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Working Paper
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Abstract

This paper builds a dynamic general equilibrium trade model, in which heterogeneous intermediate goods are produced and traded in a Melitz (2003) type sector. These intermediate goods are used in the production of a final good, which can be used for consumption, capital accumulation, or investment in market entry. Therefore capital accumulation is the direct alternative to investing in firm entry. This has a number of implications. First, in the short run following trade liberalizations a higher interest rate will prevail, which will tend to shelter incumbent firms from increased competition by discouraging investment in entry of new firms. Second, different forms of trade liberalizations with the same impact on the share of expenditure on domestic variety will have different effects on consumption, and thus welfare, both during transitions and in steady-state, due to different effects on aggregate investment. Third, the accumulation of capital will add a time dimension to the anti-variety effects of trade liberalizations.

In effect, capital accumulation generates rich adjustment dynamics both in the aggregate and at the firm level. Further, the study of transitions in this framework provides important insights regarding the effects of different forms of trade liberalizations in general and on welfare and individual firms in particular.

JEL classification: F12, F14, F41

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1 Introduction

The majority of the trade literature incorporating heterogeneous firms immediately following Melitz (2003) has focused on static comparative analysis of steady states. Recently, more consideration have been given to transitions following trade liberalization in the Melitz (2003) framework. And rightly so. In general, comparative statics are a natural and good starting point for understanding the steady-state effects of trade liberalization. But once such results are obtained, the transitions along which these effects materialize should be considered. Knowledge of the transitions is important with respect to capturing the full picture of the effects from liberalizing trade, with respect to welfare analysis and with respect to policy. Further, it is important to be aware of short-run versus long-run differences in such effects when doing empirics. In the long run we are all dead, one might be tempted to say.

Among studies considering transitions in a Melitz (2003) framework Ghironi and Melitz (2005), Costantini and Melitz (2007) and Burstein and Melitz (2011) can be mentioned. A common feature of these studies is that they maintain the assumption from Melitz (2003) of exogenous supply of input factors. Adjustment dynamics of the economy in the wake of trade liberalizations are caused by e.g. shutting down voluntary exit or introducing dynamics in firm-level productivity. In the study of dynamics, it seems natural to include capital (or any accumulable production factor) into the framework. Not only because the inclusion of capital as an accumulable production factor provides for a more realistic framework, but also because it should be obvious that endogenous capital accumulation introduces adjustment dynamics. Atkeson and Burstein (2010) rightly notes that the inclusion of capital accumulation leads to an amplification of the impact of changes in productivity on output. If this was the only effect, the adjustment dynamics induced by capital accumulation would merely be a realization of this amplification effect, and you would probably agree that such adjustment is not very interesting. However, realistically capital accumulation is not merely the expansion of a production factor, it is also the alternative to investing in firm entry.

The present paper develops a tractable model based on Melitz (2003) including capital in this way. The contribution to the understanding of transitions in the Melitz (2003) framework is that this approach captures adjustment dynamics induced solely by capital accumulation and shows that capital accumulation will definitely not only imply an amplification effect. It will also have a number of other significant implications for the effects of trade liberalizations on firms and welfare. Before developing the model, let me provide a flavor of these implications and the underlying intuition.

First, since the demand for capital will be increased by trade liberalizations, the interest rate will rise above the steady-state level in the short run following a trade liberalization. As capital accumulation is the alternative to investing in firm entry, the higher interest rate will tend to discourage entry of new firms into the market. In effect, the higher interest rate that will prevail in the short run will shelter incumbent firms against increased competition compared to the long run. As the intensified competition is the source of the increase in average productivity from reallocation among firms in the Melitz (2003) model, the full increase in average productivity will be realized only gradually as the interest rate returns to its steady-state value and investment in firm entry expands. Put another way, during the transition some of the least productive firms are not pushed out of the market immediately but rather gradually as competition intensifies because of the decreasing interest rate.

Second, when using the Melitz (2003) model and assuming Pareto distributed productivity draws, the welfare effects of trade liberalizations only depends on their effects on trade flows, as shown by Arkolakis et al. (forthcoming). This will not be the case in the present model. Different forms of trade liberalization with equivalent effects on trade flows will feature different effects on steady-state consumption, as their effects on aggregate investment will differ. The intuition is, that even though different forms of trade liberalizations acheive equivalent effects on trade flows, they work through the intensive and the extensive margins of trade to different extents. However, the extensive margin requires investment, which in the presense non-negative subjective discounting must earn a positive return in steady state. Thus, while the intensive margin affects consumption only through trade flows, the extensive margin affects consumption through trade flows *and* (return on) aggregate investment. Hence, the welfare effects of a given trade liberalization cannot be summarized by its effect on trade flows. While the discussion so far have concerned steady-state effects, there will also be implications for the transition. The cause is that achieving a higher level of steady-state consumption through a higher level of investment implies foregoing more consumption in the short run. In this way, taking consumption during the transition into account will prove crucial for comparing welfare effects of different trade liberalizations. In comparing different forms of liberalization, the welfare ranking cannot be summarized by the ranking of the implied steady-state consumption levels.

Alessandria and Choi (2011) presents a version of the Melitz (2003) model which does include capital as a production factor. However, due to their objective being different from that of the present paper, their model is much more complex and differs with respect to key assumptions. Most notably they maintain the assumption that the entry investment of firms are held in terms of labor, which makes investment in firm entry and capital accumulation less direct substitutes. Further, they dispense with the mechanism causing firms to exit the market voluntarily, implying that the steady-state distribution of productivities among incumbent firms is exogenous. Taken together, the differences blur or eliminate the clear cut effects of capital described above. Ultimately, capital does not play a major role in the adjustment dynamics of their model, which becomes apparent when they provide results abstracting from capital accumulation. This is in stark contrast to the model developed here.

Having stressed the main results of including endogenous capital accumulation, let me note that the amplification effect is not completely uninteresting when related to the anti-variety effect discussed by Baldwin and Forslid (2010). As will be shown, the amplification effect tends to accommodate a larger mass of firms and thus variety in steady-state, in spite of the labor supply being fixed. This aspect of the amplification effect is manifested slowly as capital is accumulated, thus adding a short-run versus long-run perspective to the anti-variety effects.

With the above in mind, it should be clear that adding endogenous capital accumulation to the Melitz (2003) framework has the possibility to not only generate an amplification effect in the aggregate but also to generate rich adjustment dynamics at the firm and industry level. Further, studying the transitions to which the economy is subjected in this framework will provide key insights regarding the relative evaluation of different forms of trade liberalizations.

The remainder of this paper is organized as follows. Section 2 develops the model. Section 3 consideres the steady state and build intuition of long-term effects of different forms of trade liberalizations. Section 4 provides numerical results for the transitions induced by different trade liberalization and analyzes these in depth, drawing upon the results of Section 3. Section 5 concludes.

2 The Model

The model of the present paper is developed by extending the Melitz (2003) model of international trade with heterogeneous firms into the Ramsey (1928) framework of endogenous capital accumulation. This is done by applying the Melitz (2003) extension of heterogeneous firms to the Ethier (1982) intermediate-goods interpretation of the Krugman (1979) model. A brief overview of the resulting model structure can be given as follows. The world consists of two symmetric countries.[1] Hence, in the following there will be no distinction with respect to country for any of the variables. Each country has L identical households and two production sectors; one producing a range of intermediate goods and one combining these with labor to produce a single final good. The final good will serve as the numéraire and can be used for consumption, capital accumulation or investment in firm entry. Production of intermediate goods requires capital and labor as inputs, and capital is accumulated as a result of the consumption and saving decision made by utility-maximizing households.

The remainder of this section describes the detailed setup. Subscripts will denote time for appropriate variables in the following.

2.1 Households

L identical households with an infinite horizon each provide an inelastic labor supply of one unit. The utility function has constant elasticity of intertemporal substitution,

[1] Two countries is choosen for simplicity, it is straight forward to extent the model to n countries.

and households maximize discounted instantaneous utility

$$U_t = \int_t^{\infty} e^{-\rho(s-t)} \frac{C_s^{1-\theta}}{1-\theta} \,\mathrm{d}s, \qquad \theta, \rho > 0,$$

where C is consumption. The disposable income of the representative household comprises of labor income, interest payment on the capital stock and profits of the firms in the intermediate-good sector, which add up to the value of the production of the final good, Y, less what is used for fixed costs, F. This disposable income is used for consumption, C, investment in market entry in the intermediate-good sector, I, and capital accumulation. Thus

$$\dot{K}_t = Y_t - F_t - I_t - C_t - \delta K_t, \tag{1}$$

where $\delta \geq 0$ is the depreciation rate of capital. If possible, the savings of households will be allocated between capital accumulation and investment in market entry, such that the returns from these two investments are equalized. However, in line with Melitz (2003) investment in firm entry is sunk and instant rebalancing may not be possible. If capital for some reason earns a return above that of investment in firm entry households will be at a corner solution in their investment decision until enough capital has been accumulated to realign the returns on the two types of investment. With respect to capital I will assume that investments are not sunk. This will mean that through households allocating invested resources optimally, the interest rate on capital will at any point in time reflect the highest possible return the households can earn on foregone consumption. Therefore the interest rate on capital is what appears in the Euler equation obtained from the maximization problem of the representative household. This Euler equation is given by

$$\dot{C}_t = \frac{1}{\theta} \left(r_t - \rho \right) C_t, \tag{2}$$

where r denotes the net return on capital; depreciation is thus borne by the firms.

2.2 Final-Good Sector

As mentioned above, the final good is produced using labor and intermediate goods. The specific production function of this sector is analogous to that of Romer (1990) and Rivera-Batiz and Romer (1991) albeit excluding human capital,

$$Y_t = \widehat{L}_t^{1-\alpha} \int_{\omega \in \Omega_t} \left(x_t(\omega) \right)^{\alpha} \mathrm{d}\omega, \quad 0 < \alpha < 1,$$

where $\widehat{L}$ denotes the amount of labor devoted to final good production, $x(\omega)$ denotes the amount of intermediate good ω used, and Ω is the set of available intermediate goods. Note that the model allows for labor productivity gains arising from new imported varieties.[2] The final-good sector is characterized by perfect competition, which implies that production factors are rewarded their marginal products,

$$w_t = (1-\alpha)\, Y_t/\widehat{L}_t \tag{3}$$

$$p_t(\omega) = \alpha \widehat{L}_t^{1-\alpha}(x_t(\omega))^{\alpha-1}, \tag{4}$$

where w denotes the real wage and $p(\omega)$ denotes the price of good ω. Producers in the intermediate sector faces a demand function (4) with constant price elasticity $\sigma \equiv 1/(1-\alpha)$, as is the case in Melitz (2003). The fact that demand for a given variety in the present model does not depend on the price of other varieties does not have major implications and contributes greatly to the tractability of the model, since it implies that the only channel of competition among firms in the intermediate-good sector is competition over inputs.

2.3 Intermediate-Good Sector

This sector is almost identical to that of Melitz (2003) except that the production of intermediate goods uses capital as well as labor as an input. The general setup of the sector is as follows. A mass of heterogeneous firms each produce a unique variety of the intermediate good that is used for final-good production. The sector is characterized by free entry, and there exists an unbounded pool of potential entrants. Entry into this sector requires a sunk investment of S units of the final good. When a firm has sunk this entry cost, it is able to produce a new variety ω of the intermediate good with productivity $\varphi(\omega)$, which is drawn from a known exogenous distribution $G(\varphi)$. I will use the Pareto distribution with scale parameter $\varphi_0 > 0$ and shape parameter $m > \sigma - 1$,[3]

$$G(\varphi) = \begin{cases} 1 - (\varphi_0/\varphi)^m & \text{for } \varphi > \varphi_0 \\ 0 & \text{otherwise} \end{cases} .$$

Production, x, of an intermediate good by a firm with productivity φ takes place according to a Cobb-Douglas production function of the form

$$x(k, l; \varphi) = \varphi k^{\gamma} l^{1-\gamma}, \tag{5}$$

where k and l are the amounts of capital and labor employed respectively. In addition the firm have to pay a fixed cost of f units of the final good to maintain production. Upon entry a firm faces the downward-sloping demand function (4) from both the domestic and foreign producers of the final good. However, before

[2]Broda et al. (2006) document the importance of this channel of productivity growth.

[3]The assumption of Pareto distribution is widely used in the literature, see e.g. Chaney (2008), Melitz and Ottaviano (2008), and Baldwin and Forslid (2010). Recent empirical support for the assumption is presented by e.g. Axtell (2001) and Eaton et al. (2008). The assumption $m > \sigma - 1$ ensures that expected firm profits are bounded.

a firm can sell to the foreign producers of the final good it must enter the export market, which requires an additional sunk investment of S^x units of the final good. Further, exports are subject to an iceberg trade cost, $\tau > 1$, and to maintain export production the firm has to pay a fixed cost of f^x units of the final good.

Intertemporal optimization of firm value requires maximization of instantaneous profits conditional on the markets serviced. For production to the domestic market profit maximization leads to input demand functions for capital and labor, $k^d(r, w; \varphi)$ and $l^d(r, w; \varphi)$ respectively, an indirect output function, $x^d(r, w; \varphi)$, and an indirect profit function, $\pi^d(r, w; \varphi)$.[4] Corresponding functions exists for production to the export market, and are denoted by superscript x.

During transitions firms are characterized by their productivity, φ, and their production status, z. The instantaneous profits of a firm with productivity φ and production status z are given by

$$\pi(\varphi, z) = \begin{cases} \pi^d(r, w; \varphi) + \pi^x(r, w; \varphi) & \text{if } z = x \text{ (exporter)} \\ \pi^d(r, w; \varphi) & \text{if } z = d \text{ (non-exporter)} \\ 0 & \text{if } z = n \text{ (non-producer)} \end{cases},$$

where the dependence on r and w is suppressed for notational simplicity. Optimal choice of production status is described in the following subsection.

To sum up the export costs, the exporting firms are subject to a variable, a fixed and a sunk export costs. This allows three means of liberalizing trade.

2.4 Entry and Exit Decisions of Firms

As the main contribution of this paper lies in the analysis of transition paths, optimal firm behavior outside of steady-state obviously needs to be described. That is the objective of the present subsection. An exporter can choose to leave the export market thus becoming a non-exporter and a non-exporter can choose to enter the export market thus becoming an exporter. However, once a firm has chosen to cease production all together to become a non-producer it has shut down for good. Apart from firms voluntary exiting from the sector, firms are hit by an exogenous shock which occurs at the rate η and forces the affected firms to shut down. In choosing the optimal production status at any point in time firms will consider the impact on their value over the full time horizon rather than merely the impact on current profits.

Let $V(\varphi, z)$ denote the value function of a firm with productivity φ and production status z. For $z_t \in \{d, x\}$ this value function satisfies[5]

$$(r_t + \eta) V_t(\varphi, z_t) = \max_{z^*} \left\{ \pi_t(\varphi, z^*) + \tfrac{\mathrm{d}}{\mathrm{d}t} V_t(\varphi, z^*) + V_t(\varphi, z^*) - V_t(\varphi, z_t) - \mathbf{1}_x S^x \right\}, \quad (6)$$

[4]Expressions for input demand functions as well as indirect output and indirect profit functions for both the domestic and the export market can be seen in Appendix A.

[5]For $z_t = n$ we have $V(\varphi, n) = 0$.

where $z^* \in \{n, x, d\}$ and $\mathbf{1}_x$ is an indicator variable for export entry.[6] The equation (6) can be interpreted as a no-arbitrage condition. The right hand side is the yield of maintaining the firm, which comprises of current profits, the change in value over time conditional on surviving (capital gain) and the change in value due to an optimal change in production status. In optimum this must equal the left hand side, which is the interest payments on the value of the firm using the interest rate adjusted for the probability of a death shock occurring, since the opportunity cost of investing in a firm is investment in capital, which provides the net return r. Note that when adjusting the interest rate to take account for the probability of a death shock occurring, it is implicitly assumed that firms are risk neutral. Although households (who ultimately own the firms) are risk averse, this is compatible with optimal household behavior as there is no uncertainty at the aggregate level. Hence η can be thought of as the depreciation rate on aggregate investment in firm entry.[7]

Now, let φ^q denote the lowest value of φ for which non-exporters choose to continue domestic production. Further, let φ^{qx} be the lowest value of φ for which exporters choose to continue their export production. Finally, let φ^{ex} denote the lowest value of φ for which non-exporters chooses to become exporters. I will maintain the assumption from Melitz (2003) of firms being partitioned into exporters and non-exporters, i.e. $\varphi^q < \varphi^{ex}$, which imposes restrictions on the export costs.[8] Thus, in steady state there will be a non-zero mass of non-exporters. Further it is evident that $\varphi^{qx} < \varphi^{ex}$ because of the sunk cost of entering the export market. The three cutoff productivities are determined by

$$\varphi_t^q = \inf\{\varphi : V_t(\varphi, d) > 0\} \tag{7}$$
$$\varphi_t^{qx} = \inf\{\varphi : V_t(\varphi, x) > V_t(\varphi, d)\} \tag{8}$$
$$\varphi_t^{ex} = \inf\{\varphi : V_t(\varphi, x) > V_t(\varphi, d) + S^x\}. \tag{9}$$

In the following φ^q and φ^{ex} will be referred to as the production cutoff and the export cutoff respectively. Due to the endogenous evolution of the cutoff productivities during transitions, the productivity distribution of firms and exporters, $H_t(\varphi)$ and H_t^x respectively, will in general be time varying.

Using the above value function, we can express the expected value of entering the market. Upon entry, the firm has production status d, as prospective entry into the export market is undertaken subsequently and requires an additional irreversible investment. Of course the firm can choose to shut down immediately if optimal. The expected value of entry, V^e, can thus be written as

$$V_t^e = \int_{\varphi_t^q}^{\infty} V_t(\varphi, d)\, \mathrm{d}G(\varphi) - S. \tag{10}$$

[6]Thus, $\mathbf{1}_x = 1$ if $z_t = d$ and $z^* = x$. Otherwise, $\mathbf{1}_x = 0$.

[7]The death shock is an idiosyncratic risk that can be diversified away, in the sense that it is reduced to deterministic depreciation.

[8]These restrictions are analogous to that of Melitz (2003), and can be seen in Appendix B.

Remember that the intermediate-good sector has free entry and, since firms are risk neutral, this requires the expected value of entry to be non-positive. If $V^e < 0$, the households are at a corner solution in their investment decision as described above and no firms will enter the intermediate-good sector, i.e. the mass of entrants, M^e, is zero. Otherwise M^e is determined by the condition that the returns from the two types of investment are aligned, i.e. $V^e = 0$.

Combined with the endogenous entry and exit decisions of firms this leads to movements in the mass of firms, M, which evolve according to

$$\dot{M}_t = M_t^e - \eta M_t - M_t^q \tag{11}$$

where M_t^q is the mass of firms exiting the market voluntarily. Completely analogous to (11) the mass of exporters, M^x, evolve according to

$$\dot{M}_t^x = M_t^{ex} - \eta M_t^x - M_t^{qx} \tag{12}$$

Finally, the mass of available varieties, M^a, in either country consists of the mass of firms, M, plus the mass of exporting firms, M^x, from the foreign country.

2.5 Market Clearing

Now, having described the details of the two production sectors of the economy, consider the resulting aggregate market clearing conditions which must be satisfied at all points in time. Analogous to Melitz (2003) I will define an average productivity level, $\tilde{\varphi}$, which will prove useful for this purpose. The average productivity of the intermediate-goods sector will be defined as

$$\tilde{\varphi}_t \equiv \left(\int_{\varphi_t^q}^{\infty} \varphi^{\sigma-1} \, \mathrm{d}H_t(\varphi) + \tau^{1-\sigma} \frac{M_t^x}{M_t} \int_{\varphi_t^{qx}}^{\infty} \varphi^{\sigma-1} \, \mathrm{d}H_t^x(\varphi) \right)^{\frac{1}{\sigma-1}}. \tag{13}$$

Market clearing requires that the factor reward condition (3) is satisfied and that demand for labor for production of the final good and intermediate goods equal the overall supply,[9]

$$L = \widehat{L}_t + M_t l_t^d(r, w; \tilde{\varphi}_t). \tag{14}$$

Further, market clearing implies that the demand for capital for production of intermediate goods equal the supply of capital, which is given by the capital stock,

$$K_t = M_t k_t^d(r, w; \tilde{\varphi}). \tag{15}$$

[9]Note that $M_t l^d(r, w; \tilde{\varphi}_t)$ takes total labor demand for production of intermediate goods to both the export and the domestic market into account, due to the convenient definition of $\tilde{\varphi}$. Analogously for the market clearing condition for capital.

It can be shown by combining (3) and (14) that $\widehat{L}$ is a constant fraction of L over time, and thus the time subscript is dropped. This means that, as in Melitz (2003), labor supply to the intermediate-good sector will effectively be perfectly inelastic. Exploiting this result, it is seen from (3) that final-good production is proportional to the real wage,

$$Y_t = \sigma w_t \widehat{L}. \tag{16}$$

Since $\widehat{L}$ is constant (16) implies that labor's share of output is constant.

It must hold that the total flow of fixed costs, F, which enters the dynamic budget constraint (1) is given by the sum of fixed costs for domestic and export production, thus

$$F_t = M_t f + M_t^x f^x.$$

Finally, the investment in entry, I, which comprises of investment undertaken by new firms and by firms entering the export market, is given by

$$I_t = \frac{M_t^e}{1 - G(\varphi_t^q)} S + M_t^{ex} S^x,$$

since M_t^e denotes the mass of successful new firms (with $\varphi > \varphi^q$) and unsuccessful firms (with $\varphi < \varphi^q$) also sinks the entry cost S prior to learning their productivity level.

3 Steady State

The understanding of the steady-state effects of trade liberalizations will be useful for interpreting the transitions they induce. Therefore, the present section considers the steady state before we move on to analyze the transitions in Section 4. First, I will discuss the trade liberalizations that will be considered and compared in the following. Next, I will briefly describe the steady-state effects of these liberalizations, with emphasis on the important differences relative to the Melitz (2003) model. In the interest of space, I will not present the details of the solution for the steady state and the obtained closed-form expressions here.[10]

3.1 The Considered Trade Liberalizations

The trade liberalizations that will be considered in the following will take either of three forms corresponding to a reduction in either of the three export costs. The first will be a reduction of the variable export cost, τ, the second will be a reduction of the sunk export cost, S^x and the third will be a reduction of the fixed export cost, f^x. To be able to meaningfully investigate the differences between the effects of these liberalizations they need to be conducted in a way such that they are comparable. To ensure that this is the case, I will assume in the following that the

[10] The interested reader is referred to to Appendix B

three liberalizations are conducted such that they have the same (negative) effect on the steady-state share of expenditure on domestic varieties in the intermediate-good sector, λ. This will mean that they have the same effect on "openness" as defined by Baldwin and Forslid (2010). One can show that λ is given by

$$\lambda = \left(1 + \tau^{-m} \bar{f}^{1-m/(\sigma-1)}\right)^{-1}, \qquad \bar{f} = \frac{f^x + (\rho + \eta) S^x}{f}. \tag{17}$$

That the considered trade liberalizations have the same effects on λ will by construction imply that they have the same (positive) effect on the steady-state share of expenditure on imported variety in the intermediate-good sector. Further, it will imply that the steady-state effects on e.g. final-good production and on the real wage are the same across the liberalizations. In effect, this is a natural way to make the liberalizations comparable.

That the liberalizations are comparable in the above sense will not mean that all of their effects are the same. Indeed, as we will see in the remainder of the following section and in Section 4, there will be significant and interesting differences both in steady state as well as during transitions.

All the effects of trade liberalization considered in the remainder of this section will be steady-state effects.

3.2 The Effects of Trade Liberalizations on Exports

It should come as no surprise that all of the trade liberalizations decrease the export cutoff, φ^{ex}, and increase the mass of exporters, M^x. It can be shown that the S^x- and the f^x-liberalization have the same effects on these variables. Moreover, these two liberalization will have more pronounced effects on both the export cutoff and the mass of exporters when compared to the τ-liberalization. The reason is that the S^x- and the f^x-liberalizations achieve the expansion of trade primarily through the extensive margin of trade whereas the τ-liberalization achieves the expansion primarily through the intensive margin of trade. This is qualitatively no different from the Melitz (2003) model, but it is important to keep in mind in the following. Indeed, the important differences between the three liberalizations arise partly through their different effects on the intensive and extensive margin of trade and hence on the mass of exporters.

3.3 Other Production Effects of Trade Liberalizations

Whereas the effects on the composition of exports may differ among the three liberalizations, the fact that they are conducted such that they have the same effect on the share of expenditure on domestic varieties will imply that their (positive) effects on the production cutoff, φ^q, and the average productivity in the intermediate-goods sector, $\tilde{\varphi}$, are the same. Further, the effect on final-good production, Y, and the real wage will also be the same, as noted above. Both Y and w will of course rise.

Regarding the mass of firms M, the effects of the three liberalizations will once again be the same. However, there will be a difference when compared to the original Melitz (2003) model, in which the mass of firms unambiguously decreases in the wake of a trade liberalization when assuming the Pareto distribution as pointed out by Baldwin and Forslid (2010). This need not be the case in the present model however. Due to the fixed and sunk costs of firms being held in units of the final good, the production of which expands, the result of a trade liberalization can be an increase in M, and thus in what could be called the total mass of world variety (given by $2M$ as the countries are symmetric). The steady-state mass of firms can be shown to be increasing (decreasing) following either of the trade liberalizations if $m(1-\gamma)$ is less (greater) than 1. The intuition behind this condition relates to the ability to accumulate capital. On one hand a larger value of γ means that capital is more important in intermediate-goods production, and therefore that more capital will be accumulated following the trade liberalization. This will provide a larger expansion in final-good production, Y, which is the resource used for fixed and sunk costs of firms. On the other hand trade liberalization increases the probability of unsuccessful entry – given by $G(\varphi^q)$ – and it can be shown that is does so more when m is large.[11] As a consequence, when m is larger, a given trade liberalization will tend to cause more resources to be devoted to unsuccessful entries thus resulting in less firms. The magnitude of the two parameters m and γ taken together determines whether the expansion of final-good production is enough to accommodate a larger mass of firms or whether the increase in the share of entries that are unsuccessful will dominate.

Baldwin and Forslid (2010) focuses to some extent on the possible presence of an anti-variety effect on the mass of available varieties, M^a. They show that the mass of available varieties is increasing (decreasing) in a τ-liberalization when a variable equivalent to $\bar{f}$ of the present model is less (greater) than 1. In the present model this may not be the case. Analogous to above, the intuition for this is the expansion in final-good production. In the present model, an increase in the total mass of available varieties can arise following a τ-liberalization even if the mass of firms decrease and export production has larger fixed and amortized sunk costs than the fixed cost of domestic production. This is because it is possible that the expansion of final-good production more than compensates for the higher non-variable costs of the exporters from the foreign country that "replace" domestic firms. Hence the anti-variety effect with respect to available varieties discussed by Baldwin and Forslid (2010) is less likely to arise in the present model.[12]

[11] Intuitively, when m is large there is more mass around the production cutoff and therefore the effect of the increase in φ^q is larger.

[12] Melitz and Ottaviano (2008) presents a version of the Melitz (2003) model in which the anti-variety effect with respect to the mass of available varieties never occur. However, this result relies on the assumption of perfectly-elastic labor supply to the differentiated-goods sector. In the present model the results are obtained in a setting where the labor supply to the intermediate-good sector, $L-\widehat{L}$, is perfectly inelastic.

Clearly, the steady-state effect from the expansion in final-good production will be realized slowly during the transition as capital accumulates. Immediately following the trade liberalization the magnitude of the anti-variety effects will thus be very different from the long run as we will see in Section 4. The present model will therefore add a time dimension to the anti-variety effects.

3.4 Effect on Total Investment and Consumption

So far, apart from refining the anti-variety effects of Baldwin and Forslid (2010), none of the above is really new. Further, of the effects considered so far the three liberalizations differ only with respect to the effect on the composition of exports. Even in this respect two of the liberalizations are equivalent. So where are the important differences advertised above? Let me show you. First, note that the similarity of the three liberalizations is a reflection of them being conducted in a comparable way. The difference between the three liberalizations will assert itself through their different effects on investment and thereby on aggregate consumption. Arkolakis et al. (forthcoming) shows among other things that in the Melitz (2003) model the effect of trade liberalizations on steady-state consumption (and therefore welfare) is determined solely by their effect on λ, when assuming Pareto distributed productivity draws. As will be evident, the modifications of the present model imply that this is no longer true.[13] The reason is, that due to positive subjective discounting the extensive margin of trade matters through aggregate investment and not only through trade flows.

Consider the aggregate steady-state assets, A, held by households. These comprises of the capital stock and accumulated investment in entry into the domestic and export market undertaken by producing firms. The assets will thus be given by

$$A = K + S\frac{M}{1-G(\varphi^q)} + S^x M^x, \tag{18}$$

where the last two terms are the investment in new firms and in export entry that is needed to replicate the steady-state stock of firms.[14] The three different trade liberalizations can be shown to have the same effects on the first two terms on the right hand side of (18). The effect will be positive since trade liberalization ceteris paribus raises the average reward to investing in firms and capital. However, the three trade liberalizations will have different effects on the last term, accumulated investment in export entry, partly through their different effects on export composition. Thereby they will have different effects on A. Remember the above discussion on the effect of the trade liberalizations on the mass of exporters and take the τ-liberalization as

[13]In the present model, final good production can be said to take the role of consumption in Arkolakis et al. (forthcoming). Indeed, the effect of a trade liberalization on final good production does only depend on the effect on λ in the present model.

[14]In steady-state, when the value of entry is zero, the value of incumbent firms is equal to the replacement cost.

the reference. By construction, the S^x-liberalization decreases the export investment per exporter, S^x, relative to the τ-liberalization. This effect turns out to dominate the larger increase in the mass of exporters induced by the S^x-liberalization, such that $M^x S^x$ is lower following the S^x-liberalization relative to the τ-liberalization. The f^x-liberalization on the other hand increases the accumulated export investments relative to the τ-liberalization, since it induces a larger increase in the mass of exporters (cf. above) and both liberalizations leave the investment per exporter unchanged. Thus, through different effects on $M^x S^x$, the following ranking of the level of household assets following the three trade liberalizations will prevail,

$$A_{f^x\text{-lib}} > A_{\tau\text{-lib}} > A_{S^x\text{-lib}}. \tag{19}$$

Next, consider the effects on aggregate consumption. The ranking of the effects on the assets held by households will translate into a corresponding ranking in steady-state consumption, C. To see this, consider the following simple expression that can be obtained for C.

$$C = wL + rA. \tag{20}$$

The equation (20) states that consumption is equal to labour income and net return of assets. Net return on assets is given by the steady-state interest rate r, which will be equal to the subjective discount factor $\rho > 0$. This illustrates how the investment decision of firms is compatible with the consumer preferences, since on the aggregate level the resources invested in firms provide the same net return as the capital that is accumulated. From (20), it is clear that since all three trade liberalizations have the same positive effect on w, cf. above, their effect on consumption will differ through their effects on A and the implied interest payments.[15] Now referring to the ranking in (19) of the level of assest following each of the trade liberalizations, it is obvious that a corresponding ranking of steady-state consumption levels will prevail, i.e.

$$C_{f^x\text{-lib}} > C_{\tau\text{-lib}} > C_{S^x\text{-lib}}.$$

Even though the three liberalizations are conducted such that they have the same effect on the share of expenditure on domestic varieties in the intermediate-good sector and consequently have very similar effects on the economy (cf. above), there exists an unambiguous ranking of the three liberalizations with respect to their effect on steady-state consumption. However – and this is a big however – this is *not* sufficient to conclude that a corresponding ranking of the impact on welfare exists. To asses the welfare implications, it is crucial to take consumption over the full transition into account. Indeed, the ranking of the levels of welfare following the three liberalizations that is obtained in Section 4 will not coincide with the above ranking of steady-state consumption levels.

[15]Melitz (2003) assumes zero discounting of firms profits (in excess of what accounts for the death shock). This means that net profits of firms are zero and therefore consumable income only consists of labor income.

4 Transitions Induced by Trade Liberalizations

Now, having considered the steady-state effects we will move on to analyze the transitional dynamics induced by the different trade liberalizations. These will turn out to differ in important ways, and the results will show the importance of taking the transitions induced by the liberalizations into account. I start by discussing the choice of parameter values and the three specific trade liberalizations that are considered. Then, the adjustment dynamics arising from a τ-liberalization is analyzed in detail and using this as a benchmark the effects of S^x- and f^x-liberalizations with the same impact on steady-state share of expenditure on domestic varieties in the intermediate-good sector, λ, are discussed. Finally, the welfare implications of the three liberalization when taking the transition into account is contrasted with the above steady-state results.

4.1 Baseline Parameter Values

This subsection discusses the parameter values used in the numerical solutions for transition paths. The number of households, L, and the scale parameter of the productivity distribution φ_0, are set to 1. The subjective discount rate of the households, ρ, and the depreciation rate of capital, δ, are set to 0.05 and 0.10 per year respectively, which is rather uncontroversial. This means that the total flow cost of renting a unit of capital is 0.15 per year for firms in a steady state. The rate with which firms are hit by the death shock, η, is set to 0.05 per year, which means that the expected lifetime of a firm in steady state is 20 years. The value of θ is chosen to be 2, which is standard in the literature and corresponds to an elasticity of intertemporal substitution of 0.5. The price elasticity of demand, σ, for a variety of intermediate good is set to 3, which is close to the estimate of 2.98 reported by Eaton et al. (2008) and in the ballpark of the 3.79 used by Bernard et al. (2003). The implied value of α is 2/3. The value of γ is chosen to be 0.5, implying that labor's share of output is $wL/Y = 0.56$, capitals share of output is $(r+\delta)K/Y = 0.22$, and gross profits[16] of firms as a share of output is 0.22, all in steady state. The shape parameter of the Pareto distribution of productivity draws, m is chosen to be 3.5. This means that the Pareto distribution of firm size has shape parameter $m/(\sigma-1) = 1.75$. Axtell (2001) finds this to be around 1, but the parameter restriction $m > \sigma - 1$ implies that it must be larger than one. Further, Chaney (2008) obtains an estimate of $m/(\sigma-1)$ of around 2 and Eaton et al. (2008) reports one estimate to be 1.75. In line with Burstein and Melitz (2011) and Atkeson and Burstein (2010) the sunk entry cost of firms, S, is set to 1, and the fixed cost of production f is set to 0.1. Table 1 summarizes the choice of parameter values.

This choice of parameter values implies $m(1-\gamma) = 1.75$, which means that the steady-state mass of firms will decrease following all trade liberalizations[17]. The

[16] Profits before fixed and sunk costs.
[17] See subsection 3.3.

Table 1: Baseline Parameterization.

$B = 30$	$\rho = 0.05$	$\theta = 2$	$L = 1$
$f = 0.1$	$\delta = 0.10$	$\gamma = 0.5$	$\varphi_0 = 1$
$S = 1$	$\eta = 0.05$	$\alpha = 2/3$	$m = 3.5$

steady-state mass of available varieties on the other hand will turn out to increase following all of the liberalizations.

4.2 Trade-Cost Variables and Liberalizations

In the initial steady state the values of the trade-cost variables are chosen to be $\tau = 1.25$, $f^x = 0.1$ and $S^x = 1$. Thus the export market is characterized by an entry cost and a fixed cost of the same sizes as those characterizing the domestic market. This means that the entry investment in connection with export is significant as indicated by Roberts and Tybout (1997). Further, the ratio of fixed and amortized sunk costs of export to fixed costs of domestic production is $\bar{f} = 2$ in the initial steady-state.

The first liberalization that will be considered is a τ-liberalization where τ is unexpectedly reduced to 1.20 immediately and held constant thereafter. The second and third liberalizations will be a S^x- and a f^x-liberalization respectively, which will immediately and permanently reduce the sunk and fixed cost of export. The reduction will be approximately 35% in both cases, which is chosen such that the three liberalizations all have the same effect on λ. Thus, the export entry cost requires a larger reduction than the variable cost to produce the same effect on λ, which is somewhat in line with the findings of Das et al. (2007). Following either of the trade liberalizations the steady-state share of expenditure on domestic varieties in the intermediate-good sector will change from 0.79 in the initial steady state to 0.76 in the new.

4.3 Liberalization Using Variable Export Costs

This subsection provides numerical results for the τ-liberalization described above. Solving numerically[18] for the transition path following the described liberalization gives the results shown in Figure 1. In the following, I will first describe the underlying mechanisms and then discuss the most interesting features of the transition path.

First consider what happens immediately at the time of the trade liberalization. The liberalization implies that exports become more profitable. Therefore φ^{ex} decreases and the inputs, K and L, experience a discrete increase in their marginal products causing their rewards, r and w, to increase. This increase forces the least productive firms to exit, as the value of these firms at the new higher factor prices

[18] Uses ox programme `iterate11.ox`

is negative, i.e. φ^q increases. Further, this causes the mass of firms to decrease. The decrease in the export cutoff causes a mass of firms to enter the export market immediately. The immediate net effect of the liberalization on the mass of available varieties is negative for the choosen parameter values. The lower variable export cost and the reallocation of resources from less productive firms to more productive firms is reflected in the increase in average productivity.

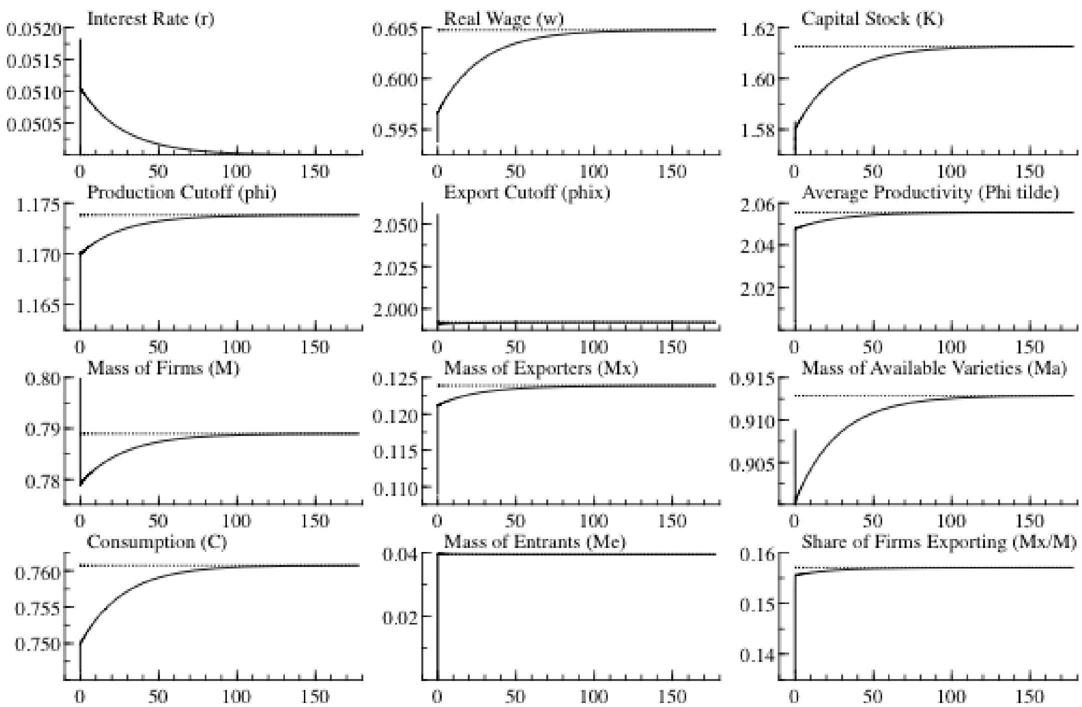

Figure 1: Transition path following a reduction in τ from 1.25 to 1.20. Dotted lines indicate the new steady state.

From an investment point of view, an immediate effect of the liberalization is that investment in entry of new firm becomes more attractive due to more profitable exports on average. Further, investment in capital becomes more attractive due to increased marginal product. However, this is not the full story. As mentioned above, the trade liberalization makes a mass of firms enter the export market immediately, their investment discretely decreasing the capital stock. This makes capital even scarcer, thus further increasing the interest rate. This does not only increase the attractiveness of capital accumulation it also discourages investment in entry of new firms, since the increased interest rate implies lower instantaneous profits but also harder discounting of future profits. As investment in firm entry cannot be recouped (it is sunk) and converted into capital stock, the result is that immediately following the liberalization capital is the more attractive investment. Households thus finds themselves at a corner solution with respect to their allocation of resources devoted to investment. I.e. the expected value of entry of new firms is negative and no such entry takes place for a period following the liberalization. As investment is focused on capital accumulation and entry into the export market in this period, capital accumulates relatively fast and the interest rate drops quickly. Zero entry takes place for about 0.10 year until the interest rate have decreased sufficiently and value of entry is zero once again. Note that due to the exogenous death shock, zero entry implies that the mass of firms decrease during this period. This relaxes the competition over inputs and therefore contributes to the decrease in the interest rate. Another implication of the decreasing mass of firms is that the real wage, the cutoffs and final-good production actually decreases slightly while entry is zero.

After the initial period of zero entry, as the interest rate has decreased sufficiently, the value of entry reaches zero again and investment in entry by new firms takes place alongside capital accumulation. This means that invested resources are now split between capital accumulation and investment in firm entry. Thus capital accumulates slower and the interest rate decreases at a much slower pace than during the period of zero entry. The most notable implication of the decreasing interest rate is that the return required on investment in firm entry decreases over time. Put another way, entry becomes cheaper relative to prospective discounted profits, which ceteris paribus makes potential entrants more eager to enter the market. Because of free entry the result is intensified competition over inputs. This means that only more productive firms can survive such that the production cutoff increases over time as capital is accumulated. Thus, in the short run some existing low-productivity firms are sheltered from intensified competition by the higher interest rate. As a result, the production cutoff only adjusts some of the way towards the new steady-state value immediately and thereafter approaches the new, higher steady-state value slowly as the interest rate decreases and competition intensifies. Analogously, the export investment also becomes smaller relative to prospective future export profits and a larger share of firms will export over time. In effect the full gain in average productivity from reallocation of resources towards the more productive firms is not realized immediately following the liberalization. A part of the gain in average productivity manifests itself over a long horizon since in the short run the higher interest rate dampens competition over inputs, which is the source of the reallocation.

At the aggregate level the accumulation of capital implies that labor becomes relatively more scarce over time, which leads to an increasing real wage, w, as time goes by. Further, the accumulation of capital makes room for more firms and exporters while also increasing final-good production. Consumption is increasing at a decreasing rate as the interest rate is above the subjective discount rate and falling. Note that households smooth consumption over time such that it is increasing even during the period of zero entry where final-good production is decreasing.

Having described the main features of the adjustment process, some characteristics are worth further inspection. First of all, due to the above mentioned sheltering effect of the interest rate, the production cutoff adjust only about two thirds of the way toward the new steady-state value immediately following the trade liberalization. This means that about 2.1 per cent of the firms are forced to exit immediately. However, 1.1 per cent of the remaining firms still have productivities below the production cutoff of the new steady-state and they stay in the market only to reap profits during the transition. As mentioned above, this is made possible by the higher interest rate to some extent sheltering them from competition from new firms. As capital is also a production factor, the immediate increase in the interest rate implies that 0.8 per cent of firms earn negative profits immediately following the liberalization. These firms stay in the market despite negative instantaneous profits because the sharp decrease in the interest rate during the period of zero entry is anticipated, and their net present value over the full horizon is positive. These short-run negative profits are to some extent a by-product of being sheltered by the higher interest rate.

Second, after the period of zero entry it takes more than 30 years for the production cutoff to adjust 80 per cent of the remaining way towards the new steady-state value. This is more than the expected lifetime of a firm. Thus a firm producing when the trade liberalization is conducted cannot expect to survive until the economy have completed most of the adjustment towards the new steady state. This highlights the importance of considering what goes on between steady states. Long-term adjustment also arises in the average productivity of the intermediate-good sector and in the share of exporters following the trade liberalization, even though most of the increase in these variables towards their new steady-state values are realized immediately. However, if the direct effect on average productivity of the decrease in τ is ignored, 73 per cent of the remaining effect (caused by reallocation) are realized immediately. A certainly non-negligible 27 per cent of the productivity gain caused by reallocation of inputs are realized slowly during the transition as the interest rate comes down and competition intensifies. All in all, the full firm-level effects from trade liberalization is manifested over a long horizon. Costantini and Melitz (2007) and Alessandria and Choi (2011) also studies the firm-level effects of trade liberalization in a model based on the Melitz (2003) framework. Costantini and Melitz (2007) find that following an unannounced abrupt liberalization using the variable cost of export, the production cutoff jumps to the new steady state value immediately. Alessandria and Choi (2011) finds that the effect of a reduction in the variable trade cost on average productivity of the differentiated goods sector is small initially, then larger in the short run, only to die out after 50 years (due to exogenous cutoff productivity). These results are in contrast with the result of the present model, which features long-term adjustment as well as a level effect in the production cutoff and the average productivity of firms.

Third, it is not only the adjustment of the above mentioned firm-level variables that takes a long time. The adjustment in most of the depicted variables following the relatively modest change in τ takes quite some time. For example it also takes more than 30 years for the real wage and the consumption level to adjust 80 per cent of the way from the old steady state to the new. Thus, as was the case with the firm-level effects from the trade liberalization, the aggregate steady-state effects on e.g. consumption are realized over a long horizon.

Fourth, the effect of the trade liberalization on the steady-state mass of firms is a reduction of about 1.4 per cent. However, after the 0.10 year of zero entry following the trade liberalization the mass of firms is about 2.6 per cent lower than the initial steady-state level. This is due to the fact that the positive effect of the expansion in final-good production is not that pronounced in the short run. This indicates that due to slow capital accumulation the anti-variety effect with respect to the mass of firms is much more pronounced in the short run than in the new steady state.

Fifth, the combined immediate effect of a decreasing mass of domestic producers and increasing mass of exporters on the total mass of available varieties is negative in this case. After the 0.10 year of zero entry following the trade liberalization, the mass of available varieties have decreased 1.0 per cent relative to the initial steady state.[19] Immediately following the trade liberalization, before the effect of capital accumulation on final-good production can be realized, the higher non-variable cost of exports relative to domestic production means that the exporters that replace domestic producers does so less than one for one. The mass of available varieties then decreases further during the period of zero entry. Only after the first 0.10 years, as firm begin to enter the market again, is this anti-variety effect with respect to available varieties mitigated. However, the mass of available varieties in the new steady state is actually 0.4 per cent higher than in the initial steady state. Thus, even though the mass of available varieties in the new steady state is higher than in the old and thus no anti-variety effect with respect to the mass of available varieties are present in the new steady state, the mass of available varieties is lower than in the old steady state during more than the first 30 years of the transition.

4.4 Liberalization Using the Sunk Export Costs

This subsection considers a trade liberalization through a reduction in the sunk export cost, i.e. the S^x-liberalization described above. Solving numerically[20] for the transition following the described liberalization gives the results depicted in Figure 2. Most of the variables behaves similarly following the trade liberalization in this subsection when compared to that of the previous subsection. In the following, I will therefore confine the discussion to those variables behaving differently enough to be interesting.

The figures for the aggregate variables in case of a S^x-liberalization look very similar to those that arise in case of a τ-liberalization (previous subsection), which is natural as the two liberalizations considered are conducted such that they have very similar steady-state effects. However, in the short run there is a difference. It turns out, that for the S^x-liberalization considered, the amount of capital sunk by firms entering the export sector immediately following the liberalization is larger

[19] The relative decline in the total mass of available varieties is of course less than that in the mass of domestic varieties as the mass of exporters increase.

[20] Using the ox programme `iterate22.ox`

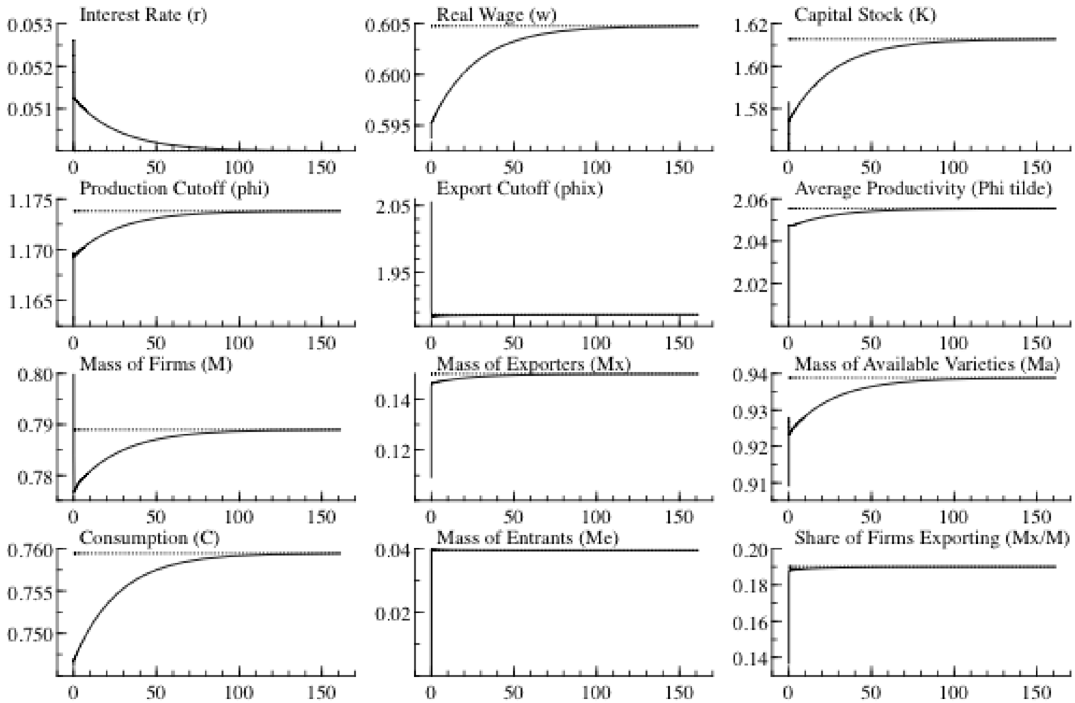

Figure 2: Transition path following a reduction in S^x from 1.00 to 0.65. Dotted lines indicate the new steady state.

than is the case with the τ-liberalization of the preceding subsection. Hence, even though the new steady-state level of export entry investment is lower with the S^x-liberalization than with the τ-liberalization (cf. above), the immediate investment in export entry is larger, which results in a larger immediate reduction of the capital stock (1.5 percent versus 0.7 per cent). As a result, the immediate increase in the interest rate is larger. This means that the sheltering effect on existing firms is more pronounced and therefore the production cutoff rises less sharply in this case. The consequence is that larger share of firms with a productivity level below the new steady-state level survive immediately following the liberalization. This share is 1.2 per cent in the present case compared to 1.1 per cent in case of the τ-liberalization. While this difference is not large, the higher interest rate increase have a more dramatic effect on instantaneous profits (due to capital being a production factor). Indeed, 1.5 per cent of firms earn negative profits immediately following the S^x-liberalization which is significantly more than the 0.8 per cent that do so in the case of the τ-liberalization. However, as described above, these firms foresee the interest rate increase, and does not shut down.[21] Another implication of the larger increase in the interest rate is an almost twice as long period of zero entry in this case, that is 0.19 years. Because of the larger initial decrease in the

[21] It is worth noting that in the present case, some firms with productivities above the production cutoff of the new steady state earns negative profits in the short run!

capital stock more time is needed to accumulate capital before it is once again attractive to invest in entry of new firms. Further, the larger decrease in the capital stock following the S^x-liberalization results in a lower capital stock throughout the transition towards the new steady state as the capital accumulation lags behind that which results from the τ-liberalization. Since capital is obviously an important input into production it follows that final-good production, Y, is larger at all times following the τ-liberalization compared to the S^x-liberalization, and the difference only disappears asymptotically. The implications for consumption are considered in a later subsection, where the relative welfare implications of the three types of liberalizations are analyzed.

Note that because of the larger immediate increase in the mass of exporters, when compared to the τ-liberalization, there is no anti-variety effect with respect to available varieties present. Neither in the short run nor in the new steady state.

4.5 Liberalization Using the Fixed Export Costs

This subsection considers the effects of the f^x-liberalization described above. Solving numerically [22] produces results that is very similar to those of the S^x-liberalization when presented in a figure similar to those above. Therefore I refer to Appendix D for this figure. In the following I will confine myself to emphasize interesting quantitative differences compared to the S^x-liberalization and then move on to considering welfare implications of the three liberalizations.

First of all, even though the effects of the f^x-liberalization comes though entry of new exporters to an extent comparable to the S^x-liberalization, it achieves this without reducing the sunk export cost per exporter. This way, the immediate entry of new exporters following the f^x-liberalization reduces the capital stock by 2.0 per cent, which is significantly more than was the case with both the S^x-liberalization (1.5 per cent) and the τ-liberalization (0.7 per cent). The implied larger increase in the interest rate means that the immediate adjustment in the production cutoff is only 58 per cent of the way towards the new steady-state value (due to to the sheltering effect being larger). Therefore, immediately following the liberalization 1.3 per cent of firms have a productivity less than the new steady state. Because of capital being a production factor, the increase in the interest rate causes 2.0 per cent of firms to earn negative profits immediately following the liberalization, which is again significantly more than the two preceding cases. However, as before, these firms stay in the market as they still have a positive net present value (they anticipate the sharp interest rate decrease during the period of zero entry). As was the case with the S^x-liberalization, even some firms with productivities above the cutoff of the new steady state earns negative profits in the short run. Another consequence of the larger reduction in the capital stock is that the period of zero entry following the f^x-liberalization is about three times that following the τ-liberalization. In

[22] Using the ox programme `iterate33.ox`

particular, zero entry last for 0.31 years in the present case. Finally, the larger initial reduction in the capital stock implies that capital accumulation and thereby production lags behind those of both of the two preceding cases.

4.6 Welfare Effects of the Three Liberalizations

Having described and compared the other important features of the three trade liberalizations, let us turn to a comparison of the induced paths of consumption and the welfare implications. The consumption paths are plotted together in Figure 3. Obviously, the results are in line with our steady-state analysis of subsection 3.4. That is, the f^x-liberalization induces the highest level of consumption in the new steady state followed by the τ-liberalization and then the S^x-liberalization, even though their effects on the steady-state level of final-good production are the same. However, as described above, the f^x-liberalization features the largest initial reduction of the capital stock, and thereby the largest increase in the interest rate, followed by the S^x-liberalization and then the τ-liberalization.

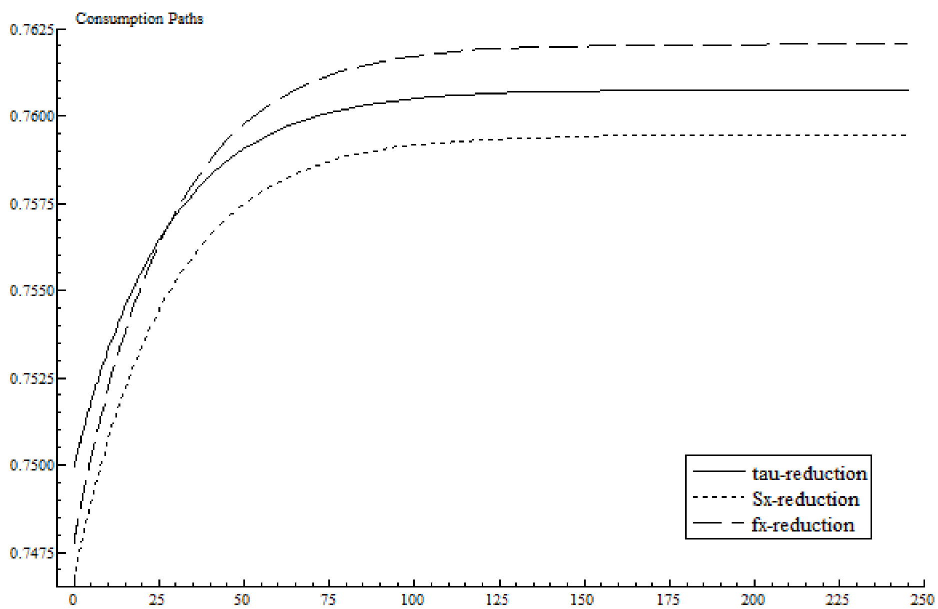

Figure 3: Path of consumption following each of the three liberalizations above.

These effects have interesting implications on the paths of consumption. First of all, because of the lower steady-state level of consumption and the larger initial reduction in the capital stock, the transition following the S^x-liberalization features lower consumption at all points in time when compared to that of the τ-liberalization. Further, because of the higher steady-state consumption and that households smooth consumption, the transition following the f^x-liberalization features higher consumption at all points in time compared to the S^x-liberalization, despite the initial reduction of the capital stock being larger. The result is that the S^x-liberalization is unambiguously the worst in terms of welfare, which can easily be seen from Figure 3. Now consider the f^x- and τ-liberalizations. Even though the f^x-liberalization induces the highest level of steady-state consumption, the larger initial reduction in capital stock implies that capital accumulation lags behind that which follows the τ-liberalization. Therefore production also lags behind and the interest rate is higher throughout the transition. This causes households to postpone consumption to a larger extent following the f^x-liberalization. The result is that the consumption is lower during the first 26 years following the f^x-liberalization compared to the τ-liberalization. After the first 26 years the consumption level induced by the f^x-liberalization is larger than that induced by the τ-liberalization as the effect of the higher steady-state consumption level dominates. This can also be seen in Figure 3. It is therefore not immediately apparent which of these two liberalizations that induces the highest welfare increase, as we are faced with a tradeoff of short-run consumption against long-run consumption. Welfare calculations reveals that the discounted values of future consumption of the representative household immediately following the τ-, f^x- and S^x-liberalization are the same as those which would arise from an immediate and permanent jump in consumption of 1.29, 1.22, and 1.01 per cent respectively. To sum up, the effects of the three liberalizations on the welfare of the representative household immediately following the liberalization can be ranked according to

$$U_{\tau\text{-lib}} > U_{f^x\text{-lib}} > U_{S^x\text{-lib}}.$$

Under the chosen parametrization the τ- and f^x-liberalizations changes place, when the transition is taken into account, compared to the ranking of steady-state welfare levels. Of the cases considered the τ-liberalization emerges as the superior form of liberalization with respect to welfare even though the f^x-liberalization induces a higher level of steady-state consumption. This clearly stresses the importance of considering the transition to which the trade liberalizations subject the economy.

The result is somewhat analogous to the golden-rule result of the Ramsey (1928) model, where the optimizing behavior of the households does not lead to the highest attainable steady-state level of consumption due to the implied short-run cost in terms of foregone consumption. In the present model, the same tradeoff between short-run and long-run consumption arises when choosing between different liberalizations. Faced with such a tradeoff it is not obvious (nor necessarily true) that higher steady-state consumption is more attractive.

5 Conclusion

By introducing capital into the Melitz (2003) framework, the present paper obtains a model featuring rich adjustment dynamics at the firm level of the economy following trade liberalizations. These arises since capital is not only a production factor but also the opportunity cost of investment in firm entry. The higher interest rate prevailing in the short run following trade liberalizations were shown to some extent to shelter low-productivity firms from increased competition, as entry of new firms are discouraged. The effect is that the full gain in average productivity of the intermediate-goods sector is realized over a long horizon. Further, following trade liberalizations the extra-high interest rate that prevails in the very short run induces a period of negative profits for low-productivity firms and zero entry of new firms.

Apart from the firm-level effects, capital accumulation also induces long-term adjustment dynamics at the aggregate level. The magnitude of these effect depends on the scarcity of capital in the short run, which in turn depends on the immediate investment in export entry. Through their different effects on investment in export entry, different trade liberalizations with the same effect on the steady-state share of expenditure on domestic varieties in the intermediate-good sector are shown to induce different transition paths. Especially the consumption paths differed and revealed a tradeoff between consumption in the short run and in the long run. Thus, steady-state analysis is not sufficient when evaluating and comparing welfare effects of different trade liberalizations.

Finally, the anti-variety effects discussed by Baldwin and Forslid (2010) appeared to be more of a short-term issue in the present model, as the expansion of final-good production over the transition tends to accommodate a larger mass of firms.

A Appendix: Firm Behaviour and Market Clearing

The input-demand functions for production to the domestic market are given by,

$$k^d(r,w;\varphi) = \alpha^{2\sigma}\widehat{L}\left(\frac{\gamma}{r+\delta}\right)^{(\sigma-1)\gamma+1}\left(\frac{1-\gamma}{w}\right)^{(\sigma-1)(1-\gamma)}\varphi^{\sigma-1}$$
$$l^d(r,w;\varphi) = \alpha^{2\sigma}\widehat{L}\left(\frac{\gamma}{r+\delta}\right)^{(\sigma-1)\gamma}\left(\frac{1-\gamma}{w}\right)^{(\sigma-1)(1-\gamma)+1}\varphi^{\sigma-1}.$$

The indirect production function for the domestic market is given by

$$x^d(r,w;\varphi) = \alpha^{2\sigma}\widehat{L}\left(\frac{\gamma}{r+\delta}\right)^{\sigma\gamma}\left(\frac{1-\gamma}{w}\right)^{\sigma(1-\gamma)}\varphi^{\sigma}.$$

The indirect profit function for the domestic market is given by

$$\pi^d(r,w;\varphi) = \alpha^{2\sigma-1}\frac{\widehat{L}}{\sigma}\left(\frac{\gamma}{r+\delta}\right)^{(\sigma-1)\gamma}\left(\frac{1-\gamma}{w}\right)^{(\sigma-1)(1-\gamma)}\varphi^{\sigma-1} - f.$$

The export-market counterparts of the above functions are related to their domestic counterparts as follows

$$k^x(r,w;\varphi) = k^d(r,w;\varphi/\tau),$$
$$l^x(r,w;\varphi) = l^d(r,w;\varphi/\tau),$$
$$x^x(r,w;\varphi) = x^d(r,w;\varphi/\tau).$$

As exporting is associated with an additional fixed cost of f^x, profits obtained from exporting are given by

$$\pi^x(r,w;\varphi) = \pi^d(r,w;\varphi/\tau) + f - f^x. \tag{21}$$

The expanded expression for the factor reward condition (3) is given by

$$w_t = (1-\alpha)\,M_t\alpha^{2(\sigma-1)}\left(\frac{\gamma}{r_t+\delta}\right)^{(\sigma-1)\gamma}\left(\frac{1-\gamma}{w_t}\right)^{(\sigma-1)(1-\gamma)}\tilde{\varphi}_t^{\sigma-1}.$$

The expanded expression for the market-clearing conditions for labor and capital is given by

$$L = \widehat{L}_t + \alpha^{2\sigma}M_t\widehat{L}_t\left(\frac{\gamma}{r_t+\delta}\right)^{(\sigma-1)\gamma}\left(\frac{1-\gamma}{w_t}\right)^{(\sigma-1)(1-\gamma)+1}\tilde{\varphi}_t^{\sigma-1}.$$
$$K_t = \alpha^{2\sigma}M_t\widehat{L}_t\left(\frac{\gamma}{r_t+\delta}\right)^{(\sigma-1)\gamma+1}\left(\frac{1-\gamma}{w_t}\right)^{(\sigma-1)(1-\gamma)}\tilde{\varphi}_t^{\sigma-1}.$$

B Appendix: Steady State

This appendix sketches the solution for the steady state and provide closed form solution for all variables of interest. In the steady state, all variables are constant over time as no source of sustained growth is introduced.[23] Thus, time subscripts will be dropped for the remainder of this section. First of all, the distribution functions of domestic and exporting firms will be time invariant and linked to the distribution from which productivities are drawn in the following way

$$H(\varphi) = \begin{cases} \frac{G(\varphi)-G(\varphi^q)}{1-G(\varphi^q)} & \text{for } \varphi > \varphi^q \\ 0 & \text{otherwise} \end{cases}, \quad H^x(\varphi) = \begin{cases} \frac{G(\varphi)-G(\varphi^{ex})}{1-G(\varphi^{ex})} & \text{for } \varphi > \varphi^{ex} \\ 0 & \text{otherwise} \end{cases}.$$

Steady-state consumption is constant, and therefore (2) implies that the net return on capital must equal the subjective discount factor, i.e. $r = \rho$. Since capital has to be constant as well, (1) leads to

$$C = Y - F - I - \delta K \tag{22}$$

which can be determined by using the steady-state values given below. Analogously to Melitz (2003) the production and export cutoffs are determined by zero cutoff profit conditions along with the free-entry condition.[24] The resulting expression for the production cutoff is given by

$$\varphi^q = \varphi_0 \left[\frac{\sigma - 1}{m - (\sigma - 1)} \frac{1}{r + \eta} \frac{f}{S} \lambda^{-1} \right]^{\frac{1}{m}}. \tag{23}$$

The export cutoff is given by[25]

$$\varphi^{ex} = \left(\frac{\lambda \bar{f}}{1 - \lambda} \right)^{\frac{1}{m}} \varphi^q. \tag{24}$$

It will be assumed that $\lambda \bar{f}/(1 - \lambda) > 1$, which implies that $\varphi^q < \varphi^{ex}$. Using the expressions for the cutoffs the average productivity, (13), is given by

$$\tilde{\varphi} = \varphi^q \left[\frac{m}{m - (\sigma - 1)} \lambda^{-1} \right]^{\frac{1}{(\sigma-1)}}, \tag{25}$$

[23] Note that because of increasing returns to scale, the model produces steady-state per capita growth in the final-good production when growth in the labor force is included. However, growth is not the concern of the present paper, and is excluded for clarity. Further, contrary to e.g. Rivera-Batiz and Romer (1991) and Baldwin and Robert-Nicoud (2008) trade liberalizations would not affect the steady-state growth rate, but rather result in a level effect.

[24] In steady state production status of a firm is uniquely determined by its productivity, such that $z = x$ for $\varphi > \varphi^{ex}$, $z = d$ for $\varphi^{ex} > \varphi > \varphi^q$ and $z = n$ otherwise.

[25] As no firms with $\varphi < \varphi^{ex}$ will export in steady state, φ^{qx} is omitted from steady-state analysis.

which is very similar to what is obtained from the Melitz (2003) model when using the pareto distribution. As the mass of firms and the mass of exporters are constant in steady state M^e and M^{ex} are given by ηM and ηM^x, respectively. M^x is given by

$$M^x = \frac{1 - G(\varphi^{ex})}{1 - G(\varphi^q)} M = \frac{1-\lambda}{\lambda \bar{f}} M. \tag{26}$$

Using steady-state values for φ^q and r, the condition $\pi^d(\varphi^q) = 0$ determines w. Then given w, (3) determines M. Finally (15) determines K and (16) determines Y. The resulting expressions are

$$w = Q_1 \lambda^{\frac{-1}{m(1-\gamma)}} \tag{27}$$

$$M = Q_2 \lambda^{1-\frac{1}{m(1-\gamma)}} \tag{28}$$

$$K = Q_3 \lambda^{\frac{-1}{m(1-\gamma)}} \tag{29}$$

$$Y = Q_4 \lambda^{\frac{-1}{m(1-\gamma)}}, \tag{30}$$

where Q_1 through Q_4 are positive constants not depending on the export costs. These constants are given by

$$Q_1 = (1-\gamma)\, \varphi_0^{\frac{1}{1-\gamma}} \left(\frac{\gamma}{\rho+\delta}\right)^{\frac{\gamma}{1-\gamma}} \left(\alpha^{2\sigma-1} \frac{\widehat{L}}{\sigma f}\right)^{\frac{1}{(\sigma-1)(1-\gamma)}} \cdot \left(\frac{\sigma-1}{m-(\sigma-1)} \frac{1}{\rho+\eta} \frac{f}{S}\right)^{\frac{1}{m(1-\gamma)}} \tag{31}$$

$$Q_2 = Q_1 \frac{\alpha \widehat{L}}{f} \frac{m-(\sigma-1)}{m} \tag{32}$$

$$Q_3 = Q_1 \alpha(\sigma-1)\widehat{L} \left(\frac{\gamma}{\rho+\delta}\right) \tag{33}$$

$$Q_4 = Q_1 \sigma \widehat{L} \tag{34}$$

C Appendix: Solving for the Transition

This appendix describes the algorithm employed to solve numerically for the transition following the liberalizations as described in the paper. The algorithm builds on the relaxation algorithm as presented by Trimborn et al. (2007). In its current form it is geared for an unannounced immediate change in trade costs and heavily utilizes knowledge of the structure of the solution. The first subsection describes the idea of the algorithm, and the next subsections describe the detailed calculations carried out.

C.1 The Algorithm

The transition path is found numerically by applying the following algorithm. A mesh of B time points, $\{t_i\}_{i=1}^{B}$, is chosen ($t_B = T$, $t_1 = 0$). This is fixed except for one point t^*, which will denote the end of the period of zero entry following the trade liberalization (needs to be determined). An initial guess is provided for t^* and for (w_t, r_t) at the B points in time. The old steady state is denoted by $t_0 = 0$. The difference in variables at t_0 and t_1 thus describes the immediate reaction to the liberalization. Given the $2B+1$ variables $(\{w_{t_i}, r_{t_i}\}_{i=1}^{B}, t^*)$, which pins down all other variables in the model, a $(2B+1)\times 1$ error vector is constructed. This error vector contains information of how far remaining equilibrium conditions are from being satisfied given the values of the $2B+1$ variables. The error vector and the initial guess is then handed to a numerical algorithm (SolveNLE in ox) solving for t^* and the path of r_t and w_t that makes this error vector zero. The error vector can be split into three parts. The first part is

$$\underline{\epsilon}_1 = \{\text{LHS}(3)_{t_i} - \text{RHS}(3)_{t_i}\}_{t_i \le t^*}$$

and thus measures how far the equilibrium condition (3) is from being satisfied. The second part of the error vector is

$$\underline{\epsilon}_2 = \{V^e_{t_i}\}_{t_i \ge t^*},$$

where $V^e_{t_i}$ is calculated using (10). The third part of the error vector is given by

$$\underline{\epsilon}_3 = \{\text{LHS}(15)_{t_i} - \text{RHS}(15)_{t_i}\}_{i=0}^{B}$$

and thus measures how far the equilibrium condition (15) is from being satisfied. Together $\underline{\epsilon}_1$ and $\underline{\epsilon}_2$ have $B+1$ entries and $\underline{\epsilon}_3$ have B entries. In total we have the $2B+1$ conditions needed. The explanation for the structure of $\underline{\epsilon}_1$ and $\underline{\epsilon}_2$ goes as follows. For $t_i < t^*$, when entry is zero, equilibrium condition (3) have to hold and the value of entry is unrestricted (apart from being negative). For $t_i > t^*$, when entry is not zero the equilibrium condition (3) is used to calculate the value of M^e, and now it is the zero entry condition that needs to be satisfied. At $t_i = t^*$, both of these conditions have to hold. Thus together $\underline{\epsilon}_1$ and $\underline{\epsilon}_2$ provides one condition for each point in the time mesh plus one additional condition for $t_i = t^*$, in total $B+1$ conditions.

To employ the above algorithm, the error vector must be evaluated given $\{w_{t_i}, r_{t_i}\}_{i=1}^B$ and t^*. This is done in a number of steps using the rest of the equilibrium conditions and described in detail below.

As already apparent, the algorithm is based on the conjecture that the solution involves a period of zero entry. It further it uses the conjecture that the cutoffs φ^q and φ^{ex} increase and decrease respectively immediately following the liberalization (from t_0 to t_1). The final conjecture used in the algorithm is that both of the cutoffs are monotonically decreasing from t_1 until t^* after which they will be monotonically increasing. The conjectures about the cutoffs simplify the computations involving the distribution of firms and exporters. Off course all of these conjectures turn out to be correct.

In the following $\bar{x}_{t_i} \equiv (x_{t_{i+1}} + x_{t_i})/2$, and let t^*_- and t^*_+ denote the first points in the time mesh before and after t^*, respectively.

C.2 The Cutoffs

The cutoffs $\{\varphi^q_{t_i}, \varphi^{ex}_{t_i}\}_{i=1}^B$, can be calculated recursively from $\{w_{t_i}, r_{t_i}\}_{i=1}^B$. At time T they can be calculated from

$$\pi^d_T(\varphi^q_T) = 0$$
$$\pi^x_T(\varphi^{ex}_T) = (r_T + \eta)S^x$$

where it is assumed that from time T and onwards everything remains constant (new steady state reached).

From time $t_i \in [t^*, t_{B-1}]$, the cutoffs can be calculated using,

$$\sum_{s=i}^{B-1} \bar{\pi}^x_{t_s}(\varphi^{ex}_{t_i}) \frac{t_{s+1} - t_s}{1 + \frac{1}{2}(\bar{r}_{t_s} + \eta)(t_{s+1} - t_s)} \prod_{k=i}^{s-1} \frac{1 - \frac{1}{2}(\bar{r}_{t_k} + \eta)(t_{k+1} - t_k)}{1 + \frac{1}{2}(\bar{r}_{t_k} + \eta)(t_{k+1} - t_k)} + \frac{\pi^x_T(\varphi^{ex}_{t_i})}{r_T + \eta} \prod_{k=i}^{B-1} \frac{1 - \frac{1}{2}(\bar{r}_{t_k} + \eta)(t_{k+1} - t_k)}{1 + \frac{1}{2}(\bar{r}_{t_k} + \eta)(t_{k+1} - t_k)} = S^x$$

for $\varphi^{ex}_{t_i}$ since it is assumed that firms entering the export market will never quit voluntarily (simplifying and it is the case). The production cutoff can be calculated from

$$\pi^d_{t_s}(\varphi^q_{t_i}) = 0$$

since it is increasing and thus there is no incentive to stay in the market for firms earning negative profits. These both have analytical solutions, that can be used for efficient calculation. For $t_i \in [t_1, t^*_-]$ the cutoffs can be calculated using

$$\pi^x_{t_i}(\varphi^{ex}_{t_i}) = S^x(r_{t_i} + \eta)$$

since the cutoff is decreasing in this interval and thus earning the required return immediately is the requirement for entering. This has an analytical solution for $\varphi^{ex}_{t_i}$. The production cutoff can be calculated using

$$\sum_{j=i}^{B-1}\left[I_{\{\varphi^q_{t_i}\geq\max\{\varphi^q_{t_s}\}^j_{s=i}\}}\frac{\bar{\pi}_{t_j}(\varphi^q_{t_i},z_{t_i})(t_{j+1}-t_j)}{1+\frac{1}{2}(\bar{r}_{t_j}+\eta)(t_{j+1}-t_j)}\prod_{s=i}^{j-1}\frac{1-\frac{1}{2}(\bar{r}_{t_s}+\eta)(t_{s+1}-t_s)}{1+\frac{1}{2}(\bar{r}_{t_s}+\eta)(t_{s+1}-t_s)}\right]=0$$

which does not have an analytical solution (i.e. a numerical solution algorithm must be used).

C.3 Value of Entry

Given the export cutoffs and future values of the wage and interest rate, it is possible to express the value of entry at all points in time after t^*. Using the conjecture that if a firm enters the export market, it never leaves it voluntarily and that the export cutoff is increasing after t^*, the value of entry for $t\in[t^*,\infty)$ can be expressed as

$$\begin{aligned}V^E_t=&\int_{\varphi^q_t}^{\infty}\int_t^{u_t(\varphi)}e^{-\int_t^s(r_v+\eta)\,\mathrm{d}v}\pi^d_s(\varphi)\,\mathrm{d}s\,\mathrm{d}G(\varphi)-S\\&+\int_{\varphi^{ex}_t}^{\infty}\int_t^{\infty}e^{-\int_t^s(r_v+\eta)\,\mathrm{d}v}\pi^x_s(\varphi)\,\mathrm{d}s\,\mathrm{d}G(\varphi)-(1-G(\varphi^{ex}_t))S^x,\end{aligned}$$

where $u_t:[\varphi^q_t,\infty)\to[t,\infty)$ gives the time of voluntary exit of a firm with productivity φ. The above expression is off course not operational in the numerical algorithm. To obtain an expression that can be utilized, differentiate the above with respect to t and note that on $[t^*,\infty)$ we have $V^E_t=0$ and $\frac{\mathrm{d}}{\mathrm{d}t}V^E_t=0$. This gives the condition

$$\int_{\varphi^q_t}^{\infty}\pi^d_s(\varphi)\,\mathrm{d}G(\varphi)+\int_{\varphi^{ex}_t}^{\infty}\pi^x_s(\varphi)\,\mathrm{d}G(\varphi)=(r_t+\eta)\left[S+(1-G(\varphi^{ex}_t))S^x\right].$$

Given the wage, the interest rate and the cutoffs all terms in this expression can be calculated analytically. Thus, the second part of the error vector, $\underline{\epsilon}_2$, can now be evaluated.

C.4 Mass of Firms Immediately Following the Liberalization

The immediate upwards jump in the cutoff φ^q following a trade liberalization will cause a mass of firms to exit the market and a mass of firms to enter the export market. Thus the mass of incumbent firms, M_{t_1}, immediately following the announcement/implementation of the policy change is related to the mass of firms in the old steady state, M_{t_0}, in the following way

$$M_{t_1}=\frac{1-G(\varphi^q_{t_1})}{1-G(\varphi^q_{t_0})}M_{t_0}\qquad(35)$$

The mass of exporting firms immediately after the policy change is calculated as

$$M_{t_1}^x = \frac{1 - G(\varphi_{t_0}^{ex})}{1 - G(\varphi_{t_0}^q)} M_{t_0} + \frac{G(\varphi_{t_0}^{ex}) - G(\varphi_{t_1}^{ex})}{1 - G(\varphi_{t_0}^q)} M_{t_0} \tag{36}$$

since $\varphi_{t_1}^{ex} < \varphi_{t_0}^{ex}$. The capital stock is discretely decreased upon the entry of exporters, meaning that the capital stock immediately following the policy change, K_{t_1} is related to the old steady state value of capital K_{t_0} in the following way,

$$K_{t_1} = K_{t_0} - \frac{G(\varphi_{t_0}^{ex}) - G(\varphi_{t_1}^{ex})}{1 - G(\varphi_{t_0}^q)} M_{t_0} S^x \tag{37}$$

C.5 Mass and Distribution of Incumbent Firms

Having obtained the value of entry at all time points, what remains is to evaluate the second part of the error vector based on the equilibrium condition (15). To do so one must be able to evaluate the mass and distribution of incumbent firms.

As no firms will enter and φ^q is decreasing until t^* the mass of firms for $t \in [t_1, t^*]$ is given by

$$M_{t_i} = e^{-\eta(t_i - t_1)} M_{t_1},$$

while the distribution of incumbent firms in this interval is given by

$$H_{t_i}(\varphi) = \frac{G(\max\{\varphi, \varphi_{t_1}^q\}) - G(\varphi_{t_1}^q)}{1 - G(\varphi_{t_1}^q)} \tag{38}$$

For the evolution of the mass of incumbent firms for $t \in [t_+^*, T]$ the following approximation is used

$$M_{t_i} = e^{-\eta(t_i - t_{i-1})} \left[M_{t_{i-1}} \left(1 - H_{t_{i-1}}\big(\max\{\varphi_{t_{i-1}}^q, \varphi_{t_i}^q\}\big)\right) + M_{t_{i-1}} \left(e^{(M_{t_{i-1}}^e / M_{t_{i-1}})(t_i - t_{i-1})} - 1 \right) \right]$$

where M_{t_1} is given above. For $t \in [t_+^*, T]$ the distribution will be

$$\begin{aligned} H_{t_i} = \sum_{t_j = t_+^*}^{t_i} & \left[e^{-\eta(t_i - t_{j-1})} \frac{M_{t_{j-1}}}{M_{t_i}} \left(e^{\left(M_{t_{j-1}}^E / M_{t_{j-1}}\right)(t_j - t_{j-1})} - 1 \right) \frac{G(\max\{\varphi, \varphi_{t_i}^q\}) - G(\varphi_{t_i}^q)}{1 - G(\varphi_{t_j}^q)} \right] \\ & + e^{-\eta(t_i - t^*)} \frac{M_{t^*}}{M_{t_i}} \frac{G\big(\max\{\varphi, \varphi_{t_1}^q, \varphi_{t_i}^q\}\big) - G\big(\max\{\varphi_{t_1}^q, \varphi_{t_i}^q\}\big)}{1 - G(\varphi_{t_1}^q)}. \end{aligned}$$

The mass of exporting firms will be

$$M_{t_i}^x = \frac{1 - G(\varphi_{t_i}^{ex})}{1 - G(\varphi_{t_1}^q)} M_{t_i} \tag{39}$$

for $t_i \in [t_1, t^*]$, while it will be

$$M_{t_i}^x = e^{-\eta(t_i - t^*)} M_{t^*}^x + \sum_{t_j = t_+^*}^{t_i} e^{-\eta(t_i - t_{j-1})} M_{t_{j-1}} \left(e^{(M_{t_{j-1}}^e / M_{t_{j-1}})(t_j - t_{j-1})} - 1 \right) \frac{1 - G(\varphi_{t_j}^{ex})}{1 - G(\varphi_{t_j}^q)}$$

for $t \in [t_+^*, T]$. The distribution of exporting firms will be

$$H_{t_i}^x = \frac{G(\max\{\varphi, \varphi_{t_i}^{ex}\}) - G(\varphi_{t_i}^{ex})}{1 - G(\varphi_{t_i}^{ex})},$$

for $t \in [t_1, t^*]$, while it will be

$$H_{t_i}^x = e^{-\eta(t_i - t^*)} \frac{M_{t^*}^x}{M_{t_i}^x} \frac{G(\max\{\varphi, \varphi_{t^*}^{ex}\}) - G(\varphi_{t^*}^{ex})}{1 - G(\varphi_{t^*}^{ex})} + \sum_{t_j = t_+^*}^{t_i} e^{-\eta(t_i - t_{j-1})} \frac{M_{t_{j-1}}}{M_{t_i}^x} \left(e^{(M_{t_{j-1}}^e / M_{t_{j-1}})(t_j - t_{j-1})} - 1 \right) \frac{G(\max\{\varphi, \varphi_{t_j}^{ex}\}) - G(\varphi_{t_j}^{ex})}{1 - G(\varphi_{t_j}^{q})}$$

for $t \in [t_+^*, T]$.

Now, it is apparent that to be able to calculate the mass of firms one has to be able to evaluate the distribution function, and that to evaluate the distribution function one has to know the distribution of incumbent firms. How this is solved is described in step C.7 below. First it is necessary to consider evaluation of $\tilde{\varphi}_{t_i}^{\sigma-1}$.

C.6 The Average Productivity

For use in evaluating the equilibrium condition (15), and determination of the mass of entrants, consider the average productivity, $\tilde{\varphi}_{t_i}^{\sigma-1}$,

$$\tilde{\varphi}_{t_i}^{\sigma-1} = \int_{\varphi_{t_i}^q}^{\infty} \varphi^{\sigma-1} \,\mathrm{d}H_{t_i}(\varphi) + \tau^{1-\sigma} \int_{\varphi_{t_i}^{qx}}^{\infty} \frac{M_{t_i}^x}{M_t} \varphi^{\sigma-1} \,\mathrm{d}H_{t_i}^x(\varphi).$$

First consider $t \in [t_1, t^*]$. In this interval we have

$$\tilde{\varphi}_{t_i}^{\sigma-1} = \frac{1}{1 - G(\varphi_{t_1}^q)} \left[\int_{\varphi_{t_1}^q}^{\infty} \varphi^{\sigma-1} \,\mathrm{d}G(\varphi) + \tau^{1-\sigma} \int_{\varphi_{t_i}^{ex}}^{\infty} \varphi^{\sigma-1} \,\mathrm{d}G(\varphi) \right].$$

Next consider the interval $t \in [t_+^*, T]$. In this interval we have,

$$\tilde{\varphi}_{t_i}^{\sigma-1} = \sum_{t_j = t_+^*}^{t_i} \frac{M_{t_{j-1}}}{M_{t_i}} \left(e^{\left(M_{t_{j-1}}^E / M_{t_{j-1}}\right)(t_j - t_{j-1})} - 1 \right) \frac{e^{-\eta(t_i - t_{j-1})}}{1 - G(\varphi_{t_j}^q)} \left[\int_{\varphi_{t_i}^q}^{\infty} \varphi^{\sigma-1} \,\mathrm{d}G(\varphi) + \tau^{1-\sigma} \int_{\varphi_{t_j}^{ex}}^{\infty} \varphi^{\sigma-1} \,\mathrm{d}G(\varphi) \right] + \frac{M_{t_1}}{M_{t_i}} \frac{e^{-\eta(t_i - t_1)}}{1 - G(\varphi_{t_1}^q)} \left[\int_{\max\{\varphi_{t_1}^q, \varphi_{t_i}^q\}}^{\infty} \varphi^{\sigma-1} \,\mathrm{d}G(\varphi) + \tau^{1-\sigma} \int_{\varphi_{t^*}^{ex}}^{\infty} \varphi^{\sigma-1} \,\mathrm{d}G(\varphi) \right]$$

The expression has a closed form solution for the pareto distribution, using that for this distribution

$$\frac{1}{1 - G(y)} \int_x^{\infty} \varphi^{\sigma-1} \,\mathrm{d}G(\varphi) = \frac{m}{m - (\sigma - 1)} y^m x^{(\sigma-1)-m}.$$

Now, combining the above expression for $\tilde{\varphi}_{t_i}^{\sigma-1}$ and the equilibrium condition (3), lets you calculate $M_{t_{i-1}}^e$ when $\{M_{t_j}^e\}_{j=1}^{i-2}$ and $\{M_{t_j}\}_{j=1}^{i-1}$ is known. Then M_{t_i} can be calculated using the above expression. This way the values $\{M_{t_j}^e\}_{j=1}^{B-1}$ and $\{M_{t_j}\}_{j=2}^{B}$ can be calculated recursively forward in time. The expression for calculating M_t^e (only relevant for $t = t^*, \ldots, t_{B-1}$) is

$$M_{t_{j-1}}^e = \frac{M_{t_{j-1}}}{t_j - t_{j-1}} \ln \left[\frac{w_{t_j} \sigma \alpha^{-2(\sigma-1)} \left(\frac{r_{t_j}+\delta}{\gamma}\right)^{(\sigma-1)\gamma} \left(\frac{w_{t_j}}{1-\gamma}\right)^{(\sigma-1)(1-\gamma)} - \Psi_{j,t_1} - \sum_{t_k=t_+^*}^{t_{j-1}} \Psi_{j,t_k}}{\frac{m}{m-(\sigma-1)} (\varphi_{t_j}^q)^m \left((\varphi_{t_j}^q)^{(\sigma-1)-m} + \tau^{1-\sigma} (\varphi_{t_j}^{ex})^{(\sigma-1)-m} \right) M_{t_{j-1}}} + e^{-\eta(t_j - t_{j-1})} \right] + \eta M_{t_{j-1}}$$

where Ψ_{j,t_k}, $t_k \geq t_+^*$ is given by

$$\Psi_{j,t_k} = \frac{e^{-\eta(t_j - t_{k-1})}}{1 - G(\varphi_{t_k}^q)} M_{t_{k-1}} \left(e^{(M_{t_{k-1}}^E / M_{t_{k-1}})(t_k - t_{k-1})} - 1 \right) \left[\int_{\varphi_{t_j}^q}^{\infty} \varphi^{\sigma-1} \, \mathrm{d}G(\varphi) + \tau^{1-\sigma} \int_{\varphi_{t_k}^{ex}}^{\infty} \varphi^{\sigma-1} \, \mathrm{d}G(\varphi) \right]$$

while

$$\Psi_{j,t_1} = \frac{e^{-\eta(t_j - t_1)}}{1 - G(\varphi_{t_1}^q)} M_{t_1} \left[\int_{\max\{\varphi_{t_1}^q, \varphi_{t_j}^q\}}^{\infty} \varphi^{\sigma-1} \, \mathrm{d}G(\varphi) + \tau^{1-\sigma} \int_{\varphi_{t^*}^{ex}}^{\infty} \varphi^{\sigma-1} \, \mathrm{d}G(\varphi) \right].$$

Now having obtained $\{M_{t_j}^e\}_{j=1}^{B-1}$ and $\{M_{t_j}\}_{j=2}^{B}$, it is easy to evaluate the distribution of incumbent firms and $\{\tilde{\varphi}_{t_i}^{\sigma-1}\}_{j=1}^{B}$ using the above expressions.

C.8 Path of Consumption and Capital Stock

The values of Y_T, K_T and thereby C_T can be calculated assuming everything is constant after the last point in the time mesh (new ss reached).

First of all this enables us to evaluate $\{C_{t_i}\}_{i=1}^{B-1}$, since

$$C_{t_{i+1}} = C_{t_i} + \frac{1}{\theta} \left(\bar{r}_{t_i} - \rho \right) \bar{C}_{t_i} (t_{i+1} - t_i) \tag{40}$$

implies that $\{C_{t_i}\}_{i=1}^{B-1}$ can be obtained from

$$C_{t_i} = C_T \prod_{j=i}^{B-1} \frac{2\theta - (\bar{r}_{t_j} - \rho)(t_{j+1} - t_j)}{2\theta + (\bar{r}_{t_j} - \rho)(t_{j+1} - t_j)} \tag{41}$$

Then $\{K_{t_i}\}_{i=2}^{B}$ can be obtained from[26]

$$K_{t_{i+1}} = K_{t_i} + \left(\bar{Y}_{t_i} - \bar{C}_{t_i} - \delta \bar{K}_{t_i} - \frac{M_{t_i}^e}{1 - \bar{G}(\varphi_{t_i}^q)} S - \bar{M}_{t_i}^{ex} S^x - \bar{M}_{t_i} f - \bar{M}_{t_i}^x f^x \right) (t_{i+1} - t_i),$$

[26] OBS: Definition of $\bar{G}$.

which can be rewritten to

$$K_{t_{i+1}} = \frac{K_{t_i} + \left(\bar{Y}_{t_i} - \bar{C}_{t_i} - \frac{\delta}{2}K_{t_i} - \frac{M^e_{t_i}}{1-\bar{G}(\varphi^q_t)}S - \bar{M}^{ex}_{t_i}S^x - \bar{M}_{t_i}f - \bar{M}^x_{t_i}f^x\right)(t_{i+1} - t_i)}{1 + \frac{\delta}{2}(t_{i+1} - t_i)} \tag{42}$$

where K_{t_1} is known from above. Note that for $t_{i+1} \in [t_2, t^*]$ $\bar{M}^{ex}_{t_i}$ can be written as

$$\bar{M}^{ex}_{t_i} = \frac{G(\varphi^{ex}_{t_i}) - G(\varphi^{ex}_{t_{i+1}})}{1 - G(\varphi^q_{t_1})}\bar{M}_{t_i} \tag{43}$$

while for $t \in [t^*_+, T]$ it can be written as,

$$\bar{M}^{ex}_{t_i} = \frac{1 - \bar{G}(\varphi^{ex}_{t_i})}{1 - \bar{G}(\varphi^q_{t_i})}M^e_{t_i}. \tag{44}$$

C.9 The Equilibrium Conditions (3) *and* (15)

At this point the equilibrium conditions (3) and (15) and thus the last two parts of the error vector can be evaluated.

D Appendix: Results for the f^x-liberalization

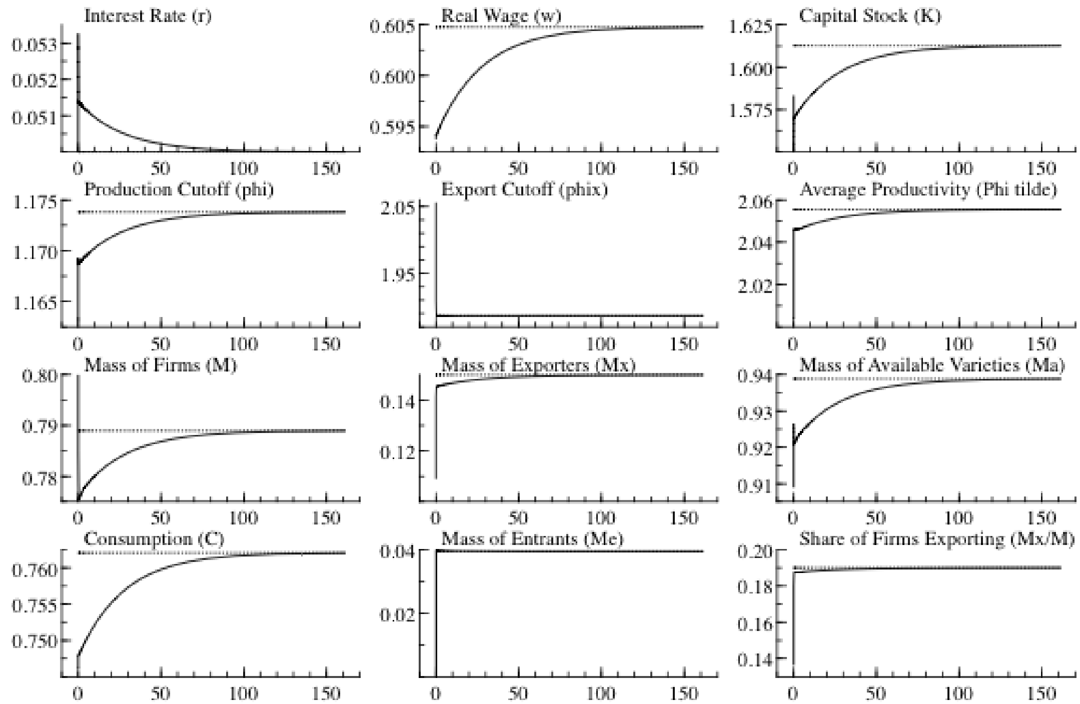

Figure 4: The transition path of various variables following a reduction in f^x from .100 to .065. Solid curves indicate the adjustment path of the variables while dotted lines indicate the new steady state.

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