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Abstract

Within a standard model of international trade with heterogeneous firms and two asymmetric countries, we derive sufficient conditions for monotone comparative statics (MCS) for the industry composition. This model outcome is defined as first-order stochastic dominance shifts in the equilibrium distributions of all activities across active firms. MCS for the industry composition occurs in a country which experiences a decline in its costs of serving the foreign market and meanwhile experiences an increase in its level of competition. In the other country, the industry-level implications are exactly opposite. These clear industry-level results hold while firms respond asymmetrically to the trade shock.

Keywords: Complementary Activities; Firm Heterogeneity; Firm-Size Distribution; Exporting; Asymmetries

JEL Classifications: D21; F12; F61; L11

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1 Introduction

In this paper, we analyse the responses of firms and industries to exogenous industry-wide trade shocks within a heterogeneous-firms model of international trade with monopolistic competition à la Melitz (2003) and Melitz and Redding (2014). Only the most productive firms serve the foreign market and are exposed to the demand levels in each of the two asymmetric countries. A demand level comprises an inverse measure of the level of competition in the industry.[1] Within this standard and general model of international trade, we derive sufficient conditions for a model outcome dubbed monotone comparative statics (MCS) for the industry composition. This model outcome is defined as first-order stochastic dominance (FSD) shifts in the equilibrium distributions of all activities across active firms. An activity refers to any choice variable at the discretion of the firm subject to the constraint that activities are complementary at the firm level.

The possibility of MCS for the industry composition relies on a class of productivity distributions that contains the commonly used Pareto distribution, namely the class of productivity distributions where log-productivity is distributed with nonincreasing hazard rate. When log-productivity is distributed with nonincreasing hazard rate, it is shown that MCS for the industry composition occurs in a model country which experiences both a decline in its costs of serving the foreign market and an induced decline in its own demand level. An induced increase in the level of competition is likely to occur after the unilateral trade shock. We also present two useful examples where it is certain to occur. In the nonliberalising country, the industry-level implications are shown to be exactly opposite. Hence, asymmetrical and unilateral trade liberalisations are likely to imply a model outcome that we name asymmetric MCS for the industry compositions when productivities are distributed for instance Pareto. Importantly, these monotone and strong industry-level implications hold while firms in a given country respond asymmetrically and nonmonotonely to the trade shocks.

One can get a flavour of the meaning of these results by looking at the benchmark model by Melitz and Redding (2014) with two asymmetrical countries. This example is nested in the model below under the common and convenient assumption of an outside industry large enough to determine

[1] Under CES preferences, the demand levels are functions of total industry spending and the price index of the industry. They are thus demand shifters.

the wage rate. The heterogeneous firms in this example face two complementary activities: labour demand for variable production and export behaviour. Within this example, the present paper shows that a unilateral trade liberalisation (through a decrease in the fixed or variable trade costs of serving the foreign market) implies MCS for the industry composition in the liberalising country if and only if log-productivity is distributed with nonincreasing hazard rate.[2] MCS for the industry composition means that the firm-size distribution makes a FSD shift to the right and that the fraction of exporters increases in the liberalising country. In the nonliberalising country, the firm-size distribution makes a shift to the left while the fraction of exporters decreases. Note that the industry-level implications are exactly opposite for the nonliberalising country implying that we obtain asymmetric MCS for the industry compositions.

Let us emphasise three important points related to the specific example above. First, the added structure obtained by analysing the specific trade model in Melitz and Redding (2014) implies that asymmetric MCS for the industry compositions now certainly occurs when log-productivity is distributed with nonincreasing hazard rate. This is because the level of competition certainly increases in the liberalising country as we argue by using parts of the analysis in Demidova and Rodriguez-Clare (2013). Second, the added structure in the example also implies that log-productivity being distributed with nonincreasing hazard rate is not only sufficient but now also necessary for obtaining asymmetric MCS for the industry compositions. This implies that the industry-level results mentioned in the example certainly do not hold when productivity is distributed Frechet or log-normally. The intuition is as follows. When log-productivity is distributed with nonincreasing hazard rate, we are assured a sufficient mass of exiting and entering low-productivity firms (with low activity levels) in the liberalising and nonliberalising countries, respectively. This assures asymmetric MCS for the industry compositions. Third, the models presented by Melitz (2003) and Melitz and Redding (2014) effectively work as a backbone in many related trade models. This implies that the results of the example above also hold for other specific trade models with complementary activities such as the model of Bustos (2011) when the liberalising country is a small open economy and wages

[2]Note that log-productivity is distributed with constant hazard rate when productivity is distributed Pareto.

are determined through an outside industry.[3] This is because the activities labour demand for variable production, exporting behaviour, and technology upgrading are complementary in the Bustos (2011) model. Furthermore, the level of competition certainly increases in the small open liberalising country, as argued below, and productivities are Pareto distributed in Bustos (2011).

The approach of the present paper reveals how one can often obtain clear-cut industry-level results during comparative statics in specific and nested trade models without having to fully solve these general-equilibrium models. The presented technique works even when (the potentially discrete) shocks and countries are asymmetric, as is most often the case, and when the model setup is flexible enough to encompass several recent and prominent trade models. The apparent complexity is handled by applying the monotonicity theorem of Topkis (1978). This monotonicity theorem, known from operations research and the work of Milgrom and Roberts (1990), does a lot of the hard work for us when it is combined with some admittedly quite simple decompositions of the comparative static effects. Interestingly, the monotonicity theorem allows for a discontinuous (and hence nondifferentiable) profit function. This is convenient when one is interested in activities such as the extensive margin export behaviour which is often modelled through a zero-one indicator function for active exporting. Further, the allowance for discontinuous profits implies that the analysis of introducing new complementary activities is straightforward. One case in point is a unilateral move from autarky to costly trade in the example above. The analysis below shows that such a move can be analysed in the exact same way as a unilateral decrease in the fixed or variable trade costs of serving the foreign market.

A large strand of recent literature within the field of international trade has recently exploited various common kinds of firm-level complementarities. We strongly relate to this literature by assuming that the activities faced by firms are complementary with: (i) each other; (ii) firm productivity; (iii) the demand level; (iv) the foreign demand level; and (v) the exogenous industry-wide trade costs parameters that we vary during the comparative statics. As argued below, this assumption very often holds in recent models of international trade. This is one reason why our approach can be used to analyse many different trade models.

The present paper should be perceived as an extension of the work by Bache and Laugesen (2014) to the more plausible case of asymmetric coun-

[3] The same can for instance be said about the model by Arkolakis (2010).

tries and asymmetric shocks which cannot be analysed in the model of Bache and Laugesen (2014) when some firms serve the foreign market. One cost of allowing for country and shock asymmetry is that wages will be determined through a perfectly competitive outside industry. One significant benefit of this extension is that this extension may better facilitate empirical analysis as we argue in Section 4. Mrazova and Neary (2013) reveal how complementarity between firm-level productivity and various activities play a key role in shaping the sorting pattern of firms in some well-known and recent models of international trade. Given that the present paper conducts comparative statics across equilibria in stead of emphasising sorting in a given equilibrium, the scope of the present paper is quite different from the scope of the paper by Mrazova and Neary (2013). The mathematics of complementarity has also been applied in the recent offshoring model of Antràs et al. (2014). Their model can be nested within our model below under the natural extension of their baseline model to nonprohibitive costs of trading final goods across countries.[4] Antràs et al. (2014) analyse the problem of heterogeneous firms determining the extensive margin offshoring and exporting strategies in a multi-country setup. The analytical comparative statics in Antràs et al. (2014) treat the levels of competition in the various countries as exogenous during comparative statics such as trade liberalisations. The results of the present paper reveal how strong industry-level results can be obtained when one allows for endogenous levels of competition.[5]

2 Model

Firms enter a monopolistically competitive industry after paying an entry cost of f_e units of labour which is the sole factor of production. Firms which enter are characterised by their productivity level, $\theta \in [\theta_0, \infty)$, realised only upon entry. θ is a realisation of the continuous random variable Θ with c.d.f. $F(\Theta)$ and lower bound $\theta_0 \geq 0$. It is assumed that $F(\Theta)$ is strictly increasing and C^1. Firm profits, π, are also assumed to be strictly increasing and continuous in θ.[6] We focus attention on a two-country model where the

[4] This extension is treated in their Section 3.5. An example based on Antràs et al. (2014) is available upon request to the author.

[5] An example based on Antràs et al. (2014) is available upon request to the author.

[6] The different nature of our approach to deriving comparative statics involves placing assumptions on endogenous objects such as firm profits. While these assumptions are clearly unorthodox, they are very likely to be satisfied in specific and recent trade models.

two wage rates (w in the home country, Home, and w_f in the foreign country, Foreign) are determined through a freely traded homogeneous outside good produced under perfect competition and constant returns to scale in both countries.

Based upon the realisation of θ, a firm decides whether to become an active producer or to exit the industry. This decision will be analysed in Section 2.2. The present section analyses firm behaviour conditional on active production. It is assumed that all firms face a decision, $x = (x_1, \ldots, x_n)$, where x_i denotes the chosen level of activity i, a choice variable. An activity refers to any variable at the discretion of the firm subject to the constraint that activities must satisfy the below Assumption 2 about four types of complementarity at the firm level. The level of an activity can be either a discrete or a continuous choice variable. We assume that $x \in X$ where $X \subseteq \mathbb{R}^n$ is the set of all decisions conceivable by firms. The set X is assumed to be a lattice. The actual choice set of firms is restricted to a subset of available decisions, $S \subseteq X$, with S being a sublattice of X. This assumption will allow us to analyse the effects of increases (under the strong set order) in the choice set S. An increase in S will exclusively be used to analyse the effects of introducing new international trading activities that are complementary to the other activities in x.

To paraphrase Milgrom and Roberts (1995), constraining the decision x to lie in a sublattice S imposes a kind of technical complementarity. To see this, recall that the sublattice constraint implies that undertaking a higher level of any activity may require but, importantly, cannot prevent undertaking a higher level of another activity. Similarly, undertaking a lower level of any activity may require but cannot prevent undertaking a lower level of another activity.[7] To provide an example, a sublattice constraint could for instance be used to model the idea that a quality (or technology) upgrade never prevents exporting and may be necessary for exporting to take place. Restricting attention to lattices and sublattices will allow complementarities between the activities in x to take effect when we later impose the before-mentioned Assumption 2 on the objective function. In models of heterogeneous firms, the lattice assumption is very often satisfied. Moreover, the lattice assumption opens the door for employing the powerful monotonicity theorem of Topkis (1978).[8] The profitability of the decisions in X and S

[7]Under the standard component-wise order, any subset of $\mathbb{R}^2$ is a sublattice if and only if its boundaries are never downward-sloping.

[8]For nice introductions to supermodular optimisation techniques and the mathematics

is influenced by a vector of exogenous industry-wide trade cost parameters, $\beta \in B$, with $B \subseteq \mathbb{R}^m$. We let β contain minus the fixed and minus the variable costs of undertaking international trade from the view point of firms in Home. Note that an increase in β amounts to an asymmetric (or unilateral) trade liberalisation.

All assumptions throughout also hold in Foreign where parameters, vectors, and sets will be denoted by $f_{ef}, \theta_{0f}, n_f, \beta_f, S_f$, and where the distribution of productivities is denoted by $F_f(\Theta)$. Notice that foreign parameters, vectors, sets, and functions appear with the subscript f and that our exposition allows for potential country asymmetry ex ante. For simplicity, foreign trade cost parameters, β_f, and the foreign choice set, S_f, are held constant throughout. The exposition below focuses on the monopolistically competitive industry in Home. The implications for the foreign industry composition are explained in Section 3.5.

2.1 Profits, Complementarities, and the Optimal Decision

Firm profits are strictly increasing in a common and endogenous aggregate statistic which captures the inverse level of competition in the industry. We will refer to this variable, $A \in \mathbb{R}_+$, as the demand level. In general, firm profits also depend on the foreign demand level, $A_f \in \mathbb{R}_+$. Firm profits are weakly increasing in this latter variable and strictly increasing for firms serving the consumers in Foreign.[9] It is assumed that some firms serve the consumers in Foreign but let us for simplicity abstract from horisontal FDI as a type of foreign market access.[10] When we derive the comparative static effects of an increase in (β, S), the implied effects on the levels of competition are captured by the endogenous changes in A and A_f.

Under these assumptions, firm profits, π, depend on the decision, x, productivity, θ, the demand level, A, the foreign demand level, A_f, the industry parameters, β, and the choice set, S. We make the following key assumption.

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of complementarity, see Topkis (1995) and Milgrom and Roberts (1995). A brief primer is also provided in Appendix A in Bache and Laugesen (2014).

[9] Under the standard assumption of CES demand, one should think of A and A_f as two demand shifters. These demand shifters are functions of total industry expenditure and the industry price index.

[10] An application to horisontal FDI, modelled along the lines of Helpman et al. (2004), is available upon request.

Assumption 1. *A, A_f, and θ only enter the profit function, π, through the products, $A\theta$ and $A_f\theta$. That is,*

$$\pi = \pi(x; A\theta, A_f\theta, \beta). \tag{1}$$

Note that the semicolon in (1) separates choice variables from arguments that are perceived as exogenous by the firms.[11] Assumption 1 is very often satisfied in models of heterogeneous firms in international trade.[12]

The next key assumption summarises the five complementarities mentioned in the introduction.

Assumption 2. *For all $(A\theta, A\theta_f, \beta)$, the profit function, $\pi(x; A\theta, A_f\theta, \beta)$, is supermodular in x on X and exhibits increasing differences in $(x, A\theta)$, $(x, A_f\theta)$, and (x, β) on $X \times \mathbb{R}_+$, $X \times \mathbb{R}_+$, and $X \times B$, respectively.*

Supermodularity in x implies that the n activities are complementary.[13] To create some intuition for this critical assumption, consider briefly the case where $\pi(x; A\theta, A_f\theta, \beta)$, perhaps after choosing some initial activities optimally, is additively separable in the products, $A\theta$ and $A_f\theta$.[14] In the recent international trade literature, this is very often the case under the standard assumptions of market segmentation and constant marginal costs of production. We will provide an example of this property in Section 3.1. Under additive separability, one could for instance write (1) as

$$\pi = \omega(x; A\theta) + \rho(x; A_f\theta, \beta) - \gamma(x; \beta),$$

where $\omega(x; A\theta)$ and $\rho(x; A_f\theta, \beta)$ denote variable profits from the home and foreign markets, respectively. $\gamma(x; \beta)$ denotes total fixed costs of the decision x. By the definition of β, $\omega(x; A\theta)$ is independent of β. Finally, recall that $\pi(x; A\theta, A_f\theta, \beta)$ is supermodular in x when $\omega(x; A\theta)$, $\rho(x; A_f\theta, \beta)$, and $-\gamma(x; \beta)$ are all supermodular in x, as in e.g. Bustos (2011) and Amiti and Davis (2012).

[11] Individual firms perceive A and A_f as exogenous variables since the market structure is monopolistic competition. In case of uncertainty after a firm has made its decision, (1) should be interpreted as expected profits; see e.g. Athey and Schmutzler (1995).

[12] The assumption is further discussed in Bache and Laugesen (2014).

[13] An extensive body of recent research within international trade relies heavily on complementary activities. Parts of this literature are surveyed in Section 9 in Melitz and Redding (2014).

[14] By Theorem 4.3 in Topkis (1978), the maximisation operation preserves supermodularity.

The assumptions of increasing differences imply that productivity, the demand level, the foreign demand level, and the elements of β are all complementary to the n activities. As mentioned in the introduction, the complementarity between θ and various activities in models of heterogeneous firms has previously been investigated by Mrazova and Neary (2013). These authors show that such complementarities are quite common. The complementarities between the activities and the domestic and foreign demand levels are implied by the complementarities between the activities and θ plus Assumption 1.

Notice that proper ordering and selection of activities and elements of β are crucial for profits to satisfy Assumption 2 in a specific model. As we let β contain minus the fixed and minus the variable costs of undertaking international trade from the view point of firms in Home, an increase in β will weakly increase the gains from the international trading activities in x. Further, an increase in β will not directly affect the gains from undertaking activities nonrelated to trade. These latter activities will however, through the complementarities among all activities, be affected by the increasing gains from trading activities. Such a mechanism describes why the carefully chosen elements of β are very often complementary to the n activities in trade models with heterogeneous firms. Note that the above effects of an increase in β abstract from the induced changes in A and A_f which are pivotal for the comparative statics below.

The reason for making the key Assumption 2 is that this assumption, together with the lattice constraints, gives rise to the later Lemma 1 in Section 2.3 which will turn out to be very useful throughout our analysis.

The optimal decision, x^*, of a firm with productivity θ is characterised by

$$x^*(A\theta, A_f\theta, \beta, S) = \arg\max_{x \in S} \pi(x; A\theta, A_f\theta, \beta).$$

Optimal firm profits are defined as

$$\pi^*(A\theta, A_f\theta, \beta, S) \equiv \max_{x \in S} \pi(x; A\theta, A_f\theta, \beta).$$

We make the following key and standard assumption which is broadly in line with the empirical evidence in e.g. Bernard et al. (2012).[15]

[15] The paper by Bache and Laugesen (2014) contains a list of nested trade models where Assumptions 1 to 3 are satisfied.

Assumption 3. *The least productive active firms do not engage in international trade.*

2.2 Entry

A firm exits the industry and forfeits the sunk cost of entry if π^* is negative. Hence, expected profits upon entry read

$$\Pi(A, A_f, \beta, S) \equiv \int \max\{0, \pi^*(A\theta, A_f\theta, \beta, S)\} \, \mathrm{d}F(\theta).$$

Under unrestricted entry and exit, the expected profits upon entry satisfy the free-entry condition

$$\Pi(A, A_f, \beta, S) = wf_e. \tag{2}$$

The foreign equivalent is

$$\Pi_f(A_f, A, \beta_f, S_f) \equiv \int \max\{0, \pi_f^*(A_f\theta, A\theta, \beta_f, S_f)\} \, \mathrm{d}F_f(\theta) = w_f f_{ef}. \tag{3}$$

Equations (2) and (3) jointly determine the demand levels, A and A_f, as functions of e.g. (β, S) and (β_f, S_f).[16] Remember that the comparative statics below hold (β_f, S_f) constant. Hence, we note that, if A is nonincreasing as a response to the increase in (β, S), then A_f is nondecreasing and vice versa. This follows readily from (3) and constant wages since the profits of the foreign firms are nonincreasing when A is nonincreasing. This implies that A_f must be nondecreasing.[17]

2.3 Industry Composition

We denote the c.d.f. of the equilibrium distribution of activity i across active firms by $H_i(x_i; \beta, S)$, $i = 1, ..., n$. To characterise these distributions, we first consider the cross-section of firms in a given equilibrium where (A, A_f, β, S) is given. Since firm profits are strictly increasing in θ, the sorting of firms into being active or exiting obeys the simple rule that all firms with productivities

[16] The reader is referred to Antràs et al. (2014) for a discussion of existence and uniqueness of the equilibrium in a specific and similar model.

[17] As some firms serve the consumers in Foreign, a similar argument ensures that a strict decrease in A induces a strict increase in A_f and vice versa.

above a certain threshold, θ_a, are active and all firms with productivities below exit. It follows that

$$\theta_a(A) \equiv \inf\{\theta : \pi^*(A\theta, A_f\theta, \beta, S) > 0\}, \tag{4}$$

where θ_a is, importantly, neither directly affected by A_f nor by (β, S). As we will soon see, these two implications of Assumption 3 greatly simplify the analysis. We assume that the lowest productivity firms are not able to produce profitably. That is, $\theta_a(A) > \theta_0$. The underlying reason could e.g. be the presence of some fixed costs of production in $\gamma(x;\beta)$, a choke price as in Melitz and Ottaviano (2008), or both.

The next step is to characterise the sorting of firms into the activities based on productivity. Assumption 2, the lattice constraints, and the monotonicity theorem of Topkis (1978) give rise to the following lemma.

Lemma 1. *The optimal decision, $x^*(A\theta, A_f\theta, \beta, S)$, is nondecreasing in $(A\theta, A_f\theta, \beta, S)$.*

In a given equilibrium, higher productivity firms thus choose weakly higher levels of all activities. Obtaining Lemma 1 is one main benefit of applying our different approach to deriving comparative statics. It will become clear that Lemma 1 and some admittedly quite simple decompositions and observations underlie all major results in what is to come. Hence, Lemma 1 makes the remaining analysis relatively easy to undertake, not at least when one takes the generality of the setup into account. A clarification might be suitable at this point. The presented approach is general in the sense that the approach can be used to derive new comparative statics for several recent trade models even though the scopes and aims of these models vary greatly. This is because these models and the present model share many key assumptions and properties. However, many of these common assumptions are arguably strong which obviously decreases the generality of our approach.

Let θ_i be the lowest level of productivity at which a firm undertakes at least level x_i of activity i. Formally, we have that

$$\theta_i(x_i; A, A_f, \beta, S) \equiv \max\{\theta_a, \inf\{\theta : x_i^*(A\theta, A_f\theta, \beta, S) \geq x_i\}\}. \tag{5}$$

Let $s_a \equiv 1 - F(\theta_a)$ denote the share of active firms and let $s_i \equiv 1 - F(\theta_i) \leq s_a$ denote the share of firms undertaking at least level x_i of activity

i. The c.d.f. of the equilibrium distribution of activity i across active firms is expressed as

$$H_i(x_i;\beta,S) \equiv 1 - \frac{s_i(x_i;A,A_f,\beta,S)}{s_a(A)}. \tag{6}$$

The industry composition refers jointly to these n distributions.

3 Comparative Statics for the Industry Composition

The present Section 3 derives sufficient conditions for MCS for the industry composition as defined by Bache and Laugesen (2014). The definition is repeated below.

Definition 1. *The industry composition exhibits MCS when increases in* (β, S) *induce first-order stochastic dominance (FSD) shifts in the equilibrium distributions of all activities across active firms. That is,* $H_i(x_i;\beta,S)$ *is nonincreasing for all levels,* x_i*, of all activities,* $i = 1,\ldots,n$*.*

Under MCS for the industry composition, the equilibrium distributions of the n activities shift towards higher values such that the share of active firms which undertake at least any (positive) level of any activity is nondecreasing. When thinking about MCS for the industry composition, it is important to bear in mind that the firm-level comparative statics are generally nonmonotone, dependent on θ, and hence asymmetric across firms in a given country. This result follows from Lemma 1, Assumption 3, and the interactions between A and A_f. Appendix A goes into details about these firm-level responses.

3.1 Example

To illustrate how our approach is useful and also to strengthen the understanding of our approach, we now provide an example. The example builds upon Melitz and Redding (2014) and analyses the effects of asymmetric trade liberalisations within a two-country Melitz (2003) model that allows for country asymmetry but relies on an outside industry to determine wages.[18]

[18] The symmetric country, symmetric shock, version of this example, without an outside industry, is analysed in Bache and Laugesen (2014).

The activities of the firms are export status, given by the zero-one indicator function, $\mathbb{1}_{ex}$, for exporting, and total labour input for variable production, l. By normalising the domestic wage rate to one, the profit function of firms in Home reads

$$\pi(l, \mathbb{1}_{ex}; A\theta, A_f\theta, \beta) = [(A\theta)^{\sigma-1} + \mathbb{1}_{ex}\tau^{1-\sigma}(A_f\theta)^{\sigma-1}]^{\frac{1}{\sigma}} l^{\frac{\sigma-1}{\sigma}} - l - f - \mathbb{1}_{ex}f_{ex},$$

where $\sigma > 1$ is the constant elasticity of substitution, $\tau > 1$ is the iceberg trade cost of exporting to Foreign, f is the fixed cost of production, and f_{ex} is the fixed cost of exporting to Foreign. Assumptions 1 and 2 are satisfied when $x = (l, \mathbb{1}_{ex})$, $X = \mathbb{R}_+ \times \{0, 1\}$, and $\beta = (-\tau, -f_{ex})$.[19] When also the standard Assumption 3 is satisfied through some implicit parameter restrictions, the present example entirely conforms to our approach implying that we can illustrate the effects of an (unilateral) increase in $\beta = (-\tau, -f_{ex})$ within the present example.[20]

To show that the firm-level comparative statics are generally nonmonotone, dependent on θ, and hence asymmetric across firms in a given country, we first present a convenient auxiliary result that follows from a small part of the analysis in Section 2.4 in Demidova and Rodriguez-Clare (2013). Within an identical model, these authors reveal (among other contributions) that an increase in $\beta = (-\tau, -f_{ex})$ induces an increase in the threshold productivity level for exporting to Home in Foreign. This finding is equivalent to an induced increase in competition in Home (A decreases; recall that β_f is constant) and an induced decrease in competition in Foreign (A_f increases). These developments imply that, on the one hand, labour demand for variable production, l, decreases for firms in Home with only domestic sales. These firms are unaffected by the increases in β and A_f but indirectly affected through the decrease in A.[21] On the other hand, the exporting activity increases for some higher productivity firms because of the increases in β and A_f.[22]

[19] Note that $\pi(\mathbb{1}_{ex}; A\theta, A_f\theta, \beta) = [(A\theta)^{\sigma-1} + \mathbb{1}_{ex}\tau^{1-\sigma}(A_f\theta)^{\sigma-1}]\frac{(\sigma-1)^{\sigma-1}}{\sigma^\sigma} - f - \mathbb{1}_{ex}f_{ex}$, where l is chosen optimally, is additively separable in $A\theta$ and $A_f\theta$.

[20] An opening to trade implies introducing a new activity (exporting) by moving from $S' = \mathbb{R}_+ \times \{0\}$ to $S'' = \mathbb{R}_+ \times \{0, 1\}$ which constitutes an increase in S (under the strong set order).

[21] Optimal labour demand for variable production is given by $l = [(A\theta)^{\sigma-1} + \mathbb{1}_{ex}\tau^{1-\sigma}(A_f\theta)^{\sigma-1}](\frac{\sigma-1}{\sigma})^\sigma$.

[22] The productivity threshold for activity 2 (exporting), θ_2, is implicitly determined by $\tau^{1-\sigma}(A_f\theta_2)^{\sigma-1}(\frac{\sigma-1}{\sigma})^\sigma = f_{ex}$.

To anticipate what is to come, the remainder of the paper contributes by deriving clear-cut industry-level results that hold even though the firm-level responses are nonmonotone and asymmetrical, as argued. These industry-level results can be illustrated within the setup of our present example. Within this setup, it can be shown that an increase in $\beta = (-\tau, -f_{ex})$ induces MCS for the industry composition in Home and exactly reverse implications in Foreign if and only if log-productivity is distributed with nonincreasing hazard rate in both countries. For the standard case where θ is distributed Pareto in both countries, we therefore have the following results relevant for the present example. In Home, an increase in $\beta = (-\tau, -f_{ex})$ implies that the firm-size distribution makes an FSD shift to the right (implying that average firm size increases) and the fraction of exporters increases.[23] This follows from Proposition 1 below. These industry-level implications are exactly reverse in Foreign implying that the firm-size distribution exhibits a shift to the left and the fraction of exporters decreases in Foreign. This follows from Proposition 2 below.

Since it is necessary for these strong industry-level results that log-productivity is distributed with nonincreasing hazard rate, these results do certainly not appear within the present example when productivity is distributed Frechet or log-normally. All this ought to become clear below.

3.2 Sufficient Conditions for MCS for the Industry Composition

Denote by ΔH_i the change in H_i induced by an increase in (β, S) from (β', S') to (β'', S'') where either β or S could remain unchanged. This change

[23] As total labour demand and total output at the firm level are both monotonically increasing in l, the FSD shift in the firm-size distribution also holds for these measures of firm size. When $\beta = -f_{ex}$, the result also holds for the distribution of total firm sales.

is decomposed as follows.

$$
\begin{aligned}
\Delta H_i = {} & \underbrace{\frac{s_i(x_i; A', A'_f, \beta', S')}{s_a(A')} - \frac{s_i(x_i; A', A'_f, \beta'', S'')}{s_a(A')}}_{\text{Total direct effect}} \\
& + \underbrace{\frac{s_i(x_i; A', A'_f, \beta'', S'')}{s_a(A')} - \frac{s_i(x_i; A'', A'_f, \beta'', S'')}{s_a(A'')}}_{\text{Total domestic indirect effect}} \\
& + \underbrace{\frac{s_i(x_i; A'', A'_f, \beta'', S'')}{s_a(A'')} - \frac{s_i(x_i; A'', A''_f, \beta'', S'')}{s_a(A'')}}_{\text{Total foreign indirect effect}},
\end{aligned} \tag{7}
$$

where the equilibrium demand levels, A' and A'_f, relate to (β', S') and the equilibrium demand levels, A'' and A''_f, relate to (β'', S'').

Start by considering the total direct effect on H_i in (7). Since an increase in (β, S) tends to increase the levels of all activities chosen by individual firms given A and A_f, it tends to increase the share of firms that undertake at least a given level of activity i, s_i.[24] This effect, which follows readily from Assumption 2 and the lattice constraints, works in favour of MCS for the industry composition. However, this might not be the whole story of the total direct effect. As the direct effect (for given A and A_f) of an increase in (β, S) on s_i might be zero for some levels of some activities, we need the direct effect on the share of active firms, s_a, to be nonpositive such that we assure a nonpositive total direct effect. The intuition is that the least productive active firms have low productivities and therefore, by Lemma 1, undertake relatively low levels of the activities. An increase in the share of active firms therefore works against MCS for the industry composition. As previously mentioned, Assumption 3 implies that the least productive active firms are not directly affected by the increase in (β, S) we analyse. The direct effect of an increase in (β, S) on s_a is therefore zero. This is exactly why we restrict increases in S to proxy introductions of new international trading activities and why we let β contain minus the trade costs parameters.[25] To sum up,

[24]By Lemma 1, x^* is nondecreasing in $(A\theta, A_f\theta, \beta, S)$. Thus, it follows from (5) that θ_i is nonincreasing in (β, S) given A and A_f. Therefore, $s_i = 1 - F(\theta_i)$ is nondecreasing in (β, S) given A and A_f.

[25]Interestingly enough, an increase in (β, S) may also proxy many alternative shocks.

the total direct effect in (7) is indeed nonpositive. The analysis below derives sufficient conditions for the two total indirect effects in (7) to be nonpositive as well.

Next, consider the total foreign indirect effect in (7) which only works through the endogenous change in s_i induced by the change in A_f. The total foreign indirect effect is nonpositive (in general) if and only if $\Delta A_f \equiv A''_f - A'_f$ induced by the increase in (β, S) is nonnegative.[26] Recall that Assumption 3 implies that s_a is independent of A_f. This typical property of trade models with heterogeneous firms makes the total foreign indirect effect straightforward to sign. Moreover, we know from Section 2.2 that $\Delta A_f \geq 0$ if and only if $\Delta A \equiv A'' - A' \leq 0$. Hence, the total foreign indirect effect is nonpositive like the total direct effect when competition intensifies in the home country (A decreases) as a result of the increase in (β, S).

Finally, consider the total domestic indirect effect on H_i in (7) which operates through the induced change in the domestic demand level, A. The total domestic indirect effect is obviously nonpositive when

$$\frac{s_i(x_i; A', A'_f, \beta'', S'')}{s_a(A')} \leq \frac{s_i(x_i; A'', A'_f, \beta'', S'')}{s_a(A'')}. \tag{8}$$

Since $F(\Theta)$ is C^1, we have that $1 - F(\theta) = e^{-\int_{\theta_0}^{\theta} \lambda_\theta(u)\,\mathrm{d}u}$ where λ_θ denotes the hazard rate of $F(\Theta)$. Using this observation, (8) can be reexpressed as

$$e^{-\int_{\theta_a(A')}^{\theta_a(A'')} \lambda_\theta(u)\,\mathrm{d}u} \leq e^{-\int_{\theta_i(x_i;A',A'_f,\beta'',S'')}^{\theta_i(x_i;A'',A'_f,\beta'',S'')} \lambda_\theta(u)\,\mathrm{d}u}.$$

After a change of integrand, we obtain the following expression for a nonpositive total domestic indirect effect

$$e^{-\int_{\log\theta_a(A')}^{\log\theta_a(A'')} \lambda_{\log\theta}(u)\,\mathrm{d}u} \leq e^{-\int_{\log\theta_i(x_i;A',A'_f,\beta'',S'')}^{\log\theta_i(x_i;A'',A'_f,\beta'',S'')} \lambda_{\log\theta}(u)\,\mathrm{d}u}, \tag{9}$$

where $\lambda_{\log\theta}$ denotes the hazard rate of the distribution of log-productivity. To tackle condition (9), we impose the following assumption.

When the least productive active firms are not directly affected by these alternative shocks, the effects of these alternative shocks are similar to the effects of the emphasised trade shocks. One example would be a decrease in the costs of technology upgrading in the Bustos (2011) model since labour demand for variable production, exporting, and technology upgrading are complementary activities in Bustos (2011).

[26]By Lemma 1 and (5), θ_i is nonincreasing in A_f given A and (β, S). Therefore, s_i is nondecreasing in A_f given A and (β, S). The only-if part can be shown using the expression in footnote 21.

Assumption 4. *The absolute value of the percentage change in θ_a induced by a change in A weakly exceeds the absolute value of the percentage change in θ_i induced by a change in A.*

In models of heterogeneous firms in international trade, the content of Assumption 4 very often follows from Assumptions 1 and 3, and that profits, perhaps after choosing some initial activities optimally, can often be written as additively separable in $A\theta$ and $A_f\theta$. As Assumption 4 can be checked through simple inspection of expressions for threshold productivity levels when a specific model is still unsolved, it is straightforward to illustrate that Assumption 4 holds in our previous example as well as in many other related trade models.[27] In our previous example, it suffices to inspect the expressions for the productivity thresholds for being active, for exporting, and for demanding a given amount of labour. When the model is still unsolved, these expressions depend on demand-level variables.

The intuition for why Assumption 4 holds in many models of heterogeneous firms is as follows. First, θ_i may not depend on A whereas θ_a always depends on A. This is for instance the case for the exporting threshold productivity level in our previous example, cf. footnote 22. Second, if θ_i depends on A, but not on A_f, then the percentage changes in θ_a and θ_i, induced by a change in A, are equal. In this case, θ_i and θ_a are both inversely proportional to A. This is the case in our previous example when one focuses on a level of labour demand chosen by firms with only domestic sales, cf. footnote 21. Third, θ_i may also depend on both A and A_f. In this case, the result in Assumption 4 very often follows from the observation that $\pi(x; A\theta, A_f\theta, \beta)$, perhaps after choosing some initial activities optimally, can often be written as additively separable in $A\theta$ and $A_f\theta$. This third possibility occurs in our previous example when one focuses on a level of labour demand chosen by firms serving the foreign market.

Leaving aside Assumption 4, let us analyse the effect of a weak decrease in A, i.e., an enhancement of competition in Home which implies that the total foreign indirect effect is nonpositive. Recall that $\theta_i \geq \theta_a$. By Assumption 4, the change in $\log \theta_a$ induced by the change in A weakly exceeds the change in $\log \theta_i$. As π being strictly increasing in A and θ and Lemma 1 imply that these changes in $\log \theta_a$ and $\log \theta_i$ are nonnegative when competition in Home is enhanced, the condition (9) is fulfilled when A is nonincreasing if the

[27] For instance, it is straightforward to show that Assumption 4 holds in Bustos (2011).

hazard rate of log-productivity is nonincreasing. The next two paragraphs explain the intuition.

Log-productivity is distributed with constant hazard rate when productivity is Pareto distributed.[28] A constant hazard rate of log-productivity implies by definition that the density at any level of log-productivity is constant relative to the probability mass above it. This means that the percentage changes (induced by a change in A) in the share of active firms, s_a, and the share of firms undertaking at least a given level of activity i, s_i, are equal if the changes in the log-thresholds, $\log\theta_a$ and $\log\theta_i$, are equal.

Relative to this case with a total domestic indirect effect equal to zero, Assumption 4 plus a nonincreasing hazard rate of log-productivity both work in the direction of a nonpositive total domestic indirect effect when A weakly decreases. To see this, note that relative to the case where productivity is Pareto distributed, a nonincreasing hazard rate puts more (relative) probability density at $\log\theta_a$ relative to $\log\theta_i$ since $\theta_a \leq \theta_i$. Moreover, by Assumption 4, the increase in $\log\theta_a$ weakly exceeds the increase in $\log\theta_i$ induced by the decrease in A. These two effects both work in favour of an unambiguously nonpositive total domestic indirect effect and MCS for the industry composition when A falls. In this case, both s_a and s_i decrease, all else equal. However, when log-productivity is distributed with nonincreasing hazard rate, and by Assumption 4, the effect on s_a will be weakly larger than the effect on s_i. In other words: we are assured a sufficient mass of exiting firms (with low activity levels) that outweighs the negative effect of the decline in A on activity levels and s_i.[29]

3.3 Increasing Hazard Rate of Log-Productivity

A strictly increasing hazard rate of log-productivity does not comply with (9) in general because, as mentioned, the change in $\log\theta_a$ induced by the change in A may equal the change in $\log\theta_i$. This rules out the possibilities

[28] $\log\theta$ being distributed with constant hazard rate, $\lambda_{\log\theta}$, implies that $F_{\log\theta}(\log\theta) = 1 - e^{-\lambda_{\log\theta}(\log\theta - \log\theta_0)}$, where $F_{\log\theta}$ denotes the c.d.f. of $\log\theta$. Rearranging gives $F(\theta) = 1 - (\theta_0/\theta)^{\lambda_{\log\theta}}$. Thus, $F(\theta)$ is given by the Pareto distribution. The normal distribution has a strictly increasing hazard rate. This also holds for the Gumbel distribution. Recall that log-productivity is distributed Gumbel when productivity is Frechet distributed as in Eaton and Kortum (2002) and Bernard et al. (2003).

[29] Absent a selection effect through variation in $\theta_a(A)$, one sees MCS for the industry composition (in general) if and only if the firm-level responses are monotone and all nonnegative. This follows from the definitions (5) and (6).

of log-normally- or Frechet-distributed productivity parameters, cf. footnote 28. In many cases, like for instance our previous example in Section 3.1, a nonincreasing $\lambda_{\log\theta}$ is moreover necessary for obtaining MCS for the industry composition. To see why this is true, remember that we need ΔH_i to be nonpositive for all levels of all activities. Next, consider a scenario where some of the firms with low productivities and only domestic sales undertake positive levels of activity j, $x_j > 0$. Let us focus attention to the level $x_j > 0$. An example could be a level of labour demanded by firms with only domestic sales in our previous example. Importantly, both the total direct effect and the total foreign indirect effect are zero in this case.[30] The reason being that the domestic firms, which undertake $x_j > 0$, are not directly affected by the increase in (β, S) and, moreover, are unexposed to the foreign demand level. Hence, the industry-level implications hinge upon the total domestic indirect effect. Finally, our choice of x_j implies that the increases in $\log\theta_a$ and $\log\theta_i$ induced by a decrease in A are equal.

3.4 The Domestic Industry Composition

The following proposition sums up the discussion, provided by Section 3.2, of the three bracket terms in (7).

Proposition 1. *Increases in (β, S) induce MCS for the industry composition in Home if log-productivity is distributed with nonincreasing hazard rate in Home and the induced change in A is nonpositive.*

In the more symmetry-oriented setting analysed by Bache and Laugesen (2014), it is very often straightforward to figure out whether an increase in (β, S) enhances or dampens competition in one country. In the current context with country asymmetries, things are a bit more complicated because of the two free-entry conditions and the implied interaction between A and A_f. However, checking the sign of the endogenous response of A should not pose big problems in the context of a specific model. After solving a specific model and deriving the effects of an increase in (β, S), one could e.g. use a known change in θ_a plus the property that θ_a is neither directly affected by A_f nor (β, S) to infer the change in A. In more complex cases, where solving the specific model turns difficult but relevant data is available, one could follow the approach of e.g. Antràs et al. (2014) in making counterfactual

[30] See footnote 21.

simulation exercises. These exercises are shown to be useful in determining the induced changes in the levels of competition.

Finally, Appendix B shows how signing the induced change in the demand level is straightforward when Home is a small open economy. Interestingly, it follows from Appendix B that increases in (β, S) induce a nonpositive change in A. This means that MCS for the industry composition appears in the small open economy Home when log-productivity is distributed with nonincreasing hazard rate in Home.

3.5 The Foreign Industry Composition

Let us henceforth focus on the case where A is nonincreasing (and thus, A_f is nondecreasing) as a result of the increase in (β, S). In this case, we see MCS for the industry composition in Home if log-productivity is distributed with nonincreasing hazard rate in Home (Proposition 1). Now, the question arises: what happens to the foreign industry composition when (β, S) increases? As formally shown in Appendix C, the answer is given by the following proposition.

Proposition 2. *Increases in (β, S) imply that $H_{if}(x_{if}; \beta_f, S_f)$ is nondecreasing for all levels, x_{if}, of all activities, $i = 1, \ldots, n_f$, if log-productivity is distributed with nonincreasing hazard rate in Foreign and the induced change in A is nonpositive.*

Note that the developments mentioned by Propositions 1 and 2 are exactly reverse and co-exist when log-productivity is distributed with nonincreasing hazard rate in both countries and competition enhances in the home country. In this case, the ex ante distributions of all activities across active foreign firms first-order stochastically dominates the ex post distributions in Foreign. Hence, the equilibrium distributions of the n_f activities shift towards lower values, i.e., to the left, such that the share of active firms which undertake at least any (positive) level of any activity is nonincreasing. These distributional shifts to the left occur even though some low-productive firms in Foreign scale up their activity levels (x_f) as a reaction to the induced increase in A_f. This is because a nonincreasing hazard rate of log-productivity assures a sufficient mass of entering low-productivity firms in Foreign with relatively low activity levels.

The reason for the reverse impacts of an increase in (β, S) on the two countries is very simple. First, the signs of the two total indirect effects in

(7) differ across countries. Second, the total direct effect in Foreign is zero since (β_f, S_f) is constant by assumption. Finally, it should be mentioned that, in many cases, like for instance our previous example in Section 3.1, a nonincreasing hazard rate of log-productivity in Foreign is necessary for obtaining the result in Proposition 2. The intuition is equivalent to the one given in Section 3.3 and details are provided in Appendix C. The next section shows how Propositions 1 and 2 and the possibility of asymmetric MCS for the industry compositions generalise to cases with multidimensional firm heterogeneity.

3.6 Multidimensional Firm Heterogeneity

To further increase the realism of models of international trade, it has become more or less customary in recent years to follow Hallak and Sivadasan (2013) in introducing multidimensional firm heterogeneity. Hence, the present section briefly discusses the effects of introducing a vector of firm-specific characteristics other than productivity, $\gamma \in \mathbb{R}^k$.[31] γ is a realisation of the random variable Γ. For simplicity and ease of notation, we let Γ be independently distributed from Θ with c.d.f. $G(\Gamma)$ in Home and $G_f(\Gamma)$ in Foreign. Upon entry to the industry, firms are now characterised by the pair of characteristics, (θ, γ). We distinguish between θ and γ because the assumptions made with respect to θ are irrelevant for γ.

Firm behaviour, firm profits, and thus also the productivity thresholds, (4) and (5), now also depend on γ. Assume that the key Assumptions 1 to 4 hold for all γ. Importantly, the sorting mentioned by Lemma 1 now only holds conditional on γ. This implication, which is one of the main benefits of introducing γ, is often attractive in a specific context where Lemma 1 might be a bit too strong for reconciling the theoretical predictions with the empirics.[32] As shown in Appendix D, we have the following result relevant for the case of multidimensional firm heterogeneity.

Proposition 3. *For any distributions $G(\Gamma)$ and $G_f(\Gamma)$, Propositions 1 and 2 remain valid if θ_a and θ_{af} are independent of γ.*

Notice that Assumption 3 implies that θ_a and θ_{af} are independent of γ when multidimensional firm heterogeneity is introduced through some firm-

[31] The exposition closely follows Bache and Laugesen (2014).

[32] Eaton et al. (2011) and Amiti and Davis (2012) resort to multidimensional firm heterogeneity for this reason.

specific trade costs as in e.g. Amiti and Davis (2012). The intuition for why Propositions 1 and 2 and the possibility of asymmetric MCS for the industry compositions generalise is that all intermediate results from Section 3.2 still hold conditional on γ. Hence, Propositions 1 and 2 generalise to multidimensional firm heterogeneity when we integrate across γ to find the effect of an increase in (β, S) on the industry compositions.

In some cases, the profits of the least productive active firms might also depend upon γ implying that θ_a and θ_{af} do so as well. For these cases, Appendix D proves the following result.

Proposition 4. *Suppose that $F(\Theta)$ and $F_f(\Theta)$ are both Pareto distributions while $G(\Gamma)$ and $G_f(\Gamma)$ are unspecified. Increases in (β, S) induce MCS for the industry composition in Home and imply that $H_{if}(x_{if}; \beta_f, S_f)$ is nondecreasing for all levels, x_{if}, of all activities, $i = 1, \ldots, n_f$, if the induced change in A is nonpositive.*

The cost of dispensing with Proposition 3's constraint that θ_a and θ_{af} are independent of γ is that the productivity distributions are further constrained in Proposition 4.

4 Concluding Remarks

This paper has contributed by extending the work of Bache and Laugesen (2014) to the more plausible case of asymmetric countries and asymmetric shocks. One benefit of this extension is that it may better facilitate empirical analysis. After all, real countries and real shocks are arguably often asymmetric. Moreover, we perceive the model outcome asymmetric MCS for the industry compositions as a strong and testable model outcome.

Under standard distributional assumptions, the example in Section 3.1 has revealed how an unilateral trade liberalisation causes (among other implications) an FSD shift in the firm-size distribution in the liberalising country and exactly reverse implications in the nonliberalising country. It has also been shown that similar implications for the firm-size distribution appear in the liberalising country when this country can be characterised as a small open economy.[33] The characterisation as a small open economy seems reasonable for many countries of the real world. Hence, Corollary 1 in Appendix

[33] As previously mentioned, this finding generalises to examples based on trade shocks in the models of Arkolakis (2010) and Bustos (2011).

B might be particularly relevant and convenient for testing empirically the results of the present paper when these results are applied to specific models.

The presented results on the firm-size distribution might also be particularly useful for empirical analysis. One reason is that a nonincreasing hazard rate of log-productivity is both sufficient as well as necessary for these FSD shifts in the firm-size distribution, for instance when the liberalising country is a small open economy. It follows that empirical analysis guided by the results in the present paper may contribute to the discussion concerning the actual distribution of firm productivity and sales.[34]

One obvious challenge for empirical analysis of the presented results when these are applied to specific models is the identification of a good exogenous trade shock. Luckily, the results of the present paper also generalise to many alternative shocks as long as the least productive active firms are not directly affected. This observation might be useful to empiricists. One example of an empirical strategy would be to test the following implications of our results, cf. footnote 25. Take a decrease in the cost of technology upgrading in a small open economy version of the Bustos (2011) model with an outside industry to determine the wage. This exogenous variation induces MCS for the industry composition in the small open economy if and only if log-productivity is distributed with nonincreasing hazard rate.[35] Hence, when this standard distributional assumption is satisfied, the fraction of firms that export and the fraction of firms that upgrade technology increase. Moreover, the firm-size distribution makes an FSD shift to the right. Finally, it should be mentioned that the natural experiment and the Norwegian data utilised by Bøler et al. (2015) seem quite suitable for testing these implications of an R&D shock in a small open economy version of the Bustos (2011) model.

A The Firm Level of Analysis

Let us define the equilibrium decision of a firm conditional on being active as

$$\tilde{x}^*(\theta, \beta, S) \equiv x^*(A\theta, A_f\theta, \beta, S). \tag{10}$$

From the RHS of (10), it is clear that changes in (β, S) have a direct effect on firm decisions for given demand levels, A and A_f, but such changes also

[34]See for instance the discussions in Bee and Schiavo (2014) and Head et al. (2014).

[35]The least productive active firms do neither export nor upgrade their technology in Bustos (2011).

induce two indirect effects through changes in the two demand levels. This insight allows us to decompose the total effect on $\tilde{x}^*$ of increasing (β', S') to (β'', S'') where either β or S could remain unchanged. Define $\Delta\tilde{x}^* \equiv \tilde{x}^*(\theta, \beta'', S'') - \tilde{x}^*(\theta, \beta', S')$ and note that

$$\begin{aligned}\Delta\tilde{x}^* = & \underbrace{x^*(A'\theta, A'_f\theta, \beta'', S'') - x^*(A'\theta, A'_f\theta, \beta', S')}_{\text{Direct effect}} \\ & + \underbrace{x^*(A''\theta, A'_f\theta, \beta'', S'') - x^*(A'\theta, A'_f\theta, \beta'', S'')}_{\text{Domestic indirect effect}} \\ & + \underbrace{x^*(A''\theta, A''_f\theta, \beta'', S'') - x^*(A''\theta, A'_f\theta, \beta'', S'')}_{\text{Foreign indirect effect}}, \end{aligned} \tag{11}$$

where A' and A'_f relate to (β', S') and A'' and A''_f relate to (β'', S''). It follows from Lemma 1 that an increase in (β, S) always has a nonnegative direct effect on $\tilde{x}^*$. The increase in (β, S) provides firms an incentive to increase their levels of at least one activity. The inherent complementarities among activities ensure that this is manifested in an increase in $\tilde{x}^*$, all else equal. Whereas the direct effect of an increase in (β, S) on $\tilde{x}^*$ is unambiguously nonnegative, the signs of the two indirect effects critically depend on whether competition is enhanced or dampened in the two countries. By Lemma 1, the sign of a particular indirect effect is equivalent to the sign of the change in the relevant demand level, A or A_f. Thus, an indirect effect is aligned with the nonnegative direct effect when competition is dampened (such that the relevant demand level increases) but opposed to the direct effect when competition is enhanced (the relevant demand level decreases).

These observations make it clear that firms in a given country respond asymmetrically to the trade shocks implied by the increase in (β, S). For instance, the least productive active firms are, by Assumption 3, solely affected by the domestic indirect effect while firms serving the foreign market are additionally affected by the remaining two effects in (11). Moreover, it follows from Section 2.2 and the discussion above that at least one of these two remaining effects has a sign that differs from the sign of the domestic indirect effect implying that the firm-level comparative statics are generally nonmonotone, dependent on θ, and hence asymmetric across firms in a given country. Section 3.1 provides an example.

B A Small Open Economy

Consider the case where Home is a small open economy with positive production of differentiated goods.[36] In this case, increases in (β, S) will, by definition, not affect A_f implying that it becomes straightforward to sign the effect on the domestic demand level via the domestic free-entry condition (2). Conveniently, it follows that trade liberalisations or introductions of new international trading activities induce a decrease in the domestic demand level under the small open economy assumption. The reason being that all firms at home make weakly larger profits after the increase in (β, S) at a constant A. Hence, in order for (2) to be satisfied, A must decrease. The following corollary of Proposition 1 summarises.

Corollary 1. *Assume that Home is a small open economy with positive production of differentiated goods. Increases in (β, S) induce MCS for the industry composition in Home if log-productivity is distributed with nonincreasing hazard rate in Home.*

As an enhancement of competition in Home is consistent with a nonpositive total foreign indirect effect, Section 3.2 focused on a decrease in A during its discussion of the total domestic indirect effect. One may also wonder whether one can obtain a nonpositive total domestic indirect effect under an induced increase in A. Hence, consider a strict increase in A and rewrite (9) such that the condition for a nonpositive total domestic indirect effect reads

$$e^{\int_{\log \theta_a(A'')}^{\log \theta_a(A')} \lambda_{\log \theta}(u)\, \mathrm{d}u} \leq e^{\int_{\log \theta_i(x_i;A'',A'_f,\beta'',S'')}^{\log \theta_i(x_i;A',A'_f,\beta'',S'')} \lambda_{\log \theta}(u)\, \mathrm{d}u}. \quad (12)$$

Recall that the change in $\log \theta_i$ induced by the change in A may easily be zero. This is often the case when one looks at the productivity threshold for exporting. Furthermore, it follows from (4) and the assumption that firm profits are strictly increasing in A and θ that $\log \theta_a$ is strictly decreasing in A. Hence, (12) cannot hold in general under a strict increase in A.

[36] As Home might specialise in the outside good when Home becomes sufficiently small relative to Foreign, it is important to check the nonspecialised nature of Home when one wants to apply the below Corollary 1 within a specific model.

C The Foreign Industry Composition

Similar to (7) we have that

$$\Delta H_{if} = \underbrace{\frac{s_{if}(x_{if}; A'_f, A', \beta_f, S_f)}{s_{af}(A'_f)} - \frac{s_{if}(x_{if}; A'_f, A', \beta_f, S_f)}{s_{af}(A'_f)}}_{\text{Total direct effect in Foreign}}$$
$$+ \underbrace{\frac{s_{if}(x_{if}; A'_f, A', \beta_f, S_f)}{s_{af}(A'_f)} - \frac{s_{if}(x_{if}; A''_f, A', \beta_f, S_f)}{s_{af}(A''_f)}}_{\text{Total domestic indirect effect in Foreign}}$$
$$+ \underbrace{\frac{s_{if}(x_{if}; A''_f, A', \beta_f, S_f)}{s_{af}(A''_f)} - \frac{s_{if}(x_{if}; A''_f, A'', \beta_f, S_f)}{s_{af}(A''_f)}}_{\text{Total foreign indirect effect in Foreign}}. \tag{13}$$

First, take the total foreign indirect effect in Foreign. This effect is clearly nonnegative when A is nonincreasing since $s_{if}(x_{if}; A''_f, A', \beta_f, S_f) \geq s_{if}(x_{if}; A''_f, A'', \beta_f, S_f)$ in this case. Next, take the total domestic indirect effect in Foreign. This effect is nonnegative if

$$\frac{s_{if}(x_{if}; A'_f, A', \beta_f, S_f)}{s_{af}(A'_f)} \geq \frac{s_{if}(x_{if}; A''_f, A', \beta_f, S_f)}{s_{af}(A''_f)},$$

or, equivalently, if

$$e^{\int_{\log \theta_{af}(A''_f)}^{\log \theta_{af}(A'_f)} \lambda_{f \log \theta}(u)\, \mathrm{d}u} \geq e^{\int_{\log \theta_{if}(x_{if}; A''_f, A', \beta_f, S_f)}^{\log \theta_{if}(x_{if}; A'_f, A', \beta_f, S_f)} \lambda_{f \log \theta}(u)\, \mathrm{d}u}, \tag{14}$$

where $\lambda_{f \log \theta}$ denotes the hazard rate of log-productivity in Foreign. By Assumption 4, it is clear that the inequality (14) is satisfied if log-productivity is distributed with nonincreasing hazard rate in Foreign while A_f is nondecreasing.[37] Finally, note that the total direct effect in Foreign is zero since (β_f, S_f) is held constant.

[37] Inequality (14) will not hold in general under a strict increase in A_f if log-productivity is distributed with strictly increasing hazard rate. This is because the log-changes in θ_{af} and θ_{if} may be equal.

D Multidimensional Firm Heterogeneity

The two free-entry conditions now read

$$wf_e = \iint \max\{0, \pi^*(\gamma, A\theta, A_f\theta, \beta, S)\}\, \mathrm{d}G(\gamma)\, \mathrm{d}F(\theta),$$

and

$$w_f f_{ef} = \iint \max\{0, \pi_f^*(\gamma, A_f\theta, A\theta, \beta_f, S_f)\}\, \mathrm{d}G_f(\gamma)\, \mathrm{d}F_f(\theta).$$

Hence, the induced directions of change in A and A_f are still of opposite sign. We now have that

$$H_i(x_i; \beta, S) \equiv 1 - \frac{\int s_i(x_i; \gamma, A, A_f, \beta, S)\mathrm{d}G(\gamma)}{\int s_a(\gamma, A)\mathrm{d}G(\gamma)}. \tag{15}$$

Making the exact same decomposition as in (7) but using instead (15) as the basis, it is clear that the total direct effect is nonpositive because of Lemma 1 and Assumption 3. Lemma 1 and Assumption 3 also imply that the total foreign indirect effect is nonpositive when competition intensifies in Home (A decreases and A_f increases) as a result of the increase in (β, S). The condition for a nonpositive total domestic indirect effect akin to (8) reads

$$\frac{\int s_i(x_i; \gamma, A', A_f', \beta'', S'')\mathrm{d}G(\gamma)}{\int s_a(\gamma, A')\mathrm{d}G(\gamma)} \leq \frac{\int s_i(x_i; \gamma, A'', A_f', \beta'', S'')\mathrm{d}G(\gamma)}{\int s_a(\gamma, A'')\mathrm{d}G(\gamma)}. \tag{16}$$

After going through the same steps as in Section 3.2, we can reexpress condition (16) as

$$\int \omega_a e^{-\int_{\log\theta_a(\gamma,A')}^{\log\theta_a(\gamma,A'')} \lambda_{\log\theta}(u)\,\mathrm{d}u}\, \mathrm{d}G(\gamma) \leq \int \omega_i e^{-\int_{\log\theta_i(x_i;\gamma,A',A_f',\beta'',S'')}^{\log\theta_i(x_i;\gamma,A'',A_f',\beta'',S'')} \lambda_{\log\theta}(u)\,\mathrm{d}u}\, \mathrm{d}G(\gamma), \tag{17}$$

where we have defined the weights $\omega_a \equiv s_a(\gamma, A')/\int s_a(\gamma, A')\, \mathrm{d}G(\gamma)$ and $\omega_i \equiv s_i(x_i; \gamma, A', A_f', \beta'', S'')/\int s_i(x_i; \gamma, A', A_f', \beta'', S'')\, \mathrm{d}G(\gamma)$ which both integrate to one. When the hazard rate of log-productivity is constant (productivity is distributed Pareto), it is clear that condition (17) is satisfied if competition in Home is enhanced. When θ_a is independent of γ, condition (17) simplifies to

$$e^{-\int_{\log\theta_a(A')}^{\log\theta_a(A'')} \lambda_{\log\theta}(u)\,\mathrm{d}u} \leq \int \omega_i e^{-\int_{\log\theta_i(x_i;\gamma,A',A_f',\beta'',S'')}^{\log\theta_i(x_i;\gamma,A'',A_f',\beta'',S'')} \lambda_{\log\theta}(u)\,\mathrm{d}u}\, \mathrm{d}G(\gamma). \tag{18}$$

Note that condition (18) is satisfied if competition enhances in Home while log-productivity is distributed with nonincreasing hazard rate. We have now managed to derive sufficient conditions for MCS for the industry composition in Home in the presence of multidimensional firm heterogeneity.

Next, we analyse the implications for the foreign industry composition. For this endeavour, we exploit a decomposition similar to (13) while allowing for the presence of γ in the foreign equivalent of equation (15). The total direct effect in Foreign is obviously again zero. It also follows, by Lemma 1 and Assumption 3, that the total foreign indirect effect in Foreign is nonnegative when the induced change in A of the increase in (β, S) in nonpositive. The condition for a nonnegative total domestic indirect effect in Foreign reads

$$\frac{\int s_{if}(x_{if};\gamma, A'_f, A', \beta_f, S_f)\mathrm{d}G_f(\gamma)}{\int s_{af}(\gamma, A'_f)\mathrm{d}G_f(\gamma)} \geq \frac{\int s_{if}(x_{if};\gamma, A''_f, A', \beta_f, S_f)\mathrm{d}G_f(\gamma)}{\int s_{af}(\gamma, A''_f)\mathrm{d}G_f(\gamma)},$$

or equivalently,

$$\int \omega_{af} e^{\int_{\log\theta_{af}(\gamma,A''_f)}^{\log\theta_{af}(\gamma,A'_f)} \lambda_{f\log\theta}(u)\,\mathrm{d}u} \mathrm{d}G_f(\gamma) \geq \int \omega_{if} e^{\int_{\log\theta_{if}(x_{if};\gamma,A''_f,A',\beta_f,S_f)}^{\log\theta_{if}(x_{if};\gamma,A'_f,A',\beta_f,S_f)} \lambda_{f\log\theta}(u)\,\mathrm{d}u} \mathrm{d}G_f(\gamma), \tag{19}$$

where $\omega_{af} \equiv s_{af}(\gamma, A'_f)/\int s_{af}(\gamma, A'_f)\,\mathrm{d}G_f(\gamma)$ and $\omega_{if} \equiv s_{if}(x_{if};\gamma, A'_f, A', \beta_f, S_f)/\int s_{if}(x_{if};\gamma, A'_f, A', \beta_f, S_f)\,\mathrm{d}G_f(\gamma)$ denote weights that again both integrate to one. Note that condition (19) is satisfied if log-productivity is distributed with constant hazard rate in Foreign and the induced change in A_f is nonnegative. When θ_{af} is independent of γ, the condition (19) simplifies to

$$e^{\int_{\log\theta_{af}(A''_f)}^{\log\theta_{af}(A'_f)} \lambda_{f\log\theta}(u)\,\mathrm{d}u} \geq \int \omega_{if} e^{\int_{\log\theta_{if}(x_{if};\gamma,A''_f,A',\beta_f,S_f)}^{\log\theta_{if}(x_{if};\gamma,A'_f,A',\beta_f,S_f)} \lambda_{f\log\theta}(u)\,\mathrm{d}u} \mathrm{d}G_f(\gamma). \tag{20}$$

Note that (20) is satisfied if log-productivity is distributed with nonincreasing hazard rate in Foreign and the induced change in A_f is nonnegative.

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