Replacement hiring and wages*

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Abstract

I structurally estimate on matched employer-employee Danish data a general equilibrium partnership model with frictions where not only the worker, but also the firm can continue to search after forming a match. The data features low returns to tenure with a significant fraction of downward wage changes within and between jobs. When the wage decreases between jobs, the wage 10 years later is still below its level in the previous job, suggesting that the transition is not motivated by the prospect of future gains. The theoretical framework is consistent with these facts. The upward pressure on wages induced by competition between firms is somewhat balanced by the downward pressure resulting from competition between workers; the process limits tenure effects and causes occasional wage cuts. These opposing forces generate substantial wage dispersion. Also the dual process of replacement hiring and job-to-job transitions induces a pull towards positive sorting which contributes to wage inequality. Replacement hiring has the potential to explain wage cuts across jobs – as some workers trade lower wages for higher job security – but the effect is quantitatively weak. Because replacement hiring mainly affects low ability workers, the model replicates the strong negative correlation between wages and the risk of unemployment in the data.

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Contents

List of Figures

List of Tables

1 Introduction

I structurally estimate on matched employer-employee Danish data a general equilibrium partnership model with frictions where not only the worker, but also the firm can continue to search after forming a match. Features of the wage data suggest that this mechanism may play an important role: a large portion of wage changes within jobs are negative and large in absolute value – even in nominal terms; tenure effects are small; and finally, job-to-job transitions are often associated with a wage decline, a loss which persists over time and does not seem to be motivated by the prospect of future higher wages.

The literature has explored extensively the implications of allowing the worker to search on the job. The extension has proved to be relevant for a variety of prominent empirical features: the extent of job-to-job transitions, the relationship between wages, tenure and the separation rate, and finally wage dispersion.[1] In particular, wages grow with tenure because workers increase the fraction of the rent that they receive by exploiting between-employer competition.[2] As workers climb the wage ladder the job-to-job transition rate declines, which helps explain the negative relationship between tenure and the separation rate. Finally, due to differences in labor market histories similar workers will be located at different points on the ladder, so that the process generates wage dispersion.

The empirical regularities concerning the dynamic of wages which I find in the Danish data suggest that, while on-the-job search for workers is an

[1]The empirical and theoretical literature on on-the-job search is extensive, and here I only lists a few works. As documented by Farber (1999) the separation rate decreases with tenure and wages, while wages increase with tenure; Fallick and Fleischman (2004) show that job-to-job transitions are twice as likely as transitions to unemployment in the U.S.. The posting model with on-the-job search by Burdett and Mortensen (1998) is the workhorse for more theoretically oriented works. See Mortensen (2003) for a review. The model has been implemented empirically by Van den Berg and Ridder (1998), Rosholm and Svarer (2004) and Christensen, Lentz, Mortensen, Neumann, and Werwatz (2005). Eckstein and Van den Berg (2007) provide a review. The celebrated paper by Postel-Vinay and Robin (2002) and Cahuc, Postel-Vinay, and Robin (2006) extend and estimate the model allowing for wage renegotiations and Burdett and Coles (2003) consider the case of wage-tenure contracts. Finally, Nagypal (2005) Postel-Vinay and Turon (2009) and Lise, Meghir, and Robin (2008) allow for idiosyncratic shocks. Bagger, Fontaine, Postel-Vinay, and Robin (2007) allow for human capital accumulation. Also, on-the-job search is important to explain features of the labor market over the business cycle, an aspect which I do not address here – see Rogerson, Shimer, and Wright (2005) for a review.

[2]Firms can compete by setting wages as in the original Burdett and Mortensen (1998) and in Burdett and Coles (2003), as well as directly, by bidding for the worker's services as in Postel-Vinay and Robin (2002), Cahuc et al. (2006) and Bagger et al. (2007).

important ingredient, a mirror mechanism for the firm might be at work as well.[3] I consider a partnership model with frictions, as in Sattinger (1993, 1995) and Shimer and Smith (2000), and allow both workers and firms to search while they are matched.[4] Search can trigger the renegotiation of the wage, as well as job-to-job transitions and replacement hiring. By "replacement hiring" I mean the replacement of an employee with a "better suited" worker – in analogy with a job-to-job transition.[5] Similarly I use "replacement search" with regard to the search activity of the firm and "on-the-job search" with regard to the search activity of the worker. Estimates for the model are consistent with values found in the literature. In addition, I find that the on-the-job search parameter for firm is economically significant. The model explains 52% of separations, with 1/5 due to replacement hiring and 4/10 to job-to-job transitions.

Wage cuts are due to the search activity of the firm, while wage increases to that of the worker; also, replacement search somewhat balances the between-employer competition induced by on-the-job search, which helps explain the small tenure effects. Replacement search has the potential to explain wage cuts between jobs: low ability workers, being aware of the high risk of being laid-off, quit their current position and join a less productive firm when the opportunity arises, thus trading higher wages for higher job security.[6] In practice this effect is quantitatively weak when the model is solved for the parameter estimates. According to the model, replacement hiring is a phenomenon that disproportionally affects lower ability workers. Because lower ability workers tend to earn lower wages, replacement hiring explains the strong negative relationship observed in the data between the wage and the likelihood of becoming unemployed.

As for wage inequality, I find that on-the-job search increases wage inequality for two reasons. First, it induces a pool towards positive sorting,

[3] Recent work by De Melo (2009) suggests that similar features might hold for U.S. data as well.

[4] By "partnership model with frictions" I essentially mean a search and matching model with heterogeneous agents and scarcity on both sides of the market, so that the allocative problem arises of which type of worker should work at which type of firm.

[5] This definition is stricter than the one adopted in descriptive work, where replacement hiring is the worker flows' in excess of the jobs' flow – see Albaek and Sørensen (1998) and Burgess, Lane, and Stevens (2000). This interpretation can be justified by viewing the hiring process as specific rather than generic, where the employer looks for specific competences associated with performing a particular task within the firm.

[6] A similar motive for job-to-job transition is proposed by Nagypal (2005), though in that work the increase in the unemployment probability is due to idiosyncratic productivity shocks hitting the firm.

so that more productive workers tend to work at more productive firms. Second, the tension arising from the dual process of search inflates wage dispersion, rather than decreasing it.

The paper is related to various strands of literature. Kiyotaki and Lagos (2007) is the first work developing a framework where both workers and firms can search on the job; agents are homogeneous ex-ante and the model is used to analyze the theoretical implications of replacement hiring for worker flows. The literature which explores the theoretical and empirical implications of allowing workers to search on the job is vast; some works which are more closely related to this paper are mentioned in footnote 1. Particularly relevant is the seminal contribution of Postel-Vinay and Robin (2002), which is an empirical study of a model with heterogeneous workers and firms where the firm has monopsony power, but wages can be renegotiated; these authors note the empirical significance of wage cuts between jobs and show how such transitions can be motivated by the prospect of future higher wages. Dey and Flinn (2005) and Cahuc et al. (2006) allow for workers to have bargaining power. Models of on-the-job search are estimated on the Danish data by Rosholm and Svarer (2004), Christensen et al. (2005) and Bagger et al. (2007). The present paper is also related to the literature on sorting initiated by Becker (1973) and extended by Sattinger (1993, 1995), Shimer and Smith (2000), Atakan (2006), Lentz (2010) and Eeckhout and Kircher (2009, 2010); empirical analysis of sorting using this framework are found in De Melo (2008), Bagger and Lentz (2008) and Lise et al. (2008). These works account for on-the-job search of workers. In particular the latter is close to this paper, though it analyzes the importance of idiosyncratic shocks rather than on-the-job search of firms.

The paper is organized as follows. In the second section I look at features of wages and in particular wage dynamics in the Danish data; in the third section I develop the model and in the fourth section I describe the estimation methodology and discuss model estimates; in the fifth section I discuss the implications of replacement hiring. The last section concludes.

2 Data

2.1 Data Description

The *Integreret Database for Arbejdsmarkedsforskning* (hereafter, IDA) is a Danish matched employer-employee dataset constructed from administrative

records and maintained by Danmarks Statistik.[7] It contains socio-economic covariates for the entire Danish population (5.3 million) as well as establishment level information for all employers in the private and public sector $(230,000)$. the data are collected annually and span the period 1980-2006. Of particular interest for this study is the wage series, which is constructed by Danmarks Statistik using gross annual earnings observed in the last week of November and hours worked during the year.[8] Hours are estimated based on mandatory pension payments made by employers for all salaried workers.

Starting in 1995 the dataset has been linked to a rich set of firm statistics, an extension referred to as Firma-IDA (hereafter, FIDA).[9] Among other covariates, FIDA includes detailed accounting information for a sample of firms based on an annual questionnaire.[10] Starting from an initial sample of $9,000$ firms the questionnaire has been gradually extended to various sectors of the economy, and, in 1999, it became representative of Danish firms with more than 4 employees.[11] Following guidelines provided by Danmarks Statistik this information can be used to construct time series for value added (VA) at the firm level.[12] The registry also contains information on a firm's full-time equivalent (FTE) labor force and on the wage bill. Full-time employment is defined as 37 hours or more per week.

Finally, IDA contains weekly information on the labor market status of every employee as well as unemployed workers receiving social security benefits or participating in activation. The take-up rate for at least some sort of benefits is 80% on average, and 85% for low income workers, Buti, Sestito, and Wijkander (2001). Therefore, the dataset provides a relatively complete record of a worker's labor market history. The LMDG group at the University of Aarhus uses this information to construct labor market histories for the Danish population age 15 to 64, spanning the period $1985 - 2003$. Roughly, four states can be distinguished. A worker can be employed, tem-

[7] http://www.dst.dk/TilSalg/Forskningsservice/Databaser/IDA.aspx provides a description of the architecture and content of the data, as well as descriptive statistics. The website can be conveniently navigated in English using Google Language Tools.

[8] http://www.dst.dk/tilsalg/forskningsservice/hkt4forsker/hkt4_variabel_liste_forsker/hkt4_variabel.aspx?fk=23416

[9] http://www.dst.dk/Vejviser/dokumentation/Varedeklarationer/emnegruppe/emne.aspx?sysrid=98591

[10] http://www.dst.dk/Vejviser/dokumentation/Varedeklarationer/emnegruppe/emne.aspx?sysrid=1033

[11] The sample includes all medium-large firms (more than 49 employees, or turnover greater than 150mil. DKK), 50% of firms with 20-49 employees, 20% of firms with 10-19 employees, and finally 10% of firms with $4 - 9$ employees.

[12] http://www.dst.dk/Vejviser/dokumentation/times/emnegruppe/emne/variabel.aspx?sysrid=921954&timespath=19|98591|

porary unemployed (e.g. sick leave, parental leave), officially registered as unemploymed and retired. A fifth residual state is defined for those instances where the worker is not observed in any (documented) state.[13] An employee can be self-employed or salaried, in the private or public sector.

I treat retirement as an absorbing state, and spells may be interval censored. I recode temporary unemployment (sick leave, maternal leave...) as continuing employment; also, if a worker returns to the same firm the unemployment/non-employment spell is recoded as continuing employment. Finally, I define as out of the labor force individuals who are in the residual state for more than a year consecutively. I record job spells at the level of the firm rather than the establishment, because movements across establishments are presumably driven by factors other than search by either the firm or the worker[14]

2.2 Sample Selection and Variables

Descriptive statistics are reported in table 1. I limit my analysis to data from 1985 to 2002, disregarding data from 2003 since there is a disproportionately small number of observations for that year. As for firms, I drop observations prior to 1999, when the sample became representative. I further limit my sample by two main criteria. First, I only consider records for workers who have accumulated at least ten years of (full time equivalent) work experience and have been out of school for at least ten years and did not return to school by 2006. The reason for these restrictions is to obtain a relatively homogeneous sample. In the first few years of labor force participation informational considerations might play a more important role in determining a worker's labor market history; also, human capital accumulation appears to be more important for wage growth – see Bagger et al. (2007). These are factors affecting transitions across labor market states and wages which the model I consider below abstracts from. Second, I only include spells corresponding to unemployment (as defined above) and employment in the

[13] "Revalidering" constitutes an additional labor market state. It is a form of activation which applies to particular cases, such as individuals with physical or mental disabilities. Individuals who are recorded in such state at least once are excluded from the analysis.

[14] A firm can obtain a tax declaration code at a relatively small cost which creates fictitious job-to-job movements from one year to the the next. Danmarks Statistik constructs a workplace identifier which remains constant over time. I use this code to check whether the worker remains in the same workplace and find that (at least) 10% of job-to-job transitions are indeed fictitious in the sense described above. I use this method to recode such (fictitious) job-to-job transitions as continuing employment.

Table 1: Descriptive statistics

	population	age≤55,exp.≥10	private sect.	vocat.tr., male
male %	54.59	59.15	70.30	—
age, mean	38.31	43.67	42.99	42.89
" std	12.83	6.80	7.03	6.85
work experience, mean	12.19	19.11	19.35	20.57
" std	9.24	5.86	5.85	5.71
primary education %	36.24	31.96	33.82	—
high school	7.59	3.06	3.23	—
vocational training	37.69	46.23	52.07	—
some further educ.	4.33	4.38	3.77	—
college degree or higher	14.15	14.37	7.11	—
manager %	11.61	13.55	12.77	9.82
skilled	47.70	50.96	52.80	63.88
clerical	13.79	16.65	16.46	13.70
unskilled	11.96	12.08	12.11	8.57
n.a.	14.95	6.76	5.85	4.02
unemployment rate %	10.5	5.38	5.08	4.21
non-private employment[(a)]	32.5	36.70	—	—
manufacturing %[(b)]	38.05	44.04	47.05	45.56
construction	9.29	10.15	10.41	15.25
trade	30.40	25.44	23.48	26.14
services	21.88	19.88	18.58	12.57
other	0.38	0.49	0.48	0.48
wage, mean[(c)]	170.17	193.12	194.89	197.57
" std	106.43	102.64	99.75	82.58
VA/FTE, mean[(d)]	438114	458171	461531	432024
" std	488473	532628	511075	316022
w.bill/FTE, mean[(d)]	304360	313171	313461	304861
" std	99382	94226	94847	77567
FTE, mean[(e)]	785	750	731	673
" std	1853	1801	1692	1692
labor share	68.11	68.21	68.21	68.13
median job duration[(f)]	83	135	116	107
median unempl. duration	19	16	12	11
n. individual	3593372	1666641	978929	343988
n. individual-year	42504656	16830072	8070340	3025099
n. firm-year[(g)]	35225	34451	32515	27899

column *1*) labor force 1985-2002 after cleaning – incomplete histories, school re-entry, etc... *2*) only observations where the worker is younger than 55, has more than 10 years of work experience and has been out of school for more than 10 years *3*) as previous and only unemployment spells and employment spells in the private sector, initiating and ending in these states or retirement *4*) as previous and only male workers with vocational training [a] employment which is not at private companies or self-employment [b] NACE rev 1.1 [c] hourly wage in 2000 (measured in DKK) [d] yearly values for year 2000 (measured in DKK); the unit of observation is an employee with associated firm characteristics [e] number of full time equivalent employees; unit of observation same as above [f] Kaplan-Meier estimates [g] 1999 to 2002.

private sector, and such that the spell starts or ends in these same states, or retirement; spells ending in retirement are treated as censored. In practice one observes few transitions between the private and the public sectors, especially compared to the rate of turnover in the private sector. Similarly, there are few spells starting or ending with non participation. 46% of jobs starting in 1985 end in another job, 28% in unemployment, 2% in retirement and 8% are right-censored; only 11% end in the public sector and 3% in out of the labor force. Given the relatively high degree of labor market attachment required in the restricted sample, these two numbers drop to 6.5% and 1.2%. The final sample contains $978,929$ individuals, $8,070,340$ individual-year observations, $418,561$ unemployment spells, $1,154,946$ job spells and $32,515$ firm-year observations.[15]

Regarding the treatment of variables, I discard wage observations that do not meet the quality criterion suggested by Danmarks Statistik. In practice, the criterion limits observations to individuals with jobs involving at least 20 hours/week. Also, I de-trend wages using the mean wage observed in the (November) cross-section for each year, thus netting out nominal as well as real trends. I refer to this variable as "w". The reason, is to focus on how search affects the relative bargaining power of workers and firms, for example as reflected in the labor share of output. The model presented below can be reconciled with productivity growth if the wage is indexed to aggregate real growth, and technical change is Hicks neutral – in this case on only needs to adjust time discounting by the growth rate.

For firm data, I use value added per FTE employee as a proxy for productivity, $av = VA/FTE$. I similarly rescale the wage bill $aw = W/FTE$. A firm's productivity and average wage are imputed to the employee observation and selected accordingly. Thus, firm data refers to the employment-position filled by an employee in the sample.[16].

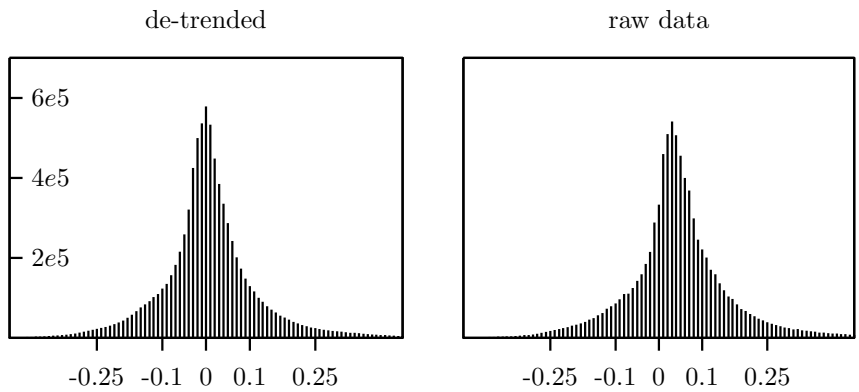
Finally, regarding duration data, I define a job-to-job transition as a transition where the worker does not experience non-employment between jobs. This choice is arbitrary and other studies have used different definitions.[17] The information available in the dataset does not allow me to

[15]The number of firm observations is overstated, because, as discussed in note 14, the firm identifier can change from one year to the next. This is not a particularly important issue because I do not make use of the longitudinal dimension of firm related variables.

[16]When computing statistics for the population of employment-positions I weight observations using the questionnaire sampling frequency – footnote 11

[17]For example Postel-Vinay and Robin (2002) define a job-to-job transition as one where the worker experiences no more than two weeks of non-employment between jobs, and, for the Danish data, Bagger et al. (2007) use a threshold of one week.

Figure 1: Year-to-Year Wage Changes



Histogram of year-to-year change in the hourly wage, conditioning on remaining at the same firm. All workers and years 1986-2002. Left panel: series detrended by the mean wage observed in the November cross-section of each year. Right panel: nominal data.

distinguish a worker's voluntary quit from a lay-off. A worker might be given notice and find a job before becoming unemployed, or a worker who finds a better job might be non-employed for some time before joining the new employer. This distinction is particularly important in this study because lay-offs are endogenous. To (partially) address this problem, I check whether a particular statistic is sensitive to alternative thresholds – 0, 2, 5, 15 weeks. Wage statistics are fairly robust, while the duration of a job *to-job* spell relatively to a job *to-unemployment* spell is sensitive. Therefore, In the estimation I do not make use of this information, and use information on wage changes instead - see section 4.3.

2.3 Wage Dynamics, Stylized Facts

In this section I present evidence concerning wage dynamics for the sample described above. First, I look at the cross-sectional distribution of wage changes from one year to the next. I then look at the "expected" wage longitudinal profile for a worker starting a job. I present figures for de-trended data (as described above) as well as for nominal data and for a number of sub-samples. The basic qualitative properties are unaffected. Note that because wages are observed only in the last week of November, wage observations for short job spells beginning earlier in the year are under-represented.

Summarizing, I find that a significant fraction of within-job wage changes

Table 2: Year-to-Year Wage Changes, Percentiles

	10th	25th	50th	75th	90th
benchmark[a]	−0.121	−0.044	0.001	0.053	0.137
raw data[b]	−0.092	−0.009	0.381	0.095	0.185
nominal defl.[c]	−0.114	−0.037	0.012	0.066	0.154
yr. 2000[d]	−0.114	−0.041	0.001	0.048	0.136
age $30-40$[e]	−0.125	−0.044	0.007	0.064	0.155
$\Delta FTE \leq 10\%$[f]	−0.125	−0.044	0.007	0.064	0.155
nom. tot. comp., full time, all year[g]	−0.044	0.003	0.037	0.081	0.144
all of the above[d][e][f][g]	−0.041	0.003	0.037	0.080	0.143
manager[h]	−0.102	−0.034	0.006	0.054	0.139
skilled	−0.128	−0.050	0.000	0.055	0.144
clerical	−0.096	−0.031	0.006	0.051	0.127
unskilled	−0.133	−0.055	0.000	0.060	0.154

[a] all workers/years, detrended series [b] nominal data [c] nominally deflated [d] changes between 1999 and 2000 [e] workers 30 to 40 years old [f] establishment size (in FTE) changes by less than 10% in previous and following year [g] workers employed full time for 52 weeks in the previous and current year, nominal annual compensation [h] by worker's category.

is negative and large in absolute value. As for the wage trajectory, growth on the job is small on average, with large wage changes mostly occurring between jobs. still, A high fraction of wage changes between jobs are negative and large, and the wage profile observed afterward in these cases, though steeper, does not appear to justify the loss experienced with the transition. The figures for wage changes within and between jobs are perhaps surprising. Postel-Vinay and Robin (2002) and De Melo (2009) document similar features for France and the U.S., respectively. As for tenure effects, the evidence is controversial. In two classic studies Altonji and Shakotko (1987) and Topel (1991) find low and high effects, respectively, for the U.S.. More recently, in a reassessment of the debate Altonji and Williams (2004) settle on a figure of 11% over 10 years, which, though higher, is comparable with the numbers reported below.

Figure 1 displays the distribution of year-to-year changes in the hourly wage received by each employed worker who remained in the same job as the previous year. The left-panel is obtained from the de-trended wage series, the benchmark, while the right-panel is from the raw wage data. On the vertical scale is the number of observations. Data are pooled together – from 1986 to 2002 – in accordance with the steady state assumption of the model. In table 2 I report deciles for these distributions.

Consider first the benchmark case. 48.1% of wage changes within a job

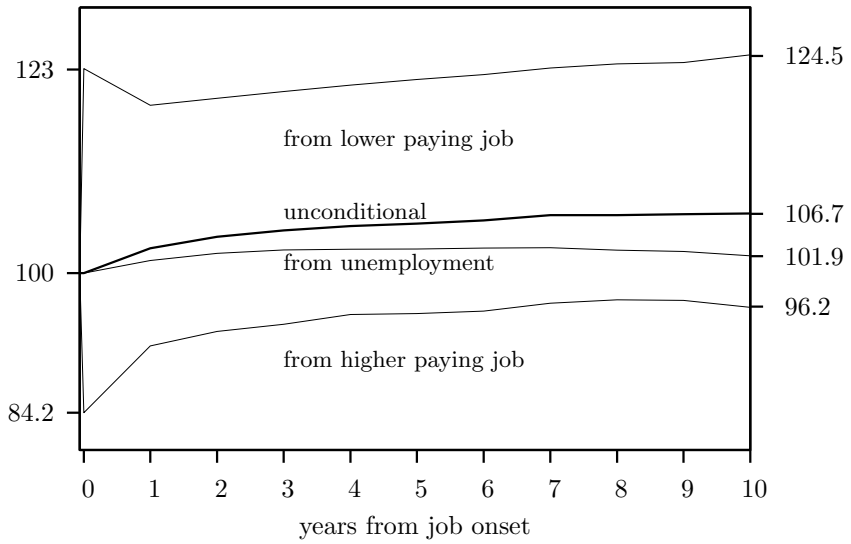
are wage cuts. Furthermore, a significant fraction of these changes are large – the 10th percentile is -0.121. The median is virtually zero, but wage increments tend to be larger in absolute value than wage decrements, resulting in a positive mean of .0136. Note that de-trending by the mean wage does not generate these features by construction. If on-the-job search is an effective means for workers to obtain wage increases, then the mean wage change is positive *conditionally* on remaining employed, while the mean wage is constant at the aggregate level. As shown in table 2, wage cuts remain a prominent feature of the data even when considering nominal wages: 10% af wages drop by more than 9% from one year to the next. The table also reports figures for nominally deflated data using the CPI deflator constructed by Denmarks Statistik.

One possible concern is time aggregation. First, wage losses might be concentrated during periods of market contraction. Second, wage cuts might reflect life-cycle trends, such as workers assuming different tasks in the latter part of their career. To address these possibilities, I present figures for wage changes between 1999 and 2000 only – the Danish economy grew by 2.56% and 3.53% respectively – and for wage changes among workers age $30 - 40$. In the first case wage volatility is attenuated, and in the second it increases, as one would expect. However, alterations are small – and remain so for a different choice of year or age segment.

Another concern is that wage losses might reflect idiosyncratic productivity shocks – Postel-Vinay and Turon (2009), Lise et al. (2008). Suppose that shocks are mainly at the level of the establishment (rather than the match) and that a firm adjusts the employment level in response to the shock. Then, if wage cuts essentially reflect productivity shocks, the distribution should significantly change when purging the sample of observations where the establishment size changed. I discard observations where the establishment size (measured in FTE) changed by more than 10% in the year preceding or following a wage observation. Percentiles for this sample are reported in the sixth row of table 2. The distribution hardly changes on the negative support. Similar figures are obtained when considering size changes of 5, 15, 20 and 25%.

The Next raw addresses measurement error. The hourly wage is constructed using estimates of hours worked which are based on social security payments made by the employer. This source of uncertainty can result in measurement error. To address this issue, I consider only observations where the worker was employed full time at the same employer during the entire year and use the total nominal compensation as wage measure. Comparing figures with those computed on the raw data (second row) one can see that

Figure 2: Wage Profile, by Originating State



All workers/years, detrended series. For each job I compute the wage change relative to the first year, conditioning on remaining employed (possibly at another firm). I Average across all jobs and construct the "unconditional" schedule by setting to 100 the wage in the first year. The "from unemployment" schedule only includes jobs initiating from unemployment. The two remaining schedules are for jobs initiating "from a lower paying job" or "from higher paying job"; in this case I set to 100 the wage in the previous job. Roughly, 1/3 of jobs begin as a transition from unemployment while the wage decreases in 47% of job-to-job transitions.

measurement error is important. However, 23% of wage changes remain negative (down from 37%) and 10% of wages decrease by more than 4% (5% by more than 9%. Finally, the eighth row displays figures for the case where all the considerations above are summed up together: I consider changes in the nominal annual compensation received in 1999 and 2000 by workers age $30-40$, who remain full time employed for the entire year, and such that the size of the establishment did not change by more than 10% between 1999 and 2000 and between 2000 and 2001.

The table also reports estimates for different occupation categories.[18] Clerical workers experience the least wage volatility, followed by managers, skilled workers and finally unskilled workers. Wage losses are pervasive and

[18] I follow Danmarks Statistik suggested aggregation based on "PSTILL": "managers and higher officials"=31-33, "clerical staff"=34, "skilled workers"=35, "unskilled workers"=36.

large in magnitude across all categories.

Next, consider the longitudinal dimension. Figure 2 displays the expected wage profile for a worker beginning a job (de-trended wages). It reports schedules for the unconditional profile as well as the profile conditioning on the originating labor market state: unemployment, employment at a lower wage and employment at a higher wage. For the "unconditional" and "from unemployment" schedules, the wage observed in period zero (i.e., in November of the first year on the job) is normalized to 100. Instead, for the two "job-to-job" schedules, I normalize the wage in the previous job to 100, so that the value reported at zero provides information on the wage change between jobs. Finally, wages in future jobs are imputed to the current job as well, when a job-to-job transition occurs – in accordance with the on-the-job search "view" of the value of a job.[19] Figures are computed using all jobs starting during the period $1985 - 2002$. Table 3 displays numbers for the first five years of a job. The first column shows the probability of a wage loss between jobs together with an indication of the sample used; in the second column is the originating labor market state and in the third the expected wage change between jobs. The remaining columns list the percentage change relative to the wage observed in the first year, after $1 - 4$ years.

The plot and the corresponding numbers reveal several patterns. First, the wage grows by roughly 1% a year in the first $5 - 6$ years but otherwise remains relatively flat. Second, contrary to the prediction of a basic model of on-the-job search with homogeneous workers, the wage profile for workers coming from unemployment is flatter, suggesting that heterogeneity and/or scarring effects of unemployment are important.[20] Third, wage changes between jobs are large in absolute value – 23.7% and -15.3% for positive and negative wage changes, respectively. Fourth, 46.86% of wage changes between jobs are negative. Fifth, the wage profile is steeper than the unconditional one following a wage drop between jobs, but after 10 years it remains below the wage in the previous job. This suggests that a large fraction of these transitions is not motivated by prospects of higher future

[19] For example, consider a worker who is unemployed, becomes employed, switches to a second job after two years, and remains 3 years in the second job before becoming unemployed again. Suppose that I observe the wage in each year: $w_{1,0}$ and $w_{1,1}$ for the first job and $w_{2,0}$, $w_{2,1}$, and $w_{2,2}$ for the second job. These observations constitute two profiles, the first one with four wage observations, and the second with two. Unemployment resets the counting.

[20] For evidence of the scarring effect of unemployment in the U.S. see Ruhm (1991) and Jacobson, LaLonde, and Sullivan (1993).

Table 3: Wage Profile, Comparison

		$E(+/-)^{(a)}$	after 1 yr.	2 yr.	3 yr.	4 yr.
benchmark$^{(b)}$	tot.		0.021	0.038	0.041	0.046
	u		0.012	0.022	0.026	0.028
	e_+	0.237	−0.033	−0.028	−0.021	−0.016
	e_-	−0.153(46.86%)	0.077	0.105	0.113	0.128
raw data$^{(c)}$	tot.		0.057	0.106	0.155	0.208
	u		0.046	0.098	0.141	0.180
	e_+	0.248	0.005	0.042	0.080	0.118
	e_-	−0.162(37.09%)	0.125	0.196	0.241	0.296
nominal$^{(d)}$	tot.		0.031	0.056	0.076	0.093
	u		0.021	0.044	0.057	0.065
	e_+	0.238	−0.022	−0.011	0.002	0.015
	e_-	−0.156(43.87%)	0.089	0.129	0.142	0.167
E adjust.$^{(e)}$	tot.		0.011	0.026	0.032	0.036
	u		0.000	0.011	0.016	0.015
	e_+	0.237	−0.029	−0.021	−0.012	−0.008
	e_-	−0.153(46.86%)	0.053	0.078	0.088	0.097
yr. 1994$^{(f)}$	tot.		0.022	0.037	0.037	0.045
	u		0.008	0.025	0.014	0.011
	e_+	0.250	−0.037	−0.042	−0.025	−0.018
	e_-	−0.149(47.29%)	0.064	0.091	0.082	0.102
age $30-40^{(g)}$	tot.		0.024	0.042	0.049	0.056
	u		0.018	0.030	0.036	0.039
	e_+	0.243	−0.029	−0.019	−0.007	0.003
	e_-	−0.156(44.88%)	0.080	0.116	0.127	0.143
manager$^{(h)}$	tot.		0.017	0.046	0.062	0.080
	u		−0.009	0.023	0.029	0.045
	e_+	0.237	−0.042	−0.024	−0.015	0.002
	e_-	−0.160(44.82%)	0.064	0.105	0.120	0.135
skilled	tot.		0.022	0.034	0.039	0.045
	u		0.005	0.016	0.017	0.021
	e_+	0.237	−0.037	−0.031	−0.029	−0.024
	e_-	−0.155(47.93%)	0.085	0.108	0.114	0.122
clerical	tot.		0.015	0.040	0.060	0.078
	u		0.006	0.028	0.048	0.059
	e_+	0.204	−0.037	−0.024	−0.010	0.004
	e_-	−0.142(45.06%)	0.062	0.104	0.122	0.150
unskilled	tot.		0.023	0.038	0.035	0.041
	u		0.006	0.023	0.018	0.022
	e_+	0.260	−0.029	−0.031	−0.031	−0.028
	e_-	−0.164(48.84%)	0.083	0.101	0.106	0.107

Change relative to the wage in the first year. Unconditional, "tot.", and by originating state: unemployment "u", employment at a lower/higher wage "e_+/e_-". [a] average change between jobs, probability of negative change in parenthesis [b] all workers/years, detrended data [c] nominal data [d] nominally deflated [e] wages in future jobs are ignored [f] jobs starting in 1994 [g] 30-40 years old at job onset [h] by worker's category.

wages, as in Postel-Vinay and Robin (2002). The table also reports figures for the raw data and nominally deflated wages. The patterns mentioned above are unchanged, with the exception of the wage profile following a wage loss between jobs. However, after accounting for inflation, the real wage hardly catches up with the wage level of the previous job after 4 years.

The fourth row shows the effect of disregarding wages earned by the worker in subsequent jobs – when the current job ends with a job-to-job transition. Profiles are flatter, suggesting that the value of future jobs is important to understand the value of the worker's current position. Next, I address time aggregation. The fifth row displays figures for jobs starting in 1994 only – Denmark grew steadily between 1994 and 2002. The patterns highlighted above are even stronger when flow sampling from 1994 forward. Finally, the sixth row shows figures for jobs starting when the worker was 30 to 40 years old – an age segment that exhibits starker wage changes as documented in table 2. Wage losses between jobs are pervasive and large even at this stage of the life-cycle, suggesting that they are not only due to workers moving to different type of jobs in the latter part of their career. The last four rows of the table present figures by worker category: managers and high officials, skilled workers, clerical staff and unskilled workers.

3 The Model

3.1 Environment and notation

Populations: Time is continuous and the economy is populated by two types of agents – workers and firms. Agents differ in terms of their productivity, indexed by $x \in [0, 1]$ and $y \in [0, 1]$ for workers and firms, respectively. Their population is described by the densities $l(x)$ and $n(y)$; capitol letters denote cumulative distributions. Workers' total mass is $\bar{L}$, while firms' total mass is $\bar{N}$. While $l(x)$ is exogenous, $n(y)$ is determined by a simple mechanism of entry and exit, which I describe below. x and y are ordinal quantities indexing whether a particular worker or firm is more productive than another, given the same partner.

Production: There is one homogeneous good and its production requires only labor – though the cost of vacancy creation can be viewed as a (sunk) investment in physical capital. When a worker x is employed at firm y he produces output flow equal to $f(x, y)$. In accordance with the interpretation for x and y, f is increasing in each argument. The consumption good is the numeraire, and w denotes the real compensation received by the worker. The total output flow produced by a firm is equal to the sum of the output

produced by each of her employees, so that firm size is undetermined and I use the terms "firm" and "employment position" interchangeably. In particular, $n(y)$ denotes the density of type y employment positions.

Search: Search is costly and purely random. $u(x)$ and $v(y)$ denote the densities of unemployed workers and vacant positions, while $m(x, y, w)$ denotes the mass of matches involving agents x and y where the worker receives compensation $\leq w$; I define $m(x, y) \equiv \lim_{w\to\infty} m(x, y, w)$. Note that $\bar{N} - \int dV(y) = \bar{L} - \int dU(x) = \iint dM(x, y)$, so that in practice $u(x)$ and $v(y)$ are trivially obtained after determining $m(x, y)$. An unemployed worker x experiences an instantaneous utility flow equal to $b(x)$.

Both workers and firms can search regardless of their employment status. The contact rate between any two agents only depends on whether they are matched, and, in particular, it is independent of their type, x and y.[21] An unemployed worker contacts a firm for a vacant employment position at rate λ_{00}. Alternatively, the contact rate is $\lambda_{01} \equiv \lambda_{00}/\kappa_y$ if the position is currently filled. If instead the worker is already employed, he contacts the vacancy at rate $\lambda_{01} \equiv \lambda_{00}/\kappa_x$. Finally, a contact between an employed worker and a firm for a position which is already filled occurs at rate $\lambda_{11} = \lambda_{00}/\kappa_x\kappa_y$, a condition which reflects a proportionality assumption in the matching technology.[22] I assume that on-the-job search is generally less efficient, $\kappa_x \geq 1$, $\kappa_y \geq 1$.

The aggregate contact rate depends on the mass of unemployed and employed workers, and on the mass of vacancies and filled positions, as determined by the matching function $\mu(U, \bar{L}-U, V, \bar{N}-V)$. Then λ_{00} is defined by the identity:

$$\lambda_{00}(U, V, \bar{N}) \equiv \frac{\mu(U, \bar{L} - U, V, \bar{N} - V)}{[U + (\bar{L} - U)/\kappa_x][V + (\bar{N} - V)/\kappa_y]}.$$

For simplicity, the dependence of λ_{00} on U, V and $\bar{N}$ is left implicit.

Rent Sharing and Values: Workers and firms are risk neutral and maximize the discounted flow of expected wages/profits at the common rate r. The worker and the firm bargain on the expected value generated by production. I follow the literature and assume efficient bargaining – Postel-

[21] This assumption can be easily relaxed by endogenizing the search intensity choice and assuming that the search effort can be contracted upon together with the wage. Lentz (2010) shows that in this case the search intensity choice maximizes the joint surplus and does not depend on the wage.

[22] Suppose the contact rate between two agents depends on the product of their search parameter (e.g. effort) and that this parameter depends on whether the agent is matched or unmatched. Then, it must be that $\lambda_{11} = \lambda_{01}\lambda_{10}/\lambda_{00} = \lambda_{00}/\kappa_x\kappa_y$

Vinay and Robin (2002), Cahuc et al. (2006) and Dey and Flinn (2005).[23] In practice this is a property which relies on three restrictions: the sharing agreement can be renegotiated and it does not affect the contact rate (so not to affect the probability of separation); the agreement does not affect sharing within an eventual new partnership (so not to alter the continuation value in case of a separation).

A renegotiation occurs when one of the two agents contact a potential alternative partner and threats to leave the current one. However, the current match is dissolved and the new one is formed, if the value required to retain the agent who received the outside offer is such to leave the current partner with a value lower than being unmatched.

Finally, I assume that the firm can commit to a wage schedule and, in particular, that the surplus sharing agreement is implemented by setting a constant wage. I denote by $W_0(x)$ the expected value for x of being unemployed, and by $W(x, y, w)$ that of being employed by y at wage w; $S^x(x, y, w) \equiv W(x, y, w) - W_0(x)$ is the individual surplus enjoyed by the employee. Regarding the firm I use $\Pi_0(y)$, $\Pi(x, y, w)$ and $S^y(x, y, w)$, respectively. Because bargaining is efficient then the joint surplus does not depend on the wage: $S(x, y) = S^x(x, y, w) + S^y(x, y, w)$.

Entry/Exit: A position is destroyed at (the exogenous) rate δ, regardless of her type or whether the position is filled by a worker or not. If it is filled by a worker, then the worker becomes unemployed. Creating a position requires an investment equal to C, which cannot be recouped. The type of the position is revealed after the investment is made and it is drawn from the (exogenous) cumulative distribution $\Phi(y)$. I restrict attention to the case where any firm type $y \in [0, 1]$ can match with a positive mass of workers.[24] The flow of entrant which I denote by $\bar{I}$ is pinned down by free entry. Because the entry and exit rate are the same across different types of position, then $N/\bar{N} = \Phi$ and free entry regulates the overall availability of jobs $\bar{N}$.

[23] Efficient bargaining means that sharing is such that match formation and match separation are efficient, i.e. the total value that is created exceeds the total value that is forsaken/destroyed.

[24] A vacancy type which does not satisfy this requirement would be scrapped immediately after entry in the presence of an arbitrarily small maintenance cost. Then, provided that $W_0(y)$ is weakly increasing, one can redefine the entry cost as $C/[1 - \Phi(z)]$ and the firm index as $(y - z)/(1 - z)$, where z is the marginal viable type. The important caveat with this interpretation is that counter-factual analysis will ignore selection effects on the range of viable types.

3.2 Bargaining

In this section I describe a solution concept that disentangles bargaining situations involving three or four parties. This problem was first analyzed by Kiyotaki and Lagos (2007) who consider a screening game where nature assigns the worker or the firm the right to make a take-or-leave offer. The share received by each agent is uniquely determined in expected terms, but his/her actual payoff depends on nature's draw ex-post. Given the focus on wages in this paper rather than flows, I consider an alternative approach. Also, the equilibrium concept is parsimonious in terms of behavioral assumptions, in the sense that it hinges on rationality rather than on a particular game structure; however, the derivation relies on an ad-hoc feasibility restriction – see below. In practice, I find the same result (in terms of ex-ante values) as Kiyotaki and Lagos (2007), with the exception of renegotiations triggered by meetings involving two agents already matched to another partner.[25]

In a typical bilateral bargaining game, the sharing agreement depends on the outside option available to each party – Rubinstein (1982), Binmore, Rubinstein, and Wolinsky (1986) and Osborne and Rubinstein (1990). Suppose that an agent can choose to bargain with different parties. She cannot bargain with more than one agent at the same time, but she can interrupt negotiations with one agent and resume negotiations with another as she pleases. Thus, her outside option when bargaining with a particular agent is the maximum value she can obtain by bargaining with any other available party.

I derive the outcome of the multilateral bargaining problem by requiring that the bargaining positions of each of the agents involved be mutually consistent. In particular I show that a unique solution always exists for bargaining situations arising in the environment described above, provided that the bargaining power parameter lies in the interior of the unit interval.

Consider the case of a worker and a firm x and y seeking to split the quantity $W + \Pi$ and having outside options O_x and O_y respectively. Let β denote the bargaining power of the worker. Then, following for example Cahuc et al. (2006) or Lentz (2010), one obtains the familiar result:

$$\begin{aligned} W_b^x(y) &= O_x + \beta[W + \Pi - O_x - O_y] \\ \Pi_b^y(x) &= O_y + (1-\beta)[W + \Pi - O_x - O_y]. \end{aligned}$$

[25] In Kiyotaki and Lagos (2007) nature assigns the right to make the take-or-leave offer to the worker or to the firm with equal probability. It is easy to generalize that setting to the case of an arbitrary probability which can be reinterpreted as the worker's bargaining power parameter.

$W_b^x(y)$ ($\Pi_b^y(x)$) is the payoff obtained by x (y) when bargaining with y (x); the payoff equals the agent's outside option plus a fraction of the extra value generated by the agreement which reflects his (her) bargaining power. Note that the result can be expressed in terms of values net of the value of non-production:

$$S_b^x(y) = \tilde{O}_x + \beta[S - \tilde{O}_x - \tilde{O}_y]$$
$$S_b^y(x) = \tilde{O}_y + (1-\beta)[S - \tilde{O}_x - \tilde{O}_y],$$

where $S_b^x(y) \equiv W_b^x(y) - W_0(x)$, $\tilde{O}_x \equiv O_x - W_0(x)$ ($S_b^y(x)$ and $\tilde{O}_y$ are similarly defined).

In order to apply the solution concept described above, the bilateral bargaining game must address circumstances in which the outside option of one of the two parties is larger than the amount they seek to split. I impose an ad hoc feasibility restriction: in those cases where $\tilde{O}_x$ and $\tilde{O}_y$ imply a payoff larger than S for one of the two agents, that agent receives the whole amount S:

$$S_b^x(y) \equiv \max\left\langle \min\left\langle \tilde{O}_x + \beta[S - \tilde{O}_x - \tilde{O}_y], S\right\rangle, 0\right\rangle \tag{3.1a}$$
$$S_b^y(x) \equiv \max\left\langle \min\left\langle \tilde{O}_y + (1-\beta)[S - \tilde{O}_x - \tilde{O}_y], S\right\rangle, 0\right\rangle \tag{3.1b}$$
$$= S - S_b^x(y), \tag{3.1c}$$

where the last equality follows from $S \geq 0$.

Two unmatched agents: When an unemployed worker x contacts a firm y for a vacant employment position, the parties' outside option is the value of non-production, O_x and O_y. Then, $\tilde{O}_x$, $\tilde{O}_y = 0$ and the sharing arrangement is such that $S_a^x(x,y) = \beta S(x,y)$ and $S_a^y(x,y) = (1-\beta)S(x,y)$. The two parties form the partnership i.f.f. $S(x,y) \geq 0$ and I use $a(x)$ to denote the set of "agreeable" matches $a(x) = \{y : S(x,y) \geq 0\}$ – from which the subscript on the payoff function.

One matched agent and one unmatched agent: Consider the case in which the worker is employed and contacts a firm y' for a vacant position. Suppose that the surplus associated with the perspective job is higher, $S(x,y') > S(x,y)$. Then, using (3.1) the requirement for mutual consistency between $S_b^x(y)$ and $S_b^x(y')$ is summarized by the system:

$$S_b^x(y) = \max\left\langle \min\left\langle S_b^x(y') + \beta[S(x,y) - S_b^x(y')], S(x,y)\right\rangle, 0\right\rangle \tag{3.2a}$$
$$S_b^x(y') = S_b^x(y) + \beta\left[S(x,y') - S_b^x(y)\right], \tag{3.2b}$$

where the second condition is simplified by exploiting the fact that the first equation implies $0 \leq S_b^x(y) \leq S(x,y) < S(x,y')$. These inequalities and (3.2b) imply that $S_b^x(y') > S_b^x(y)$ when $\beta > 0$. Then, from (3.2a) it follows that $S_b^x(y) = S(x,y)$ and therefore $S_b^x(y') = S(x,y) + \beta[S(x,y') - S(x,y)] \geq 0$. Because $S_b^x(y') > S_b^x(y) = S(x,y)$, the solution sustains an equilibrium in which x leaves the current employer and moves to y' to form a new match. Note that match dissolution is efficient. I assume that an indifferent agent prefers to remain with the current partner. Kiyotaki and Lagos (2007) refer to this outcome as a "single" breach and, accordingly, I use $s_m^x(x,y)$ to denote the set of y' such that x moves to y' when he is matched to y: $s_m^x(x,y) \equiv \{y' : S(x,y') > S(x,y)\}$. The subscript "$m$" indicates that the meeting triggers the formation of a new "match", rather than a renegotiation of the terms of trade within the current partnership. The payoffs for x, y and y' are:

$$S_{s_m^x}^x(x,y,y') = S(x,y) + \beta[S(x,y') - S(x,y)] \tag{3.3a}$$

$$S_{s_m^x}^y(x,y,y') = 0 \tag{3.3b}$$

$$S_{s_m^x}^{y'}(x,y,y') = (1-\beta)[S(x,y') - S(x,y)], \tag{3.3c}$$

respectively.

The same logic follows if x is matched with y', but he receives a portion of the surplus which is smaller than $S_{s_r^x}^x(x,y',y) \equiv S(x,y) + \beta[S(x,y') - S(x,y)]$; in this case the worker exploits the outside offer to renegotiate the wage and obtains $S_{s_r^x}^x(x,y',y)$. I denote the set of y' which trigger a "renegotiation" of the sharing agreement between x and y by $s_r^x(x,y,S^x) \equiv \{y' \in a(x) : S(x,y') \leq S(x,y), S^x \leq S_{s_r^x}^x(x,y,y')\}$. The associated payoffs are:

$$S_{s_r^x}^x(x,y',y) = S(x,y) + \beta[S(x,y') - S(x,y)] \tag{3.4a}$$

$$S_{s_r^x}^y(x,y',y) = 0 \tag{3.4b}$$

$$S_{s_r^x}^{y'}(x,y',y) = (1-\beta)[S(x,y') - S(x,y)]. \tag{3.4c}$$

Finally, the same logic applies to the case where the firm contacts an unemployed worker and has the option of replacing her current employee. Similar to the notation introduced above, the set $s_m^y(x,y) \equiv \{x' : S(x',y) > S(x,y)\}$ summarizes encounters leading to replacement hiring, while $s_r^y(x,y,S^x) \equiv \{x' \in a(y)^{-1} : S(x',y) \leq S(x,y), S^x \geq S_{s_r^y}^x(x,y,x')\}$ meetings which only trigger the renegotiation of the wage – the superscript "-1" indicates the inverse

of the set a. The corresponding payoffs for x, y, and x' are:

$$S^x_{s^y_m}(x,y,x') = S^x_{s^y_r}(x',y,x) = 0$$
$$S^y_{s^y_m}(x,y,x') = S^y_{s^y_r}(x',y,x) = S(x,y) + (1-\beta)[S(x',y) - S(x,y)]$$
$$S^{x'}_{s^y_m}(x,y,x') = S^{x'}_{s^y_r}(x',y,x) = \beta[S(x',y) - S(x,y)],$$

respectively.

Two matched agents: Finally consider the case in which a worker x is currently employed at firm y and contacts firm y' for an employment position which is currently filled by worker x'. The bargaining problem is described by the system:

$$S^x_b(y) = \min\left\langle S^x_b(y') + \beta[S(x,y) - S^x_b(y'), S(x,y)\right\rangle \tag{3.6a}$$
$$S^x_b(y') = \max\left\langle \min\left\langle S^x_b(y) + \beta[S(x,y') - S^x_b(y) - S^{y'}_b(x')], S(x,y')\right\rangle, 0\right\rangle \tag{3.6b}$$
$$S^{y'}_b(x) = S(x,y') - S^x_b(y') \tag{3.6c}$$
$$S^{y'}_b(x') = \min\left\langle S^{y'}_b(x') + (1-\beta)[S(x',y') - S^{y'}_b(x)], S(x',y')\right\rangle, \tag{3.6d}$$

where the non-negativity constraint in the first and last condition is redundant because the second guarantees $S^{y'}_b(x)$, $S^x_b(y') \geq 0$. In the appendix I show that this system always admits one and only one solution, provided that $\beta \in (0,1)$. Let $D(x,y,x',y') \equiv S(x,y') - S(x,y) - S(x',y')$. The solution is such that when $D > 0$, x and y' form a new partnership and the surplus is split according to:

$$S^x_{d_m}(x,y,x',y') = S(x,y) + \beta D(x,y,x',y') \tag{3.7a}$$
$$S^y_{d_m}(x,y,x',y') = 0 \tag{3.7b}$$
$$S^{x'}_{d_m}(x,y,x',y') = 0 \tag{3.7c}$$
$$S^{y'}_{d_m}(x,y,x',y') = S(x',y') + (1-\beta)D(x,y,x',y'). \tag{3.7d}$$

Similar to the notation introduced above $S^i_{d_m}$ denotes agent i's payoff and $d^x_m(x,y) \equiv \{x', y' \in a(y')^{-1}a(x) : D(x,y,x',y') > 0\}$ is the set of matches (x',y') such that x quits job y and firm y' lays-off x' to hire x in his place. The choice of the notation reflects Kiyotaki and Lagos (2007) terminology, who refer to this occurrence as a "double" breach. Note that match formation and separation are efficient. Similarly, I define $d^y_m(x,y) \equiv \{x' \in$

$a(y)^{-1}, y' \in a(x') : D(x', y', x, y) > 0\}$. When instead $D \leq 0$, the solution is:

$$S^x_{d^x_r}(x, y, x', y') = S(x, y) + \beta[S(x, y) - X(x, y, x', y')] \tag{3.8a}$$

$$S^y_{d^x_r}(x, y, x', y') = (1 - \beta)[S(x, y) - X(x, y, x', y')] \tag{3.8b}$$

$$S^{x'}_{d^x_r}(x, y, x', y') = \beta[S(x', y') - (S(x, y') - X(x, y, x', y'))] \tag{3.8c}$$

$$\begin{aligned} S^{y'}_{d^x_r}(x, y, x', y') = {} & S(x, y') - X(x, y, x', y') \\ & + (1 - \beta)[S(x', y') - (S(x, y') - X(x, y, x', y'))], \end{aligned} \tag{3.8d}$$

where:

$$X(x, y, x', y') \equiv \max\left\langle \min\left\langle S(x, y) + \frac{D(x, y, x', y')}{2}, S(x, y') \right\rangle, 0 \right\rangle$$

is the portion of $S(x, y')$ that x could obtain if she were to bargain and match with y'. In the equilibrium sustained by this solution x and y remain matched together, but x and y' use the outside offer to renegotiate the sharing agreement with their respective partner, if they *both*can gain from doing so. The conditions for a renegotiation are summarized by the set $d^x_r(x, y, S^x) \equiv \{x', y' \in a(y')^{-1}a(x), S^{x'} : D(x, y, x', y') \leq 0, S^x \leq S^x_{d^x_r}(x, y, x', y'), S^{x'} \geq S^{x'}_{d^x_r}(x, y, x', y')\}$. Similarly, I define $d^y_r(x, y, S^x) \equiv \{x' \in a(y)^{-1}, y' \in a(x'), S^{x'} : D(x', y', x, y) \leq 0, S^x \geq S^x_{d^{x'}_r}(x', y', x, y), S^{x'} \leq S^{x'}_{d^{x'}_r}(x', y', x, y)\}$.

3.3 Values

A type x unemployed worker experiences an exogenous and constant utility flow equal to $b(x)$. As a result of search activity he can either contact a firm for a vacancy or for a position which is already filled by another worker. In the first case he is hired if and only if the surplus is (weakly) positive (i.e., if the value generated by production is (weakly) greater than that of continuing to search); in the second case she is hired in place of the firm's current employee if he can generate a greater surplus – see section 3.2. Formally, the value of a type x unemployed worker is determined by the continuous Bellman equation:

$$\begin{aligned} rW_0(x) = {} & b(x) + \lambda_{00} \int_{a(x)} S^x_a(x, y') dV(y') \\ & + \lambda_{01} \int_{a(x)} \int_{\overline{s}^{y'}_m(x, y')} S^x_{s^{y'}_m}(x', y', x) dM(x', y'), \end{aligned} \tag{3.9}$$

where the over score "$\bar{\ }$" denotes the complement of a set.

Similarly, when employed, a worker can contact a firm for either a vacancy or for a position which is already filled by another worker; the contact rate is κ_x times lower. However, her current employer might contact another worker as well, unemployed or currently employed. And, a meeting can not only affect the match through separation/match formation, but also by triggering the renegotiation of the wage. Correspondingly, one can define four terms reflecting the impact of search activity on the employee's value. First, a worker can move to another firm, which occurs at rate:

$$\lambda_{x,m}(x,y) \equiv \lambda_{10} \int_{s_m^x(x,y)} dV(y') + \lambda_{11} \iint_{d_m^x(x,y)} dM(x',y'),$$

and increases the value of employment – net of $W_0(x)$) – to:

$$\begin{aligned} E(S^x|s_m^x, d_m^x) \equiv \lambda_{10} &\int_{s_m^x(x,y)} S^x_{s_m^x}(x,y,y')dV(y') \\ &+ \lambda_{11} \iint_{d_m^x(x,y)} S^x_{d_m^x}(x,y,x',y')dM(x',y'). \end{aligned}$$

in expectation. Second, the worker can exploit the outside offer to renegotiate the sharing agreement $S^x(x,y,w)$, as indexed by w:

$$\begin{aligned} E(S^x|s_r^x, d_r^x) \equiv \lambda_{10} &\int_{s_r^x(x,y,w)} S^x_{s_r^x}(x,y,y')dV(y') \\ &+ \lambda_{11} \iiint_{d_r^x(x,y,w)} S^x_{d_r}(x,y,x',y')dM(x',y',w'). \end{aligned}$$

where "w" is short for $S^x(x,y,w)$ in s_r^x and d_r^x. I define $\lambda_{x,r}(x,y,w)$ as the rate at which the renegotiation occurs.

Regarding the search activity of the firm, she replaces the worker at rate:

$$\lambda_{y,m}(x,y) \equiv \lambda_{01} \int_{s_m^y(x,y)} dU(x') + \lambda_{11} \iint_{d_m^y(x,y)} dM(x',y'),$$

in which case the worker's value net of $W_0(x)$ is zero. On the other hand the, firm can exploit the outside offer to renegotiate the wage, which lowers the worker's individual surplus to:

$$\begin{aligned} E(S^x|s_r^y, d_r^y) \equiv \lambda_{01} &\int_{s_r^y(x,y,w)} S^x_{s_r^y}(x,y,x')dU(x') \\ &+ \lambda_{11} \iiint_{d_r^y(x,y,w)} S^x_{d_r^{x'}}(x',y',x,y)dM(x',y',w'). \end{aligned}$$

This event occurs at rate $\lambda_{y,r}(x,y,w)$ (defined as above).

Using these expressions, the Bellman equation characterizing the value of a worker x when he is employed at firm y and receives wage w can be compactly written in terms of his individual surplus:

$$\begin{aligned}[r(x,y)+\lambda_r(x,y,w)]S^x(x,y,w) &= w - rW_0(x) \\ &+ E(S^x|s^x_m,d^x_m) + E(S^x|s^x_r,d^x_r) + E(S^x|s^y_m,d^y_m) + E(S^x|s^y_r,d^y_r).\end{aligned} \tag{3.10}$$

Here $r(x,y)$ denotes the discount rate incorporating the time preference parameter, r, and the rate at which the match dissolves due to the destruction of the position – δ – a job-to-job transition – $\lambda_{x,m}(x,y)$ – or replacement hiring – $\lambda_{y,m}(x,y)$. Because of efficient bargaining, $r(x,y)$ is the total discount rate for the joint surplus. Instead, $\lambda_r(x,y,w)$ denotes the rate at which the wage is renegotiated in response to outside offers made to the worker – $\lambda_{x,r}(x,y,w)$ – or the firm – $\lambda_{y,r}(x,y,w)$.

The derivation of the value equations for the firm corresponding to (3.9) and (3.10) follows the same logic, except that the exogenous shock destroys the position, whether it is vacant or not. The value of a vacancy is:

$$\begin{aligned}(r+\delta)\Pi_0(y) &= \lambda_{00}\int_{a(y)^{-1}} S^y_a(x',y)dU(x') \\ &+ \lambda_{10}\int_{a(y)^{-1}}\int_{\bar{s}^{x'}_m(x',y)} S^y_{s^{x'}_m}(x',y',y)dM(x',y'),\end{aligned} \tag{3.11}$$

while that of the same position when operated by worker x and paying wage w is:

$$\begin{aligned}[r(x,y)+\lambda_r(x,y,w)]S^y(x,y,w) &= f(x,y) - w - (r+\delta)\Pi_0(y) \\ &+ E(S^y|s^x_m,d^x_m) + E(S^y|s^x_r,d^x_r) + E(S^y|s^y_m,d^y_m) + E(S^y|s^y_r,d^y_r).\end{aligned} \tag{3.12}$$

The terms $E(S^y|s^\bullet_\bullet,d^\bullet_\bullet)$ are defined analogously to $E(S^x|s^\bullet_\bullet,d^\bullet_\bullet)$.

When a renegotiation occurs, the worker and the firm review the sharing agreement; after the renegotiation S^x and S^y must still sum up to S, therefore $E(S^x|s^x_r,d^x_r,s^y_r,d^y_r)+E(S^x|s^x_r,d^x_r,s^y_r,d^y_r)=\lambda_{y,r}(x,y,w)S(x,y)$. Thus, the terms reflecting the impact of a renegotiation drop when summing (3.10) and (3.12):

$$(r+\delta)S(x,y) = f(x,y) - rW_0(x) - (r+\delta)\Pi_0(y) + \Sigma(x,y), \tag{3.13}$$

Here $\Sigma(x,y)$ is the *net* contribution of search activity to the joint surplus: $\Sigma(x,y) \equiv E(S^x|s^x_m,\ d^x_m,\ s^y_m,d^y_m)\ + E(S^y|s^x_m,\ d^x_m,s^y_m,d^y_m)\ -\lambda_m(x,y)S(x,y)$.

As seen in section 3.2, when an agent leaves the current partner he/she extracts from the new partner a payoff equal to the surplus generated by the previous partnership, plus a portion of the additional surplus corresponding to his/her bargaining power. As a result the net contribution of search to the current surplus, $\Sigma(x, y)$, simplifies to:

$$\begin{aligned}
&\lambda_{10} \int_{s_m^x(x,y)} \beta[S(x, y') - S(x, y)]dV(y') \\
&+\lambda_{11} \iint_{d_m^x(x,y)} \beta[S(x, y') - S(x, y) - S(x', y')]dM(x', y') \\
&+\lambda_{01} \int_{s_m^y(x,y)} (1-\beta)[S(x', y) - S(x, y)]dU(x') \\
&+\lambda_{11} \iint_{d_m^x(x,y)} (1-\beta)[S(x', y) - S(x, y) - S(x', y')]dM(x', y').
\end{aligned}$$

The wage does not enter (3.13) because of efficient bargaining. In particular, the contact rate with other partners does not depend on the wage by assumption (however, see footnote 21); and the value that an agent can extract from the new partnership does not depend on the wage because the wage can be renegotiated.[26]

3.4 Allocation

In this section I derive the conditions characterizing the population of firms $n(y)$ and the steady state allocation $m(x, y)$. $n(y)$ depends on the equilibrium between entry and exit. While exit is exogenous – δ – the creation of positions satisfies free-entry, and as such it depends on the value of a vacancy (3.11). Regarding the allocation, match formation and match separation are efficient and are determined by the surplus function (3.13), which is independent from the wage. Therefore, $m(x, y)$ and $n(y)$ can be solved independently from wages, together with the value equations (3.13), (3.11) and (3.9).

First, consider the problem of determining the steady state allocation. The flow of newly created matches involving x and $y \in a(x)$ must equal the

[26] Shimer (2006) analyzes the on-the-job search model with bargaining in the case where the wage cannot be renegotiated.

flow of those which are destroyed. Thus, $m(x, y)$ solves:

$$\begin{aligned}[\delta + \lambda_m(x,y)]m(x,y) = \quad & \lambda_{00}u(x)v(y) \\ & + \lambda_{01}\int_{\overline{s}^y_m(x,y)} u(x)m(x',y)dx' \\ & + \lambda_{10}\int_{\overline{s}^x_m(x,y)} v(y)m(x,y')dy' \\ & + \lambda_{11}\int_{a(x)} m(x,y')\int_{d^x_m(x,y')|y} m(x',y)dx'dy',\end{aligned} \tag{3.14}$$

Given $n(y)$, the distribution of unemployed workers and vacancies is trivially derived using the identities $u(x) = l(x) - \int_{a(x)} m(x, y')dy'$ and $v(y) = n(y) - \int_{a(y)^{-1}} m(x', y)dx'$.

Then, consider the problem of determining the population of firms, $n(y)$. As mentioned in section 3.1, there is no firm selection, because the creation and destruction rate is the same across different types of jobs. As a result, $n(y)/\bar{N}$ equals the distribution at entry $\Phi(y)$ and free-entry determines the overall availability of jobs, $\bar{N}$:

$$\bar{N} : C = \int_0^1 \Pi_0(y|\bar{N})d\Phi(y). \tag{3.15}$$

The flow of entrant required to sustain this stock of positions is $I = \delta\bar{N}$.

3.5 Wages

Here I derive the expression for the stationary distribution of wages across matches (x, y). Combined with the equation for the employee's individual surplus, (3.10) – and the solution to the allocation problem, $S(x, y)$, $n(y)$ and $m(x, y)$ – this condition permits characterization of wages. The derivation heavily exploits the monotonicity of $S^x(x, y, w)$ in w – see appendix.

It is useful to introduce some additional notation. When a new match between a type x worker and a type y position is formed, the wage the two parties negotiate is smaller than w if and only if the associated value for the worker is smaller than $S^x(x, y, w)$. In this case the formation of the match contributes to $m(x, y, w)$ – recall that $m(x, y, w)$ is cumulated with respect

to w. Accordingly, define the sets:

$$\Xi_{s^x_m}(x,y,w) \equiv \{y' : S^x_{s^x_m}(x,y',y) \leq S^x(x,y,w)\}$$
$$\Xi_{s^y_m}(x,y,w) \equiv \{x' : S^x_{s^y_m}(x',y,x) \leq S^x(x,y,w)\}$$
$$\Xi_{d^x_m}(x,y,w) \equiv \{x',y' : S^x_{d^x_m}(x,y',x',y) \leq S^x(x,y,w)\}.$$

For example, $\Xi_{s^x_m}(x,y,w)$ summarizes job-to-job transitions where x leaves the current employer y' to take the vacant job y, and such that the compensation he obtains from the new employer is less than w.

Second, consider the case in which the worker x is matched to position y and the worker (firm) receives an outside offer which triggers the renegotiation of the wage w. Then, the same offer would have triggered the renegotiation of any wage lower (higher) than w, so that the occurrence contributes negatively (positively) to $m(x,y,w)$. In the case of meetings involving two matched agents, a renegotiation requires that both agents are willing to renegotiate the wage with their respective partner. Analogous to d^x_r, I denote by $\widetilde{d}^x_r$ the set of (x',y') such that the worker x contacting firm y' wants to exploit the outside offer to renegotiate the wage w (with y):

$$\widetilde{d}^x_r(x,y,w) \equiv \Big\{x' \in a(y')^{-1}, y' \in a(x) : D(x,y,x',y') \leq 0, \\ S^x(x,y,w) \leq S^x_{d^x_r}(x,y,x',y')\Big\}.$$

And, I use $\widetilde{w}_{d^{y'}_r}$ to denote the threshold such that, for any wage w' greater or equal than $\widetilde{w}_{d^{y'}_r}$, firm y' wants to renegotiate the wage (with x') as well:

$$S^{x'}(x',y',\widetilde{w}_{d^{y'}_r}(x',y',x,y)) \equiv S^{x'}_{d^x_r}(x,y,x',y').$$

Similarly, regarding outside offers to y I define $\widetilde{d}^y_r(x,y,w) \equiv \{x' \in a(y)^{-1}, y' \in a(x') : D(x',y',x,y) \leq 0, S^x(x,y,w) \geq S^x_{d^{x'}_r}(x',y',x,y)\}$ and $\widetilde{w}_{d^{x'}_r}$ such that $S^{x'}(x',y',\widetilde{w}_{d^{x'}_r}(x',y',x,y)) \equiv S^{x'}_{d^{x'}_r}(x',y',x,y)$.

Using this notation, and denoting $\overline{m}(x,y,w) = m(x,y) - m(x,y,w)$, the

condition characterizing $m(x,y,w)$ in steady state can be written as:

$$
\begin{aligned}
m(x,y,w)[\delta &+ \lambda_m(x,y) + \lambda_{10} V(s_r^x(x,y,w)) \\
&+ \lambda_{11} \iint_{\widetilde{d}_r^x(x,y,w)} \overline{m}(x',y',\widetilde{w}_{d_r^{y'}}(x',y',x,y))dx'dy'] \\
= \quad & 1_{\{S_a^x(x,y) \leq S^x(x,y,w)\}} u(x)v(y) \\
&+ \lambda_{10} v(y) M(x, \Xi_{s_m^x}(x,y,w)) + \lambda_{01} u(x) M(\Xi_{s_m^y}(x,y,w), y) \\
&+ \lambda_{11} \iint_{\Xi_{d_m^x}(x,y,w)} m(x,y')m(x',y)dx'dy' \\
&+ \lambda_{01} \overline{m}(x,y,w) U(s_r^y(x,y,w)) \\
&+ \lambda_{11} \overline{m}(x,y,w) \iint_{\widetilde{d}_r^y(x,y,w)} m(x',y',\widetilde{w}_{d_r^{x'}}(x',y',x,y))dx'dy'.
\end{aligned}
$$

The "out-flow" results from exogenous and endogenous separations (first two terms on the left-hand-side) and renegotiations triggered by the worker (third and fourth term). The "in-flow" results from match formation (first four terms on the right-hand-side) and renegotiations triggered by the firm (last two terms). Note that the order of integration is at most two. Together with (3.10) this expression determines the individual surplus function of the worker – and the firm, since $S^y = S - S^x$ – and the wage distribution across matches (x,y).

4 Estimation

4.1 Estimation Method

An observation in the panel is $\psi_{i,t} = \{lms_{i,t}, w_{i,t}, aw_{j_{i,t}}, va_{j_{i,t}}\}$, where i indexes the worker, t the week and j the firm; lms denotes the labor market state, "employed" or "unemployed". If the worker is employed, then w denotes the hourly wage observed in November of that year; aw is the wage bill paid by the firm over the year normalized by the size of her workforce in FTE, and, similarly, va is the value added per FTE produced by the firm employing the worker.

I estimate the model by indirect inference – Gourieroux, Monfort, and Renault (1993). This technique involves finding the vectors of model parameters minimizing the distance between a set of statistics computed on the data (the "auxiliary model") and the same set of statistics computed on data generated by simulating the model for a specific vector of model parameters. Formally, let $\Gamma(\psi_N)$ be a vector of statistics computed on the

original sample, having size N. I denote by ω the vector of model parameters and by $\psi_N^s(\omega)$ the sample generated by simulating the model for the parameter vector ω and the sequence s of pseudo-random numbers. The artificial sample has the same size as the original data, N. Define $\Gamma^S(\omega)$ the mean vector of statistics obtained by simulating the model S times:

$$\Gamma^S(\omega) \equiv \frac{1}{S}\sum_{s=1}^{S}\Gamma(\psi_N^s(\omega)).$$

The vector of model parameter estimates $\hat{\omega}$ is the one that minimizes the distance between $\Gamma(\psi_N)$ and $\Gamma^S(\omega)$ for a given weighting matrix A:

$$\hat{\omega} = \arg\min_{\omega}\left[\Gamma^S(\omega) - \Gamma(\psi_N)\right] A \left[\Gamma^S(\omega) - \Gamma(\psi_N)\right]. \tag{4.1}$$

Gourieroux et al. (1993) derive the asymptotic theory for the estimator above and show that it is consistent and normally distributed. In the just identified case – which I implement below – the asymptotic variance is independent of the weighting matrix and it is equal to:

$$\Sigma_{\hat{\omega}} = \left(1 + \frac{1}{S}\right) J(\omega_0)^{-1}\Sigma_\Gamma J(\omega_0)'^{-1}. \tag{4.2}$$

In this expression Σ_Γ is the asymptotic variance of $\sqrt{N}[\Gamma(\psi_N) - \Gamma(\psi_\infty)]$ for N going to infinity; $J(\omega_0)$ is the Jacobian of $\Gamma^S(\omega)$ evaluated at the true parameter vector, $J(\omega_0) = \partial\Gamma^S(\omega)/\partial\omega\big|_{\omega=\omega_0}$. I estimate Σ_0 by block re-sampling across workers the original data: I draw from the set of worker identifiers and include the entire time-series for that particular worker. I assume that this sampling protocol preserves the joint distribution of the data. Then, substituting $J(\hat{\omega})$ for $J(\omega_0)$ the result above can be used to derive standard errors for the estimates, $\hat{\omega}$. In the estimation I simulate a sample of $448,352$ workers for 18 years so that the number of worker-year observations is equal to the one in the original sample, $8,070,340$. The original data is an unbalanced panel, a discrepancy which could be relevant for duration statistics; however in practice the number of observations is large and statistics are computed very precisely. S is set equal to 1. Finally, the model does not account for firm size. When simulating the model, an issue arises for the definition of value added per worker and the average wage paid by the firm. As in Cahuc et al. (2006), I assume that the law of large number holds, so that, for example, the productivity of a firm y is $E(f(x,y)|y)$. The assumption is justified by the fact that most workers work at large firms – see table 1.

4.2 Model Specification

I set the time preference parameter to .05, corresponding to a real risk-free rate of roughly 5% per year. The remaining unknowns of the model are: the entry cost C, the worker's bargaining power parameter, β, the position destruction rate, δ, the search technology, heterogeneity/production, and the unemployment instantaneous utility schedule $b(x)$.

Consider first the search technology. I assume that the total contributions of search coming from matched and unmatched agents enter the matching function additively with weights κ_x and κ_y. I restrict the matching function to the Cobb-Douglas form with exponent 1/2, which is consistent with estimates found in the empirical literature reviewed by Petrongolo and Pissarides (2001) – between 0.5 and 0.7:

$$\mu(U, \bar{L} - U, V, \bar{N} - V) = \lambda \left[U + \frac{\bar{L} - U}{\kappa_x} \right]^{.5} \left[V + \frac{\bar{N} - V}{\kappa_y} \right]^{.5}. \tag{4.3}$$

The search technology depends on three parameters: λ indexes the overall level of frictions in the economy, while κ_x and κ_y capture the relative (in)efficiency of on-the-job search and replacement search. Due to the assumption of constant returns to scale for the matching function, population size can be normalized to $\bar{L} = 1$ without loss of generality. Then, $\bar{N}$ indexes the relative availability of jobs and can be interpreted as a proxy for market tightness (every agent can contact any other agent).

Next consider heterogeneity. As mentioned in the previous section, x and y are ordinal quantities which only index whether a worker or firm is more productive than another. Then, heterogeneity can be modeled equivalently through the production function or the population densities – see for example Eeckhout and Kircher (2009). I fix $l(x)$ and $n(y)$ to be uniform on the unit interval and assume that the production function takes the multiplicative form used by Lu, McAfee, and Greenwich (1996), Postel-Vinay and Robin (2002) and Cahuc et al. (2006):[27]

$$f(x, y) = [a_x + (1 - a_x)x][a_y + (1 - a_y)y]. \tag{4.4}$$

I augment the technology by introducing the parameters a_x and a_y, which are intended as an index of the amount of (ex-ante) heterogeneity for workers and firms, respectively. I constrain these two parameters to the unit interval so that the output of any match is bounded by one. This is just

[27] Bagger and Lentz (2008) explicitly address the problem of estimating the sign and the strength of sorting.

a normalization – provided that the unemployment utility flow is rescaled accordingly – a fact that can be verified via an inspection of the value equations (3.9), (3.10), (3.11) and (3.13).

Finally, I assume that all workers experience the same utility flow $b(x) = b$ when unemployed, regardless of their type. In Denmark, unemployment benefits vary with earnings, however in practice while the replacement rate is roughly 90% for low income workers, it rapidly declines as income increases; in addition the take up-rate of unemployment benefits is lower for higher income workers, as discussed in Buti et al. (2001). For this reason, a parametrization which is linear in the type of the worker as in Postel-Vinay and Robin (2002) or Cahuc et al. (2006) seems inappropriate for the Danish data.

Summarizing, I estimate 9 parameters: the entry cost, the worker's bargaining power, the position destruction rate, the three parameters of the search technology, the two parameters for heterogeneity/production, and finally the utility flow of unemployment. Note, that given a particular value of $\bar{N}$ one can solve the model and find ex-post the appropriate level of the entry cost C satisfying the free-entry condition (3.15). I redefine the vector of model parameters substituting $\bar{N}$ for C so that ω becomes:

$$\omega = (\bar{N}, \delta, \lambda, \kappa_x, \kappa_y, \beta, b, a_x, a_y). \tag{4.5}$$

This allows to disregard the free-entry condition, which simplifies the computation of the criterion function (4.1) in the minimization procedure.

4.3 Identification

In this section I provide an intuitive argument for identification by linking each parameter in (4.5) to a particular statistic.[28]

Consider first the basic search friction parameters, λ, δ and the newly introduced market tightness parameter, $\bar{N}$. For identification I rely on duration data and the unemployment rate (u%) which results from the equilibrium between flows into and out of unemployment. Regarding duration data, I select the median duration of unemployment (T_u^{50th}) and the median duration of a job (T_e^{50th}), Kaplan-Meyer estimates. In particular, one can view market tightness as determining the job offer arrival rate and, therefore, unemployment duration. The exogenous separation rate determines the flow into unemployment and, thus, the equilibrium unemployment level.

[28] Eckstein and Van den Berg (2007) provide a review of the empirical implementation of the search model and a discussion of identification.

Finally, match separation has an endogenous component which depends on the rate of job-to-job transitions and replacement hiring, provided that there is on-the-job and replacement search. Given values for κ_x and κ_y (and assuming that at least one of them is not "too large"), the higher λ, the higher the rate at which an agent finds a better partner, which decreases T_e^{50th}.

Next, consider the two parameters capturing the relative efficiency of search for workers and firms, κ_x and κ_y. As discussed by Eckstein and Van den Berg (2007), identification of κ_x can be obtained from the rate of job-to-job transitions. Similarly κ_y could be identified using information on transitions to unemployment. However, in practice, it is difficult to distinguish a job to-job transition from a job to-unemployment transition which is followed by another job: the worker might be given a notice and find an alternative job before actually becoming unemployed, or a worker finding a better job might spend some time out of the labor force before joining the new employer. In the context of this model the distinction is crucial because one must distinguish separations initiated by the worker from those initiated by the firm. Instead of relying on duration data, I consider the implications of search for wage dynamics: Outside offers received by the worker induce wage increases while outside offers to the firm trigger wage cuts. Thus, I target the 10th and the 90th percentile of the distribution of year-to-year wage changes to identify κ_y and κ_x, respectively.

Now, consider the problem of identifying the heterogeneity/production parameters. When a_x is 1, workers are homogeneous. Suppose firms are homogeneous as well and suppose search on-the-job is excessively costly. Then the distribution of wages becomes degenerate. Firm heterogeneity and on-the-job search both increase wage dispersion, but, other things equal, the lower a_x the higher is wage dispersion. Thus, I include the inter-quartile range of the distribution of wages (IQR_w) as a statistic to identify a_x. Similarly the inter-quartile range of the distribution of value added per worker (IQR_{va}) can be used to identify a_y.

Next, consider workers' bargaining power β and the instantaneous unemployment utility level b. Abstract from on-the-job search and suppose $a_y < 1$ so that firms are heterogeneous. Also, suppose for simplicity that workers are homogeneous. Then, when $\beta = 0$, all firms pay the same wage, while as β increases more productive firms pay higher wages. Thus, β can be identified from the joint distribution of output and wages and, in particular, I use a regression of wages (aw) on value added (va) – in fact this is the approach used in regression studies of rent sharing, e.g. Blanchflower, Oswald, and Sanfey (1996). The wage bill paid by a firm increases with value added, but not as fast, resulting in a concave relation – Mortensen (2003);

I regress log wages on log value added and include the slope coefficient in the vector of auxiliary statistics (β_{va}). Similarly, b affects the bargaining position of the worker and can be identified through the joint distribution of output and wages. Abstracting from on-the-job search and heterogeneity, the higher b the higher the reservation wage and therefore the labor share ($ls = \sum_{i,j} aw_{j_i} / \sum_{i,j} va_{j_i}$).

Summarizing, the vector of auxiliary statistics includes 9 elements:

$$\psi_N = (u\%, T_u^{50th}, T_e^{50th}, \Delta w^{10th}, \Delta w^{90th}, \beta_{va}, ls, IQR_w, IQR_{va}).$$

The time dimension is aggregated by averaging statistics across years.

4.4 Estimation Results

I estimate the just identified model outlined in the identification argument given above. Table 4 lists the statistics which constitute the base for the estimation, together with standard errors (in parenthesis) computed by bootstrapping the original data.

The average unemployment rate for the sample over the period is 5.1%. This is the result of a relatively short median unemployment duration, 11.65 weeks, and a very short median job duration, 115.88 weeks.[29]

The inter-quartile range for wages is .348. Wages are normalized by the mean wage so the figure indicates that the wage at the third quartile is 35% higher than the wage at the first quartile, in mean-wage units. The 90th to 10th percentile ratio is 2.15. Overall, wage dispersion for the sample is low, a feature that applies to the Danish data in general.[30] Dispersion is higher for value added per worker: the inter-quartile range is .505, with a 90/10 ratio of 2.57.

The two last figures in table 4 concern the joint distribution of value added per worker and wages. The log-linear regression of the average wage paid by a firm on value added per worker gives the set of coefficients: $\beta_0 = -.158$, $\beta_{va} = .410$. The slope coefficient is below one, which reflects the concave relation between value added and wages. Finally the labor share

[29] By comparison, the average unemployment rate in the U.S. over the same period and for workers 16 years of age and older, is 5.73%, with a median unemployment duration equal to 7.15 weeks (Bureau of Labor Statistics, CPS data, tables A-7 and A-9, respectively). Using supplements to the CPS, Farber (1995, table A3) computes median job duration figures for different years and across different age categories. For 1987 he reports: 1.7 (age 25-34), 3.4 (age 35-44), 4.4 (age 45-54) and 0.2 years (age 55-64); and for 1993: 2.1, 3.8, 5.2 and 1.3 years (same age groups as above).

[30] The average 90/10 ratio in the U.S. over the same period is 5.25 for males 15 years of age or older, and 4.24 for women (Bureau of Labor Statistics, CPS data, table IE-2).

Table 4: Statistics Used for the Estimation

$u^{(a)}$	.051	$IQR_w{}^{(e)}$	0.348
	(0.00014)		(0.00055)
$T_u^{50th\,(b)}$	11.65	$IQR_{va}{}^{(f)}$	0.505
	(0.04325)		(0.00136)
$T_e^{50th\,(c)}$	115.88	$\beta_{va}{}^{(g)}$	0.410
	(0.21774)		(0.00268)
$\Delta w^{90th\,(d)}$	0.132	$ls^{(h)}$	0.682
	(0.00015)		(0.00155)
Δw^{10th}	−0.115		
	(0.00011)		

All workers in private sector, 1985-2002, at least 10 yrs. of experience, age< 55. Detrended series, figures averaged across years – except for duration data. [a] unemployment rate [b] unemployment duration, median (Kaplan-Meier) [c] employment duration, median (Kaplan-Meier) [d] year-to-year wage changes, percentiles [e] wages (w), inter-quartile range [f] value added (va), inter-quartile range [g] log-linear regression of wage (aw) on value added (va), slope coeff. [h] labor share.

is 0.682, which is comparable with values found for other OECD countries.

In table 5 I report estimates for the model parameters together with standard errors in parenthesis. Standard errors are computed using the asymptotic variance-covariance matrix for the estimator, (4.2). The model matches the vector of auxiliary statistics reported in table 4. I discuss the estimates in relation to the literature and postpone to the remaining of the paper a more thorough discussion of the estimates most relevant to this work – the relative efficiency of on-the-job search (κ_x) and replacement search (κ_y), the worker's bargaining power parameter (β) and heterogeneity (a_x and a_y).

The mass of employment positions is estimated at 1.096 which, given an unemployment rate of 5.08%, implies a ratio of almost three vacant positions per unemployed worker. This number is difficult to interpret. Job openings data are available for the U.S. through the Bureau of Labor Statistics starting with 2001 – Job Openings and Labor Turnover Survey (JOLTS). According to JOLTS, the stock of openings was roughly half the number of job seekers between 2003 and 2007. However, it is hard to translate figures for "openings" into an absolute measure of the employment positions – Nagypál (2009) – and the dataset is better suited for analysis which exploit the variation over time of the data.

Next, consider the search friction parameters: the exogenous separation rate, δ, and the parameters of the matching function, λ, κ_x and κ_y. The first number can be viewed as the residual separation rate which is not explained by the model. Discrepancies across different studies reflect differences in job

Table 5: Model Estimates

$\bar{N}^{(a)}$	1.096	$\kappa_x{}^{(d)}$	10.854	$a_y{}^{(g)}$	0.509
	(0.00185)		(0.01076)		(0.00190)
$\delta^{(b)}$	0.146	$\kappa_y{}^{(e)}$	7.493	$\beta^{(h)}$	0.469
	(0.00322)		(0.01806)		(0.00534)
$\lambda^{(c)}$	4.320	$a_x{}^{(f)}$	0.668	$b^{(i)}$	0.104
	(0.00797)		(0.00482)		(0.03517)

Standard errors in parenthesis. [a] mass of positions [b] destruction rate [c] baseline search friction [d] on-the-job search (in)efficiency [e] replacement search (in)efficiency [f] worker heterogeneity [g] firm heterogeneity [h] worker's bargaining power [i] unemployment utility.

duration across datasets and samples, as well as differences in the mechanism leading to the dissolution of the match. Job duration in Denmark is comparable to that in the U.S. – footnote (29) – and lower than in France, where Postel-Vinay and Robin (2002) estimate job duration at $10-20$ years. On the second point, in the present work separations are the result of the worker quitting his job, as much as of the firm laying-off the worker. The estimate for the exogenous separation rate, δ, is .146 which corresponds to an annual probability of separation equal to 13.6%.[31] This number compares favourably with estimates found by Rosholm and Svarer (2004), Christensen et al. (2005) and Bagger et al. (2007) – between 7% and 25% – who estimate on-the-job search model on the same data.

The estimate for the baseline parameter of the matching function, λ, is 4.32, which translates into a weekly job offer probability for the unemployed equal to 10%. Rosholm and Svarer (2004) and Bagger et al. (2007) find values between 3 and 6%. In the present setting not all workers accept all offers. While for these specific estimates an unemployed worker is always hired to fill a vacant position, he is hired 40% of cases if the position is already filled. These two components sum up to a 72% acceptance probability for the unemployed.

κ_x is estimated at 10.854 which is larger than values found in the literature – Eckstein and Van den Berg (2007). In particular Dey and Flinn (2005) estimate the ratio between the arrival rate of offer off and on-the-job at 6 for the U.S., and Postel-Vinay and Robin (2002) between 2 and 3 for France. For the Danish data analyzed in this paper, Rosholm and Svarer (2004) find values between 4 and 7 and Bagger et al. (2007) between 9 and 12. κ_y is estimated at a lower level, 7.493; however, due to the relatively high market tightness ($\bar{N}/\bar{L} = 1.096$), an employee is more likely to receive

[31] $1 - exp(-\delta \times 1)$.

an outside offer than the employer – 32.5% versus 23.5% annual probability.

Worker and firm heterogeneity is important. As mentioned above, for these particular parameter estimates any match is acceptable by an unemployed worker and a vacant position, $a(x) = [0, 1] \forall x \in [0, 1]$. Thus a worker produces $1/a_y = 1.96$ times more when employed at the most productive firm rather than at the least productive – and a position generates $1/a_x = 1.50$ times more output when operated by the most able worker.

The worker's bargaining power parameter β is estimated at .469. Cahuc et al. (2006) find values around $.15 - .38$ for higher skilled workers (with the exception of manufacturing for which they find $\beta = .98$) and approximately zero otherwise. Dey and Flinn (2005) estimate the bargaining power parameter for the case of homogeneous workers and firms at .25, but are unable to estimate the parameter for the more general case with heterogeneous agents due to lack of information on firm productivity – for this specification they assume the symmetric case, $\beta = .5$. These studies only allow for on-the-job search. When allowing for replacement search, the competition between workers counters the competition between employers, and a higher value for the worker's bargaining power parameter is required to match the joint distribution of output and wages.

The utility flow experienced by an unemployed worker is estimated at $b = .104$, which is 24% the mean wage for the artificial sample and 35% the wage at the first decile. The number is higher than what the evidence suggests for the replacement rate in Denmark – Buti et al. (2001). Dey and Flinn (2005) estimate b at a negative value, which is needed to match wage dispersion. In this case the estimate is driven by the high dispersion of value added relative to wages, which requires a high level of firm heterogeneity relative to worker heterogeneity. If b were higher, no worker would accept employment at a type 0 firm.

Finally, given parameter estimates for $\bar{N}$ and the other parameters of the model I can recover the underlying entry cost, C. Imposing the free entry condition (3.15) I obtain $C = .9011$. This number can be used to run the counter-factual for a policy prohibiting replacement hiring. When firms cannot replace the worker the value of a vacancy rises instead of decreasing, because market tightness effectively drops for vacant positions (the vacancy does not have to compete with filled positions when seeking to hire a worker) and in addition the pool of unemployed contains a higher fraction of high types. Thus, the equilibrium value for $\bar{N}$ rises from 1.096 to 1.103.

5 Replacement Hiring, Discussion

5.1 The Equilibrium Allocation

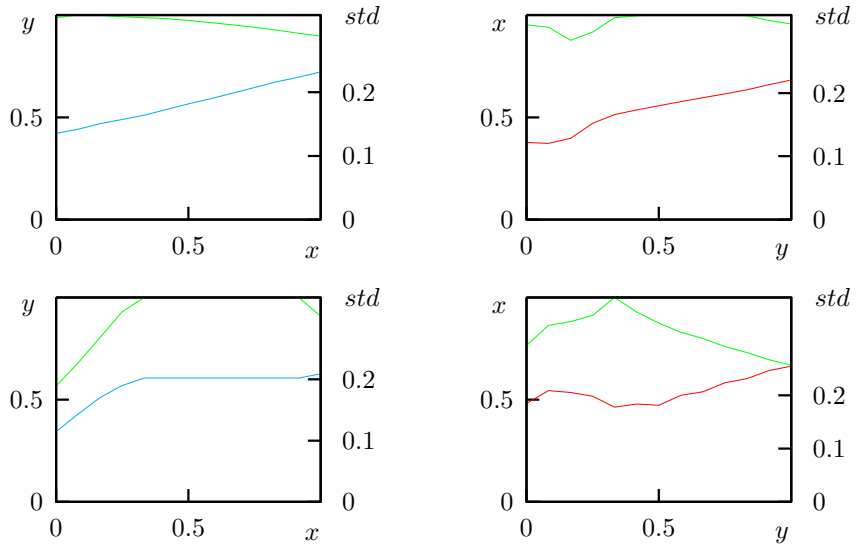
I first discuss heterogeneity and the equilibrium allocation because this allows to describe some features of the equilibrium which will be useful in the remaining of the paper.

As mentioned in section 4.4 heterogeneity is estimated to be significant both for workers and for firms: a worker produces 1.96 times more when employed at the most productive rather than the least productive firm. The corresponding figure for the firm is 1.50. In addition, more productive workers tend to be employed at more productive firms in equilibrium, because the production function is super-modular.[32] In Figure 3 I plot the average employer type for each type of worker – in blue, left panel – and the average worker type for each type of firm – in red, right panel – and the corresponding standard deviations – in green, right scale. The two top panels are for the case where firms can search on the job – estimates in table 5 – while the two bottom panels are for the counter-factual where a hypothetical policy prohibits replacement hiring – κ_y large – and the availability of positions, $\bar{N}$, adjusts accordingly (3.15). The figure shows that the allocation exhibits strong positive sorting. More interestingly, this feature is strengthened when the search activity of firms complements that of workers. This statement requires some further discussion.

In his seminal paper Becker (1973) shows that in a setting with heterogeneous agents, the efficient allocation is to pair similar types together when the production technology is super-modular. Sattinger (1993, 1995) and Shimer and Smith (2000) extend the framework to allow for search frictions. Finally, Eeckhout and Kircher (2010) demonstrate that a partnership model with frictions exhibit a tendency towards negative sorting. This property can be understood as the result of an "insurance" motive. Consider the case of en economy with a modular production technology $f(x, y) = x + y$, where the amount produced does not depend on how agents are sorted. A pool of low type unemployed workers ensures – at a "relatively low cost social cost" – that a high type worker can be quickly matched back into production if he becomes unemployed. Over time, the mass of asymmetric matches involving a high and a low type agent builds up.

[32] "super-modularity" means that types are strongly complementary. Formally, f is super-modular if: $x' > x$ and $y' > y \Rightarrow f(x', y') + f(x, y) > f(x', y) + f(x, y')$. If f is twice differentiable, then the condition is equivalent to requiring that the cross partial being increasing in the partner's type.

Figure 3: Expected Partner Type, and Variance

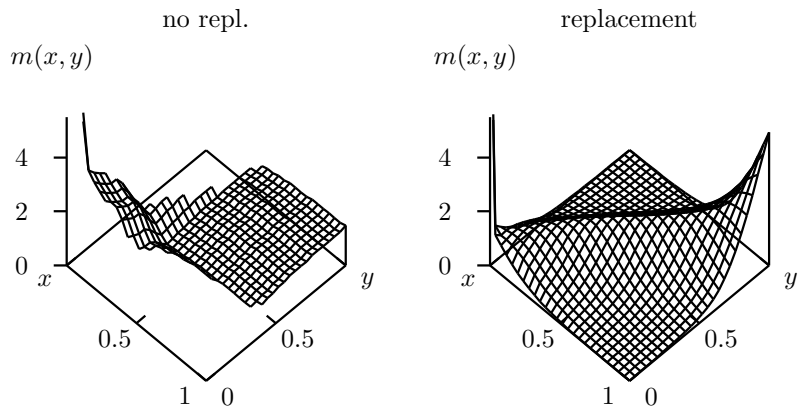


Left panels, worker; right panels, firm; top panels, replacement search; bottom panels, shutting down replacement search. Worker's expected partner type $E(y|x)$ in blue, firm's expected partner type $E(x|y)$ in red (left-scale), partner type standard deviation in green (right-scale).

However, when both workers and firms can search on-the-job, they can contact a matched party as well, which creates an additional margin for improvement: if two asymmetric pairs contact one another the two high type agents can match together and remain into production, while the two low types enter unemployment expanding the "insurance" pool. The result of this process can be seen in figure 4, where I contrast the stationary distribution of matches for these two cases, "off-the-match" search only – left panel – and on-the-job search and replacement search – right panel. The combined process of replacement hiring and job-to-job transitions gradually moves the mass at the "odd" corners towards the diagonal, and flattens the "flaps" of the distribution on the left-panel into the saddle-shape of the distribution on the right panel.

Sorting accentuates differences in wages and output across workers. The highest type worker is $1/a_x = 1.50$ times more productive than the lowest type, but he tends to work at higher productivity firms, .72 versus .42, so that on average he produces 1.78 times more. As for wages, the unemployment instantaneous utility flow, b, strengthens the bargaining position of

Figure 4: On-The-Job Search and Sorting



$f(x,y) = x + y$ and $r = .05$, $\delta = .1$, $\bar{N} = 1$, $\lambda = 5$, $\beta = .5$, $b = 0$. Right, with on-the-job and replacement search, $\kappa_x = \kappa_y = 1$; left, without, $\kappa_x = \kappa_y = 10^{15}$.

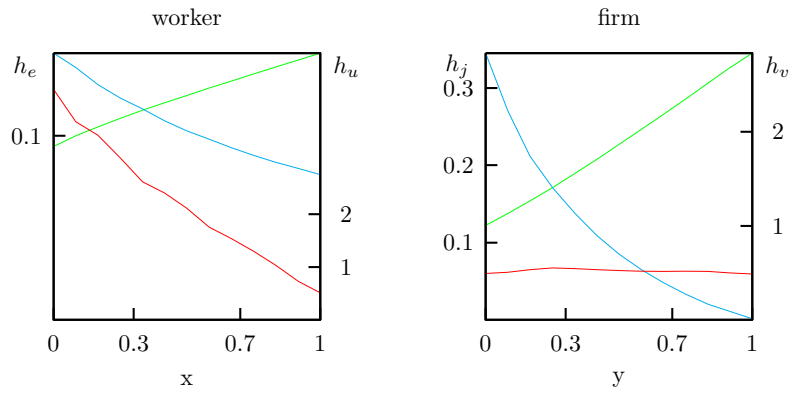
low productivity workers proportionally more so that the ratio between the average wage earned by the most productive worker relative to the least productive is 1.69.

5.2 Flows

Because agents are heterogeneous, they have a different opportunity cost of searching and their matching set will differ as well. This translates into systematic differences in transition rates across different types of agent. In particular, due to replacement search higher worker types will escape unemployment faster.

Next, I discuss in detail how replacement hiring affects flows between jobs and back into unemployment. As mentioned above the arrival rate of offers for an employed worker is higher than that for a filled position: despite the fact that $\kappa_x > \kappa_y$, the high level of $\bar{N}$ means that there is a high ratio of vacancies to unemployed workers. As a result, an employed worker is almost 50% more likely to receive an outside offer – 32.5% annual probability – than the firm is to contact a worker for the same position – 23.5% annual probability. Conditional on receiving an outside offer numbers are comparable

Figure 5: Unemployment and Employment Hazard

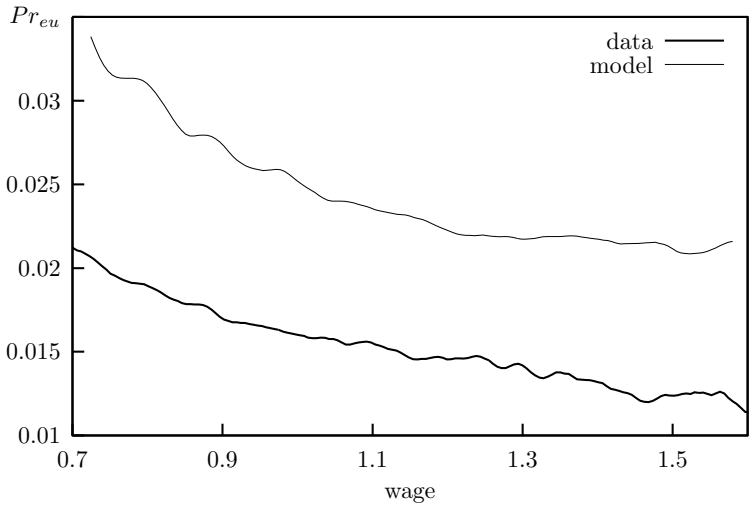


Hazard across types, net of δ. Left panel, worker: job finding rate in green (right-scale); lay-off rate in red; job-to-job in blue (left-scale). Right panel, firm: vacancy filling rate in green (right-scale), worker's quit rate in blue; worker's replacement rate in red (left-scale).

across workers and firms: the worker moves to the new employer with 18.1% probability and renegotiates the wage 45.3% of times; as for the firm the numbers are 15.0% and 43.2%, respectively. Combining these figures, a wage increase is 1.45 times more likely than a wage cut and a job-to-job transition is 1.69 times more likely than replacement hiring. This latter number is consistent with values found by Fallick and Fleischman (2004) using survey data for the U.S., and by Rosholm and Svarer (2004) and Christensen et al. (2005) for the Danish data used in this paper. Overall, combining job-to-job transitions and lay-off the model accounts for 52% of separations; the rest is captured by the residual separation rate, δ.

Next consider match formation and separation rates across different types of workers and firms. In the left panel of figure 5 I plot for each worker type the hazard of unemployment (green), and the hazard of employment to another job (blue) or to unemployment due to replacement (red) – i.e. net of δ. In the right-panel I report the same information for the firm. As mentioned, lower type workers face a lower unemployment hazard. However, the most striking feature is the variability of the lay-off rate across different types of worker. Replacement appears to be a phenomenon which mainly affects low type workers. Because low type workers receive lower wages as well, this feature of the model is consistent with the stylized fact of a strong negative relationship between the wage and the probability of unemployment. This relationship is displayed in figure 6 for the original

Figure 6: Wage And Unemployment Probability



Hourly wage in November and weekly probability of unemployment: local linear regression, Epanechnikov kernel, Silverman's rule of thumb for bandwidth choice. Data, thick line.

sample, together with the same relationship computed for the model at the parameter estimates. The curve is computed as the local linear regression of exit into unemployment on the last wage observed, in the current or previous year (remember that the wage is observed in November).[33] The intercept for the model is higher, but as I discussed in section 2.3 the distinction between job-to-job and job-to-unemployment-to-job is uncertain in the data.

5.3 Search, Bargaining and Wages

The model fits the features of within-job wage changes discussed in section 2.3. On average, the employee receives an outside offer with annual probability equal to 32.5% and as a result he renegotiates the wage (upward) in 45.3% of the instances. The figures for the firm are 23.5% and 43.2%, respectively. Combining these numbers, the probability that the wage raises in a given year is 14.7%, while the probability that it decreases is 10.1%. Accordingly, the competition between workers induced by the prospect of replacement hiring is an economically important force which somewhat bal-

[33] I use the Epanechnikov kernel, and I select the bandwidth according to Silverman's rule of thumb.

ances the upward pressure on wages induced by on-the-job search.

It has been argued that in the presence of rents, on-the-job search is used by the employee to increase the wage, and that this can help explain important features of the data, particularly wage dispersion across workers, and the relation between employment duration and wages.[34] However, in the Danish data a large fraction of within-job wage changes from one year to the next is negative (1) and the wage does not grow very much over the employment spell (2); also, wages often decrease between jobs and the wage stream experienced by the worker in the following years does not appear to compensate for the initial loss (3).

The introduction of replacement search potentially reconciles these facts with the view of wage dispersion as determined by search behavior. A firm search activity counters the upward pressure induced by the search activity of workers (2) and causes occasional downward renegotiations (1). The tension between these two forces, however, increases wage dispersion, rather than decreasing it. When making replacement search prohibitive the interquartile range decreases from .348 to .283. Also, the introduction of replacement search resolves the tension between the high level of the labor share, and wage dispersion: on-the-job search can only lead to wage increases and wage dispersion requires a low value of the worker's outside option and/or bargaining power parameter. Dey and Flinn (2005) estimate b to be negative, Postel-Vinay and Robin (2002) assume $\beta = 0$ and Cahuc et al. (2006) generally find low values for β. When allowing for replacement hiring, wage dispersion does not hinges on a low value for b and β. Indeed, as I discussed in section 4.4, β is estimated at a higher levels than in Cahuc et al. (2006).

Next, consider the longitudinal dimension of wages. In table 6 I report numbers for the expected wage growth since job onset, together with figures computed conditioning on the originating labor market state – as for the evidence presented in figure 2 and table 3. The first block (horizontally) is for the data, the second for the benchmark model with replacement hiring, the third for the counter-factual where search on the job is made prohibitive for firms.

The model fits the unconditional wage profile and low tenure effects rather well. As for its decomposition, it does not reproduce the lower wage growth following unemployment – see the discussion in section 2.3. As noted there, this feature of the data suggests the importance of heterogeneity and scarring effects. The model does not account for scarring, but in principle it

[34] Burdett and Mortensen (1998), Burdett and Coles (2003), Postel-Vinay and Robin (2002) and Cahuc et al. (2006).

Table 6: Wage Profile, Data and Model

		$E(+/-)$	1 yr.	2 yr.	3 yr.	4 yr.
data	tot.		0.021	0.038	0.041	0.046
	u		0.012	0.022	0.026	0.028
	e_+	0.237	−0.033	−0.028	−0.021	−0.016
	e_-	−0.153(46.86%)	0.077	0.105	0.113	0.128
estim. model$^{(a)}$	tot.		0.034	0.055	0.068	0.077
	u		0.058	0.093	0.115	0.129
	e_+	0.202	−0.037	−0.058	−0.068	−0.074
	e_-	−0.169(11.26%)	0.132	0.204	0.244	0.271
	$x < .25$	−0.159	0.106	0.166	0.186	0.209
count.-fact.$^{(b)}$	tot.		0.019	0.034	0.046	0.056
	u		0.029	0.052	0.070	0.084
	e_+	0.071	0.007	0.013	0.018	0.023
	e_-	−0.014(2.91%)	0.006	0.012	0.016	0.021

Columns as in table 3. ^{a} with replacement search ^{b} shutting down replacement search

could reproduce this feature of the data by exploiting heterogeneity. Lower type workers are more likely to be unemployed, and, due to the asymmetric effect of the unemployment instantaneous utility parameter, these workers experience a lower wage growth. However the effect is not strong enough.

Next, consider job-to-job transitions. The model does fairly well explaining the magnitude of wage changes between jobs but severely under-predicts the probability of a wage cut. When the wage increases, the model captures the fact that in the following years the wage tends to remain stable, or decline. Finally, consider the case where the wage drops. Postel-Vinay and Robin (2002) show that a model with on-the-job search and wage renegotiations can explain wage cuts between jobs, if the wage then grows sufficiently fast. This does not appear to be the case in the Danish data. Allowing for replacement hiring mitigates the growth rate of wages in the following periods. Low type workers facing a high risk of replacement escape unemployment by switching to less productive firms, and trade higher wages for higher job security.[35] In the ninth raw of the table I report figures for wage cuts experienced by worker in the first skill quartile, $x < .25$. In these cases the wage rises less in the following years. Replacement hiring mitigates the growth of wages through this channel but the model is still far from matching the data.

[35] This feature of the model resembles Nagypal (2005) explanation of the "extent of job to job transitions". In that paper workers switch job to escape unemployment due to negative productivity shocks.

6 Conclusions

Using matched employer-employee Danish data I find that a large portion of wage changes within and between jobs are negative and large in absolute value; tenure effects are small; finally, when the wage decreases between jobs the wage remains below its level in the previous job after 10 years. To explain these facts I construct and structurally estimate a partnership model with frictions where not only the worker, but also the firm can continue to search after forming a match.

Allowing for replacement search helps explain these facts, particularly within job wage changes and tenure effects. Concerning between job wage changes, the extension increases the likelihood of a wage drop between jobs and mitigates the wage growth observed in the following periods. This is due to low ability workers switching to less productive jobs, trading lower wages for higher job security. However, given the parameter estimates, this effect is weak and the model does not adequately fit the data along this dimension. The model explains 52% of job separations, with replacement hiring being responsible for 1/5 and job-to-job transitions explaining 4/9. The lower number found for replacement hiring relative to job-to-job transitions is the result of a lower contact rate and a lower probability of replacement conditinal on the firm contacting another worker. Replacement hiring mainly affects low ability workers, which explains the strong negative correlation between wages and the probability of unemployment. Finally, I find that allowing firms to search while matched increases wage inequality, due to the tension between replacement search and on-the-job search, as well as due to the combination of replacement hiring and job-to-job transitions which creates a pool towards positive sorting.

Overall, I find that replacement hiring is important to understand flows and wages, and that it has strong general equilibrium implications concerning the allocation of resources across production possibilities and the wage distribution.

A Bargaining: a matched worker and a matched position

I study system (3.6) and determine under what conditions it admits a solution and the solution is unique. I first consider the case $S(x,y') > S(x,y) + S(x',y')$ and then $S(x,y') \leq S(x,y) + S(x',y')$.

Suppose:

$$S\left(x,y'\right) > S(x,y) + S\left(x',y'\right) \tag{A.1}$$

The system always admits the solution:

$$\begin{aligned}
S_b^x(y) &= S(x,y) \\
S_b^x\left(y'\right) &= (1-\beta)\,S(x,y) + \beta\left[S\left(x,y'\right) - S\left(x',y'\right)\right] \\
S_b^{y'}(x) &= \beta S\left(x',y'\right) + (1-\beta)\left[S\left(x,y'\right) - S(x,y)\right] \\
S_b^{y'}\left(x'\right) &= S\left(x',y'\right)
\end{aligned}$$

since (A.1) implies $S_b^x(y') \geq S(x,y)$ and $S_b^{y'}(x) \geq S(x',y')$ and (together with $S(x,y') = S_b^x(y') + S_b^{y'}(x))$ $S_b^x(y'), S_b^{y'}(x) < S(x,y')$. In addition, when $\beta \in (0,1)$ $S_b^{y'}(x) > S_b^{y'}(x')$ and $S_b^x(y') > S_b^x(y)$ and therefore this solution sustains the double breach outcome summarized by (3.7). I now verify that (3.6) does not admit any other solution when $\beta \in (0,1)$ and (A.1) holds.[36] Suppose $S_b^x(y') = 0$, then $S_b^x(y) = \beta S(x,y)$, $S_b^{y'}(x) = S(x,y')$ and $S_b^{y'}(x') = S(x',y')$; substituting inside the expression for $S_b^x(y')$ I obtain $(1-\beta)\,\beta S(x,y) + \beta\left[S(x,y') - S(x',y')\right] > 0$. This also rules out $S_b^{y'}(x) = S(x,y')$. By symmetry $S_b^x(y') = S(x,y')$ and $S_b^{y'}(x) = 0$ cannot be part of a solution either. Now suppose $S_b^x(y) < S(x,y)$, which according to (3.6a) requires $S_b^x(y') < S_b^x(y) < S(x,y)$; because I showed $0 < S_b^x(y') < S(x,y')$ then (3.6b) can be rewritten as $0 = (1-\beta)\left[S_b^x(y) - S_b^x(y')\right] + \beta[S(x,y') - S_b^{y'}(x') - S_b^x(y')]$ which is incompatible with the previous inequality when (A.1) holds, since $S_b^{y'}(x') \leq S(x',y')$. Finally, by symmetry $S_b^{y'}(x') < S(x',y')$ cannot be either.

Then, suppose:

$$S\left(x,y'\right) \leq S(x,y) + S\left(x',y'\right) \tag{A.2}$$

[36] When β equals 0 or 1 and (A.1) holds then (3.6) admits a continuum of solutions indexed respectively by $0 \leq S_b^x(y) \leq S(x,y)$ $\{S_b^x(y') = S_b^x(y),\ S_b^{y'}(x) = S(x,y') - S_b^x(y),\ S_b^{y'}(x') = S(x',y')\}$ and by $0 \leq S_b^{y'}(x') \leq S(x',y')$ $\{S_b^{y'}(x) = S_b^{y'}(x'),\ S_b^x(y') = S(x,y') - S_b^{y'}(x'),\ S_b^x(y) = S(x,y)\}$.

Consider the case in which none of the constraints is binding; then (3.6) is linear and can be rewritten as:

$$\begin{bmatrix} 1 & -(1-\beta) & 0 & 0 \\ -(1-\beta) & 1 & 0 & \beta \\ 0 & 1 & 1 & 0 \\ 0 & 0 & -\beta & 1 \end{bmatrix} \begin{pmatrix} S_b^x(y) \\ S_b^x(y') \\ S_b^{y'}(x) \\ S_b^{y'}(x') \end{pmatrix} = \begin{pmatrix} \beta S(x,y) \\ \beta S(x,y') \\ S(x,y') \\ (1-\beta) S(x',y') \end{pmatrix}$$

The determinant of the matrix on the left hand side is $2\beta(1-\beta)$. Assuming $\beta \in (0,1)$ the system above admits one and only one solution:

$$\begin{aligned} S_b^x(y) &= S(x,y) + (1-\beta)\frac{S(x,y') - S(x',y') - S(x,y)}{2} \qquad \text{(A.3)} \\ S_b^x(y') &= S(x,y) + \frac{S(x,y') - S(x',y') - S(x,y)}{2} \\ S_b^{y'}(x) &= S(x',y') + \frac{S(x,y') - S(x',y') - S(x,y)}{2} \\ S_b^{y'}(x') &= S(x',y') + \beta\frac{S(x,y') - S(x,y) - S(x',y')}{2} \end{aligned}$$

where note that (A.2) implies $S_b^x(y') \leq S_b^x(y) \leq S(x,y)$ and $S_b^{y'}(x) \leq S_b^{y'}(x') \leq S(x',y')$. However $S_b^x(y') < 0$ if:

$$S(x,y') - S(x',y') + S(x,y) < 0 \qquad \text{(A.4)}$$

and $S_b^{y'}(x) < 0$ if:

$$S(x,y') + S(x',y') - S(x,y) < 0 \qquad \text{(A.5)}$$

Thus (A.3) solves the original system (3.6) only if $\beta \in (0,1)$ and if (A.4) and (A.5) are violated. Suppose $\beta \in (0,1)$.[37] (A.4) and (A.5) are mutually exclusive, as it can be seen by summing one with the other, and as I show below each leads to a solution which is continuous to each of the two "sides" of (A.3). Suppose $S_b^x(y') = 0$ and in particular:

$$\begin{aligned} S_b^x(y) &= \beta S(x,y) \qquad \text{(A.6)} \\ S_b^x(y') &= 0 \\ S_b^{y'}(x) &= S(x,y') \\ S_b^{y'}(x') &= \beta S(x,y') + (1-\beta) S(x',y') \end{aligned}$$

[37] If β equals 0 or 1 and (A.2) holds then (3.6) admits a continuum of solutions indexed respectively by $0 \leq S_b^x(y) \leq \min\langle S(x,y'), S(x,y)\rangle$ $\{S_b^x(y') = S_b^x(y),\ S_b^{y'}(x) = S(x,y') - S_b^x(y),\ S_b^{y'}(x') = S(x',y')\}$ and by $0 \leq S_b^{y'}(x') \leq \min\langle S(x,y'), S(x',y')\rangle$ $\{S_b^{y'}(x) = S_b^{y'}(x'),\ S_b^x(y') = S(x,y') - S_b^{y'}(x')$ and $S_b^x(y) = S(x,y)\}$.

For this to be a solution to (3.6) it must be that $(1-\beta)\, S_b^x(y) + \beta[S(x,y') - S_b^{y'}(x')] \leq 0$ and $\beta S(x,y') + (1-\beta)\, S(x',y') \leq S(x',y')$. Substituting for $S_b^x(y)$ and $S_b^{y'}(x')$ the first inequality becomes $\beta(1-\beta)[S(x,y') + S(x,y) - S(x',y')] \leq 0$ which holds provided that (A.4) holds; the second inequality also follows from (A.4). Similarly:

$$\begin{aligned} S_b^x(y) &= (1-\beta)\, S(x,y') + \beta S(x,y) \\ S_b^x(y') &= S(x,y') \\ S_b^{y'}(x) &= 0 \\ S_b^{y'}(x') &= (1-\beta)\, S(x',y') \end{aligned} \tag{A.7}$$

is a solution to (3.6) provided that (A.5) holds. Putting together (A.3), (A.6) and (A.7) I conclude that if $\beta \in (0,1)$ and (A.2) holds, then (3.6) always admits the solution:

$$\begin{aligned} S_b^x(y) &= \beta S(x,y) + (1-\beta)\, X(x,y,x',y') \\ S_b^x(y') &= X(x,y,x',y') \\ S_b^{y'}(x) &= S(x,y') - X(x,y,x',y') \\ S_b^{y'}(x') &= (1-\beta)\, S(x',y') + \beta\left[S(x,y') - X(x,y,x',y')\right] \end{aligned}$$

This solution sustains the equilibrium underlying (3.8) i.f.f. $S_b^x(y) \geq S_b^x(y')$ and $S_b^{y'}(x') \geq S_b^{y'}(x)$; these two inequalities always hold under (A.2): let $\tilde{X}(x,y,x',y') = [S(x,y') - S(x',y') + S(x,y)]/2$ and note that (A.2) implies $S(x,y) \geq \tilde{X}(x,y,x',y')$ and $S(x',y') \geq S(x,y') - \tilde{X}(x,y,x',y')$.

This proves existence – when $\beta \in (0,1)$; I finally show uniqueness by verifying that there are no other solutions to (3.6) when $\beta \in (0,1)$ and (A.2) holds. There are two families of candidates left, one where $(1-\beta)\, S_b^x(y') + \beta S(x,y) > S(x,y)$, and the other where $\beta S_b^{y'}(x) + (1-\beta)\, S(x',y') > S(x',y')$; by symmetry it is enough to prove that $(1-\beta)\, S_b^x(y') + \beta S(x,y) > S(x,y)$ cannot be part of a solution. This inequality can be rewritten as $S_b^x(y') > S(x,y)$ and it implies $S_b^x(y) = S(x,y)$; also, this requirement makes the max operator in (3.6b) and the min operator in (3.6d) "superfluous" – regarding the latter $\beta S_b^{y'}(x) + (1-\beta)\, S(x',y') \geq S(x',y')$ cannot be under (A.2) since it implies $S(x,y') - S_b^x(y') - S(x',y') \geq 0$. Thus (3.6)

can be rewritten as:

$$\begin{aligned}
S_b^x(y) &= S(x,y) \\
S_b^x(y') &= \min\left\langle (1-\beta) S_b^x(y) + \beta\left[S(x,y') - S_b^{y'}(x')\right], S(x,y')\right\rangle \\
S_b^{y'}(x) &= S(x,y') - S_b^x(y') \\
S_b^{y'}(x') &= \beta S_b^{y'}(x) + (1-\beta) S(x',y')
\end{aligned}$$

Suppose $(1-\beta) S_b^x(y) + \beta[S(x,y') - S_b^{y'}(x')] \geq S(x,y')$; substituting for $S_b^x(y)$ this inequality can be rewritten as $(1-\beta)[S(x,y) - S(x,y')] - \beta S_b^{y'}(x') \geq 0$ which is incompatible with $S(x,y) < S_b^x(y') = S(x,y')$. Thus one is left with a linear system which can be reduced to two equations (second and fourth) in two unknowns ($S_b^x(y')$ and $S_b^{y'}(x')$); its solution is such that $S_b^x(y') = \{S(x,y) + \beta[S(x,y') - S(x',y')]\}/(1+\beta)$ which however cannot be greater than $S(x,y)$ under (A.2).

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