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An Interpretation of the Gini Coefficient in a Stiglitz Two-Type Optimal Tax Problem

Bo Sandemann Rasmussen

AARHUS UNIVERSITY
BUSINESS AND SOCIAL SCIENCES
DEPARTMENT OF ECONOMICS AND BUSINESS

An Interpretation of the Gini Coefficient in a Stiglitz Two-Type Optimal Tax Problem*

Bo Sandemann Rasmussen†

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Abstract

In a two-type Stiglitz (1982) model of optimal non-linear taxation it is shown that when the utility function relating to consumption is logaritmic the shadow price of the incentive constraint relating to the optimal tax problem exactly equals the Gini coefficient of the second-best optimal income distribution of a utilitarian government. In this sense the optimal degree of income redistribution is determined by the severity of the incentive problem facing the policy-maker. Extensions of the benchmark model to allow for more general functional forms of the utility function and for more than two types of workers reveal that also in these cases the desired degree of income redistribution is positively correlated with the shadow prices of the incentive constraints.

Keywords: Optimal taxation, income distribution, incentive constraint, Gini coefficient.

JEL: H21, H23, H24.

1 Introduction

The Gini coefficient is an often used measure of the degree of income inequality. It basically states how much of total income in the economy that has to be redistributed from high to low income agents for a fully egalitarian income distribution to follow. In economies where income redistribution policies are

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†Department of Economics and Business, Aarhus University, Fuglesangs Alle 4, DK-8210 Aarhus V, Denmark. e-mail: brasmussen@econ.au.dk. Phone: +45 8716 5551.

pursued the realized size of the Gini coefficient will both depend on the redistributive preferences of the policy-maker and on the structural properties of the underlying economy.

In the standard optimal tax literature emanating from Mirrlees (1971) the government cannot observe the innate abilities of households so it has to levy taxes on observable measures like income. In doing so in a socially optimal way the tax system must induce the various types of household to earn income at the level intended for their type, so the equilibrium behaviour will reveal the underlying productivities of the households. Thus, the optimal tax system must respect incentive constraints ruling out that households of one type mimics the behaviour of other types of households. The difficulty facing the government in separating the various types of households can be formally measured by the shadow price of the incentive constraints entering the optimal tax problem, and the conjecture is then that there is a negative relation between this shadow price and level of income redistribution generated by the optimal policy.

To show formally that the shadow price of the incentive constraints facing the government affects the degree of income redistribution a standard two-type Stiglitz (1982) model is set up. Under utilitarianism it is shown that when the utility function relating to consumption of goods is logaritmic the equilibrium shadow price of the incentive constraint exactly equals the Gini coefficient of the second-best optimal income distribution. Hence, the more binding the incentive constraint is - and therefore its shadow price is higher - the more unequal is the final income distribution. In this particular case we can provide a new interpretation of the Gini coefficient: It measures how easily the government can redistribute income among households without providing them with inadequate incentives to supply labour. Subsequently, the model is extended first to allow for a more general constant elasticity utility function for consumption, and subsequently also to have more than two types of households. In these cases the specific result of equality of the Gini coefficient and the shadow price of the incentive constraint no longer holds in its strict form, but it is still the case that the Gini coefficient and the values of the shadow prices of the incentive constraints are highly positively correlated. This suggests quite generally, that relaxation of the incentive constraints facing the government's optimal tax-transfer problem will lead to a more equitable distribution of consumption possibilities.

The rest of the paper is organized as follows. Section 2 presents the benchmark model while section 3 provides the analytical result. Section 4 considers some numerical examples while section 5 provides a couple of extensions of the benchmark model. Finally, Section 6 offers some concluding remarks.

2 The Model

Consider a standard Stiglitz (1982) model with two types of households differing in productivity only. The technology for producing output is linear in labour input and the output price is normalized at unity. Let a fraction $\pi_L \in [0,1]$ of the population (of a fixed size) have productivity w_L while the remaining fraction $\pi_H = 1 - \pi_L$ has productivity $w_H > w_L$. Individual labour supplies are determined endogenously and depend on productivities and the tax system. As individual productivities are private information the tax must be levied on income - the product of productivity and labour supply. This leads to second-best optimal tax policies that distort labour supply decisions and redistribute income from high to low productivity households.

There are essentially three assumptions needed for the main result of the analysis to go through: Household utility is additively separable in consumption and leisure; the sub-utility function associated with consumption is logaritmic; and the government has utilitarian preferences. Of course, this limits the generality of the results. However, the assumptions stated here are often utilized in this literature, and extensions of the model will reveal that the result can also qualitatively carry over to other settings, making it of more general interest.

Hence, household preferences can be described by the additive separable utility function

$$U(c_i, l_i) = \ln(c_i) - v(l_i), \qquad i = L, H \tag{1}$$

where $v(\cdot)$ is increasing and convex. Consumption equals after-tax income

$$c_i = Y_i - T_i(Y_i), \qquad i = L, H \tag{2}$$

where the tax functions, T_i, are general non-linear functions of labour income, $Y_i \equiv w_i l_i$.

Expressed in terms of the observables (c_i, Y_i) we have that

$$U(c_i, l_i) \equiv V(c_i, Y_i; w_i) = \ln(c_i) - v\left(\frac{Y_i}{w_i}\right), \qquad i = L, H \tag{3}$$

so household behaviour can be described by the usual first-order condition that the marginal rate of substitution between leisure and consumption equals the marginal after-tax wage

$$v'(l_i)c_i = w_i(1 - T_i'). \tag{4}$$

The utilitarian social welfare function is

$$W = \sum_{i \in \{L,H\}} \pi_i \left[\ln(c_i) - v\left(\frac{Y_i}{w_i}\right)\right], \tag{5}$$

while the government budget constraint (assuming the tax policy is purely redistributive) reads

$$\sum_{i\in\{L,H\}} \pi_i T_i(Y_i) = 0. \tag{6}$$

The aggregate resource constraint simply states that total consumption equals total income

$$\sum_{i\in\{L,H\}} \pi_i (Y_i - c_i) = 0, \tag{7}$$

so given Walras' Law this (and the budget constraints of the households) makes the government budget constraint redundant in the optimal tax problem.

The second-best optimal tax policy consists of specification of two income-consumption bundles, (Y_L, c_L) and (Y_H, c_H), that maximize social welfare conditional on the aggregate resource constraint and the incentive constraint that the high-productivity household does not prefer to mimic the low-productivity household:[1]

$$\ln(c_H) - v\left(\frac{Y_H}{w_H}\right) \geq \ln(c_L) - v\left(\frac{Y_L}{w_H}\right), \tag{8}$$

making the equilibrium outcome fully revealing.

3 Analytical Results

The Lagrangian associated with the social welfare maximization problem is

$$\begin{aligned} \mathcal{L} \;=\; & \sum_{i\in\{L,H\}} \pi_i \left[\ln(c_i) - v\left(\frac{Y_i}{w_i}\right)\right] + \mu \left[\sum_{i\in\{L,H\}} \pi_i (Y_i - c_i)\right] \\ & + \lambda \left[\ln(c_H) - v\left(\frac{Y_H}{w_H}\right) - \ln(c_L) + v\left(\frac{Y_L}{w_H}\right)\right], \end{aligned} \tag{9}$$

where μ and λ are the Lagrange multipliers on the aggregate resource constraint and the incentive constraint, respectively.

[1] Given the additively separable utility function (and the utilitarian social welfare function) it follows from Arnott, Hosios and Stiglitz (1988) that this is the only incentive constraint binding in equilibrium.

The first-order conditions are:

$$\frac{\partial \mathcal{L}}{\partial c_L} = \frac{\pi_L}{c_L} - \mu\pi_L - \frac{\lambda}{c_L} = 0 \tag{10a}$$

$$\frac{\partial \mathcal{L}}{\partial Y_L} = -\pi_L v'\left(\frac{Y_L}{w_L}\right)\frac{1}{w_L} + \mu\pi_L + \lambda v'\left(\frac{Y_L}{w_H}\right)\frac{1}{w_H} = 0 \tag{10b}$$

$$\frac{\partial \mathcal{L}}{\partial c_H} = \frac{\pi_H}{c_H} - \mu\pi_H + \frac{\lambda}{c_H} = 0 \tag{10c}$$

$$\frac{\partial \mathcal{L}}{\partial Y_H} = -\pi_H v'\left(\frac{Y_H}{w_H}\right)\frac{1}{w_H} + \mu\pi_H - \lambda v'\left(\frac{Y_H}{w_H}\right)\frac{1}{w_H} = 0, \tag{10d}$$

which together with the two constraints determine the optimal allocation (Y_L, c_L, Y_H, c_H) and the two multipliers, μ and λ.

As usual, it is not possible to obtain an explicit analytical solution for the optimal allocation - even if a specific functional form for the subutility function for leisure were specified - due to the non-linearity of the optimality conditions. However, it turns out that combining the first-order conditions for the consumption levels, equations (10a) and (10c), we can obtain a reduced form solution for the shadow price on the incentive constraint, λ:

$$\lambda = \frac{\pi_L\pi_H\,(c_H - c_L)}{\pi_L c_L + \pi_H c_H}, \tag{11}$$

where, of course, the two consumption levels are endogenous themselves.

An often used measured for income inequality is the Gini coefficient that essentially measures the share of total income (net or gross) that would have to be transferred from high income households to low income households for the income distribution to be totally equitable. In our case with a redistributing government the natural inequality measure is the Gini coefficient based on after-tax income (equal to consumption). For an economy with households divided into N groups, and ranked according to increasing consumption levels, the Gini coefficient can be calculated as

$$G = 1 - \frac{\sum_{i=1}^{N} h(c_i)(D_{i-1} + D_i)}{D_N}, \tag{12}$$

where $h(c_i)$ is the share of group i in the population and

$$D_i \equiv \sum_{j=1}^{i} h(c_j)c_j,$$

with $D_0 = 0$. In our case $h(c_i) = \pi_i$, $i = L, H$ so

$$\begin{aligned} D_L &= \pi_L c_L \\ D_H &= D_N = \pi_L c_L + \pi_H c_H. \end{aligned}$$

The Gini coefficient then becomes

$$\begin{aligned} G &= 1 - \frac{\pi_L^2 c_L + \pi_H (2\pi_L c_L + \pi_H c_H)}{\pi_L c_L + \pi_H c_H} \\ &= \frac{\pi_L \pi_H (c_H - c_L)}{\pi_L c_L + \pi_H c_H}. \end{aligned} \tag{13}$$

Obviously, comparing equations (11) and (13) reveals that the Gini coefficient equals the shadow price of the incentive constraint, $G = \lambda$. Thus, the second-best optimal level of income inequality - as measured by the Gini coefficient - is exactly equal to the measure of how costly is it to redistribute income from high to low income households. Hence, the amount of income a utilitarian government will redistribute depends on how easily income can be transferred from high ability to low ability households without inducing the high ability households to masquerade as low ability households and supply less labour than intended by the optimal redistributive policy.

In the present setting a crucial parameter for how costly income redistribution policies are is the elasticity of labour supply. To verify that larger elasticities of labour supply leads to higher Gini coefficients some numerical analyses are helpful.

4 Numerical Results

To perform numerical analyses of the model we need to fully parameterize the model. A quite standard functional form for the subutility function of leisure is the constant elasticity function

$$v(l_i) = \frac{(l_i)^{1+1/\varepsilon}}{1+1/\varepsilon}, \qquad i = L, H,$$

where $\varepsilon > 0$ is the Frisch elasticity of labour supply. To obtain the second-best optimal allocation, parameter values for productivities (w_L, w_H), population shares (π_L) and the Frisch elasticity of labour supply (ε) must be chosen and then the system of six non-linear equations can be solved for the consumption levels (c_L, c_H), the income levels (Y_L, Y_H) and the two multipliers (μ, λ).

For the productivity levels we choose $w_L = 3$ and $w_H = 8$, equal shares of the two types are assumed, $\pi_L = \pi_H = \frac{1}{2}$ and then we consider the consequences of having different labour supply elasticities. Specifically, we let ε take on six different values ranging from $\varepsilon = 2.0$ to $\varepsilon = 0.05$. The results are given in the table below.[2]

	$\varepsilon = 2.0$	$\varepsilon = 1.0$	$\varepsilon = 0.5$	$\varepsilon = 0.25$	$\varepsilon = 0.1$	$\varepsilon = 0.05$
c_L	3.25	3.78	4.33	4.79	5.18	5.34
c_H	7.36	7.04	6.66	6.28	5.89	5.71
Y_L	1.16	1.74	2.23	2.57	2.82	2.91
Y_H	9.45	9.09	8.77	8.50	8.25	8.14
μ	0.19	0.18	0.18	0.18	0.18	0.18
$\lambda = G$	0.19	0.15	0.11	0.067	0.032	0.017

As is evident from the table a more elastic labour supply reduces the incentives of the government to redistribute income from high to low productivity households. Hence, costly redistribution results in a less equitable outcome.

5 Generalizations

That the Gini coefficient exactly equals the shadow price of the incentive constraint of the optimal tax-transfer problem is, of course, a result that rests on the specific assumptions made so far in this paper, and although these assumptions are quite commonly used in this literature it is of interest to investigate how the result will differ under slightly different assumptions. First, we will let the sub-utility function related to consumption have constant elasticity (being different from one as it is in the logaritmic case), and subsequently we will consider the case of three types of households differing in unobservable productivity.

5.1 Constant Elasticity of Consumption

The utility function of the households is now assumed to be

$$U(c_i, l_i) = \frac{c_i^{1-\sigma} - 1}{1 - \sigma} - v(l_i), \qquad \sigma \neq 1,\ i = L, H \tag{14}$$

[2] The numerical results are obtained using the fsolve.m routine in MATLAB. Files are available upon request.

where $\sigma > 0$ is the constant elasticity of marginal utility.[3] The Lagrangian is now

$$\mathcal{L} = \sum_{i\in\{L,H\}} \pi_i \left[\frac{c_i^{1-\sigma}-1}{1-\sigma} - v\left(\frac{Y_i}{w_i}\right) \right] + \mu \left[\sum_{i\in\{L,H\}} \pi_i (Y_i - c_i) \right] \tag{15}$$
$$+\lambda \left[\frac{c_H^{1-\sigma}-1}{1-\sigma} - v\left(\frac{Y_H}{w_H}\right) - \frac{c_L^{1-\sigma}-1}{1-\sigma} + v\left(\frac{Y_L}{w_H}\right) \right],$$

so the first-order conditions for the optimal consumption choice become

$$\frac{\partial \mathcal{L}}{\partial c_L} = \frac{\pi_L}{c_L^\sigma} - \mu\pi_L - \frac{\lambda}{c_L^\sigma} = 0 \tag{16a}$$
$$\frac{\partial \mathcal{L}}{\partial c_H} = \frac{\pi_H}{c_H^\sigma} - \mu\pi_H + \frac{\lambda}{c_H^\sigma} = 0. \tag{16b}$$

Solving for the shadow price of the incentive constraint gives

$$\lambda = \frac{\pi_L \pi_H \left(c_H^\sigma - c_L^\sigma\right)}{\pi_L c_L^\sigma + \pi_H c_H^\sigma}, \tag{17}$$

while the Gini coefficient still is given as

$$G = \frac{\pi_L \pi_H (c_H - c_L)}{\pi_L c_L + \pi_H c_H}.$$

Of course, for $\sigma \neq 1$ the Gini coefficient and the shadow price of the incentive constraint are no longer equal. To establish how the two might be related we have to resort to solving the model numerically. In these simulations we use the same specific functional form for $v(l_i)$ as before and then derive the numerical solutions of the model for different combinations of the elasticity of marginal utility, σ, and the Frisch elasticity of labour supply, ε. Specifically, we choose $\sigma = \{0.5; 0.8; 1.25; 2.0\}$ and $\varepsilon = \{0.05; 0.1; 0.25; 0.5; 1.0; 2.0\}$. The equilibrium values of λ and G for the various parameter combinations are given in the table below.[4]

[3] For $\sigma = 1$ we get the logaritmic case already studied.

[4] The numerical results are obtained using the fsolve.m routine in MATLAB. Files are available upon request.

	$\varepsilon = 2.0$	$\varepsilon = 1.0$	$\varepsilon = 0.5$	$\varepsilon = 0.25$	$\varepsilon = 0.1$	$\varepsilon = 0.05$
$\sigma = 0.5; \lambda$	0.13	0.090	0.060	0.036	0.016	0.0086
$\sigma = 0.5; G$	0.24	0.18	0.12	0.072	0.032	0.017
$\sigma = 0.8; \lambda$	0.17	0.13	0.089	0.055	0.026	0.014
$\sigma = 0.8; G$	0.21	0.16	0.11	0.069	0.032	0.017
$\sigma = 1.25; \lambda$	0.22	0.17	0.13	0.081	0.039	0.021
$\sigma = 1.25; G$	0.18	0.14	0.10	0.065	0.031	0.017
$\sigma = 2.0; \lambda$	0.27	0.23	0.17	0.12	0.060	0.033
$\sigma = 2.0; G$	0.15	0.12	0.090	0.060	0.030	0.016

Two observations can be made from this table. First, higher values of σ induce the government to redistribute more income from the high ability to the low ability households. Intuitively, this follows from the impact of σ on the curvature of the indifference curves in consumption-leisure space. Higher values of σ make the indifference curves more convex, making it easier for the government to set highly redistributive taxes and still separate the high from the low ability households. Therefore, more redistribution is optimal when σ is high. Secondly, for a given value of σ the shadow price of the incentive constraint, λ, and the Gini coefficient, G, are highly positively correlated (in all the cases above the correlation coefficient between λ and G exceeds 0.99). Hence, the shadow price of the incentive constraint is a strong predictor of the optimal degree of income redistribution. Notice also, that quite systematically the Gini coefficient exceeds the shadow price of the incentive constraint for $\sigma < 1$, while the opposite holds for $\sigma > 1$ (while, of course, the two are equal for $\sigma = 1$ as our main result showed).

So even outside the logaritmic case the desired degree of income redistribution is highly correlated with the shadow price of the incentive constraint.

5.2 A Three Type Model

Another - and a more challenging - extension of the model is to have three types of households with unobservable productivities $w_L < w_M < w_H$ (low, middle and high) with corresponding shares of the population of π_L, π_M and π_H (where $\sum_{i\in\{L,M,H\}} \pi_i = 1$). With three types of households the optimal policy is still one that leads to separation of the three types (this is the revelation principle, see Salanié (2011)). Hence, we now need (at least) two incentive constraints:[5] An incentive constraint preventing the high type

[5] In principle, an incentive constraint preventing the high type from mimicking the low type could be imposed, but such a restriction will typically not be binding once the other two incentive constraints are. In the simulations stated below this type of incentive constraint can be shown not to be binding.

from mimicking the middle type, and an incentive constraint preventing the middle type from mimicking the low type.

Formally, the Lagrangian of the optimization problem of the government is:

$$\begin{aligned}\mathcal{L} &= \sum_{i\in\{L,M,H\}} \pi_i \left[\ln(c_i) - v\left(\frac{Y_i}{w_i}\right)\right] + \mu\left[\sum_{i\in\{L,M,H\}} \pi_i (Y_i - c_i)\right] \\ &\quad + \lambda_H \left[\ln(c_H) - v\left(\frac{Y_H}{w_H}\right) - \ln(c_M) + v\left(\frac{Y_M}{w_H}\right)\right] \\ &\quad + \lambda_M \left[\ln(c_M) - v\left(\frac{Y_M}{w_M}\right) - \ln(c_L) + v\left(\frac{Y_L}{w_M}\right)\right],\end{aligned} \tag{18}$$

where we now have two shadow prices, λ_H and λ_M. The first-order conditions for the optimal consumption choices are

$$\frac{\partial \mathcal{L}}{\partial c_L} = \frac{\pi_L}{c_L} - \mu\pi_L - \frac{\lambda_M}{c_L} = 0 \tag{19a}$$

$$\frac{\partial \mathcal{L}}{\partial c_M} = \frac{\pi_M}{c_M} - \mu\pi_M - \frac{\lambda_H}{c_M} + \frac{\lambda_M}{c_M} = 0 \tag{19b}$$

$$\frac{\partial \mathcal{L}}{\partial c_H} = \frac{\pi_H}{c_H} - \mu\pi_H + \frac{\lambda_H}{c_H} = 0. \tag{19c}$$

Solving for the two shadow prices gives (after some tedious manipulations)

$$\lambda_M = \frac{\pi_L\pi_H(c_H - c_L) + \pi_L\pi_M(c_M - c_L)}{\pi_L c_L + \pi_M c_M + \pi_H c_H} \tag{20}$$

$$\lambda_H = \frac{\pi_L\pi_H(c_H - c_L) + \pi_M\pi_H(c_H - c_M)}{\pi_L c_L + \pi_M c_M + \pi_H c_H}. \tag{21}$$

In the three type case we can at best get that the Gini coefficient is a function of the two shadow prices. Using equation (12) for the three type case we get after simple manipulations that the Gini coefficient can be written as

$$G = \frac{\pi_L\pi_H(c_H - c_L) + \pi_L\pi_M(c_M - c_L) + \pi_M\pi_H(c_H - c_M)}{\pi_L c_L + \pi_M c_M + \pi_H c_H}. \tag{22}$$

Comparing equations (20), (21) and (22) reveals that

$$G = \lambda_M + \lambda_H - \frac{\pi_L\pi_H(c_H - c_L)}{\pi_L c_L + \pi_M c_M + \pi_H c_H}. \tag{23}$$

Hence, also in this case the Gini coefficient is positively related to the shadow prices of the incentive constraints. However, this result is not quite as useful as in the two type case where a higher shadow price of the incentive constraint unambiguously leads to less income redistribution. The problem is that the last term in the expression for the Gini coefficient is itself endogenous so if redistribution becomes more costly - leading to higher λ_M and/or λ_H - it is likely that both the numerator and the denominator of the final term will increase.[6]

To illustrate the possible relation between the shadow prices of the incentive constraints and the Gini coefficient we turn to solving the model numerically. Productivities are set at $w_L = 3$, $w_M = 5$ and $w_H = 8$, while the three groups are assumed to be of equal size, $\pi_L = \pi_M = \pi_H = 1/3$. The Frisch elasticity of labour supply varies from $\varepsilon = 0.05$ to $\varepsilon = 2.0$. The equilibrium allocations, shadow prices and the Gini coefficient are presented in the table below.[7]

	$\varepsilon = 2.0$	$\varepsilon = 1.0$	$\varepsilon = 0.5$	$\varepsilon = 0.25$	$\varepsilon = 0.1$	$\varepsilon = 0.05$
c_L	3.08	3.48	3.93	4.39	4.87	5.09
c_M	3.70	4.11	4.53	4.86	5.13	5.23
c_H	7.38	7.07	6.70	6.30	5.84	5.61
Y_L	1.72	2.06	2.37	2.62	2.84	2.92
Y_M	3.05	3.55	4.04	4.44	4.75	4.87
Y_H	9.39	9.05	8.74	8.49	8.26	8.14
μ	0.211	0.205	0.198	0.193	0.189	0.188
λ_M	0.116	0.0962	0.0739	0.051	0.0257	0.0139
λ_H	0.188	0.149	0.109	0.0717	0.0350	0.0187
G	0.202	0.163	0.122	0.0818	0.0405	0.0217

To assess how the shadow prices, λ_M and λ_H, and the Gini coefficient, G, are related the correlation coefficient, r, between $(\lambda_M + \lambda_H)$ and G can be calculated showing a strong, positive correlation between the sum of the shadow prices and the Gini coefficient:

$$r = 0.98.$$

Hence, we still have that the desired degree of income redistribution is strongly related to the values of the shadow prices of the incentive constraints, so anything that can alleviate the incentive constraints is likely to lead to a more egalitarian distribution of consumption possibilities.

[6] With more costly redistribution $c_H - c_L$ is likely to increase, while the denominator - being equal to average consumption - is also likely to increase. Hence, the total effect on the final term is ambiguous.

[7] As before, the numerical results are obtained using the fsolve.m routine in MATLAB. Files are available upon request.

It also follows from the table above that the incentive constraint preventing the high type from masquerading as a low type (which is not imposed in the optimization problem) is not binding in equilibrium. E.g. in the case of $\varepsilon = 0.05$ the utility of the high type household when choosing the income-consumption bundle intended for him is

$$V(c_H, Y_H; w_H) = 1.66,$$

while the utility of the high type household mimicking the low type household is

$$V(c_L, Y_L; w_H) = 1.63,$$

justifying the exclusion of this incentive constraint on the government optimization problem.

Of course, there may be cases where the shadow prices of the incentive constraints will be less precise predictors of the desired degree of income redistribution. If, e.g., in the three type version of the model the equilibrium is a pooling equilibrium where two types are not being separated by the optimal tax-transfer policy[8] then one of the shadow prices will equal zero, and the value of the other shadow price may be less informative about the total amount of redistribution taking place. Hence, our results are mostly relevant in cases where a fully revealing second-best equilibrium exists.

6 Concluding Remarks

The basic result of the paper is that when the government is utilitarian, household preferences are additively separable in consumption and leisure, and the subutility function of consumption is logaritmic, the shadow price of the incentive constraint in the optimal tax problem exactly equals the Gini coefficient of the net income distribution. Thus, we can interpret the Gini coefficient as a measure of how easily the government can redistribute income among households without providing them with inadequate incentives to supply labour.

Extensions of the benchmark model shows that although the specific result does not carry over to settings with either other functional forms for preferences or when there are more than two types of households, qualitatively similar results still apply in the sense that the Gini coefficient of the net income distribution will be highly positively correlated with the shadow prices of the incentive constraints.

[8] That would typically require that the single crossing property - which is satisfied in our numerical examples - is violated.

As a corollary to these results it follows that anything that can alleviate the incentive constraints facing the government's attempt to redistribute income will help the government in securing a more equitable outcome. As an example of measures that can alleviate the incentive constraints using tags in the tax function in the spirit of Akerlof (1978) is an obvious choice (see also Blomquist and Micheletto (2008) for the use of age as an efficient tag). Hence, if individual characteristics are correlated with innate ability and observable to the government using these characteristics in the tax function will generally relax the incentive constraint and allow for a more equitable outcome to be obtained.

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