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Abstract

We study the optimal taxation of labor income in the presence of search frictions. Heterogeneous workers undertake costly search off- and on-the-job in order to locate more productive jobs that pay higher wages. More productive workers search harder, resulting in equilibrium sorting where low-type workers are overrepresented in low-wage jobs while high-type workers are overrepresented in high-wage jobs. Absent taxes, worker search effort is efficient, because the social and private gains from search coincide. The optimal tax system balance efficiency and equity concerns at the margin. Equity concerns make it desirable to levy low taxes on (or indeed, subsidize) low-wage jobs including unemployment, and levy high taxes on high-wage jobs. Efficiency concerns limit how much taxes an optimal tax system levy on high-paid jobs, as high taxes distort the workers’ incentives to search. The model is simulated for reasonable parameter values. The model is also extended to allow for amenities that are unobservable to the tax authorities and therefore cannot be taxed. Ultimately, we want to estimate the model using a Danish matched employer-employee data set.

Key words: Optimal taxation, Search, Job ladder.

JEL codes: H21, H31 J63, J64.

1 Introduction

The equilibrium allocation of resources does not materialize costlessly in a market with search frictions, as it does in a perfectly competitive market. That is, improving the allocation of resources in a frictional market requires resources in itself, and is therefore value creation. This value creation may be distorted by taxation. In the context of a frictional labor market, workers and firms undertake costly search in order to identify the jobs in which their productivity (and therefore, wage) is the highest. Taxes on
labor income will have an allocative effect, as it reduces the incentives of workers to
search for more high-productive, well-paid jobs.

We study the optimal taxation of labor income in a frictional labor market where
workers search off- and on-the-job. Workers are \textit{ex ante} heterogeneous in their produc-
tivity, and hence face different job prospects. A benevolent planner has a preference for
income equality, where income is measured as (expected) lifetime discounted income.
On the one hand, in order to redistribute from high- to low-productive workers, the
planner wants to levy high taxes on individuals with a high current income (which tend
to be high-productive workers) and redistribute the tax revenues towards agents with
a low current income (which tend to be low-productive workers).

On the other hand, workers of all types expend resources to find more produc-
tive jobs that pay better. All workers enter as unemployed, and through job search,
first as unemployed and then as employed, gradually climb the wage/productivity
ladder. Hence, search effort yields both private and social returns, but is costly
and—crucially—not deductible. Increasing taxes on wages at a given rung of the
wage/productivity ladder reduces the incentives to search at lower rungs, but does not
affect the incentives to search on higher rungs. Hence, in order to protect the creation
of value, the planner wants to levy lower taxes on higher wage levels. The optimal tax
system trades off the equity and efficiency concerns of the planner\footnote{We assume that labor supply is fixed, both on the intensive and the extensive margin. Hence our focus is solely on the workers’ search decisions. In a richer model with endogenous labor supply, the distortionary effects of taxes on the workers’ search decisions will come in addition to (the well-studied) distortions in labor supply.}

We first construct a model of on-the-job search. In this model, search is random,
however in the absence of taxes the equilibrium is constrained efficient. Workers of
different types face different wage distributions when they search. The role of firms
are very much played down, they do not contribute actively to the search process,
and wages are equal to productivity. We first derive analytically conditions for an
optimal tax system. We then extend the model by allowing for amenities, which are
not observable by the planner and hence cannot be taxed. When choosing between jobs,
a worker takes both wages net of taxes and amenities into account, and this creates a
new margin that is distorted by taxes. Equity concerns imply that the planner levy
taxes that increase in gross wages. After successful search, a worker therefore has a
tendency to accept too few jobs that offer a higher wage and too many jobs that offer
a lower wage than the wage in the current job, relative to what the planner would like
the worker to do. This is because workers, in contrast to the planner, do not take taxes
into account when choosing between jobs. In isolation this effect tends to increase the efficiency loss of taxes. Again we derive analytical conditions for an optimal tax system. We solve the basic model (without amenities) numerically, and give examples of optimal tax systems. Our next step will be to calibrate the model to Danish register data, and derive an optimal taxation system based on the calibrated model.

The existing literature on optimal taxation and search is mostly concerned with the search decisions of unemployed workers. Hungerbühler, Lehmann, Parmentier, and van der Linden (2006) analyze optimal taxation in a one-shot unemployment search model. In their model, firms use resources to open vacancies and wages are determined by wage bargaining. They assume (like us) that workers are risk neutral, while the planner has preferences over the (expected) income distribution over different worker types. They show how a revelation mechanism can be applied at the bargaining stage, so that the worker and the firm bargain over which type to reveal to the planner. As a result, the revelation principle can be used to derive the optimal mechanism. Under the optimal taxation scheme, the employment level is optimal for the most productive worker-firm pairs, while there is over-employment for the lower types that do search.

Golosov, Maziero, and Menzio (2013) study optimal taxation in a one-shot competitive search equilibrium model with identical, risk averse workers and heterogeneous firms. There is a fixed cost for workers from sending an application. The equilibrium without taxation is inefficient, as optimal risk sharing requires that workers are compensated for applying to jobs they do not get. In the constrained efficient equilibrium, the unemployment insurance is set so that workers are indifferent between searching for any job and not searching, as this gives maximum insurance given workers’ incentive compatibility constraint. There is no transfers between workers searching for different firm types; the firms in effect finance the unemployment benefit of the workers they attract but do not hire. As a result, taxes are regressive.

Shi and Wen (1999) analyze the effect of taxes in a model of random unemployment search, in which workers accumulate capital. Higher labor taxes discourage working, and leads to lower investments by firms and lower wages. Capital taxation on the other hand increases labor supply, as workers get a lower return on their capital. Hence capital taxation may improve the allocation of resources. Domeij (2005) analyzes optimal taxation within the same modeling framework, and find that the optimal capital tax is zero if and only if the Hoisio’s condition is satisfied. Jiang (2012) uses a similar setup to analyze the welfare effects of a UK tax reform. Arseneau and Chugh (2012) studies taxation in a calibrated DSGE model with search frictions, and argues that
cyclical variations in the search-based labor wedge call for taxes that vary over the business cycle. Wilemme (2017) studies taxation in a model of mismatch, and shows that taxes should be regressive to correct for workers not being sufficiently selective. Geromichalos (2015) study optimal taxation with risk averse workers in a one-shot urnball model of the labor market.

A couple of recent papers analyze taxation and on-the-job search. Sleet and Yazici (2017) studies optimal taxation in a Burdett and Mortensen (1998) model of on-the-job search. A tax on labor income reduces net wages for workers, and hence increases their pre-tax reservation wage. As a result, the entire wage distribution shifts, and this influences the division of rents between workers and employers. Bagger, Hejlesen, Sumiya, and Vejlin (2017) evaluates equilibrium effects on labor allocation of a series of tax reforms in Denmark and also analyze optimal tax reforms using an equilibrium on-the-job search model with Burdett and Mortensen (1998) wage setting.

On a conceptual level, our paper is also related to papers outside the search literature. Saez (2002) analyses a model of taxation in which taxes influence participation (the extensive margin) as well as which firm type (level) to work for (the intensive margin). Working for a firm at a higher level gives higher income, but this may come at a cost. If the extensive margin is sufficiently important, taxes for low-income employed workers may be lower than for unemployed workers. This is studied in more detail in Christiansen (2015). Although the Saez (2002) model is very different from ours, there are interesting similarities between the two: In Saez’ model, reducing taxes at a given level induces some workers who previously were choosing an occupation one level above or below switches to that level. In our model, by contrast, reducing taxes at a given job type reduces the incentives to search for workers in that job type, increases the incentives to search for all workers further down the job ladder, and leaves the incentives unchanged for all workers higher up in the job ladder.

Another related paper is Best and Kleven (2013). They study an environment with learning by doing, so that the wage of an old worker depends on his labor supply as young. A higher marginal income tax on old workers thereby reduces the labor supply of young workers. The model is calibrated to wage data for young and old workers from the PSID. Estimates of the effect of labor supply on future wages are obtained from existing studies of the effect of experience on wages. The authors find that with age-dependent taxes, the presence of on-the-job learning implies that an optimal tax system prescribes lower taxes for old workers than for young workers. With age-independent taxes, it implies that the optimal tax system prescribes a lower marginal tax rates
across the board.

2 Basic Model

There are $J$ worker types in the economy. The fraction of workers of type $j$ is denoted by $\tau_j$, i.e. $\sum_{j=1}^{J} \tau_j = 1$. Workers discount the future at a pure discount rate $\bar{r}$ and exit the market at rate $\lambda$ to be replaced by new unemployed workers. The effective discount rate is thus $r \equiv \bar{r} + \lambda$. Workers search equally efficient off- and on-the-job. The number of firms is exogenously given.

The job ladder has $n+1$ rungs with rungs indexed by $i \in \{0, 1, ..., n\}$. Each rung $i$ is associated with a productivity level $y_i > y_{i-1}$, with the lowest rung $i = 0$ being the value of home production (i.e. productivity during unemployment). After successful search, the probability that a worker of type $j$ draws a job of productivity $y_i$ is denoted $f_j^i$, and define $F^i_j \equiv \sum_{k=1}^{i} f_k^j$. Our assumption that productivities are discretely distributed is made for tractability only, extending the analysis to allow for a continuum of job productivities is straight-forward. The offer distribution is independent of the search intensity and acceptance decisions of workers, and is a primitive of the model.

Our modeling of the productivity distribution of workers and firms is general, and may capture different assumptions regarding the productivity distribution of firms, complementarities between worker and firm types, and of match-specific productivity components. Let us give an example of an underlying structure that gives rise to a job distribution of our type: Suppose there are $L$ firm-types in the economy. The fraction of firms of type $\ell$ is $\xi_\ell$. All firms exhibit constant returns to scale in production. The productivity of a type-$j$ worker and a type-$\ell$ firm is stochastic and revealed at the point when they meet. Denote by $\tilde{f}(j, \ell) \geq 0$ the probability that the realized productivity is $y_i$. It follows that $f_j^i = \sum_{l=1}^{L} \xi_l \tilde{f}(j, l)$. This production structure, without any restrictions on the probabilities, are sufficiently rich to capture the production functions of most on-the-job search models in the literature.

Worker types are indexed so that a higher $j$ means a higher type. More precisely we assume that if $F^{j+1}$ stochastically dominates $F^j$ for all $j$. In addition we assume that $f_i^{j+1}/f_i^j$ is strictly increasing in $i$ whenever properly defined ($f_i^{j} \neq 0$) and different from zero.

An employed worker may be hit by a negative employment shock and enter unemployment. The rate at which this happens is $s$. In principle, $s$ may depend on both worker- and job type, in which case we write $s = s_i^j$. 

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The arrival rate of job offers, $p$, to a worker is proportional to the number of efficiency units of search, $e$, provided by the worker. That is, $p = A e$, where $A$ is an exogenous constant. We may think of $A$ as proportional to the number of firms $K$ in the economy, $A = a K$. The aggregate number of matches is $NA\bar{e}$, where $N$ is the aggregate number of workers in the economy (which we will normalize to 1), and $\bar{e}$ their average search intensity. Hence our search technology differs from the standard search technology in that a worker’s search intensity does not create congestion effects and thereby lower job finding rates for other workers. This seems a natural assumption if workers search for firms, and firms have a constant-return-to-scale production technology and give a job offer to all workers that approach them. The cost of search effort is $\kappa_j c(e)$, where the coefficient $\kappa_j$ may be worker-type dependent. If not, we normalize $\kappa$ to 1.

The wage a worker receives gross of taxes is equal to his productivity $y_i$. This is consistent with a model of wage bargaining in which the worker has all the bargaining power. It follows that absent taxes, there are no externalities associated with the workers’ choice of search intensity. Higher search intensity of a worker does not create congestion externalities for other workers, and although it increases the hiring probability of firms, this does not create positive externalities as the worker’s wage is equal to his productivity.\footnote{The search-and wage determination processes are special cases of those developed by [Diamond (1982), Mortensen (1982), and Pissarides (1985), with an exogenous number of vacancies and with the workers’ bargaining power equal to 1. See also Pissarides (1994) for a model with on-the-job search.}

We assume that the government cannot observe worker types directly and restrict the tax system to be contingent on current wages. Hence the government, through taxation, determines the net-of-tax wage $w_i$ that a worker (irrespective of his or her type) receives in a job of productivity $y_i$. Note that $w_0$ is the net-of-tax unemployment income. We require that $w_i$ is nondecreasing in $i$, so that income taxation leaves the ranking of job unaltered. It follows that the probability distribution of $w_i$ is the same as that of $y_i$.\footnote{A simpler way to proceed is to impose a particular functional form of the tax function, for instance by following [Heathcote, Storesletten, and Violante (2017)] and assume that the tax function is given by $z - \lambda z^{1-\tau}$. It follows that the after-tax-income of a worker is $\lambda z^{1-\tau}$ and the elasticity of the after-tax-income is $1 - \tau$.}

An important issue is the preferences of the agents. As in Golosov, Maziero, and Menzio (2013), Best and Kleven (2013) and others we assume that agents in the economy are risk neutral, while the planner’s welfare weights on the different groups of workers depend on their expected net present income.\footnote{One way of justifying this assumption is that idiosyncratic risk is shared in families of workers, where...}
income of an unemployed worker of type \(j\) less of search costs. Note that \(W^j_0\) is also the lifetime income of a worker that enters the market. An entering type-\(j\) worker’s objective is to maximize \(W^j_0\). A planner’s objective is to maximize the welfare function

\[
\Omega = \sum_{j=1}^{J} \kappa_j \Phi(W^j_0) \tag{1}
\]

where \(\Phi(W^j_0)\) is strictly increasing and concave.

A driving assumption in our analysis is that search effort is unobservable to the planner, and cannot be contracted upon. Furthermore, we assume that search cost is associated with distress when searching for a job and reduced leisure, not reduced income. Hence the search cost is born fully by the worker, while the gain in terms of higher income is partly taxed away. We also assume that the search cost is independent of the worker’s income. Hence the level of wages in the different jobs do not influence search intensity, only the wage differentials. At this point we follow Saez (2002) when he assumes that the choice of sectors do not depend on the income levels in the different sectors, only the differences in income between them.

3 Optimal taxation

In this section we will first solve the model for a given set of taxes, and then derive the optimal tax system.

3.1 Asset values

For any variable \(X^j_i\), define \(EX^j_i = \sum_{k=i+1}^{n} f^j_k X^j_k\) and the operator \(\Delta\) as \(\Delta X^j_i = \sum_{k=i+1}^{n} f^j_k (X^j_k - X^j_i)\) for any variable \(X^j\). Let \(W^j_i\) denote the expected discounted income of a type-\(j\) worker in a type-\(i\) job. Analogously, let \(M^j_i\) denote the expected discounted future income gross of taxes for a type-\(j\) worker in a type-\(i\) job. It follows that

\[
(r + s)W^j_i = w_i + Ae^j_i \Delta W^j_i + sW^j_0 - c(e^j_i) \tag{2}
\]

\[
c'(e^j_i) = A\Delta W^j_i \tag{3}
\]

\[
(r + s)M^j_i = y_i + Ae^j_i \Delta^j M^j_i + sM^j_0 - c(e^j_i) \tag{4}
\]

all the family members are of the same type.
For a given vector \((w_0, \ldots w_n)\), and a given \(W_0^j\), the model can be solved recursively for each type separately:

1. At the top:

\[
\begin{align*}
W_n^j &= \frac{w_n + sW_0^j}{r + s} \\
e_n^j &= 0 \\
M_n^j &= \frac{y_n + sM_0^j}{r + s}
\end{align*}
\]

2. Recursively further down:

\[
\begin{align*}
W_i^j &= \frac{w_i + Ae_i^j EW_i^j + sW_0^j - c(e_i^j)}{r + s + Ae_i^j (1 - F_i^j)} \\
c'(e_i^j) &= Ae_i^j \Delta W_i^j \\
M_i^j &= \frac{y_i + Ae_i^j EM_i^j + sM_0^j - c(e_i^j)}{r + s + Ae_i^j (1 - F_i^j)}
\end{align*}
\]

**Lemma 1** *For any vector \((w_0, w_1, \ldots w_n)\) the vector \(W_0^j, \ldots W_n^j\) exists and is unique.*

**Proof.** For a given \(\hat{W}_0^j\), equation (2)-(4) uniquely define \(W_i^j = W_i^j(\hat{W}_0^j)\) for all \(i\). In particular, \(W_i^j = W_i^j(\hat{W}_0^j)\). Equilibrium is a fixed-point to this mapping. As the mapping is continuous and defined on a compact and convex set, Brouwer’s fixedpoint theorem ensures existence. Furthermore, Maxwell’s sufficiency condition is satisfied \((W_i^j(\hat{W}_0^j + \Delta) < W_i^j(\hat{W}_0^j) + k\Delta)\) for some \(k < 1\), which implies that the fixed-point is unique.

Consider now a change in \(w_i\). For a worker in a job \(k > i\), this only influences the npv income \(W_k^j\), and since the transition rate to unemployment is \(s\) independently of the job type, it follows that

\[
\frac{dW_k^j}{dw_i} = \frac{s}{r + s} \frac{dW_0^j}{dw_i} \quad \forall k > i.
\]

From the envelope theorem it follows that the effect of an increase in \(w_i\) on \(W_i^j\) is

\[
\frac{dW_i^j}{dw_i} = \frac{1 + s \frac{dW_0^j}{dw_i}}{r + s + Ae_i^j (1 - F_i^j)}.
\]
The effect of a change in \( w_i \) on \( W_{i-l} \) is recursively defined as

\[
\frac{dW_{i-l}}{dw_i} = \frac{A e^j_{i-l} + s \frac{dW^j_0}{dw_i}}{r + s + A e^j_{i-l}(1 - F_{i-l})} \sum_{k=i-l+1}^{i} f_i^j \frac{dW^j_{i-k}}{dw_i}
\]  

(7)

An increase in \( w_i \) reduces search effort at \( w_i \), and increases search effort at lower wages. From (3) it follows that

\[
\frac{de^j_{i-l}}{dw_i} = A \frac{dW^j_{i-l}}{dw_i} / c''(e^j_{i-l})
\]

(8)

**Remark 1** \( \frac{d\Delta W^j_{i-l}}{dw_i} > 0 \) for all \( l \geq 1 \).

**Proof.** We know that this is true for \( l = 1 \). Suppose it is true for all \( l < l' \). Suppose it is not true for \( l' \). Then we know from (2) that \( \Delta W^j_{l'} \) decreases, but that can only be true if \( W^j_{l'} \) increases, a contradiction. ■

**Remark 2** The higher is the worker type, the higher is the search intensity at a given current wage level \( w_i \).

**Proof.** Consider worker types \( j \) and \( j + 1 \). Since \( F_{j+1} \) stochastically dominates \( F_j \). Hence for all \( i \), \( W^j_i < W^{j+1}_i \). From equation (2) it follows that \( \Delta W^{j+1}_i > \Delta W^j_i \).

**Remark 3** Note that \( \Delta W^j_i \) is independent of \( W^j_0 \). Define \( \tilde{W}^j_i \) by (2), but with 0 substituted in for \( W^j_0 \). It follows that \( W^j_i = \tilde{W}^j_i + \frac{r}{r+s} W^j_0 \). This simplifies the derivatives, as it is sufficient to calculate the derivative of \( \tilde{W}^j_i \) and then add the derivative of \( W^j_0 \) at the end. In particular it follows that we can write \( rW^j_0 = w_0 + A e^j_i(\tilde{W}^j_i + \frac{r}{r+s} W^j_0) \). This simplifies the calculation of the derivative of \( \tilde{W}^j_i \).

Finally, consider \( M^j_i \). Note that a change in \( w_i \) does not affect \( M^j_i \) directly, only through its effect on \( e^j_i \). From (4) we have that

\[
(r + s)M^j_i = y_i + e^j_i A \Delta M^j_i + s M^j_0 - c(e^j_i) = y_i + e^j_i A \Delta W^j_i - c(e^j_i) + e^j_i A \Delta (M^j_i - W^j_i) + s M^j_0
\]

(9)

\[5\] Recall that, due to the envelope theorem, marginal changes in \( e^j_i \) has no effects on \( W^j_i \). Note the similarity with the literature on sufficient statistics for welfare analysis, see Chetty (2009).
From the envelope theorem it follows that \( \frac{dM_j^i}{dw_i} \) is given by

\[
(r + s) \frac{dM_j^i}{dw_i} = \frac{de_j^i}{dw_i} A \Delta(M_i - W_i) + s \frac{dM_j^0}{dw_i}
\]  

(10)

For \( k > i \), it follows that

\[
\frac{dM_k}{dw_i} = \frac{s}{r + s} \frac{dM_0}{dw_i}
\]  

(11)

Further down the job ladder, it follows that

\[
(r + s) \frac{dM_{i-l}}{dw_i} = A_{i-l} e_{i-l}^j \sum_{k=i-l+1}^i \frac{dM_{i-k}}{dw_i} f_i^j + \frac{de_{i-l}^j}{dw_i} \Delta(M_{i-l}^i - W_{i-l}^i) + s \frac{dM_j^0}{dw_i}
\]  

(12)

A change in \( w_i \) does not directly influence \( M_i^j \), only indirectly through \( e_i^j \). A change in \( e_i^j \) influences both the return to search and the cost of search. Taking the derivative of (9), using the first order condition for the worker’s choice of \( e_i^j \) (and include the effect of unemployment) gives (10).

The effects of a change in \( w_i \) on \( M_{i-l}^j \) are first that effects of changes in the values \( M_{i-l+1}^j, \ldots M_i^j \) this is reflected in the first term. Second, a change in \( w_i \) also influences the search effort, which is captured by the second term. The last term captures the effects through \( M_0^j \).

Finally, note that the equivalent of Remark 3 applies for \( M_j^i \).

**Remark 4**. Suppose taxes \( (y_i - w_i) \) are constant (in dollar) at and above \( w_i \) and that \( s = 0 \). Then \( \Delta(M_i^j - W_i^j) = 0 \), and hence \( \frac{dM_j^i}{dw_i} = 0 \). If taxes are proportional, then \( \frac{dM_j^i}{dw_i} < 0 \).

### 3.2 The planner’s problem

Suppose the planner in steady state needs to raise an amount \( \bar{M} \) in net present value (NPV) income.\(^6\) The planner’s problem then reads

\(^6\)Equivalently, suppose the planner needs to raise a steady state flow revenue of \( r \bar{M} \).
\[
\max_{w_0, \ldots, w_n} \sum_{j=1}^{n} \tau_j \Phi(W^j_0)
\]

S.T.
\[
\sum_{j=1}^{n} \tau_j M^j_0 - \bar{M} \geq \sum_{j=1}^{n} \tau_j W^j_0
\]
\[
c'(e^j_i) = A\Delta W^h_i
\]
\[
w_i \leq w_{i+1}
\]

The first constraint says that the planner cannot use more than the total value of resources available. Note that as \(c(e)\) enters linearly in both \(W\) and \(M\), it cancels out. Hence the constraint requires that the income available must be equal to the total income generated. In what follows we assume that wages are increasing in types, and then check that they actually do in optimum afterwards. The Lagrangian reads

\[
L = \sum_{j=1}^{n} \tau_j \Phi(W^j_0) + \lambda \left( \sum_{j=1}^{n} \tau_j M^j_0 - \sum_{j=1}^{n} \tau_j W^j_0 - \bar{M} \right) - \sum_{i,j} \mu^j_i (A\Delta W^j_i - e^j_i),
\] (13)

where \(\lambda\) and \(\mu^j_i\) are the Lagrangian parameters associated with the constraints. The first order condition for \(w_i\) reads (where \(\frac{dM^j_i}{dw_i}\) is given by 12 and thus includes the effects of changes in effort levels)

\[
\sum_{j=1}^{n} \tau_j (\Phi'(W^j_0) - \lambda) \frac{dW^j_0}{dw_i} = -\lambda \sum_{j=1}^{n} \tau_j \frac{dM^j_0}{dw_i}.
\] (14)

The left-hand side is the welfare effect of increasing \(w_i\) over and above the shadow value of income. The right-hand side is the cost in terms of reduced search effort.

Intuitively, the value of \(\frac{dW^j_0}{dw_i}\) depends on how large fraction of the time (discount rate weighted) that a worker of type \(j\) spends in a job of type \(i\). The lower is the worker type, the more time the worker spends in unemployment, as inflow to unemployment
typically is independent of or decreasing in a worker’s type \( j \) type while outflow is higher for high-type workers with a higher search effort. This bias may be even stronger in low-type jobs. Low-type workers have a higher probability of meeting a low-type job, and hence may have a higher inflow rate to these jobs than high-type workers. At the same time the outflow rate is still higher for the high-type workers.

Let \( g_j = \Phi'(W_{j0}) \), \( W_i = \sum_{j=1}^{n} \tau_j \frac{dW_j}{dw} \), and \( M_i = \sum_{j=1}^{n} \tau_j \frac{dM_j}{dw} \), and finally let \( \omega_i^j = \frac{W_{j0}}{dW_i} / \dot{W}_i \). Then we can rewrite (14) as

\[
\sum_{j=1}^{n} \tau_j (g_j - \lambda) \omega_i^j = -\lambda \frac{\dot{M}_i}{W_i} \tag{15}
\]

From the planner’s perspective, increasing \( w_i \) has two effects: a distributional effect and an incentive effect. The left-hand side of (15) reflects the distributional effect, it shows how an NPV-dollar used on increasing \( w_i \) is distributed on the different worker types. A high correlation between the welfare weights \( g_i \) and the distribution weights \( \omega_i^j \) implies a positive distributional effect of increasing \( w_i \). The right-hand side shows the efficiency loss of increasing \( w_i \), i.e., the effect on the total amount of available resources per npv dollar spent on increasing \( w_i^j \). As we have seen, the efficiency loss tends to be positive for low values of \( i \) and negative for high values of \( i \). The efficiency concerns thus puts a limit on how much the planner wants to redistribute.

At the lower end of the wage hierarchy, low-type workers are overrepresented. Hence for lower wages, \( \omega_i^j \) tend to be decreasing in worker type. Hence the correlation between \( g_j \) and \( \omega_i^j \) tends to be positive, and this calls for low (or negative) taxes at low wage levels. On the other hand, high wages /low taxes at the lower end of the wage distribution tends to reduce search effort, while higher wages at high wage levels tend to increase search effort. Efficiency considerations therefore set a limit on how much the planner wants to redistribute.

The unemployment state is somewhat special, as all agents start as unemployed, and all “restart” as unemployed after a negative employment shock. Hence it may well be that the fraction of high-types among the unemployed is higher than the fraction among workers in the low-paid jobs. In addition, increasing the income of unemployed workers clearly has a negative incentive effects for unemployed workers. Hence it is not \textit{a priori} clear that the planner will set the unemployment benefit particularly high, it may be more efficient to obtain redistribution by subsidizing low-wage jobs. This will
be explored numerically below.

\textbf{Remark 5} Suppose the planner has linear preferences, so that the welfare weights $\omega^j_i$ are equal for all $j$ independently of $W^j_i$. Then the planner will set $y_j - w_j = M/r$ for all $j$. With such a tax policy, the planner will not distort the workers’ search effort, and efficiency is obtained. This is equal to a poll tax.

4 Including amenities

Suppose now that a job comes with two attributes, productivity $y_i$ and other qualities, which we denote by $z$ and refer to as amenities. Amenities cannot be observed by the planner, and hence cannot be taxed. The utility flow of a job is thus the sum of wages net of taxes and amenities. The joint distribution of amenities and productivity types for a person of type $j$ can be written as $F^j(i, z)$. Let $F^j_i$ denote the marginal probability that a worker of type $j$ draws a job of productivity type (or just type) $i$ or lower. Let $f^j_i$ denote the probability that $y = y_i$. Let $F^j(z|i)$ denote the conditional distribution of $z$ given $i$. In order to simplify the analysis we assume that the draws of productivities and amenities are independent. This means that we can write the distribution of $z$ as $G(z)$. We will modify this assumption in future work.

Above, we required that that taxes will not influence the workers’ ranking of jobs. This is consistent with the assumption that marginal taxes are less than 100 percent. However, with amenities the tax system will influence the ranking of jobs, and lead to new inefficiencies as the agents in the presence of high marginal taxes may turn down job offers with a high pre-tax wage that scores low on amenities. We still require that taxes do not alter the ordering of the pecuniary returns of jobs.

The utility flow of the current job is equal to the sum $w_i + z$. The worker will accept a new job if and only if the characteristics of the new job ($i', z'$) satisfies $w_{i'} + z' > w_i + z$. Hence $w_i + z$ is a sufficient statistics for the npv value of the job.

Define $\tilde{z}_{i,k}(z) = w_i + z - w_k$. A worker in a job with characteristics $(w_i, z)$ accepts a new job offer at wage level $w_k$ if and only if the level of amenities $z'$ in the new job satisfies $z' > \tilde{z}_{i,k}(z)$. The NPV income of a worker of type $j$ in a job with characteristics $(w_i, z)$ can thus be written as

$$ (r + s)W^j_i(z) = w_i + z + A\epsilon^j_i \Delta W^j_i(z) + sW_0^j - c(e^j_i) \quad (16) $$
where
\[ \Delta W^j_i(z) = \sum_{k=1}^{n} f^j_k \int_{\tilde{z}_{i,k}(z)} (W^j_k(\tilde{z}) - W^j_i(z))g(\tilde{z})d\tilde{z} \]  
(17)

As above we have that
\[ c'(e) = A\Delta W^j_i(z) \]  
(18)

Let \( \hat{F}^j_i(z) \) denote the probability that a worker of type \( j \) in a job at level \( i \) with amenities \( z \) does not accept a job offer when it arrives. It follows that \( \hat{F}^j_i(z) = \sum_{k=1}^{n} f^j_k G(\tilde{z}_{i,k}(z)) \).

From the envelope it follows that
\[ W^j_i(z) = \frac{1}{r + s + Ae^j(1 - \hat{F}^j_i(z))} \]  
(19)

**Lemma 2** for a given distribution \( F^j_i \) with finite support, the equilibrium exists.

**Proof.** Let us sketch the proof. The value functions \( W^j_i(z) \) are real functions defined on a closed subset of \( \mathbb{R}^{n+1} \). Let \( D \) denote the set of continuous functions defined on that subset and bounded above by \( (y_n + z^{\text{max}})/r \). We know that \( D \) is complete under the sup norm. Equation (16)-(18) define a mapping \( \Gamma : D \to D \). It follows trivially that \( \Gamma \) is continuous, and increasing. Let \( \bar{e} < \infty \) denote the supremum of \( e \). Since the set of value functions is bounded it follows that \( \bar{e} \) is bounded. We want to show that Blackwell’s sufficient condition holds. Let \( \delta \) be a strictly positive constant. For any \( W \in D \) it follows that \( \Gamma(W + \delta) \leq \Gamma W + \frac{r}{r+\bar{e}A} \delta = W + \beta \delta \) with \( \beta < 1 \). Hence the Blackwell sufficient condition is satisfied, and \( \Gamma \) is a contraction mapping. From the contraction mapping theorem it follows that \( \Gamma \) has a unique fix-point.  

Let us consider the effect on \( W^j_i(z) \) of a change in \( w_i \). Without amenities, this would only influence the NPV values in firms with wage at or below \( w_i \) (in addition to the effects through \( W_0 \)). With amenities this is no longer the case, as workers may accept jobs with lower wages. As the envelope theorem still applies, the effect on effort level and acceptance decisions do not have first order effects on \( W^j \) and can be ignored. Hence

\[ (r + s)\frac{dW^j_i(z)}{dw_i} = 1 + \sum_{k=1}^{n} f^j_k \int_{\tilde{z}_{i,k}(z)} \left( \frac{dW^j_k(\tilde{z})}{dw_i} - \frac{dW^j_i(z)}{dw_i} \right)g(\tilde{z})d\tilde{z} + s \frac{dW^j_0}{dw_i} \]

The first term on the rhs is the direct effect of an increase in the wage. The second term is the indirect effect, showing the effect on asset values after a job switch, which
is influenced by a change in \( w_i \) as the worker may switch back to a job at level \( w_i \) in the future. Note that \( \frac{dW^j_i}{dw_i} \neq W^j_i'(z) \), as the latter does not take into account that the value of staying in a job of type \( i \) will be higher if the worker is rehired at a job on this level in the future.

**Remark 6** In the limit, as the number of job types go to infinity and \( f_j^i \rightarrow 0 \ \forall \ k \), the probability that a worker returns to a job of type \( i \) if accepting a better job offer goes to zero. In this case \( \frac{dW^j_i}{dw_i} = W^j_i'(z) \) given by (19). By defining \( e \) as a function of \( w + z \), taking the derivative of \( e \) and inserting for \( \frac{dW^j_i}{dw_i} \) gives a second order differential equation in \( e \) that can be solved, at least numerically.

The effect of an increase in \( w_i \) on workers hired in other job types read

\[
(r + s) \frac{dW^j_i(z)}{dw_i} = \sum_{k=1}^{n} f^j_k \int_{\tilde{z}_{i,k}(z)} \left( \frac{dW^j_k(\tilde{z})}{dw_i} - \frac{dW^j_i((\tilde{z}))}{dw_i} \right) g(\tilde{z}) d\tilde{z} + s \frac{dW^j_0}{dw_i}
\]

The expression is more complicated than one may first expect, and this is due to the endogenous search effort, which varies between states. As an example, suppose \( i \) is a high-type job and \( w_i + z \) is low. The search effort initially is is high, and hence an increase in \( w_i \) has a relatively high impact on \( W^j_i(z) \). If the worker transfers to a job of type \( k \neq i \) with a higher \( w_k + z \), the search effort falls, and the effect of an increase in \( w_i \) is smaller. This feeds back into \( \frac{dW^j_i(z)}{dw_i} \). Note also that in this case, we cannot abstract from the third term when the number of types increase. True, the probability of moving to state \( i \) goes to zero, but that is also true for \( \frac{dW^j_i(z)}{dw_i} \). Relatively speaking, the third term does not vanish.

The total income a worker earns gross of taxes can be written as

\[
rM^j_i(z) = y_i + z + Ac^j_i(z)\Delta^j M^j(z + y_i) - c(e^j(z)) \tag{20}
\]

where \( \Delta^j M^j_i(z) \) is defined analogous with \( \Delta^j W^j_i(z) \), i.e.,

\[
\Delta M^j_i(z) = \sum_{k=1}^{n} f^j_k \int_{\tilde{z}_{i,k}(z)} \left( M^j_k(\tilde{z}) - M^j_i(z) \right) g(\tilde{z}) d\tilde{z}
\]

Note that the threshold \( \tilde{z}_{i,k}(z) \) is determined so as to maximize \( W^j \), not \( M^j \).

Consider an increase in \( w_i \). Above this influenced \( M^j_i \) through its effect on search effort of workers at ring \( i \) and below. Now it may potentially influence search intensity
in jobs above ring $i$ as well. In addition we get a new effect through the workers’ ranking of alternatives. Taxes drive a wedge between productivity and wage, and this may distort the worker’s job acceptance decision. If taxes (in levels) are increasing with the wage – as optimal taxes typically prescribe – a worker will put less weight on gross wages (i.e. productivity) and more on amenities than the planner will. Hence a worker will have a tendency to accept too few jobs that offer a higher wage than his current wage, and too many jobs that offer a lower wage.

Formally, the derivative of (20) reads (after adding and subtracting $W_i$ as above), see equation (9)

$$
(r + s) \frac{dM_i^j(z)}{dw_i} = \frac{dce_i^j(z)}{dw_i} A\Delta(M_i - W_i) + Ac_i^j(z) \sum_{k=1}^{n} f_k g(\tilde{z}_{i,k}(z)) (t_i - t_k) \\
+ Ac_i^j(z) \sum_{k=1}^{n} \int_{\tilde{z}_{i,k}(z)} \left( \frac{dM_k^j(\tilde{z})}{dw_i} - \frac{dM_i^j(\tilde{z})}{dw_i} \right) g(\tilde{z}) d\tilde{z} \\
+ s \frac{dM_0^j}{dw_i}
$$

(21)

The first third and fourth terms are as above, the new term is the second term. It shows the effect of a wage change on the acceptance decision of job offers. An increase in $w_i$ decreases the acceptance rate of other job offers, and this will give a gain/loss that depends on the difference in tax rate. This difference can be written as $\sum_{k=1}^{n} f_k g(\tilde{z}_{i,k}(z)) (M_k^j(\tilde{z}_{i,k}(z)) - M_i^j(z))$. Note that $(r + s)M_i^j(z) = w_i + t_i + \Delta^j M_i^j(z + y_i) - c(e_i^j(z))$. Since we are looking for the marginal switch, the NPV income and search and acceptance behaviour is the same before and after the switch. It follows that the flow change in value created is the difference in taxes, and that the flow difference ends at rate $Ac_i^j(z)(1 - \bar{F}_i^j(z))$. The third term reflects the impact on the value of future jobs, which again reflects that the worker may return to a job of type $i$. The final term shows the effect of an increase in $w_i$ on the value of being unemployed.

**Remark 7** *In the limit, as the number of job types go to infinity and $f_k^j \to 0 \ \forall \ k$, the probability that a worker returns to a job of type $i$ if accepting a better job offer goes to zero. In this case the third and fourth terms vanish.*

Finally, consider a change in $w_i$ on $M_i^j(z)$ the value of being in job $l \neq i$. The first

---

$^7$An alternative formulation of the total income is the following.: Let $T_i^j(z)$ denote the expected discounted future tax payments of a worker of type $j$ in a job of type $i$ with amenities $z$. It follows that $M_i^j(z) \equiv W_i^j(z) + T_i^j(z)$. With this formulation, the derivatives derived above follows easily.
thing to note is that an increase in \( w_i \) will not influence the choice between two jobs \( w_l, w_k, k, l \neq i \). It follows that

\[
(r + s) \frac{dM_i^j(z)}{dw_i} = \frac{dc_i(z)}{dw_i} A \Delta (M_l - W_l) + Ae_i(z) \frac{f_i(g(\tilde{z}_{il}(z))(t_i - t_l))}{r + s + Ae_i(z)(1 - F_i(z))} \\
+ \sum_{k=1}^{n} f_k^j \int \frac{dM_k^j(\tilde{z})}{dw_i} - \frac{dM_i^j(z)}{dw_i})g(\tilde{z})d\tilde{z} \\
+ s \frac{dM_0^j}{dw_i}
\]

Finally, consider the planner’s problem. Formally, the problem is equivalent to the planner’s maximization problem without amenities, with the old asset values replaced with the new ones. In particular, the trade-off between equity and incentives to search is still present. However, there are new elements in the trade-off related to the workers’ acceptance decision of firms.

With the possible exception of the state of being unemployed, taxes will typically be increasing in job types because of equity considerations. As a result, workers will have a tendency to accept too few high-wage job offers (with low amenities) and accept too many low-wage offers (with high amenities). This effect is particularly prevalent at the highest paid job, jobs for which unambiguously too few workers employed in other job types accept and too many workers quit from after successful on-the-job search. Hence the introduction of amenities increases the social cost of redistribution, as there is a new margin, acceptance rate of jobs, that taxes distort (in addition to search intensity).

5 Simulation of Model Without Amenities

In this section we show a simulation of the model without amenities.

5.1 Parameterization

The unit of time is a year. Following Christensen, Lentz, Mortensen, Neumann, and Werwatz (2005) we assume that the search cost function \( c(e) \) is given by

\[
c(e) = \frac{c_0}{1 + 1/c_1} e^{1+1/c_1}
\]

We assume that \( y_i \) is uniformly distributed on the interval \([y_l, y_h]\) with \( y_1 = y_l \) and
Each worker type $j$ draw from a worker specific offer distribution, $F^j$. This is approximated by a beta distribution with parameters $\alpha_j$ and $\beta_j$.

The social welfare function is given by

$$\Gamma = \sum_{j=1}^{J} \begin{cases} \tau_j \log(W^j_0) & \text{if } \gamma = 1 \\ \tau_j \frac{(W^j_0)^{1-\gamma}}{1-\gamma} & \text{if } \gamma \neq 1 \end{cases}$$

where $W^j_0$ is the value function for a type $j$ worker in unemployment. To make the illustration easier we set the number of worker types to 3 ($J = 3$) and the number of firm types to 10 ($n = 10$). Each type is equally likely so $\tau_j = 1/3$.

Table 1 shows the the parameter values. We set $s = 0.2$ such that the average employment length is 5 years. $A$ is set to hit an unemployment rate of around 10 in the no-tax case. We normalize $c_0 = 1$, since this cannot be separately identified from $A$. Following the results in Christensen, Lentz, Mortensen, Neumann, and Werwatz (2005) we set $c_1 = 1$ such that the search cost function is quadratic. We normalize $y_l = 1$ and set the highest level of productivity to be 5 times higher.
Table 1: Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_i = s$</td>
<td>Job destruction rate</td>
<td>0.2</td>
</tr>
<tr>
<td>$r$</td>
<td>Effective discount rate</td>
<td>0.05</td>
</tr>
<tr>
<td>$A$</td>
<td>Scale parameter in search technology</td>
<td>0.85</td>
</tr>
<tr>
<td>$\kappa_j = \kappa$</td>
<td>Worker type efficiency</td>
<td>1</td>
</tr>
<tr>
<td>$c_0$</td>
<td>Scale parameter in search cost function</td>
<td>1</td>
</tr>
<tr>
<td>$c_1$</td>
<td>Elasticity of search cost function</td>
<td>1</td>
</tr>
<tr>
<td>$y_l$</td>
<td>Lower bound on productivity distribution</td>
<td>1</td>
</tr>
<tr>
<td>$y_h$</td>
<td>Upper bound on productivity distribution</td>
<td>5</td>
</tr>
<tr>
<td>$UI_{fact}$</td>
<td>$y_0 = UI_{fact} \times y_l$</td>
<td>0.5</td>
</tr>
<tr>
<td>$\bar{M}$</td>
<td>Amount to raise</td>
<td>(0.3 \times \sum_{j=1}^{J} \tau_j \sum_{i=1}^{n} g_j^i M_i^j)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Constant relative risk aversion</td>
<td>1</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>Worker type 1, Offer Distr., Alpha Parameter</td>
<td>0.75</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>Worker type 2, Offer Distr., Alpha Parameter</td>
<td>0.85</td>
</tr>
<tr>
<td>$\alpha_3$</td>
<td>Worker type 3, Offer Distr., Alpha Parameter</td>
<td>0.95</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>Worker type 1, Offer Distr., Beta Parameter</td>
<td>5</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>Worker type 2, Offer Distr., Beta Parameter</td>
<td>4</td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>Worker type 3, Offer Distr., Beta Parameter</td>
<td>3</td>
</tr>
</tbody>
</table>

$\bar{M}$ is set such that the government set taxes to raise 30 of the output in the no-tax equilibrium.

We are going to refer to workers of type 1, 2, and 3 as low, medium, and high, respectively, since $F_3 \leq F_2 \leq F_1$.

5.2 Results

In this section we compare four different equilibria; 1) one without any taxation, 2) one with a poll-tax, 3) one with proportional taxation, and 4) one with optimal taxation. In the first equilibrium $\bar{M} = 0$, since there is no taxation to finance it, while in the last three cases we set $\bar{M} > 0$ and to the same amount such that in each of the tax schemes the government collects the same revenue.

Table 2 shows some key numbers for the different equilibria. We can see that the unemployment rate differs across worker types with workers of type 1 being most
Table 2: Key Equilibrium Objects

<table>
<thead>
<tr>
<th></th>
<th>Worker Type 1</th>
<th>Worker Type 2</th>
<th>Worker Type 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Tax ($M = 0$)</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>- Unemployment Rate</td>
<td>0.129</td>
<td>0.119</td>
<td>0.108</td>
</tr>
<tr>
<td>- Average Before Tax Income</td>
<td>2.252</td>
<td>2.590</td>
<td>3.002</td>
</tr>
<tr>
<td>- Average Net of Tax Income</td>
<td>2.252</td>
<td>2.590</td>
<td>3.002</td>
</tr>
<tr>
<td>- $W_0^j$</td>
<td>35.134</td>
<td>40.513</td>
<td>47.457</td>
</tr>
<tr>
<td>- $\Omega$</td>
<td>3.707</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Poll Tax ($M &gt; 0$)</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>- Unemployment Rate</td>
<td>0.129</td>
<td>0.119</td>
<td>0.108</td>
</tr>
<tr>
<td>- Average Before Tax Income</td>
<td>2.252</td>
<td>2.590</td>
<td>3.002</td>
</tr>
<tr>
<td>- Average Net of Tax Income</td>
<td>1.636</td>
<td>1.974</td>
<td>2.387</td>
</tr>
<tr>
<td>- $W_0^j$</td>
<td>22.824</td>
<td>28.202</td>
<td>35.147</td>
</tr>
<tr>
<td>- $\Omega$</td>
<td>3.342</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Proportional Tax ($M &gt; 0$)</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>- Unemployment Rate</td>
<td>0.153</td>
<td>0.141</td>
<td>0.128</td>
</tr>
<tr>
<td>- Average Before Tax Income</td>
<td>2.135</td>
<td>2.460</td>
<td>2.864</td>
</tr>
<tr>
<td>- Average Net of Tax Income</td>
<td>1.576</td>
<td>1.816</td>
<td>2.114</td>
</tr>
<tr>
<td>- $W_0^j$</td>
<td>24.259</td>
<td>27.979</td>
<td>32.856</td>
</tr>
<tr>
<td>- $\Omega$</td>
<td>3.337</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Optimal Tax ($M &gt; 0$)</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>- Unemployment Rate</td>
<td>0.132</td>
<td>0.123</td>
<td>0.113</td>
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<tr>
<td>- Average Before Tax Income</td>
<td>2.221</td>
<td>2.544</td>
<td>2.934</td>
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<tr>
<td>- Average Net of Tax Income</td>
<td>1.647</td>
<td>1.925</td>
<td>2.242</td>
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<tr>
<td>- $W_0^j$</td>
<td>23.806</td>
<td>28.317</td>
<td>33.850</td>
</tr>
<tr>
<td>- $\Omega$</td>
<td>3.345</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

unemployed. However, the unemployment rates do not differ that much, since all workers have similar job destruction rates. Imposing a poll-tax does not distort choices, but average net of tax income decreases and thus the value of unemployment decreases. Proportional taxation clearly distorts search choices as illustrated by the increase in unemployment and the decrease in average net of tax income. The optimal tax schedule decrease unemployment and increase net of tax income compared to the proportional tax scheme. Finally, Table 2 shows $\Omega$, the welfare as evaluated by the planner. We note that the welfare ranking is as expected. The proportional tax regime delivers the lowest welfare, with the poll tax regime begin second. The optimal tax regime delivers the highest welfare by balancing equity and efficiency concerns at the margin.

In figure 1, we show the output, which is equal to the wage income before taxation, of each state. Output is uniformly distributed across firms and we imposed a UIB
level of 50 percent of the lowest output. We also show the net of tax income. In the poll-tax case all workers pay the same and unemployed workers actually receives negative payments. In the proportional tax case all workers pay 26 percent of their income. Finally, in the optimal tax case workers in all states pay taxes with unemployed worker paying all their income. Our simulations are set up such that all worker types face (almost) the same unemployment risk as shown in Tables 1 and 2. Thus, taxing workers in this state is a relatively efficient way of collecting taxes while adding more incentives to search for higher paying jobs that are also taxed. This effect may be watered down or vanish altogether depending on the precise specification of the job destruction process.

![Figure 1: Gross and Net Income](image)

Figure 1: Gross and Net Income

Figure 8 shows tax rate on the right side y-axis. This is just calculated as the percentage difference between the net of tax income and the before tax income as
shown in figure 1. The bars show the distribution of worker types conditional on the state of the worker. The state conditional distributions are quite stable across the different tax regimes. Note, however, as we shall see further below, this does not imply taxation has no impact on the allocation of labor. However, we can conclude that the response to the different tax regimes is similar across worker-types. For the optimal tax scheme, the tax is decreasing up until the 80th percentile of firms. After the 80th percentile the tax rate start to increase. The reason is that in those firms around 50-65 percent of the workers are of the highest type. Thus, taxing workers in this state is a very good way of directing taxation at the highest type.

In figure 3 we show the CDF’s of the offer ($F$) and steady state ($G$) distributions for each taxation scheme. The difference between the offer and steady state distributions is largest for the high worker type. This reflects that the incentives to search for a better job is highest for this type, which we show in figure 4. It is clear that taxation only influences search intensity in the proportional case, whereas it does not change search intensity that much in the optimal tax case. This is a clear indication that designing the tax scheme is important in order to minimize the cost.
This point is further illustrated by Figure 4 which shows the search intensities for every possible worker-firm type combination (including unemployment) and for each of the four tax regimes we consider. We note that the proportional tax regime depresses search for every worker types at all rungs at the job ladder relative to the nondistorted search choices in panels (a) and (b). Under the optimal tax regime, workers search harder at the bottom and middle rungs, but less at the higher rungs, reflecting the progressivity of the optimal tax system for very high wages that allow the planner to target high-type workers.
Figure 5 illustrates how different tax regimes impact the allocation of labor. Figures 5a, 5b, and 5c show the steady state distribution of type 1 (low), type 2 (medium) and type 3 (high) workers across firm types, i.e., across the job ladder. We note that for every worker type, the labor allocation under the optimal tax regime strictly and clearly dominates the allocation under a (revenue neutral) proportional tax. The allocation under the non-distorted poll-tax regime of course dominates the allocation under the optimal tax regime.
Finally, in figure 6, we show how the optimal tax schedule depends on the coefficient of relative risk-aversion in the welfare function. Recall that $\gamma = 1$ was the baseline value.
5.3 Robustness Checks

In this section we show results for the optimal taxation if $\bar{M} = 0$ and $UI_{fact} = 0.95$.

First, we set $\bar{M} = 0$ in order to see how this affects optimal policy. This corresponds to the case, where all tax-income is redistributed through transfers. The state conditional tax rate is decreasing from unemployment as it was previously, but the drop is much larger than in the baseline case with $\bar{M} > 0$. The reason is that redistributing taxes to workers in low firm types is now the most efficient way of allocating resources to the low worker type.
We now set $UI_{frac} = 0.95$ instead of 0.5. This does not change that much, which is only natural since all the income was taxed away for the unemployed in the baseline case. This is still optimal to do, since it is a good way of taxing the high type workers without distorting search incentives.

6 The way forward

Above we have simulated the model without amenities. Simulating the model with amenities is a relatively straightforward extension.

In a separate note [Bagger, Moen, and Vejlin, 2017] we extend the model and allow for directed search and entry of firms as in [Moen, 1997] and [Garibaldi and Moen, 2016] in order to incorporate the effect of taxes on wage formation.

We may also extend the model by assuming that the search costs are partly deductible for tax purposes. In this case, the tax system both reduces the costs and the
gains from search, and the incentives to search will depend on the slope of the marginal tax rates.

Our ultimate goal is to structurally estimate the model using a Danish employer-employee data set. We will use a similar data source as in Bagger and Lentz (2018). Information regarding job-to-job transitions and productivity differences and wage differences between firms give information regarding the economic importance of job-to-job transitions, the individual costs associated with search (broadly defined) and job-to-job transitions, and hence the sensitivity of the job-to-job transitions on net wage differences. This will make us able to gauge the economic impact of taxation on job-to-job transitions and hence to taxation and the dead-weight loss of taxes.
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