e_0, \hat{e}]$. ■

A.3 Proof of Proposition 8: Quantity Discount with General Distribution

Proof. For $e \in [e_0, \hat{e}]$, q_e is solved from (20). Integrating both sides from e_0 to e, we have

$$\int_{e_0}^{e} \left[x - \frac{1 - F(x) - \Sigma_{\hat{e}}}{f(x)} \right] u'(q_x) dx = \int_{e_0}^{e} c'(q_x) dx$$

$$\rightarrow \left[e - \frac{1 - F(e) - \Sigma_{\hat{e}}}{f(e)} \right] u(q_e) - \int_{e_0}^{e} u(q_x) \left\{ 1 - \frac{-[f(x)]^2 - [1 - F(x) - \Sigma_{\hat{e}}] f'(x)}{[f(x)]^2} \right\} dx = c(q_e)$$

$$\rightarrow \int_{e_0}^{e} u(q_x) dx = \left[e - \frac{1 - F(e) - \Sigma_{\hat{e}}}{f(e)} \right] u(q_e) - \int_{e_0}^{e} u(q_x) \frac{[f(x)]^2 + [1 - F(x) - \Sigma_{\hat{e}}] f'(x)}{[f(x)]^2} dx - c(q_e).$$

The associated money payment is

$$z_e = eu(q_e) - \int_{e_0}^{e} u(q_x) dx$$

The real unit price is

$$\frac{z_e}{q_e} = \frac{eu(q_e) - \int_{e_0}^{e} u(q_x) dx}{q_e}$$

Differentiating z_e/q_e with respect to q_e, we have

$$\begin{aligned}
\frac{\partial \left(\frac{z_e}{q_e} \right)}{\partial q_e} &= \frac{\left[\frac{\partial e}{\partial q_e} u(q_e) + eu'(q_e) - \frac{\partial e}{\partial q_e} u(q_e) \right] q_e - \left[eu(q_e) - \int_{e_0}^{e} u(q_x) dx \right]}{q_e^2} \\
&= \frac{eu'(q_e) q_e - eu(q_e) + \int_{e_0}^{e} u(q_x) dx}{q_e^2} \\
&\propto eu'(q_e) q_e - eu(q_e) + \int_{e_0}^{e} u(q_x) dx \\
&= eu'(q_e) q_e - eu(q_e) + \left[e - \frac{1 - F(e) - \Sigma_{\hat{e}}}{f(e)} \right] u(q_e) \\
&\quad - \int_{e_0}^{e} u(q_x) \frac{[f(x)]^2 + [1 - F(x) - \Sigma_{\hat{e}}] f'(x)}{[f(x)]^2} dx - c(q_e) \\
&= eu'(q_e) q_e - \frac{1 - F(e) - \Sigma_{\hat{e}}}{f(e)} u(q_e) \\
&\quad - \int_{e_0}^{e} u(q_x) \frac{[f(x)]^2 + [1 - F(x) - \Sigma_{\hat{e}}] f'(x)}{[f(x)]^2} dx - c(q_e) \\
&= \frac{1 - F(e) - \Sigma_{\hat{e}}}{f(e)} [u'(q_e) q_e - u(q_e)] + c'(q_e) q_e \\
&\quad - \int_{e_0}^{e} u(q_x) \frac{[f(x)]^2 + [1 - F(x) - \Sigma_{\hat{e}}] f'(x)}{[f(x)]^2} dx - c(q_e)
\end{aligned}$$

where "$\propto$" represents "have the same sign as".

If $c(q_e)$ is linear, $c'(q_e) q_e = c(q_e)$. We have shown earlier that $\Sigma_{\hat{e}} < 1 - F(e)$ or $1 - F(e) - \Sigma_{\hat{e}} > 0$. Since $u'' < 0$, we have $u'(q_e) q_e - u(q_e) < 0$. To prove the quantity discounts result, we only need to

show that $[f(x)]^2 + [1 - F(x) - \Sigma_{\hat{e}}]f'(x) > 0$.

$$\begin{aligned}
[f(x)]^2 + [1 - F(x) - \Sigma_{\hat{e}}]f'(x) &> 0 \text{ is equivalent to} \\
x\frac{[f(x)]^2}{f(x)} + x\frac{[1 - F(x) - \Sigma_{\hat{e}}]f'(x)}{f(x)} &> 0 \text{ or} \\
xf(x) + x\frac{[1 - F(x) - \Sigma_{\hat{e}}]f'(x)}{f(x)} &> 0
\end{aligned}$$

Using $x > \frac{1-F(x)-\Sigma_{\hat{e}}}{f(x)} > 0$ for $x > e_0$ (note the first-order condition with respect to q_e), we show that

$$\begin{aligned}
& xf(x) + x\frac{[1 - F(x) - \Sigma_{\hat{e}}]f'(x)}{f(x)} \\
> \quad & 1 - F(x) - \Sigma_{\hat{e}} + x\frac{[1 - F(x) - \Sigma_{\hat{e}}]f'(x)}{f(x)} \\
= \quad & [1 - F(x) - \Sigma_{\hat{e}}]\left[1 + \frac{xf'(x)}{f(x)}\right] \\
= \quad & [1 - F(x) - \Sigma_{\hat{e}}]\frac{f(x) + xf'(x)}{f(x)} \\
> \quad & 0 \text{ if } xf(x) \text{ increases in } x. \blacksquare
\end{aligned}$$

A.4 Proof of Proposition 9: Effects of Inflation

Proof. The proof is similar to the proof of Proposition 4. As $S'(\bar{z}) = 0$, one can show that $S'(\bar{z}) + 1 - \frac{\gamma}{\beta} < 0$ for $\gamma > \beta$ and $\gamma \to \beta$ from above. We again focus on $\hat{z} \in [0, \bar{z}]$. For a generically unique $\hat{z}$, it must be true that $S''(\hat{z}) < 0$. So $\frac{d\hat{z}}{d\gamma} = \frac{1}{\beta S''(\hat{z})} < 0$. The rest of the results follow from Proposition 7.

A.5 Proof of Proposition 10: Hot Potato Effect with General Distribution

Proof. As $d\hat{e}/d\gamma < 0$, proving $dq_e/d\gamma > 0$ and $dz_e/d\gamma > 0$ is equivalent to proving $dq_e/d\hat{e} < 0$ and $dz_e/d\hat{e} < 0$. We do this by proving $d\left(\frac{dq_e}{de}\right)/d\hat{e} < 0$ and $d\left(\frac{dz_e}{de}\right)/d\hat{e} < 0$, or $q_e(e;\hat{e})$ and $z_e(e;\hat{e})$ are steeper for lower $\hat{e}$.

We first show $d\left(\frac{dq_e}{de}\right)/d\hat{e} < 0$. Use equation (20) that characterizes q_e for $e \in [e_0, \hat{e}]$ to calculate dq_e/de as (to simplify notation, we omit the arguments of the functions $u(\cdot)$ and $c(\cdot)$):

$$\frac{dq_e}{de} = \frac{u'^2}{c''u' - c'u''}\left[1 + \frac{f^2 + (1 - F - \Sigma_{\hat{e}}]f')}{f^2}\right].$$

The first term $\frac{u'^2}{c''u'-c'u''} > 0$ decreases in $\hat{e}$ if $c''' \leq 0$ and $u''' \geq 0$:

$$\begin{aligned}
\frac{d\left(\frac{u'^2}{c''u'-c'u''}\right)}{d\hat{e}} &= \frac{d\left(\frac{u'^2}{c''u'-c'u''}\right)}{dq_e}\frac{dq_e}{d\hat{e}} \\
&\propto \frac{2u'(c''u'-c'u'')-u'^2(c'''u'+c''u''-c''u''-c'u''')}{(c''u'-c'u'')^2}\frac{dq_e}{d\hat{e}} \\
&= \frac{2u'(c''u'-c'u'')-u'^3c'''+u'^2c'u'''}{(c''u'-c'u'')^2}\frac{dq_e}{d\hat{e}} \\
&< 0 \text{ if } c''' < 0 \text{ and } u''' > 0.
\end{aligned}$$

The second term $1+\frac{f^2+[1-F-\Sigma_{\hat{e}}]f'}{f^2} > 0$ decreases in $\hat{e}$ if $f' \leq 0$:

$$\begin{aligned}
\frac{d\left[1+\frac{f^2+(1-F-\Sigma_{\hat{e}})f'}{f^2}\right]}{d\hat{e}} &= -\frac{d\Sigma_{\hat{e}}/d\hat{e}}{f^2}f' \\
&\leq 0 \text{ if } f' \leq 0.
\end{aligned}$$

Since both the first term and the second term of dq_e/de (are positive and) decrease in $\hat{e}$, $d\left(\frac{dq_e}{de}\right)/d\hat{e} < 0$.

Now we show $d\left(\frac{dz_e}{de}\right)/d\hat{e} < 0$. Remember that $z_e = eu'(q_e)dq_e/de$, so

$$\frac{dz_e}{de} = \frac{eu'^3}{c''u'-c'u''}\left[1+\frac{f^2+(1-F-\Sigma_{\hat{e}}f')}{f^2}\right]$$

The first term $\frac{eu'^3}{c''u'-c'u''} > 0$ decreases in $\hat{e}$ if $c''' \leq 0$ and $u''' \geq 0$:

$$\begin{aligned}
&\frac{d\left(\frac{eu'^3}{c''u'-c'u''}\right)}{dq_e} \\
= \; & e\frac{3u'^2(c''u'-c'u'')-u'^3(c'''u'+c''u''-c''u''-c'u''')}{(c''u'-c'u'')^2} \\
= \; & e\frac{3u'^2(c''u'-c'u'')-u'^4c'''+u'^3c'u'''}{(c''u'-c'u'')^2} \\
> \; & 0 \text{ if } c''' < 0 \text{ and } u''' > 0.
\end{aligned}$$

The second term is positive and decreases in $\hat{e}$ if $f' \leq 0$. Since both the first term and the second term of dz_e/de (are positive and) decrease in $\hat{e}$, $d\left(\frac{dz_e}{de}\right)/d\hat{e} < 0$.

As $dz_e/d\gamma > 0$ and $d\hat{z}/d\gamma < 0$, $d(z_e/\hat{z})/d\gamma > 0$.■

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