Liquidity Constraints and Fiscal Stabilization Policy

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June 8, 2011

Abstract

It is often claimed that the presence of liquidity constrained households enhances the need for and the effects of fiscal stabilization policies. This paper studies this in a model of a small open economy with liquidity constrained households. The results show that the consequences of liquidity constraints are more complex than previously thought: The optimal stabilization policy in case of productivity shocks is independent of the liquidity constraints, and the presence of liquidity constraints tends to reduce the need for an active policy stabilizing productivity shocks.

JEL classification: E32, E63, F41

Keywords: Liquidity constraints; Stabilization policy; Fiscal policy; Small open economy

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*Comments from Torben M. Andersen and participants in a seminar at Aarhus University are gratefully acknowledged.

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1 Introduction

It is often claimed that the presence of liquidity constrained households enhances both the need for and the effects of fiscal stabilization policies. According to IMF (2009) fiscal policy is more effective when economic agents face tighter liquidity constraints, a conclusion partly based on the findings in Tagkalakis (2008). The basic intuition is that intertemporal mobility is lower, when liquidity constraints are tight, and therefore the response to shocks and policy stimulus is larger. However, despite the large literature on liquidity constraints originating from Jappelli & Pagano (1989) and Campbell & Mankiw (1991), see e.g. Mankiw (2000) and Galí, López-Salido & Vallés (2007), very few studies support these presumptions.

The financial and economic crisis has emphasized that access to credit is an important determinant of economic fluctuations and aggregate demand. Also, Sarantis & Stewart (2003) finds that the average proportion of current income consumers is 70.6% across the 20 OECD countries considered, and it varies from 33.1% (in the UK) to 99.3% (in the Netherlands). In light of this, it is surprising that the economic literature is largely silent regarding the interactions between liquidity constraints and stabilization policies.

The aim of this paper is to provide new insights on the effects of introducing liquidity constraints in intertemporal models, and surprising results are derived for the need and role of an active fiscal stabilization policy.

In case of productivity shocks it is shown that an optimal stabilization policy exists, which is independent of the liquidity constraints. Furthermore, the results indicate that the need for stabilization of productivity shocks is actually decreasing in the fraction being liquidity constrained. Hence, the consequences of liquidity constraints are not as straightforward as usually argued.

The rest of the paper is organized as follows: The model is set up in Section 2, while Section 3 considers the steady state, and Section 4 analyzes a supply shock. Finally, Section 5 offers some concluding remarks.

2 Model

This paper considers a two-sector model for a small open economy, where one sector produces a tradeable good and the other sector produces a non-tradeable good. The model set-up largely follows Andersen & Holden (2002). The price of tradeables (in domestic currency) is given exogenously from the world market, whereas the price of non-tradeables is determined endogenously. A fraction $\lambda$ of the households is hand-to-mouth consumers (liquidity constrained) since
they simply spend their current income, whereas a fraction $1 - \lambda$ has full access to saving and borrowing. Therefore, Ricardian Equivalence does not prevail in this setup. Agents are risk averse, and the households own the firms. Furthermore, capital markets are incomplete, i.e., there exists an internationally traded bond but equities are not traded internationally. The public sector collects taxes and there is a public demand for non-tradeable goods. Finally, business cycle fluctuations are generated by supply shocks.

### 2.1 Households

It is assumed that the households inelastically supply a given amount of labour ($L$). Each household has an infinite horizon, and their objective is to maximize expected lifetime utility given by

$$U_t = E_t \left[ \sum_{j=0}^{\infty} (1 + \rho)^{-j} u(b_{t+j}) \right], \quad \rho > 0$$

(1)

where $E_t$ denotes the expectation operator given information at time $t$, $\rho$ is the subjective rate of time preference, and $u(\cdot)$ is the instantaneous utility function given by

$$u(b_{t+j}) = b_{t+j} - \frac{k}{2} \left( b_{t+j} \right)^2, \quad k > 0$$

(2)

where $b_{t+j}$ is a composite index of consumption of non-tradeables $c_{NT}^{t+j}$ and tradeables $c_{T}^{t+j}$ defined as

$$b_{t+j} = \frac{1}{\Omega} \left( c_{NT}^{t+j} \right)^{\alpha} \left( c_{T}^{t+j} \right)^{1-\alpha}, \quad 0 < \alpha < 1$$

(3)

where $\Omega \equiv \alpha^\alpha (1 - \alpha)^{1-\alpha}$. Hence, there is risk aversion with respect to the composition of the consumption bundle.

Since preferences are homothetic, the optimal consumption decision can be split into two, i.e. first the household maximizes the value of the composite consumption bundle for a given level of nominal expenditures $M_{t+j}$ in period $t+j$, and secondly the household chooses how much to spend each period. Denoting the price of non-tradeables (tradeables) by $P_{NT}^{t+j}$ ($P_{T}^{t+j}$) nominal expenditures in period $t+j$ are defined as

$$M_{t+j} \equiv P_{NT}^{t+j} c_{NT}^{t+j} + P_{T}^{t+j} c_{T}^{t+j}.$$  

(4)

\footnote{Distinguishing between tradeables and non-tradeables is a very simple way to model a small open economy since the impacts of foreign shocks are collected into the price of tradeables. Tinbergen (1965) was the first to use this method (in a Keynesian model). He referred to these goods as international and national goods, respectively.}
Now, consider the maximization of the value of the consumption bundle for \( M_{t+j} \) given. Since the composite index is of the Cobb-Douglas type, optimal consumption implies

\[
\begin{align*}
\hat{c}_{t+j}^{NT} &= \alpha \frac{M_{t+j}}{P_{t+j}^{NT}} \\
\hat{c}_{t+j}^{T} &= (1 - \alpha) \frac{M_{t+j}}{P_{t+j}^{T}}
\end{align*}
\]

and therefore the optimal value of the consumption bundle can be written

\[
b_{t+j} = \frac{M_{t+j}}{Q_{t+j}}
\]

where the consumer price index \( Q_{t+j} \) is defined as

\[
Q_{t+j} \equiv (P_{t+j}^{NT})^\alpha (P_{t+j}^{T})^{1-\alpha}.
\]

Next, we consider the choice of nominal expenditures \( M_{t+j} \).

### 2.1.1 Liquidity constrained households

An exogenous fraction of the households (\( \lambda \)) is assumed to be liquidity constrained (denoted \( c \)) in the sense that they simply spend their current income. These are sometimes referred to as hand-to-mouth consumers.

As these households do not have access to saving and borrowing, they maximize (1) by letting

\[
M^{e}_{t+j} = I_{t+j}
\]

where \( I_{t+j} \) denotes the after-tax nominal income in period \( t + j \), i.e. the level of nominal expenditures is given by the nominal income. Nominal income is assumed to be independent of the liquidity status and is determined as

\[
I_{t+j} \equiv P_{t+j}^{NT}y_{t+j}^{NT} + P_{t+j}^{T}y_{t+j}^{T} - T_{t+j}
\]

where \( y_{t+j}^{NT} (y_{t+j}^{T}) \) denotes output from the non-tradeables (tradeables) sector, and \( T_{t+j} \) is a lump-sum tax paid by all households.

Thus, the constrained households’ consumption of non-tradeables and tradeables, respectively, is

\[
\begin{align*}
\hat{c}_{t+j}^{NT,e} &= \alpha \frac{I_{t+j}}{P_{t+j}^{NT}} \\
\hat{c}_{t+j}^{T,e} &= (1 - \alpha) \frac{I_{t+j}}{P_{t+j}^{T}}
\end{align*}
\]

\({}^2\)In the rest of the paper it is assumed that marginal utility is always positive, i.e., \( k \hat{b}_{t+j} < 1 \forall j \).
Finally, using (9) in (7), the value of the optimal consumption bundle can be written

\[ b_{t+j}^c = i_{t+j} \]  

(13)

where \( i_{t+j} \equiv I_{t+j}/Q_{t+j} \).

### 2.1.2 Non-liquidity constrained households

The remaining households (fraction \( 1 - \lambda \)) have access to saving and borrowing (denoted \( nc \)), and therefore they maximize (1) subject to the intertemporal budget constraint

\[
\sum_{j=0}^{\infty} \prod_{k=0}^{j} (1 + r_{t+k})^{-1} M_{t+j} \leq \sum_{j=0}^{\infty} \prod_{k=0}^{j} (1 + r_{t+k})^{-1} I_{t+j} + F_t
\]  

(14)

where \( F_t \) is nominal wealth at the beginning of period \( t \), \( r_{t+k} \) is the nominal interest rate, and nominal income \( I_{t+j} \) is defined in (10). It is assumed that equities are not traded internationally, and therefore the risk associated with variations in domestic production (and thus income) cannot be fully diversified via the international capital market. Thus, households are subjected to uninsurable risk due to incomplete capital markets, which leaves a role for an active stabilization policy, cf. Andersen (2001).

However, we still allow for some risk diversification since the non-liquidity constrained households have access to an internationally traded bond, which is assumed to offer a rate of return specified in terms of the consumption bundle, that is

\[
\frac{(1 + r_{t+1}) Q_t}{Q_{t+1}} = 1 + \delta_t.
\]  

(15)

To prevent the country from accumulating or decumulating foreign debt forever, it is assumed that \( \delta_t = \rho \forall t \), i.e., the objective and the subjective discount rates are equal, which implies that the real rate of return on the bond is riskless, and henceforth denoted \( 1 + \delta \).

These assumptions enable us to write the intertemporal budget constraint (14) as

\[
\sum_{j=0}^{\infty} (1 + \delta)^{-j} b_{t+j} \leq \sum_{j=0}^{\infty} (1 + \delta)^{-j} i_{t+j} + f_t
\]  

(16)

where \( i_{t+j} \equiv I_{t+j}/Q_{t+j} \) and \( f_t \equiv F_t/Q_t \) are measured in real terms. To simplify notation it is useful to define

\[
A_t \equiv \sum_{j=0}^{\infty} (1 + \delta)^{-j} E_t[i_{t+j}] + f_t
\]  

(17)

which is the expected present value of the household’s wealth – measured in real terms. Maximization of (1) subject to (16) yields

\[
b_{t}^{nc} = \frac{\delta}{1 + \delta} A_t
\]  

(18)
and the associated no-Ponzi game condition is
\[
\lim_{T \to \infty} (1 + \delta)^{-T} f_{t+T} = 0. 
\] (19)
Furthermore, we have that
\[
E_t [b_{t+j}^{ne}] = b_t^{ne}, \quad j \geq 0
\] (20)
\[
E_t [A_{t+j}] = A_t, \quad j \geq 0
\] (21)
and hence, consumption and wealth follow random walks.

Finally, for the non-constrained households the consumption of non-tradeables and trade-ables is
\[
c_t^{NT,ne} = \alpha \frac{\delta}{1 + \delta} A_t \frac{Q_t}{P_t^{NT}}
\] (22)
\[
c_t^{T,ne} = (1 - \alpha) \frac{\delta}{1 + \delta} A_t \frac{Q_t}{P_t^{T}}
\] (23)

2.1.3 Aggregation

Aggregate demand for the two goods is
\[
c_t^{NT,Agg} = \lambda c_t^{NT,c} + (1 - \lambda) c_t^{NT,ne}
\] (24)
\[
c_t^{T,Agg} = \lambda c_t^{T,c} + (1 - \lambda) c_t^{T,ne}
\] (25)
with \( \lambda \in [0, 1] \).

2.2 Firms

Firms are either producing tradeables or non-tradeables, and all firms are price and wage takers. The production function of a firm of type \( h = NT, T \) is
\[
y_t^h = \frac{\eta_t}{\beta} (L_t^h)^{\beta}, \quad 0 < \beta < 1
\] (26)
where \( L_t^h \) is labour input, and \( \eta_t \) is a productivity parameter. Firms maximize profits, which yields the following demand for labour
\[
L_t^h = \left( \eta_t \frac{P_t^h}{W_t} \right)^{\frac{1}{1-\beta}}
\] (27)
where \( W_t \) is the nominal wage rate, and therefore the output supply function is
\[
y_t^h = \frac{1}{\beta} \eta_t^{\frac{1}{\beta}} \left( \frac{P_t^h}{W_t} \right)^{\frac{\beta}{1-\beta}}
\] (28)
with \( h = NT, T \).
2.3 Wages

It is assumed that the labour market is competitive, and therefore the (nominal) wage is determined from the market clearing condition

\[ \bar{L} = N^{NT} \left( \frac{\eta_t P^{NT}}{W_t} \right)^\frac{1}{1-\beta} + N^T \left( \frac{\eta_t P^T}{W_t} \right)^\frac{1}{1-\beta} \] (29)

where \( N^{NT} \) (\( N^T \)) is the number of firms producing non-tradeables (tradeables). Therefore, the equilibrium wage can be written

\[ W_t = W \left( \frac{P^{NT}_t, P^T_t, \eta_t}{+} \right) \] (30)

and using this in the output supply functions yields

\[ y_{t}^{NT} = s^{NT} \left( \frac{P^{NT}_t, P^T_t, \eta_t}{+} \right) \] (31)
\[ y_{t}^{T} = s^{T} \left( \frac{P^{NT}_t, P^T_t, \eta_t}{+} \right) \] (32)

where the signs of the partial derivatives follow from (28) and (29). Hence, an increase in the price of tradeables (non-tradeables) decreases the output supply of non-tradeables (tradeables) since the wage increases. However, an increase in the own price increases the output supply, since the wage increases less than proportional to the increase in the price, and thus the sector-specific real wage decreases.

2.4 Public sector

The public sector demands non-tradeables \( (g_{t}^{NT}) \), and this is financed via lump-sum taxes. It is assumed that the public sector runs a balanced budget, i.e.,

\[ P^{NT}_t g_{t}^{NT} = T_t. \] (33)

Public demand is assumed not to affect directly the utility of households to focus on the pure demand effects\(^3\). Furthermore, it is assumed, for simplicity, that the public sector does not demand tradeables, as this would only affect the domestic economy through increasing the tax burden and worsening the trade balance.

\(^3\)It would suffice to assume that public demand is separable from private consumption in the household utility function.
2.5 Equilibrium

The market for non-tradeables is in equilibrium when

\[ y_t^{NT} = c_t^{NT,Agg} + y_t^{NT}. \]  

(34)

The trade balance \((x_t)\) is determined by the excess supply of tradeables

\[ x_t = y_t^T - c_t^{T,Agg}. \]  

(35)

This closes the model.

2.6 Consumption risk

In Appendix A it is shown that the present value of expected household income can be rewritten as

\[ A_t = \frac{1 - \lambda \alpha}{1 - \alpha} f_t + \frac{1}{1 - \alpha} \sum_{j=0}^{\infty} (1 + \delta)^{-j} E_t \left( \frac{P_{t+j}^T y_{t+j}^T}{Q_{t+j}} \right) \]  

and combined with (18) this shows that the risk in the private consumption bundle of the non-constrained households arises from variability in the real income generated in the tradeables sector, \(\frac{P_{t+j}}{Q_{t+j}} y_{t+j}^T\). The same is true for the constrained households. This is seen by combining (13) with (see Appendix A for a derivation)

\[ i_t = \pi_0 f_t + \pi_1 \frac{P_t}{Q_t} y_t^T + \pi_2 \sum_{j=0}^{\infty} (1 + \delta)^{-j} E_t \left( \frac{P_{t+j}^T y_{t+j}^T}{Q_{t+j}} \right), \]  

(37)

where \(\pi_0 \equiv \frac{(1-\lambda)\alpha}{1-\alpha} \frac{\delta}{1+\delta}\), \(\pi_1 \equiv \frac{1}{1-\lambda \alpha}\), and \(\pi_2 \equiv \frac{(1-\lambda)\alpha}{1-\lambda \alpha} \frac{\delta}{1+\delta}\). Hence, as in Andersen & Holden (2002) consumption risk stems from real income generated in the tradeables sector, and clearly there is a potential role for an active stabilization policy aimed at stabilizing real income from the tradeables sector.

3 Steady state

Assuming that the non-constrained households have no initial wealth, \(F_t = f_t = 0\), the steady state behaviour of the non-constrained households is the same as that of the constrained households, which leads to the following proposition.

**Proposition 1** The (initial) steady state is independent of the fraction of liquidity constrained households, \(\lambda\).
Proof. In steady state we have from (17) (dropping the time subscripts) \( A = f + \sum_{j=0}^{\infty} (1 + \delta)^{-j} i = \frac{1+\delta}{\delta} i \), assuming \( f = 0 \). Hence, \( c^{NT,c} = c^{NT,nc} \) using (11) and (22), and thus \( \frac{\partial c^{NT,Agg}}{\partial \lambda} = \frac{\partial y^{NT}}{\partial \lambda} = 0 \) from (24) and (34), respectively. \( \square \)

Andersen & Holden (1998) show that their model has a well-defined steady state. Proposition 1 implies that the model in this paper is identical to their model in steady state, and therefore the same is true for this model. Also, see Andersen & Holden (1998) for comparative static results of the steady state.

Proposition 1 is intuitive, since the non-constrained households with a constant real income flow \( i \) simply choose to spend their current income, as the objective and subjective discount rates are equal and the time horizon is infinite.

4 Productivity shocks

This section considers a supply shock. In particular it is assumed that productivity behaves according to

\[ \eta_t = \bar{\eta} + \varepsilon_t \]

where \( \varepsilon_t \) is the deviation of productivity from its long-run value, \( \bar{\eta} \), and we assume that these deviations are serially uncorrelated and unexpected, i.e., \( E_t [\varepsilon_{t+j}] = 0 \) \( \forall j > 0 \) and \( E_t [\varepsilon_{t+j} \varepsilon_{t+k}] = 0 \) \( \forall j \neq k \).

To obtain analytical solutions we rely on linearizations around the initial steady state. To ease notation let \( rx^h_t \equiv \frac{P^h x^h_t}{q^h_t} \) denote the deflated measure of a variable \( x^h_t \) with \( h = NT, T \), and let \( \tilde{r}x^h_t \) denote the deviation of \( rx^h_t \) from its steady state value.

It is assumed that public demand for non-tradeables follows the rule

\[ \tilde{r}g^{NT}_t = \kappa \varepsilon_t \]  

(38)

where \( \kappa \) is the stabilization parameter chosen by the government. Note that the policy rule only specifies the change in government spending following a shock.

At first, consider the case of no stabilization, i.e., \( \kappa = 0 \). In Appendix B it is shown that real income generated in the tradeables sector evolves as

\[ \tilde{r}y^T_t = \chi_1 \left( - \chi_2 \tilde{r}y^T_{t-1} - \chi_3 \sum_{j=1}^{\infty} (1 + \delta)^{-j} E_t [\tilde{r}g^T_{t+j}] + \chi_4 \varepsilon_t \right) \]  

(39)

where the \( \chi \)'s are defined in the appendix, \( \chi_1, \chi_2, \chi_3 > 0 \), and \( \chi_4 > 0 \) for \( \kappa = 0 \). Hence, the immediate effect of a positive productivity shock is that the real income generated in the tradeables sector exceeds its steady state level, since more is produced of both the tradeable
and the non-tradeable good, cf. (31) and (32), and the latter implies a drop in the relative price of non-tradeables, cf. (34).

Since $\frac{\partial x_1}{\partial \lambda} < 0$ (and $\frac{\partial x_4}{\partial \lambda} = 0$), the direct effect of a productivity shock is diminishing in the fraction being liquidity constrained. Hence, there is a lower variation in real income from the tradeables sector, and thus in the consumption bundles, cf. (36) and (37), when liquidity constraints are tight. The immediate consequence of this result is that the need for an active stabilization policy following a supply shock is decreasing in the fraction being liquidity constrained.

This result may at first come as a surprise, but intuition is straightforward. The rise in real income from the tradeables sector following a positive productivity shock will raise savings of the non-constrained households and thus aggregate consumption in the following periods. Hence, the relative price of non-tradeables will increase and real income from the tradeables sector will decrease in future periods. This effect tends to reduce the expected present value of the household’s wealth, and the non-constrained households smooth consumption out by lowering consumption today (relatively). This leads to a (further) drop in the relative price of non-tradeables, which then implies an even larger increase in real income generated in the tradeables sector this period. When more households are hand-to-mouth consumers, and thus not able to smooth out consumption, fewer will reduce consumption today (relatively) in expectation of lower income in the future, i.e., there is less intertemporal mobility. This will reduce the drop in the relative price of non-tradeables. Thus, the rise in real income from the tradeables sector will be reduced. Vice versa for a drop in productivity.

Now, considering the possibility of an active fiscal stabilization policy via the public demand for non-tradeables leads to the following proposition:

**Proposition 2** Following a productivity shock, there exists a choice of the stabilization parameter, $\kappa = \kappa^* > 0$, which ensures perfect stabilization of the values of the consumption bundles, i.e., $\bar{l}^n_c = \bar{l}^c = 0$.

**Proof.** See Appendix B. ■

Hence, the optimal stabilization policy implies that public demand for non-tradeables increases following a positive productivity shock since the relative price of non-tradeables (and thus the wage) has to increase enough to offset the direct effect of the productivity gain on real income generated in the tradeables sector. Since households are risk averse, this stabilization policy will on average increase welfare because consumption is stabilized.

**Proposition 3** The optimal stabilization policy is independent of the fraction of liquidity constrained, i.e., $\frac{\partial \kappa}{\partial \lambda} = 0$. 

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Proof. See Appendix B. ■

The optimal stabilization policy implies that the steady state level of consumption is attained in all periods, and therefore Proposition 3 follows directly from Proposition 1. Hence, in this model fiscal policy is effective, but it is not true that fiscal policy is more effective in stabilizing the economy when liquidity constraints are tight.

Finally note that the discussion above assumes that the supply shock affects productivity in both sectors. However, the same qualitative results are obtained in the case of asymmetric shocks, i.e., by assuming that only one of the sectors is affected, cf. Appendix B.

5 Concluding remarks

This paper introduced liquidity constraints in a model of a small open economy. Following a productivity shock it was shown that there exists an active fiscal stabilization policy which is able to perfectly stabilize the consumption bundles. This policy is independent of the liquidity constraints in the sense that the optimal reaction of public consumption does not depend on the fraction being liquidity constrained. Furthermore, the presence of liquidity constraints actually reduces the need for an active policy stabilizing productivity shocks.

Hence, the consequences of liquidity constraints are more involved than previously thought, and there is clearly a need for more work in this area. Also note that some of the usual arguments carry over to this model: From a policy perspective it will be possible to temporarily boost (reduce) aggregate demand by making a positive (negative) temporary transfer from the non-constrained to the constrained households. Note that the policy does not have to be unexpected in order to achieve this reaction in aggregate demand. On the other hand, the temporary nature of the transfer is crucial, since there are no activity effects of permanent transfers, but only redistributional effects. Future research should therefore dedicate a particular focus to the modelling of the aggregate demand structure.

Footnotes:

4Formally this is seen by replacing (10) with $i_{t+j}^c = P_{NT,NT}^c y_{NT}^c + P_{NT,NT}^c y_{NT}^c - T_{t+j}^c + \frac{1}{1+\lambda} u_t$ and $i_{t+j}^{nc} = P_{NT,NT}^{nc} y_{NT}^{nc} + P_{NT,NT}^{nc} y_{NT}^{nc} - T_{t+j}^{nc} - \frac{1}{1-\lambda} u_t$, where $u_t$ is the total real amount transferred from the non-constrained households to the constrained, and $\lambda \in (0,1)$. Note that ceteris paribus the transfer does not affect aggregate income, but only the distribution of income. Since the non-constrained households smooth out the temporary windfall gain/loss but the constrained do not, aggregate consumption of non-tradeables will, ceteris paribus, change by $\alpha_{t+j} \frac{1}{1+\lambda} Q_{NT} u_t$, cf. (11), (22) and (24).

5To see this let $w_{t+j} = \pi \forall j \geq 0$. Then, the economy will immediately reach a new steady-state, where $c_{t}^{NT,c}$ has changed by $\alpha_{t+j} \frac{Q_{NT}}{1+\lambda} u_t$, cf. (11), and $c_{t}^{NT,nc}$ has changed by $\alpha_{t+j} \frac{Q_{NT}}{1-\lambda} u_t$, cf. (22). Hence, aggregate consumption is unchanged, cf. (24).
References


IMF (2009), ‘World economic outlook’.


APPENDICES

A Consumption risk

In this appendix (36) and (37) are derived. Using (10), (33) and (34) in \( i_{t+j} = I_{t+j}/Q_{t+j} \) yields

\[
i_{t+j} = \frac{P_{t+j}^{NT} A_{t+j}}{Q_{t+j}^{NT}} + \frac{P_{t+j}^{T} y_{t+j}}{Q_{t+j}}
\]

\[
= \lambda \alpha i_{t+j} + (1 - \lambda) \alpha \frac{\delta}{1 + \delta} A_{t+j} + \frac{P_{t+j}^{T} y_{t+j}}{Q_{t+j}} \Leftrightarrow \\
i_{t+j} = \frac{1}{1 - \lambda \alpha} \left[ (1 - \lambda) \alpha \frac{\delta}{1 + \delta} A_{t+j} + \frac{P_{t+j}^{T} y_{t+j}}{Q_{t+j}} \right]
\]

(40)

where the second line uses (11), (22) and (24). Inserting in (17) implies

\[
A_t = f_t + \sum_{j=0}^{\infty} (1 + \delta)^{-j} E_t \left[ (1 - \lambda) \alpha \frac{\delta}{1 + \delta} A_{t+j} + \frac{P_{t+j}^{T} y_{t+j}}{Q_{t+j}} \right] \frac{1}{1 - \lambda \alpha}
\]

\[
= f_t + \frac{(1 - \lambda) \alpha}{1 - \lambda \alpha} A_t + \sum_{j=0}^{\infty} (1 + \delta)^{-j} E_t \left[ \frac{P_{t+j}^{T} y_{t+j}}{Q_{t+j}} \right] \frac{1}{1 - \lambda \alpha} \Leftrightarrow \\
A_t = \frac{1 - \lambda \alpha}{1 - \alpha} f_t + \frac{1}{1 - \alpha} \sum_{j=0}^{\infty} (1 + \delta)^{-j} E_t \left[ \frac{P_{t+j}^{T} y_{t+j}}{Q_{t+j}} \right]
\]

where the second line uses the random walk property of wealth, (20). Finally, inserting in (40) yields

\[
i_t = \frac{(1 - \lambda) \alpha}{1 - \alpha} \frac{\delta}{1 + \delta} f_t + \frac{1}{1 - \lambda \alpha} P_{t}^{T} y_{t}^T + \frac{1}{1 - \alpha} (1 - \lambda) \alpha \frac{\delta}{1 + \delta} \sum_{j=0}^{\infty} (1 + \delta)^{-j} E_t \left[ \frac{P_{t+j}^{T} y_{t+j}}{Q_{t+j}} \right].
\]

B Productivity shocks

This appendix proves Propositions 2 and 3. Using (11), (22) and (24) in (34) and rewriting yields

\[
ry_t^{NT} = \lambda \alpha i_t + (1 - \lambda) \alpha \frac{\delta}{1 + \delta} A_t + ry_t^{NT}. \tag{41}
\]

Furthermore, by making a first-order Taylor approximation of (31) and (32) around the initial steady state, we get

\[
\tilde{r}_y^{NT} = \gamma_0 \tilde{P}_t^{NT} + \gamma_1 \varepsilon_t \tag{42}
\]

\[
\bar{r}_y = -\rho_0 \bar{P}_t^{NT} + \rho_1 \varepsilon_t \tag{43}
\]

13
where $\gamma_0, \gamma_1, \rho_0, \rho_1 > 0$ follows directly from (31) and (32). Combining the linearized version of (41) with (42) and inserting (38) yields

$$
\tilde{P}^{NT}_t = \lambda \frac{\alpha}{\gamma_0} \tilde{t}_t + (1 - \lambda) \frac{\lambda}{\gamma_0} \frac{\delta}{1 + \delta} \tilde{A}_t + \frac{\kappa - \gamma_1}{\gamma_0} \tilde{\varepsilon}_t
$$

Inserting this and the linearized versions of (36) and (37) in (43) implies

$$
\tilde{r}y_t^T = \chi_1 \left( -\chi_2 \tilde{f}_t - \chi_3 \sum_{j=1}^{\infty} (1 + \delta)^{-j} E_t \left[ \tilde{r}y_{t+j}^T \right] + \chi_4 \tilde{\varepsilon}_t \right)
$$

where $\chi_1 \equiv \left( 1 + \frac{\rho_1}{\gamma_0} \frac{1}{1 + \delta} \left[ \frac{\lambda}{1 - \lambda} + \delta \frac{\alpha}{1 - \alpha} \right] \right)^{-1} > 0$, $\chi_2 \equiv \frac{\rho_1}{\gamma_0} (1 - \lambda) \frac{\alpha}{1 - \alpha} \frac{\delta}{1 + \delta} > 0$, $\chi_3 \equiv \frac{\rho_0}{\gamma_0} \frac{\alpha}{1 - \alpha} \frac{\delta}{1 - \lambda} > 0$, and $\chi_4 \equiv \rho_1 - \rho_0 \frac{\kappa - \gamma_1}{\gamma_0} > 0$.

Choosing $\kappa = \gamma_1 + \frac{\rho_1}{\rho_0} \gamma_0 \equiv \kappa^* > 0$ implies $\tilde{r}y_t^T = 0$. To see this note that initially $\tilde{f}_t = 0$. Furthermore, $\tilde{A}_t = \tilde{t}_t = 0$ from (36) and (37) when $E_t \left[ \tilde{r}y_{t+j}^T \right] = 0 \forall j > 0$. To see the latter point, note that $\tilde{f}_{t+1} = (1 + \delta) \left( \tilde{f}_t + \tilde{t}_t - \tilde{b}_t \right) = 0$, and therefore there are no effects in future periods. Hence, using these results in (13) and (18) we have proven $\tilde{b}_t^c = \tilde{b}_t^{nce} = 0$.

Thus, $\frac{\partial \kappa^*}{\partial \lambda} = \frac{\partial \left( \gamma_1 + \frac{\rho_1}{\rho_0} \gamma_0 \right)}{\partial \lambda} = 0$, since the fraction of liquidity constrained households does not affect the steady state (recall that the $\gamma$’s and the $\rho$’s are partial derivatives evaluated in steady state), cf. Proposition 1.

Asymmetric shocks can be analyzed either by setting $\gamma_1 = 0$ (only shocks to the tradeables sector) or by setting $\rho_1 = 0$ (only shocks to the non-tradeables sector), which yields the optimal stabilization parameters $\kappa^*|_{\gamma_1=0} = \frac{\rho_1}{\rho_0} \gamma_0 > 0$ and $\kappa^*|_{\rho_1=0} = \gamma_1 > 0$, i.e. the qualitative results are not altered.
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