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Optimal Unemployment Insurance: How Important is the Demand Side?

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Optimal Unemployment Insurance: How Important is the Demand Side?*

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Abstract

I develop and calibrate an equilibrium model of search with endogenous savings and search intensity. The wage offer distribution is endogenized by firms making vacancy and entry choices. This allows me to conduct a counterfactual analysis of the optimal unemployment insurance (UI) level. The provision of UI is motivated by the worker's inability to perfectly insure against income shocks, but at the same time UI introduces a distortion to the level of search intensity of the worker and vacancy intensity of firms.

I find that equilibrium effects are important to take into account. Making policy from a partial model can introduce large welfare losses. It is also shown that different kinds of taxes have different implications on welfare.

JEL Classification: D3, D9, E2, E61, J6.

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1 Introduction

In this paper I argue that equilibrium effects should be taken into account when trying to assess the optimal level of unemployment insurance (UI). Primarily, economists have been concerned with two aspects of UI. First, unemployment insurance distorts incentives to search for a job since higher insurance makes the gain of getting a job smaller. Second, if financial markets are incomplete in the sense that a worker cannot insure against future income losses, unemployment insurance can be used to smooth consumption.¹ I develop and calibrate an equilibrium model taking into account not only how workers change search behavior, but also how firms react to the change in worker behavior.

The economic literature regarding unemployment insurance has several strands; One part of the literature deals with the design of optimal schemes for duration dependent unemployment insurance, allowing the worker to self-insure, but assuming that jobs last forever, see Shimer and Werning (2008), Coles (2008), Kocherlakota (2004), and Hopenhayn and Nicolini (1997). Another part also takes the demand side of the labor market as given, but allows for the possibility of later job destruction, see Lentz and Tranæs (2005), Rendon (2006), Lise (2010), and Lentz (2009). These models allow the worker to self-insure. Lise (2010) develops a continuous model of workers who have the possibility to self-insure and face uncertainty about their future labor market state. The firms are modelled as agents making a passive wage offer, so essentially the workers face an exogenous wage offer distribution. Lentz and Tranæs (2005) consider a Markov model with two states, employed or unemployed. Hence, there is no on-the-job search, but the search effort when unemployed is endogenous. They introduce a lottery in order to characterize the workers optimal search effort and savings. Both Lise (2010) and Lentz and Tranæs (2005) take the wage offer distribution as given, i.e., wage setting and firm behavior are not influenced by the parameters of the model. I endogenize these features in order to take equilibrium effects into account. Rendon (2006) also develops a partial on-the-job search model where workers live a finite period of time. Rendon

¹Acemoglu and Shimer (1999) argue that higher unemployment insurance will generate more productive matches, since workers are more willing to take more productive, but also more uncertain jobs. The model presented in this paper will not be able to incorporate this type of effect. Although, differences in search technologies between employed and unemployed workers could potentially generate this phenomenon.

shows that borrowing constraints are important. A third strand of models allows for equilibrium effects through free entry of firms and wage determination, see Fredriksson and Holmlund (2001) and Coles and Masters (2006). But this literature ignores the possibility of self-insurance through savings. Thus, in this paper I try to combine the last two strands of the literature on optimal UI. The paper closest to it in setup is perhaps Krussel et al. (2010). They introduce an equilibrium setting where workers are risk-averse and only have access to incomplete insurance. However, Krussel et al. do not allow workers to choose search intensity, which induces workers to save too much when assets are close to the credit constraint compared to a world where they can choose search intensity. Using Danish data I show that unemployment duration is not independent of the worker's wealth level, which is the implication of the model in Krussel et al. The main focus in their paper is not the moral hazard problem that UI introduces, which is one of the cornerstones of this paper, but rather matching business cycle fluctuations.

The main contribution of this paper is to endogenize the demand side of the labor market, while keeping the features of endogenous savings and search intact, and thus endogenizing the wage offer distribution. This enables me to make a counter-factual analysis regarding unemployment insurance while taking into account not only the change in the behavior of the workers when changing unemployment insurance, but also the change in firm behavior. When unemployment insurance increases, workers will search less intensely for a job. Firms will realize this and post fewer vacancies and the unemployment rate will increase even more. Increasing unemployment insurance will also pressure marginal profit firms to shut down. So far, these effects have been ignored in the literature. This will tend to underestimate the effect of increasing benefits on the unemployment rate, thereby overestimating the return to high benefits.

More specifically, this paper extends the model in Lise (2010) to an equilibrium model. Workers choose their search effort as well as how much to save and consume at each moment in time both while unemployed and employed. I assume that no central financial market trades sets of contingent claims for a constant level of consumption. Thus, the worker has to self-insure by saving, but this is imperfect in the sense that he cannot perfectly smooth consumption. Firms are modelled as a collection of jobs with constant and firm-specific productivity and a possibility to advertise for

vacancies, where the advertising rate is a choice of the firm. A matching process governed by a standard matching function brings workers and firms together.

The model captures several key aspects of the data. In the model workers with more wealth have longer unemployment and employment durations. Because wealthier workers can afford to wait longer for better opportunities, they search less intensively. There is empirical evidence that wealth affects labor market decisions. Algan et al. (2003) use three years of survey data from France and focus on liquid assets, which they define as time deposits, i.e., omitted wealth data are financial securities, house savings and life insurance. They find that more wealth increases the unemployment duration and the probability of a voluntary job quit. Bloemen and Stancanelli (2001) use Dutch survey data to estimate a simultaneous equations model where wealth can affect the reservation wage directly, but wealth is also allowed to affect the job offer arrival rate. They find that wealth has a positive effect on reservation wages and a negative effect on the employment probability. This paper will show that these relationships also pertain to the Danish data used in this study.

Also the positive relationship between wealth and wages in the data is replicated in the model since, in general, workers who are placed higher on the productivity ladder have been longer in employment and therefore have been saving more.

The only motive for holding savings in this model is to insure against future income fluctuations. This approach ignores saving for retirement. Although an interesting question, retirement behavior is probably better studied in an OLG-type model.

A simple version of the model with no wage dispersion is calibrated to match Danish data and otherwise standard parameter values. The calibrated model is used to illustrate in which way equilibrium effects work. It is shown that equilibrium effects are just as important as the partial equilibrium effects for the response in unemployment to an increase in UI. Based on the simple model the optimal replacement rate is calculated to be 47 percent. It is also shown that the use of the optimal UI from the partial equilibrium model has large adverse effects on welfare. Furthermore, the model is used to evaluate four different kinds of tax schemes. It is found, applying a utilitarian welfare function, that income taxation and value added (VA) tax results in higher

welfare than that of both taxes on capital gains and wealth taxes. The last two types of taxes greatly distort the self-insurance of the worker resulting in relatively large welfare losses.

The structure of the paper is as follows. In section 2, the model is presented. In section 3, I describe the steady state conditions. Section 4 describes the numerical solution. The data used is presented in section 5. In section 6 the calibration is described and I present the results. Section 7 shows robustness results for different parameter values. Section 8 concludes.

2 Model

The model is an on-the-job search model with risk averse workers who have the option to self-insure against future income loss by saving in a single asset. The model is cast in continuous time. A firm consists of a collection of jobs, and the possibility to create new vacancies and hire new workers. Firms are heterogeneous and decide how much to advertise for vacancies. Workers and firms are brought together by a standard matching function. At a cost potential firms can draw an exogenous productivity type and enter the market. Thus how many vacancies a firm advertises and how many firms exist is determined in equilibrium. Interest rates are taken as exogenous in the model. Since the model is to be calibrated on Danish data, this is a reasonable assumption.

2.1 Workers

Workers are ex ante identical and infinitely lived agents. I normalize the measure of workers to one. They have the possibility to save at an exogenous interest rate r . The employed worker receives a wage $w(p)$, where p is the productivity of the firm at which the worker is employed, while the unemployed worker receives unemployment benefits b . Letting unemployment insurance be a constant and independent of the length of the unemployment spell is not a bad approximation to reality. The Danish system, to which the model will be calibrated, does in many ways have unlimited unemployment benefits. Shimer and Werning (2008) study the optimal design of unemployment insurance. They find that a flat scheme is optimal when workers have constant absolute risk aversion, and that a near constant scheme is optimal if the worker has constant relative risk

aversion. Optimal here refers to the scheme minimizing the cost of providing a given utility to the worker. Given this, a constant unemployment insurance level is not a bad approximation.

Employed and unemployed workers make contact with random firms as a function of their own search effort and the aggregated level of job search and vacancy advertising. Firms cannot make counteroffers when their workers are contacted by outside firms as in the Postel-Vinay and Robin (2002) setting. Employed workers face the risk of the job terminating exogenously and becoming unemployed. The worker's life is therefore one of transitions between different jobs and unemployment. The worker's decision problem is twofold; he decides how much to consume and save and how much search effort to exert. Saving is determined by the desire to consume and to save for future consumption. The search effort level is determined by a convex search cost and an increasing probability of getting a new job offer in the level of search effort exerted. No central financial market trades sets of contingent claims for a constant level of consumption, so the worker has to self-insure. At most, a worker can borrow \underline{a} .

The unemployed worker receives a flow income of b and the value function is²

$$\varphi U(a) = \max_{c \in Y_0(a), s \geq 0} u(c(a, b)) - e(s) + U'_a(a) da/dt + \lambda(\theta) s \int_{\underline{p}}^{\bar{p}} [W(a, p') - U(a)] d\Gamma(p') \quad (1)$$

where $Y_0(a)$ is the feasible set of consumption possibilities, i.e.,

$$Y_0(a) = \begin{cases} [0, \infty) & \text{for } a > \underline{a} \\ [0, ra + b] & \text{for } a = \underline{a} \end{cases}$$

φ is the discount rate, $u(\cdot)$ is the instant utility function, a is the asset level, s is the endogenous search effort, $e(s)$ is a strict convex cost function of search effort, Γ is the cumulative distribution function (CDF) of productivities of vacancies, and $\lambda(\theta)$ is the arrival rate of vacancies per unit of search effort, where θ is market tightness, which will be determined in equilibrium. The first two terms in the Bellman equation for unemployed workers are the instantaneous utility from consumption and disutility from searching. The third term is the expected change in the value of being unemployed. This value changes since accumulated assets change, and comprise two terms;

²See Appendix for a delta-type derivation of the Bellman equation

how much the value function changes with respect to assets and how fast assets change. The last term is flow rate of job offers times the expected value of such an offer.

The employed worker at a type p firm has the following value function³

$$\varphi W(a, p) = \max_{c \in Y(a), s \geq 0} u(c(a, p)) - e(s) + W'_a(a, p) da/dt - \delta(W(a, p) - U(a)) \quad (2)$$

$$+ \lambda(\theta) s \int_p^{\bar{p}} [W(a, p') - W(a, p)] d\Gamma(p') \quad (3)$$

δ is the job destruction rate. Assets evolve according to the differential equation

$$da = [w(p) + ra - c]dt \quad (4)$$

where $w(p)$ is the wage as a function of the productivity of the firm, r is the interest rate, and c is consumption. Search technology is assumed to be the same for employed and unemployed workers, i.e., being unemployed is equivalent to being employed at a type b firm.

2.1.1 F.O.N.C.

The first order necessary condition for s is

$$e'(s) = \lambda(\theta) \int_p^{\bar{p}} W(a, p') - W(a, p) d\Gamma(p') \quad (5)$$

The marginal cost from searching is equal to the marginal gain. The first order necessary condition for c is

$$u'(c(a, p)) = W'_a(a, p) \quad (6)$$

This is the standard result that the marginal value of consumption should equal the marginal value of assets, so the worker is indifferent between saving and consuming. Notice that at the borrowing limit, \underline{a} , consumption is also determined by equation (6) since it is never optimal for consumption

³See Appendix for a delta-type derivation of the Bellman equation

to jump and $W'_a(a, p)$ is continuous.

Optima The first order approach clearly finds a maximum for search effort since

$$-e''(s) < 0$$

which always holds by assumption of strict convexity of $e(s)$.

Proving that the first order approach is valid for consumption, i.e. the optima found are global maxima and not minima, amounts to the condition $W''_{aa}(a, p) < 0$. It is a well-known problem that it is generally difficult to show that the value functions of the worker are concave in assets if an endogenous search effort is present, see Lentz and Tranæs (2005). The reason for the result is that the value function is a convex combination between two concave functions where the degree of convexity is endogenous. Lentz and Tranæs (2005) solve this by introducing a lottery in assets. They also note that they have never found a solution that was not concave in any of their simulations. Chetty (2008) simply assumes that the value functions are concave and notes that the assumption is never violated in simulations for reasonable parameter values. I will take this approach too, i.e., I will check if the value functions are indeed concave in the solution. Lise (2010) claims that the value functions are concave, but he does not give any formal proof.

Characterization of optimal search effort and savings It is hard to characterize the value functions in these types of models, see Lentz and Tranæs (2005). I therefore highlight some assumptions that are necessary for further characterization of the solution. These assumptions have never been violated in any of the simulations in this paper.

A1) The value functions are concave in assets, i.e., $W''_{aa}(a, p) < 0$ and $U''_{aa}(a) < 0$

A2) The marginal value of assets is declining in p , i.e., $W''_{ap}(a, p) < 0$

Proposition 1 *Under assumption A2) consumption is increasing in the productivity of the firm, and, under assumption A1) consumption is increasing in the level of assets.*

Proof. Implicit differentiation of equation (6) wrt. a yields

$$\begin{aligned} u''(c(a, p))c'_a(a, p) &= W''_{aa}(a, p) \\ c'_a(a, p) &= \frac{W''_{aa}(a, p)}{u''(c(a, p))} > 0 \end{aligned}$$

since $u''(c(a, p)) < 0$ and, given assumption A1), consumption is increasing in assets.

Implicit differentiation of equation (6) wrt. p yields

$$\begin{aligned} u''(c(a, p))c'_p(a, p) &= W''_{ap}(a, p) \\ c'_p(a, p) &= \frac{W''_{ap}(a, p)}{u''(c(a, p))} > 0 \end{aligned}$$

Under assumption A2) consumption is increasing in the productivity of the firm. The analysis for the unemployed is essentially the same since being unemployed is the same as being employed at a type b firm. ■

Workers with higher asset levels consume more. This is a quite natural result. Workers in a higher productivity firm also consume more for a given level of assets. This is because the income is higher in higher productivity firms.

Proposition 2 *Under the assumption of a strictly convex search cost function, $c(s)$, search effort is decreasing in the level of assets and productivity of the firm. Employees at the highest productivity firm do not search for a new job.*

Proof. Implicit differentiation of equation (5) wrt. a yields

$$\begin{aligned} e''(s)s'_a(a, p) &= \lambda(\theta) \int_p^{\bar{p}} W'_a(a, p') - W'_a(a, p) d\Gamma(p') \\ &\quad \lambda(\theta) \int_p^{\bar{p}} u'(c(a, p')) - u'(c(a, p)) d\Gamma(p') \\ s'_a(a, p) &= \frac{\int_p^{\bar{p}} (W'_a(a, p') - W'_a(a, p)) d\Gamma(p')}{\int_p^{\bar{p}} (u'(c(a, p')) - u'(c(a, p))) d\Gamma(p')} < 0 \end{aligned}$$

Since $e''(s) > 0$ and $u'(c(a, p')) < u'(c(a, p))$ for $p' > p$, i.e., marginal utility given assets are

lower in a higher productivity firm since, as shown above, consumption is increasing in p . Implicit differentiation of equation (5) wrt. p yields

$$e''(s)s'_p(a, p) = \lambda(\theta) \int_p^{\bar{p}} -W'_p(a, p)d\Gamma(p')$$

$$s'_p(a, p) = -\frac{\lambda(\theta) \int_p^{\bar{p}} W'_p(a, p)d\Gamma(p')}{e''(s)} < 0$$

So search effort decreases when workers move to more productive firms. From equation (5) it is easy to see that

$$\lim s(a, p) \rightarrow 0 \text{ for } p \rightarrow \bar{p}$$

The analysis for the unemployed is the same. ■

A worker's search effort is decreasing in assets. Since low wealth workers are closer to the budget constraint, and thereby closer to getting constrained in their consumption possibilities, they will put more effort into getting a job. A worker's search effort is also decreasing in the productivity of the firm he is working in. This is a standard result in models with endogenous search effort. At the limit workers in the highest productivity firm have no gain of searching. Christensen et al. (2005) show that endogenous search is important. Workers in low productivity firms find jobs faster than workers in high productivity firms. The difference is more pronounced than a model with constant search effort across productivities would suggest.

2.1.2 Upper and lower bounds on asset distribution

Lower bound The lower bound on assets in the model, \underline{a} , is difficult to set. Aiyagari (1994) shows that imposing a present value budget balance together with a non-negative consumption is the same as imposing a borrowing constraint at $\underline{a} = -b/r$. Since the consumption cannot be negative, the most a worker can borrow is such that he uses all his income to pay the interests, i.e., $-\underline{a}r = b$. This will result in a very low level of assets in the current model. Lise (2010) solves this by imposing a minimum consumption level, where $u'(c) \rightarrow \infty$ for $c \rightarrow \underline{c}^+$, and \underline{c} is estimated. This

makes the value function hard to solve for values near the lower limit since the derivative of the value function with respect to assets also goes to infinity. I therefore choose to set the borrowing limit higher than $-b/r$ and \underline{c} to zero. I will later calibrate \underline{a} .

Upper bound

Proposition 3 *Under the assumption that the worker is sufficiently patient, $\varphi > r - \delta$, the upper bound on assets is finite and given by*

$$w(\bar{p}) + r\bar{a} = \phi \left(\frac{\lambda \delta u'(c(\bar{a}, b))}{\varphi - r + \delta} \right)$$

Proof. See Lise (2008) ■

If the assumption of $\varphi > r - \delta$ is dropped, there is no guarantee of an upper limit on the asset distribution, see Bayer and Wälde (2009). That is, there are no steady state in levels. This would make the computational solution of the model much harder. The later analysis is therefore limited to the assumption that $\varphi > r - \delta$. Lise (2010) also invokes this assumption.

2.2 Wage Bargaining

Solving Nash bargaining over value functions to determine the wage is not feasible since the problem might be non-convex because a wage increase gives longer employment durations.⁴ But a circumvention of this problem still leaves a wage that is dependent on the asset level of the worker.⁵ It is hard to imagine a real life situation where the employer demands to see the level of assets of the worker and that this has an impact on the wage negotiated. Also one would need to address how wages are renegotiated when assets increase or decrease.

I follow a more tractable approach. The worker and the firm bargain over the joint instantaneous surplus $p - b$. The game is set in artificial time. The bargaining is similar to that of Binmore, Rubinstein and Wolinsky (1986), and Shaked and Sutton (1984) where workers and firms have time preferences over the outcome, but the worker and the firm bargain over instant surplus and not value functions. The worker and the firm take turns making offers that the other party can accept

⁴See Shimer (2006).

⁵Conley & Wilkie (1996) make an extension of the Nash solution to cover non-convex bargaining sets.

or decline. If the other party declines the offer, that party makes a new proposition and so on. Let ξ_1 and ξ_2 be the discount rates for player 1 and 2, respectively, in this game.

The solution to this game is easy to show, see Shaked & Sutton (1984). Let Q be the supremum of payoffs that player 1 gets in any SPE starting at time 2. Then at time 1, player 1 will accept any offers of at least $\xi_1 Q$ so player 2 will get at most $(p - b) - \xi_1 Q$. At time 0, player 2 will therefore accept offers of at least $\xi_2 [(p - b) - \xi_1 Q]$, and so player 1 will get a maximum of $(p - b) - \xi_2 [(p - b) - \xi_1 Q]$ at time 0. Since the games at time 0 and time 2 are the same, the following must hold

$$\begin{aligned} Q &= (p - b) - \xi_2 [(p - b) - \xi_1 Q] \\ &\Updownarrow \\ Q &= \frac{(1 - \xi_2)(p - b)}{1 - \xi_1 \xi_2} \equiv \beta (p - b) \end{aligned}$$

where β is just a function of the underlying time preferences. An analogous argument can be made when Q is the infimum instead of the supremum. Thus, the unique solution is Q . Without loss of generality, let the worker be player 1, i.e., the wage equation is

$$w(p) = \beta (p - b) + b$$

Lentz and Mortensen (2008a) and Mortensen (2009) also use a splitting of instantaneous surplus, although the games are differently motivated. Lentz and Mortensen (2008a) show a strong positive and almost linear relationship between wage and productivity using the Danish data also used in this paper. This supports the instantaneous rent-sharing in this model. Bargaining protocols over value functions would most likely lead to a more convex relationship between wage and productivity since workers in low productivity firms would have lower average wealth holdings. Thereby, the outside option of the worker would be smaller, and the firm would get a bigger cut of the surplus.

Endogenizing the wage using instantaneous surplus splitting has several advantages. First, it is very tractable and eases the solution of the model a lot which is important in the numerical solution of the problem. Second, it maintains many reasonable features. The wage is increasing in

the productivity of the firm and the benefit level. The firm surplus is increasing in productivity and decreasing in the benefit level. And finally, the wage is not directly affected by the asset level of the worker. If the outcome of the wage negotiation depended on the level of assets, the worker would have an incentive to save in order to pressure his employer into giving him a higher wage. This feature seems unattractive.

2.3 Firms

Let there be a measure M of firms, where M is determined in equilibrium by a free entry condition. Firms comprise different jobs with equal productivity p and the possibility of creating new jobs by advertising vacancies.⁶ Individual jobs terminate at an exogenous rate, $\lambda(\theta)s(a, p)\bar{\Gamma}(p) + \delta_j$. Firms choose a vacancy intensity, v , at a cost of $k(v)$, where $k(v)$ is increasing and strictly convex. For a given choice of v , firms meet new workers at a rate $q(\theta)v$. If the worker is unemployed, he accepts the job offer by the firm. If the worker is employed he only accepts the job offer if the productivity of the new firm is higher than the productivity of the incumbent firm. Firms are risk neutral. The value of a job for a type p firm employing a worker with asset level a is $J(a, p)$ defined by the Bellman equation⁷

$$rJ(a, p) = (1 - \beta)(p - b) - \delta J(a, p) - \lambda(\theta)s(a, p)\bar{\Gamma}(p)J(a, p) + J'_a(a, p)da/dt$$

where β is the workers bargaining power as a function of the time preferences of workers and firms, δ_j is the job specific destruction rate, and r is the interest rate. The value of a vacancy for a firm with productivity p is $V(p)$ ⁸

$$(\delta_f + r)V(p) = \max_{v \geq 0} -k(v) + q(\theta)v \left[\frac{uS_0}{(1-u)S_1 + uS_0} \int_a^\infty J(x, p)d\Lambda_0(x) + \frac{(1-u)S_1}{(1-u)S_1 + uS_0} \int_a^p \int_a^\infty J(x, p)d\Lambda_1(x, y) \right] \quad (7)$$

⁶A CRTS production technology and reversible capital adjustments will suffice.

⁷See Appendix for a delta-type derivation of the Bellman equation

⁸See Appendix for a delta-type derivation of the Bellman equation

where $q(\theta)$ is the arrival rate of workers per unit of vacancy intensity, δ_f is the firm destruction rate, i.e., $\delta = \delta_f + \delta_j$, and

$$\Lambda_1(a, p) = \int_{\underline{p}}^p \int_{\underline{a}}^a \frac{s(x, y)}{S_1} g_1(x, y) dx dy$$

$$\Lambda_0(a) = \int_{\underline{a}}^a \frac{s(x, b)}{S_0} g_0(x) dx$$

where $S_1 = \int_{\underline{p}}^{\bar{p}} \int_{\underline{a}}^{\infty} s(a, p) dG_1(a, p)$ and $S_0 = \int_{\underline{a}}^{\infty} s(a, b) dG_0(a)$ are the average search efforts of employed and unemployed workers respectively. $G_1(a, p)$ is the joint CDF of workers in a type p firm with assets a . $G_0(a)$ is the CDF of unemployed workers with assets a . This reflects a proportionality in the matching process, i.e., a worker who searches twice as hard meets firms at twice the rate.

This way of modeling the firms is not standard. In the Burdett and Mortensen (1998) framework the number of firms are not endogenous, but firms can potentially have different productivities. In the standard Mortensen and Pissarides (1994) model firms are a very vague concept, see Fredriksson and Holmlund (2001) for an application. The reason is that there are no firm heterogeneity and the value of a vacancy is driven down to zero. So essentially, there are no difference between firms. The above formulation of the firm is based on the fact that I would like to have firm heterogeneity in productivities. This is based on the fact that there is a positive empirical correlation between the wage rate of a firm and value added per worker, i.e., more productive firms pay higher wages. If one wishes to model firm heterogeneity, one cannot let the value of a vacancy be driven down to zero. If this was the case, only the highest productivity firms would exist. Therefore, firms enter this economy at a cost i and draw a productivity value from the distribution $f(p)$ with support on $[\tilde{p}, \bar{p}]$. If $p < b$, the firm exits the market. Let the value of drawing a firm type be

$$X = \int_{\underline{p}}^{\bar{p}} \max\{V(p), 0\} dF(p) = \int_{\underline{p}}^{\bar{p}} V(p) dF(p) \quad (8)$$

where $\underline{p} = \min(b, \tilde{p})$. Firms pay a cost of i to draw a type, so free entry is equivalent of $X = i$. When firms enter the economy, they have zero workers.

2.3.1 F.O.N.C.

From equation (7) the first order condition for vacancy creation is

$$k'(v) = q(\theta) \left[u \int_{\underline{a}}^{\infty} J(x, p) d\Lambda_0(x) + (1 - u) \int_{\underline{p}}^p \int_{\underline{a}}^{\infty} J(x, p) d\Lambda_1(x, y) \right] \quad (9)$$

Since $k(v)$ is strictly convex, this is the optimal vacancy choice of the firm.

Characterization of the optimal vacancy choice

Proposition 4 *A firm's vacancy choice monotonically increases in the productivity of the firm.*

Proof. Implicit differentiation of equation (9) yields

$$k''(v)v'(p) = q(\theta) \left[u \int_{\underline{a}}^{\bar{a}} J'_p(x, p) d\Lambda_0(x) + (1 - u) \int_{\underline{p}}^p \int_{\underline{a}}^{\bar{a}} J'_p(x, p) d\Lambda_1(x, y) \right. \\ \left. + (1 - u) \int_{\underline{a}}^{\bar{a}} J'_p(x, p) d\Lambda_1(x, p) \right]$$

$$v'(p) = \frac{q(\theta) \left[u \int_{\underline{a}}^{\bar{a}} J'_p(x, p) d\Lambda_0(x) + (1 - u) \int_{\underline{p}}^p \int_{\underline{a}}^{\bar{a}} J'_p(x, p) d\Lambda_1(x, y) \right. \\ \left. + (1 - u) \int_{\underline{a}}^{\bar{a}} J'_p(x, p) d\Lambda_1(x, p) \right]}{k''(v)} > 0$$

since $k''(v) > 0$. More productive firms have higher vacancy rates since the value of a job is higher for them and they therefore are larger in equilibrium. ■

2.4 Government

The government is modeled as just collecting a flat income tax, tax , from employed and unemployed workers and distributing the revenue among workers currently unemployed, i.e., the government sets the tax such that the tax revenue equals the total cost of the unemployment insurance scheme.

$$b \cdot u = tax(1 - u) \int_{\underline{p}}^{\bar{p}} \int_{\underline{a}}^{\bar{a}} w(y)g_1(x, y)dxdy + tax \cdot b \cdot u$$

$$tax = \frac{b \cdot u}{(1 - u) \int_{\underline{p}}^{\bar{p}} \int_{\underline{a}}^{\bar{a}} w(y)g_1(x, y)dxdy + b \cdot u}$$

3 Steady State Equilibrium

The model is solved in steady state. A constant returns to scale matching function determines the total number of matches in the economy as a function of total search effort and vacancy intensity

$$m(S, V)$$

where S is the aggregate search effort by workers and V is the total vacancy intensity by the firms defined by,

$$V = M \int_{\underline{p}}^{\bar{p}} v(p)dF(p|p \geq \underline{p}) \quad (10)$$

$$S = uS_0 + (1 - u)S_1 = u \int_{\underline{a}}^{\infty} s(a, b)dG_0(a) + (1 - u) \int_{\underline{p}}^{\bar{p}} \int_{\underline{a}}^{\infty} s(a, p)dG_1(a, p) \quad (11)$$

The flow rate of matches per unit of search effort is the total number of matches divided by the aggregate search effort.

$$\frac{m(S, V)}{S} = m(1, V/S) = m(1, \theta) = \lambda(\theta) \quad (12)$$

The flow rate of matches per unit of advertising intensity is the total number of matches divided by the aggregate vacancy intensity

$$\frac{m(S, V)}{V} = \lambda(\theta)/\theta = q(\theta) \quad (13)$$

where $\theta = V/S$ is market tightness. The distribution of productivities of vacancies facing the worker is

$$\Gamma(p) = \int_{\underline{p}}^p \frac{Mv(p)}{V} dF(p|p \geq \underline{p}) \quad (14)$$

i.e., this distribution is different from the distribution of productivities of firms, $F(p)$, since high productivity firms advertise more than low productivity firms.

3.1 Steady State Distribution of Assets and Productivities

In equilibrium the joint distribution of assets and productivities is characterized by flow equations. It is necessary to track these distributions since the firms maximization problem depends on it. Recall that $G_1(a, p)$ is the joint CDF of workers in a type p firm with assets a . $G_0(a)$ is the CDF of unemployed workers with assets a . The flow equation describing the joint distribution of productivities and assets among the employed, where $g_1(a, p)$ is the PDF, is

$$\begin{aligned} u\lambda(\theta)\Gamma(p) \int_{\underline{a}}^a s(x, b)g_0(x)dx + (1-u) \int_{\underline{p}}^p \left[1\left[\frac{da(a, y)}{dt} < 0\right] \frac{dG_1(a, y)}{da} \right] \frac{da(a, y)}{dt} dy = \quad (15) \\ (1-u)\delta \int_{\underline{p}}^p \int_{\underline{a}}^a g_1(x, y)dx dy + (1-u)\lambda(\theta)\bar{\Gamma}(p) \int_{\underline{p}}^p \int_{\underline{a}}^a s(x, y)g_1(x, y)dx dy \\ - (1-u) \int_{\underline{p}}^p 1\left[\frac{da(a, y)}{dt} > 0\right] \frac{dG_1(a, y)}{da} \frac{da(a, y)}{dt} dy \end{aligned}$$

The flow equation describing the asset distribution of the unemployed, where $g_0(a)$ is the PDF, is

$$(1-u)\delta \int_{\underline{p}}^{\bar{p}} \int_{\underline{a}}^a g_1(x,y) dx dy = \tag{16}$$

$$u\lambda(\theta) \int_{\underline{a}}^a s(x,b)g_0(x)dx + ug_0(x)1\left[\frac{da(a,b)}{dt} < 0\right]\frac{da(a,b)}{dt}$$

This implies that the unemployment rate is

$$u = \frac{\delta}{\lambda(\theta) \int_{\underline{a}}^a s(x,b)g_0(x)dx + \delta} \tag{17}$$

Proof. See Appendix A ■

The flow equations have a nice intuitive interpretation. Looking at the flow equation for the unemployed, the first two terms are standard except for the fact that search effort is integrated over the asset distribution in term two. The last term is unusual. It comprises the mass of unemployed workers times the measure of workers just on the limit of the CDF times the rate at which these workers flow into the CDF, i.e., this is the flow coming from workers changing assets. The inflow into the CDF can be high for three reasons. First, there are a lot of unemployed workers. Second, the measure of unemployed workers is large at this point in the distribution. Third, the savings rate is very low (high in absolute value).

The steady state equilibrium is defined as

Definition 5 *A steady state equilibrium in this model is a set of search intensities, $s(a,p)$, consumption choices, $c(a,p)$, vacancy decisions, $v(p)$, joint distributions, $G_1(a,p), G_0(a)$, an unemployment rate, u , and a mass of firms, M , that satisfy equation (5),(6),(8),(9),(15),(16), and (17).*

The proof of existence and uniqueness of the equilibrium is beyond the scope of this paper.⁹

⁹Numerical solutions have always converged with a single solution.

4 Solving the Model

The model described above cannot be solved analytically. To solve it numerically, it is necessary to specify the functional form of the utility function, $u(c)$, the search effort cost function, $e(s)$, the vacancy cost function, $k(v)$, and the matching function, $m(S, V)$.

4.1 Function specifications

The utility function is specified as a CRRA utility function

$$u(c) = \begin{cases} \frac{c^{1-\gamma}}{1-\gamma}, & \gamma \neq 1 \\ \log(c), & \gamma = 1 \end{cases}$$

where γ determines the degree of risk aversion. Both the search effort cost function and the vacancy cost function are power functions

$$\begin{aligned} e(s) &= \alpha_s \frac{s^{1+1/\eta_s}}{1+1/\eta_s} \\ k(v) &= \alpha_v \frac{v^{1+1/\eta_v}}{1+1/\eta_v} \end{aligned}$$

where $\alpha_s, \alpha_v > 0$ are scaling parameters and $\eta_v, \eta_s > 0$ to ensure strict convexity. The matching function is CRTS

$$m(S, V) = \pi S^\chi V^{1-\chi} \tag{18}$$

where $\chi \in (0, 1)$ is the matching function elasticity.

4.2 Procedure

The model is more complicated to solve than an ordinary search model. This is so because of the introduction of savings and a demand side for labor. The following procedure, inspired by the equilibrium proof, is used in solving the model.

1. Make a guess of market tightness, θ , and distribution of productivities of vacancies, $\Gamma(p)$.

2. Solve the worker's problem by iteration on the mappings for $W(a, p)$ and $U(a)$. This gives the optimal search effort level $s(a, p)$ and optimal consumption $c(a, p)$.
3. In the flow equations for employed and unemployed workers, equation (15) and (16), use $s(a, p)$ and solve for the joint distribution of assets and productivities, $G_0(a)$, $G_1(a, p)$, and u .
4. Given $s(a, p)$, $G_0(a)$, $G_1(a, p)$, and u , solve $J(a_i, p)$, and $V(p)$ by iterating on the mappings.
5. Solve for the value of M by the free entry condition.
6. Use $v(p)$ and equation (14) to get the distribution of productivities of vacancies, $\Gamma(p)$, and equation (10) and (11) to get θ .
7. Using the new distribution of productivities of vacancies and market tightness, repeat 2-6 until convergence.

The model is specified in continuous time. However, I solve it in discrete time using discretization of the state space and value function iteration procedures.¹⁰ These methods have some limitations. Most importantly, all workers in a type p firm have a "steady state" asset level that the worker would slowly converge to if he never moved from the type p firm. Especially, a worker in the highest type firm will define the upper bound of the asset distribution. Workers near this upper asset level will only be willing to make limited savings, i.e., the upper limit can never be approximated very well. With these limitations in mind, I continue the analysis.

In order to obtain reasonable precision, I can only solve the model for one productivity level, so either the worker is employed or the worker is unemployed.¹¹ This does not change much with respect to the main qualitative features of the model, although it means that the model will not be able to fit the wage distribution and asset distribution. However, whether or not equilibrium effects are important does not really depend on the fitting of these distributions.

¹⁰I have tried to solve the model in continuous time using both Chebyshev polynomials and spline approximations. However, the solutions did not have a satisfactory degree of accuracy which is very important when solving for the integral equations governing the joint distribution of assets and productivities.

¹¹I use 8,000 points in the asset distribution. Allowing the unemployed worker to choose savings and consumption at a level of 1 per cent of his income. Adding 10 per cent more points in the asset distribution, the calibrated moments only change very little (max. of E-4).

5 Data

The data used in the empirical analysis is the Danish register-based matched employer-employee data set IDA covering the period 1987 to 2003.¹² IDA contains annual socioeconomic information on workers and background information on employers, and it covers the entire Danish population. In addition to the worker and firm identifiers the data contains earnings information which consists of the annual average hourly wage in the job occupied in the last week of November. This data set is merged with detailed spell data on individual labor market histories.

The spell data consists of a worker and an employer id, start and end date of the spell, a variable describing the state that the worker is in and four different measures of hourly wages if the worker is employed. The spell data is constructed from administrative registers with information on public transfers, earnings, as well as start and end dates for all jobs reported by firms to the Danish Tax Authorities, and mandatory employer pension contributions. To make the data more suitable for this study, I manipulate it in the following ways. There are sixteen states the worker can occupy in the raw data, these are aggregated into five states; employed (E), unemployed (U), nonparticipating (N), self-employed (S), and retirement (O). Temporary non-participation and unemployment spells (shorter than 5 weeks) where the previous and next employer are identical are perceived as one employment spell. Similarly, non-participation and unemployment spells that are shorter than 3 weeks where the previous and the next employer are different, are recorded as two employment spells where the in-between unemployment spell is included in the later employment spell.

Wealth data is collected in this period because Denmark had a wealth tax which was in effect until 1997. In this period individuals' reporting of assets was audited by the tax authorities, while banks, mortgage institutions etc. reported the holdings of individuals to the tax authorities directly. The wealth data is therefore considered to be of good quality. In 1997 the tax was abolished, but the automatic reporting systems did not change. There are no breaks in the wealth series over time.

Total wealth can be divided into assets and liabilities which can then be further decomposed. Assets consist of housing assets, shares, deposited mortgage deeds, cash holdings in banks, bonds,

¹²IDA: *Integreret Database for Arbejdsmarkedsforskning* (Integrated Database for Labor Market Research) is constructed and maintained by Statistics Denmark.

and other assets. Housing assets are defined as the value of property set by the tax authorities because Denmark has a property tax. Shares, bonds, and deposited mortgage deeds contain the market value of each of these, respectively. Cash is cash in the bank. The last category contains self-reported information about non-deposited bonds, cash holdings, a particular type of unquoted shares (in ships) as well as the value of investment objects and high value objects such as cars and boats. Liabilities consist of four different categories: mortgage debt, bank debt, secured debt and other debt.

The primary deficiency of the wealth data is that public and union retirement savings are not available. In Denmark most pension savings are either public or regulated through the unions forcing people to save for future retirement. However, since the model has no retirement aspect, I would like to disregard savings for retirement.

Since housing assets are defined as the value set by the tax authorities, which might not be the true value of the asset, I try to correct the values for differences between the estimated value and the actual value in the market. This is done using data from the Customs and Tax Administration¹³ on the trading value of houses sold compared to the value set by tax authorities.¹⁴ I use data from the period 1987-2003 since the correction of housing prices can only be done for this period.

5.1 Sample

I disregard workers with invalid information, such as gaps in their spell history or missing variables. These do not constitute a large number of individuals. Next, I define labor market entry to be the month of graduation from the highest completed education recorded.¹⁵ I delete spells that start before this date. If the worker is observed in education after the date of highest completed education, the worker is disregarded. For instance, high school graduates who attend college

¹³Told- og Skattestyrelsen.

¹⁴ $ADJ_KOEJD = KOEJD * (\text{mean}(\text{traded houses}) / \text{mean}(KOEJD | \text{on being traded}))$. I.e., I correct for differences in the value of traded houses and the value set by the tax authorities. This implicitly assumes that houses being sold are independent of the value set by the tax authorities, and more importantly that the difference is the same across Denmark.

¹⁵We only have information on the highest completed education back to 1969, so it is missing for workers who took it before 1969. Also, immigrants and workers who never finished primary school have missing values. We keep these workers in the data set since we believe that the problems with immigrants and workers who never finished primary school are quite small, and workers who finished their education before 1969 have already entered the labor market in 1988.

are deleted since their highest education is high school but they still are observed in education (college). I censor individuals who enter retirement or self-employment. The resulting states are employment, unemployment, non-participation. Since wage determination and mobility is very different between the public and private sector, I censor individuals at the beginning of the spell before entry into public employment. This is done since the job that workers hold prior to their entry into public employment might be special in some way.

In the data, being unemployed means receiving some form of unemployment benefit. It is not completely clear how to treat non-participating individuals. If a worker does not have unemployment insurance but has amassed some savings, he/she cannot receive social assistance, i.e., the worker becomes non-participating if he becomes unemployed. Also non-participation is clearly not an absorbing state in the data. The transition patterns are not that different from the transition patterns of unemployed individuals. Therefore, unemployment and non-participation are aggregated into non-employment. This is also done in Taber and Vejlin (2011). This makes the data consistent with the model, since the model does not have a non-participating state.

To reduce the impact of outliers in the data, I trim a half percent at the bottom and top of both the wage and wealth distribution. Wealth and wages are log detrended using the wage inflation in the data to 2003 levels. This is done in order to be able to compare individuals over time.

5.2 Descriptive features

Table 1 contains descriptive statistics for wealth and hourly wages for repeated November cross sections. The average hourly wage and wealth holdings are 196 and 86,300 DKK, respectively. Wealth is more dispersed and more skewed than wages. The distribution of wealth is very dense around zero, implying that plotting the density would give little information about the distribution. Instead figure 1 shows the Lorenz curve for total wealth. Approximately 35-40 percent of individuals have negative wealth, 30-35 percent have very small wealth holdings, either positive or negative, and 30 percent own almost all the accumulated wealth. This gives a very skewed distribution as is also indicated by table 1. Lise (2010) finds that almost no individuals in the NLSY79 data set have negative wealth and that those who have negative wealth have a very small

amount of it. This is in sharp contrast to the Danish data used in this study. There could be several reasons for this difference. One could be misreporting in the NLSY79 data set, but it could also be because public and union retirement savings are not available in the Danish data.

Figure 2 shows a non-parametric regression using local polynomials of hourly wages on wealth. There is, perhaps not surprisingly, a strong positive relationship between hourly wages and wealth. At low wage levels there seems to be zero correlation between wages and wealth. At medium and higher wage levels there seems to be a strong positive relationship between wages and wealth holdings.

Table 2 shows estimates of wealth in a proportional hazard model of employment, unemployment, and non-employment with and without covariates. Looking at employment levels there is a strong effect on employment durations. Increasing the wealth level by one standard deviation decreases the hazard rate by 18 percent. This estimate drops to 12 percent when worker characteristics are controlled for. This primarily reflects the fact that the older workers have higher wealth holdings, make fewer job transitions and are less likely to get fired. Turning to unemployment durations there are differences between controlling for worker characteristics or not. Not controlling for anything else, wealth has a negative effect on unemployment durations. However, once worker characteristics are included in the regressions, wealth has a positive effect on unemployment durations. There is probably a great extent of unobserved heterogeneity in the data. One would expect that more "able" workers have higher wealth holdings and also exit unemployment faster. Finally, the estimates of wealth from a non-employment hazard model are negative whether or not I control for worker characteristics. Although, the estimate gets smaller once controls are included.

6 Simulations

In this section, I will perform a series of different simulations to highlight different features of the model. In order to do so, I need to pick values for the parameters in the model.

6.1 Parameters

Some of the parameters in the simulation are taken from the literature and others are taken from the Danish data presented above. A sensitivity check regarding key parameters is presented in the robustness section.

Table 3 shows the parameters of the model that are set exogenously. The subjective discount rate of workers and the interest rate is set at standard levels in this literature, see Lise (2010) and Lentz (2009). The bargaining power is set to 0.5, which is standard in this type of models, see Lentz and Mortensen (2008a). The unemployment insurance is normalized to 0.1. The productivity level is set to 0.4 to reflect the fact that the wage income for an employed worker on average is approximately two and a half times larger than unemployment benefits.¹⁶ The relative risk aversion parameter is very important in this type of models as highlighted in Lentz (2009). Lise (2010) estimates the risk aversion parameter γ to be in the range of 1.5–2.3, while Lentz (2009) estimates this parameter to be 2.2. Chetty (2008) uses a value of 1.7 in his simulations. I have chosen to use a value of 2. The curvature of the search cost function is taken from Lise (2010) and set to 1.5. Christensen et al. (2005) estimate an almost quadratic cost function, while Lentz (2009) estimates a cost function with a much higher curvature.

The parameter values for the vacancy cost function is set such that it is quadratic as in Garibaldi and Moen (2009). Lentz and Mortensen (2008a) set η_v to 0.5 in their simulations, which results in a more steep cost function. Petrongolo and Pissarides (2001) report estimates from several studies of matching functions. The elasticity with respect to search is reported to be between 0.4 and 0.6 in most studies. The lower estimates are typically from studies incorporating employed search. The matching function elasticity is set to 0.5 in these simulations.

Table 4 lists the parameters of the model that I am going to calibrate and the moments that are used to this end. In order to match the entry cost in the model, I use the mass of firms. The number of firms relative to the number of workers in the data is 0.0967. From table 1 it can be inferred that the average wealth holdings of non-employed individuals are approximately 82

¹⁶Wages are inflated to the 2003 level. As seen in Table 1, the average hourly wage is 196.31. An employed worker typically works 37 hours a week, so the weekly income is DKK 7263. In 2003, the maximum unemployment benefits were DKK 3,364 a week. However, not all unemployed workers received the maximum amount.

percent of the average monthly wage¹⁷, i.e., since the monthly wage is set to 0.25, the target of the average wealth holdings of non-employed is set to 0.204. The monthly job destruction rate is 0.0113 in the data. This results in a job destruction rate of 0.010 and a firm destruction rate at 0.0013¹⁸. To calibrate the scale parameter of the matching function, I use the employment share, which is 80.42 percent in the data.

Table 5 shows equilibrium quantities of the calibrated model. The model does not fit the average asset holding of the employed very well. The workers in the model accumulate too many assets while employed. The variance of the assets is matched a little bit better, but the fit still leaves something to be desired. It is not that strange that the variance does not match since all employed workers receive the same wage. This generates a lot less dispersion in savings than a model that allows for a continuous distribution of wages. This can also be seen if I compare the minimum and maximum values of assets in the model and in the data. Turning to the Lorenz curve of wealth in the model in figure 3, one can also see that the model has difficulties fitting the upper and lower part of the wealth distribution. However, the purpose of this paper is not to fit the wealth distribution since there are other factors behind it than just precautionary savings.

6.2 Decomposition of Unemployment

Using the calibrated model parameters, I try to decompose how an increase in UI affects unemployment duration. Table 6 shows the results. First, I let workers change search behavior. This simulation of the model is essentially a partial equilibrium model taking the demand side as given. Workers will exert less search effort for two reasons. First, the expected wage increase is smaller. This is the moral hazard effect of higher UI. Second, they will be able to uphold a higher level of consumption since UI is now higher. The latter effect also decreases search effort irrespective of the moral hazard channel as pointed out by Chetty (2008). The two effects combined give an increase in unemployment of 2.3 percentage points. Secondly, I allow the wage to change. Since wages are set in a bargaining between workers and firms over the instantaneous surplus of the match, $p - b$,

¹⁷The average wealth holding is 24,000 DDK. And the average monthly wage is $\frac{196.31DKK/Hour \cdot 1800Hours/Year}{12Months/Year} = 29,447DKK$.

¹⁸The firm destruction rate is rather low, since in this model size and firm destruction are independent. I have chosen to match the number of workers fired where the firm has closed.

the wages now increase because the UI level has increased. This results in increased search effort from the workers. The result is that unemployment decreases, but only by 0.1 percentage point, which is a relatively small decrease in unemployment. The reason is that search effort is not very responsive to changes in the wage. The primary determinant of workers search effort is their ability to have a reasonable consumption level today. That is, the main determinant is the asset holding of the worker and not the potential wage gain from finding a job. Next, I let the job offer rate, determined by θ , adjust to the lower level of search effort holding the firm side fixed. Given that firms create the same vacancies and workers now search less, the job offer arrival rate per unit of search effort increases. This increases the benefit from search, which result in a further decrease in unemployment of 0.4 percentage points. Workers realize that all other workers are also searching less which makes it easier for them to find a job. Fourth, I allow firms to adjust their vacancy rates to the lower level of search effort, thereby decreasing the arrival rate of job offers per unit of search intensity. This increases unemployment by 0.9 percentage point. Finally, I allow the mass of firms to adjust to the free entry condition. This results in a final increase in the unemployment rate of 1.4 percentage points. The final result is an increase in unemployment by 4.2 percentage point or 21 percent. 2.3 percentage point of the increase is due to the supply side adjustments, while the remaining increase is due to the demand side adjustments, i.e., the equilibrium effects. This illustrates that equilibrium effects are potentially large and should be taken into account when trying to asses the optimal benefit level.

6.3 Optimal UI - Welfare Analysis

Even though the unemployment response to a change in benefits seems be responsive to the inclusion of the demand side, this is not necessarily the case for the overall welfare. In this section I will try to asses the optimal level of UI in the calibrated model. It is not possible to derive a closed form solution for the optimal benefit level in the presented model. I therefore simulate the model with different benefit levels.

Define the welfare function as a utilitarian welfare function, i.e.,

$$W = \sum_{i=1}^N (1-u)W(a_i)g_1(a_i) + uU(a_i)g_0(a_i)$$

Since the total entry cost of firms is equal to the total discounted expected profit and firms are risk neutral, these terms cancel each other out.

Figure 4 displays how different welfare measures and the unemployment rate react to different UI levels in steady states. The unemployment rate is steadily increasing in the UI level. In order to maximize total welfare as defined above, the UI level should be 0.115, i.e., a replacement rate of 46 percent.¹⁹ The welfare function is rather flat near the maximum, but increasing UI to a replacement rate of more than 60 percent seems to have large negative effects on welfare. In general, unemployed workers prefer a higher level of benefits than employed workers, but the welfare function is relatively flat.

In order to investigate whether or not demand side considerations are important, I solve the model, where the job arrival rate, $\lambda(\theta)$, is taken to be a parameter, but the link between UI and wages still exists. This gives quite different welfare implications. The link between UI and wages is not broken because this will answer a different question related to the Hosios condition in the search literature, see Hosios (1990). When the firm sets a vacancy rate, it implies two externalities. First, it is easier for the workers to find a job the higher the vacancy rate. This is a positive externality that implies that firms set too low vacancy rates. Second, higher vacancy rates for one firm implies that other firms have a harder time finding workers. This is a negative externality which implies that firms tend to set too high vacancy rates. The Hosios condition states that the externalities have to be equal in order to attain efficiency. That is, setting a lower or higher wage implies a welfare effect by the Hosios argument. In order to see if the equilibrium effects are important, I let wages in both the equilibrium and partial equilibrium model vary with the same amount. Figure 5 shows the results. The optimal replacement rate in the equilibrium model is around 46 percent, whereas the optimal replacement rate in the partial equilibrium model is 65.5 percent.

In this simulation the optimal replacement rate is higher in the partial model. The reason

¹⁹The replacement rate is $\frac{b}{w}$

is that in the partial model increasing benefits, and thereby decreasing search effort, will not imply a different vacancy posting by firms. In the equilibrium model the firms realize that the probability of encountering an unemployed worker is now smaller. Therefore firms decrease their vacancy intensities. This in turn reduces the search effort of the workers and lead to prolonged unemployment. This effect is not captured by the partial equilibrium model. The effect going through vacancy intensities is unambiguous. Given that a firm wants to hire a worker and that the costs of hiring workers are decreasing, one might expect that firms in some cases will decrease their vacancy intensity rates. That is, what might be seen as an income effect dominates the substitution effect. The reason why this is not the case in this model is that firms are not trying to hire a particular number of workers. Rather the firm can hire any amount of workers and it will therefore always increase the vacancy intensity rate if it gets easier to hire workers.

However, there are equilibrium effects. When firms start to decrease their vacancy intensity rates as workers decrease their search effort as argued above, this will increase the arrival rate of workers per unit of vacancy intensity. This effect will in turn increase the firms vacancy intensity rates. If the vacancy choice is very elastic, this effect will be large. There is also an effect going through the free entry condition. When it gets harder to hire workers, the value of the firm will decrease. In equilibrium this will result in fewer firms entering the economy. This will decrease the total vacancy intensity and increase the arrival rate of workers. The model does not yield any closed form solutions, so one cannot derive under which conditions the equilibrium effects are positive or negative.

In the current model parameters erroneously implementing the optimal replacement rate from the partial equilibrium model results in large welfare losses as seen in figure 4. It is clear that the unemployment level is much less responsive to changing UI levels in the partial equilibrium model, precisely due to the effects mentioned above. In order to have an understanding of how big the welfare changes are, I compare them to different decreases in welfare caused by decreases in productivity. These are shown in figure 6. Implementing the 65.5 percent replacement rate, which is optimal according to the partial equilibrium model, leads to a decrease in aggregate welfare similar to that caused by a decrease in productivity of 10 percent in a model with a replacement

rule of 40 percent (the baseline model). This is a substantial difference and it highlights the fact that drawing conclusions from a partial equilibrium model can be costly in terms of welfare.

6.4 Taxation

UI is government financed, but the government can choose the financing scheme. In the baseline model presented in this paper, UI is financed by a tax on income. One could also finance expenditures by a value added (VA) tax, a tax on capital gains (interests) or a tax on wealth holdings. Table 7 shows the results. The simulations are comparisons of different steady states.

The first column shows the results from the baseline model where the only type of taxation is an income tax. In the second column, the results using only a VA tax on consumption is presented. There is little difference between an income tax and a VA tax in this model. The small difference that are noticeable between these schemes arise since the taxation happens at different points in time. In the case of the income tax, the taxation occurs when income is received by the worker. This makes the employed worker's savings smaller than when the VA tax is employed, where the taxation occurs when savings are converted into consumption. This mechanism helps the worker smooth consumption over jumps in income. The higher level of assets is also the reason for the slightly higher unemployment level since wealthier workers exert less search effort. Turning to column 3, a tax on capital gains, i.e., positive interest payments, of 30 percent is introduced. The rest of the public budget is financed by an income tax. This type of tax has a large distortion on the accumulation of assets. Lowering the net return on assets makes it less attractive for the worker to self-insure. The same effect is found in Lentz (2009), where the optimal replacement rate depends on the difference between the discount rate and the interest rate. Finally, column 4 shows the case where there is a wealth tax of 0.2 percent of positive wealth holdings. Not surprisingly, this form of taxation has a huge effect on the worker's willingness to self-insure. The average asset holdings decrease dramatically, since holding any positive amount of assets is very expensive. The measures of welfare used also decrease a lot. Even a small wealth tax has large negative effects on the accumulation of assets. The revenue generated from both the interest rate tax and the wealth tax constitutes around 3 percent of the public budget. In comparison, the introduction of a wealth

tax of 0.2 percent has the same effect on welfare as lowering productivity by 2 percent, which is not an unsubstantial amount.

7 Robustness

Some of the parameter choices in the baseline model can be discussed. In this section a robustness check is performed in order to see if any of the choices made for the non-calibrated parameters are driving the results. In order to say something about the effect of different parameter choices, I calibrate the model to the new set of parameters.

There are three parameters that one might expect to have the most significant impact. First, the degree of relative risk aversion. This parameter plays a critical part in the saving behavior of the worker. It was chosen to be set at 2. As already discussed the literature sets this parameter in the range of 1.5 to 2.3. I have chosen to solve the model for these values.

Secondly, the curvature of the search cost curvature determines the size of the response in search effort to for instant changes in the UI level. The parameter was set at 1.5 in the baseline model. Christensen et al. (2005) estimate the cost function to be quadratic implying a parameter of 1 making search effort less responsive. Finally, the curvature of the vacancy cost function is important since it determines the size of the response from firms when worker change search effort. The value was set to 1 following Garibaldi and Moen (2009). Lentz and Mortensen (2008a) set η_v to 0.5 in their simulations, and so I will try to solve calibration for this value as well.

Table 8 reports the new calibrated values given the four new set of parameters. Figure 7 reports the results from changing these three parameters on the unemployment decomposition from above. Overall the patterns are similar in all versions of the model. The most striking feature is that changing the search cost function curvature seems to consistently make unemployment less responsive to UI changes. This happens since the choice of search effort is set by the worker to equate the marginal search cost and the marginal gain by search. When the cost function becomes steeper in search effort, smaller changes in search effort will appear for a given change in the marginal gain, thus making the response smaller. Overall, I conclude that the decomposition of unemployment does not hinge on the parameter choices taken in the baseline model.

8 Conclusion

This paper develops an equilibrium model where workers choose search effort and consumption levels. Firms comprise a range of different jobs with equal productivity p and the possibility to create new jobs by advertising vacancies. Firms enter the market by drawing a firm specific productivity. Once in the market, firms choose vacancy intensity. Wages are set such that firms and workers split the instantaneous surplus of the match. A simple model with one type of firm is calibrated. The model fit is reasonable given that it is only a very simple model with only one type of firm.

It is shown that equilibrium effects are important when trying to assess the optimal UI level. The increase in the unemployment rate is about double the size if one takes into account demand side changes as opposed to taking the demand side as given. First, higher wages caused by the increased outside option and higher job offer arrival rates caused by the fact that workers realize that all other workers search less tend to increase search effort compared to the partial equilibrium model. This tends to decrease the unemployment rate. However, these effects are dominated by the effect of the firm's decisions. Firms realize that workers now search less, which will make firms post fewer vacancies and fewer firms will enter in equilibrium. These two later effects dominate the equilibrium effects of increased wages and higher job offer arrival rates. In total, the equilibrium effects are just as important as workers changing search behavior.

Further, the conclusions based on a partial equilibrium model that do not take the demand side responses into account can be erroneous. A partial model will overestimate the beneficiary effects of a high replacement rate since it will not take into account the demand side response to the lower level of search effort exerted by the workers. Implementing the erroneous replacement rate from a partial equilibrium model will lead to welfare losses corresponding to those associated with a decrease in productivity of 10 percent.

When trying to assess the optimal replacement rate, it is important to take into account the entire dynamic transition path of the economy, otherwise one will tend to underestimate the optimal UI level, since moving to higher levels of UI reduces the need for precautionary savings thereby allowing higher consumption possibilities in the transition between the old and the new

steady state.

Finally, I also show that even small wealth taxes or taxes on capital gains can be harmful to worker welfare. This happens since workers reduce the accumulation of assets since assets are not taxed. The reduced amount of assets severely reduce self-insurance against income shocks. If a policymaker thus want to tax assets in some form, it would also be beneficial to have a higher unemployment insurance, since this would offset some of the welfare losses from the reduced self-insurance

References

- [1] Algan, Y., Chéron, A., Hairault, J.-O. and F. Langot, 2003. "Wealth effect on labor market transitions." *Review of Economic Dynamics* 6: 156–178.
- [2] Aiyagari, S., 1994. "Uninsured idiosyncratic risk and aggregate saving." *The Quarterly Journal of Economics* 109: 659–684.
- [3] Bayer, C. and K. Wälde, 2009. "General Equilibrium Island-Matching and Saving in Continuous Time: Theory". Unpublished Manuscript.
- [4] Binmore, K., Rubinstein, A., and A. Wolinsky, "The Nash Bargaining Solution in Economic Modeling," *RAND Journal of Economics*, Summer 1986, 17 (2), pp. 176–188
- [5] Bloemen, H. and E. Stancanelli, 2001. "Individual wealth, reservation wages, and transitions into employment." *Journal of Labor Economics* 19: 400–439.
- [6] Burdett, K. and D. T. Mortensen, 1998. "Wage Differentials, Employer Size and Unemployment". *International Economic Review*, 39, 257-73.
- [7] Chetty, R., 2008. "Moral Hazard vs. Liquidity and Optimal Unemployment Insurance." *Journal of Political Economy* 116(2): 173-234
- [8] Christensen, B. J., Mortensen, D., Neumann, G., and A. Werwatz, 2005. "On the Job Search and the Wage Distribution". *Journal of Labor Economics*, January 2005, vol. 23(1), p. 31-58.

- [9] Coles, M., 2008. "Optimal Unemployment Insurance with Hidden Search Effort and Endogenous Savings". Unpublished Manuscript.
- [10] Coles, M. and A. Masters, 2006. "Optimal Unemployment Insurance in a Matching Equilibrium," *Journal of Labor Economics*, University of Chicago Press, vol. 24(1), pages 109-138, January.
- [11] Conley, J.P., and S. Wilkie, 1996. An extension of the Nash bargaining solution to nonconvex problems. *Games and Economic Behavior* 13 (1), 26–38.
- [12] Garibaldi, P., and E. Moen, 2009. "Industry Dynamics and Search in the Labor Market", Unpublished Manuscript.
- [13] Fredriksson, P. and B. Holmlund, 2001. "Optimal Unemployment Insurance in Search Equilibrium", *Journal of Labor Economics* 19, 370-399.
- [14] Hopenhayn, H. and P. Nicolini, 1997. "Optimal Unemployment Insurance", *Journal of Political Economy*, 105, 2, 412-438.
- [15] Hosios, A., 1990. "On the Efficiency of Matching and Related Models of Search and Unemployment," *Review of Economic Studies*, Blackwell Publishing, vol. 57(2), pages 279-98, April
- [16] Kocherlakota, N. 2004: "Figuring Out the Impact of Hidden Savings on Optimal Unemployment Insurance", *Review of Economic Dynamics*, Volume 7, Issue 3, July 2004, Pages 541-554
- [17] Krusell, P., Mukoyama, T. and A. Sahin, "Labor-Market Matching with Precautionary Savings and Aggregate Fluctuations", *Review of Economic Studies*, 77:4.
- [18] Lentz, R., 2009. "Optimal unemployment insurance in an estimated job search model with savings", *Review of Economic Dynamics*, January 2009, vol. 12(1), pp. 37-57.
- [19] Lentz, R., and D. Mortensen, 2008a. "Labor Market Friction, Firm Heterogeneity, and Aggregate Employment and Productivity", Unpublished Working Paper

- [20] Lentz, R., and T. Tranæs, 2005. "Job search and savings: Wealth effects and duration dependence." *Journal of Labor Economics* 23: 467–489.
- [21] Lise, J., 2010. "On-the-Job Search and Precautionary Savings: Theory and Empirics of Earnings and Wealth Inequality", Unpublished Manuscript.
- [22] Merton, R., 1971. "Optimum consumption and portfolio rules in a continuous-time model." *Journal of Economic Theory* 3: 373–413.
- [23] Meyer, B., 1990. "Unemployment Insurance and Unemployment Spells." *Econometrica* 58: 757-782.
- [24] Mortensen, D. and C. Pissarides, 1994. "Job Creation and Job Destruction in the Theory of Unemployment," *Review of Economic Studies*, Blackwell Publishing, vol. 61(3), pages 397-415, July
- [25] Mortensen, D., 2009. "Wage Dispersion in the Search and Matching Model with Intra-Firm Bargaining", Unpublished Working Paper
- [26] Petrongolo, B. and C. Pissarides 2001. "Looking into the Black Box: A Survey of the Matching Function", *Journal of Economic Literature* 38 (June 2001) 390-431
- [27] Postel-Vinay, F., Robin, J.-M., 2002. "Equilibrium wage dispersion with worker and employer heterogeneity". *Econometrica* 70 (6), 2295–2350.
- [28] Rendon, S, 2006. "Job Search and Asset Accumulation under Borrowing Constraints". *International Economic Review*, 47(1) pp.233-263.
- [29] Sennewald, K., "Controlled stochastic differential equations under Poisson uncertainty and with unbounded utility," *J. Econ. Dynam. Control*, 2007, 31 (4), 1106-1131.
- [30] Shaked, A, and J. Sutton, 1984. "Involuntary unemployment as a perfect equilibrium in a bargaining model". *Econometrica*, 52(6), 1351-1364.
- [31] Shimer, R., 2006. "On-the-job search and strategic bargaining." *European Economic Review* 50: 811–830

- [32] Shimer, R., and I. Werning, 2008. "Liquidity and Insurance for the Unemployed." FRB Minneapolis Staff Report #366
- [33] Taber, C. and R. Vejlin, 2011. "Estimation of a Roy/Search/Compensating Differentials Model of the Labor Market", Unpublished Manuscript.

A Derivation of Bellman equations

A.1 Bellman equation for the worker

The Bellman equation is derived by writing up the discrete time equivalent and letting the period length go to zero. Let Δ be the period length

$$\begin{aligned}
 W(a, p, t) = & \max_{c \in Y(a), s \geq 0} u(c(a, p, t))\Delta - e(s(a, p, t))\Delta + \frac{1}{1 + \Delta\varphi}[\delta\Delta U(a + \Delta a, t + \Delta)] \quad (19) \\
 & + \Delta\lambda(\theta)s \int \max\{W(a + \Delta a, p', t + \Delta), W(a + \Delta a, p, t + \Delta)\}d\Gamma(p') \\
 & + (1 - \Delta\lambda(\theta)s - \delta\Delta)W(a + \Delta a, p, t + \Delta) + o(\Delta)
 \end{aligned}$$

where $o(\Delta)$ is a term that goes to zero faster than Δ such as the probability of receiving two job offers in one period. Arrange to get

$$\begin{aligned}
 \Delta\varphi W(a, p, t) = & \max_{c \in Y(a), s \geq 0} u(c(a, p, t))(1 + \Delta\varphi)\Delta - e(s(a, p, t))(1 + \Delta\varphi)\Delta \\
 & - \delta\Delta(W(a + \Delta a, p, t + \Delta) - U(a + \Delta a, t + \Delta)) \\
 & + \Delta\lambda(\theta)s \int_p^{\bar{p}} W(a + \Delta a, p', t + \Delta) - W(a + \Delta a, p, t + \Delta)d\Gamma(p') \\
 & + W(a + \Delta a, p, t + \Delta) - W(a, p, t) + o(\Delta)
 \end{aligned}$$

Now divide by Δ and let $\Delta \rightarrow 0$.

$$\begin{aligned}
 \varphi W(a, p) = & \max_{c \in Y(a), s \geq 0} u(c(a, p)) - e(s) + W'_a(a, p)da/dt - \delta(W(a, p) - U(a)) \\
 & + \lambda(\theta)s \int_p^{\bar{p}} [W(a, p') - W(a, p)] d\Gamma(p')
 \end{aligned}$$

where $\frac{W(a + \Delta a, p, t + \Delta) - W(a, p, t)}{\Delta} \rightarrow \frac{dW(a, p, t)}{dt} = W'_a(a, p)da/dt$ as $\Delta \rightarrow 0$ and $\frac{o(\Delta)}{\Delta} \rightarrow 0$ as $\Delta \rightarrow 0$. The

Bellman equation can also be derived from the original problem using Ito's calculus, see Merton

(1971) and Sennewald (2007).

A.2 Bellman equation for jobs

$$J(a, p, t) = (1 - \beta)(p - b) + \frac{1}{1 + \Delta r} [\delta \Delta \cdot 0 + \lambda(\theta) s(a, p, t) \Delta \bar{\Gamma}(p) \cdot 0 \\ + (1 - \delta_j \Delta - \lambda(\theta) s(a, p, t) \Delta \bar{\Gamma}(p)) J(a + \Delta a, p, t + \Delta) + o(\Delta)]$$

Rearrange to get

$$\Delta r J(a, p, t) = (1 - \beta)(p - b)(1 + \Delta r) - \delta_j \Delta (J(a + \Delta a, p, t + \Delta) \\ - \lambda(\theta) s(a, p, t) \Delta \bar{\Gamma}(p) J(a + \Delta a, p, t + \Delta) \\ + J(a + \Delta a, p, t + \Delta) - J(a, p, t) + o(\Delta))$$

Divide by Δ and let $\Delta \rightarrow 0$.

$$r J(a, p, t) = (1 - \beta)(p - b) - \delta J(a, p, t) - \lambda(\theta) s(a, p) \bar{\Gamma}(p) J(a, p, t) + J'_a(a, p, t) da/dt$$

where $\frac{J(a+\Delta a, p, t+\Delta) - J(a, p, t)}{\Delta} \rightarrow \frac{dJ(a, p, t)}{dt} = J'_a(a, p, t) da/dt$ as $\Delta \rightarrow 0$ and $\frac{o(\Delta)}{\Delta} \rightarrow 0$ as $\Delta \rightarrow 0$.

A.3 Bellman equation for vacancies

$$V(p, t) = \max_{v \geq 0} [-k(v) + (1 - \Delta \delta_f) V(p, t + \Delta) + o(\Delta)] \\ + \frac{1}{1 + \Delta r} [q(\theta) v \Delta \left[\frac{u S_0}{(1 - u) S_1 + u S_0} \int_{\underline{a}}^{\infty} J(x, p) d\Lambda_0(x) + \frac{(1 - u) S_1}{(1 - u) S_1 + u S_0} \int_{\underline{p}}^p \int_{\underline{a}}^{\infty} J(x, p) d\Lambda_1(x, y) \right]]$$

Rearrange to get

$$\begin{aligned} \Delta rV(p, t) + \Delta \delta_f V(p, t + \Delta) &= \max_{v \geq 0} -k(v)(1 + \Delta r) \\ &+ q(\theta)v\Delta \left[\frac{uS_0}{(1-u)S_1 + uS_0} \int_{\underline{a}}^{\infty} J(x, p) d\Lambda_0(x) + \frac{(1-u)S_1}{(1-u)S_1 + uS_0} \int_{\underline{p}}^p \int_{\underline{a}}^{\infty} J(x, p) \right. \\ &\left. + V(p, t + \Delta) - V(p, t) + o(\Delta) \right] \end{aligned}$$

Divide by Δ and let $\Delta \rightarrow 0$.

$$(r + \delta_f)V(p, t) = \max_{v \geq 0} -k(v) + q(\theta)v \left[\frac{uS_0}{(1-u)S_1 + uS_0} \int_{\underline{a}}^{\infty} J(x, p) d\Lambda_0(x) + \frac{(1-u)S_1}{(1-u)S_1 + uS_0} \int_{\underline{p}}^p \int_{\underline{a}}^{\infty} J(x, p) d\Lambda_1(x, y) \right]$$

B Derivation of flow equations

Proof. Let Δ be a small time interval and let

$$\tilde{a} : \tilde{a}(a, p, \Delta) = a_t - \frac{da(a_t, p)}{dt} \Delta$$

i.e., \tilde{a} defines the marginal worker who will be accumulating enough assets to move out of or into the CDF.

Let $G_{1,t}(a, p)$ be the CDF of workers in firm type p with assets a . Let $\Gamma(p)$ be the distribution of offers from which the worker is drawing. Then

$$\begin{aligned}
& G_{1,t+\Delta}(a, p) - G_{1,t}(a, p) = \\
& u\lambda(\theta)\Delta\Gamma(p) \int_{\underline{a}}^a s(x, b)g_{0,t}(x)dx - (1-u)\delta\Delta \int_{\underline{p}}^p \int_{\underline{a}}^a g_{1,t}(x, y)dx dy - \\
& (1-u)\lambda(\theta)\Delta\bar{\Gamma}(p) \int_{\underline{p}}^p \int_{\underline{a}}^a s(x, y)g_{1,t}(x, y)dx dy \\
& + (1-u) \int_{\underline{p}}^p 1\left[\frac{da(y)}{dt} < 0\right] \int_a^{\tilde{a}} g_{1,t}(x, y)dx dy - (1-u) \int_{\underline{p}}^p 1\left[\frac{da(y)}{dt} > 0\right] \int_{\tilde{a}}^a g_{1,t}(x, y)dx dy \\
& \quad \Downarrow
\end{aligned}$$

$$\begin{aligned}
& \frac{G_{1,t+\Delta}(a, p) - G_{1,t}(a, p)}{\Delta} = \\
& u\lambda(\theta)\Gamma(p) \int_{\underline{a}}^a s(x, b)g_{0,t}(x)dx - (1-u)\delta \int_{\underline{p}}^p \int_{\underline{a}}^a g_{1,t}(x, y)dx dy \\
& - (1-u)\lambda(\theta)\bar{\Gamma}(p) \int_{\underline{p}}^p \int_{\underline{a}}^a s(x, y)g_{1,t}(x, y)dx dy \\
& + (1-u) \int_{\underline{p}}^p \frac{1\left[\frac{da(y)}{dt} < 0\right] \int_a^{\tilde{a}} g_{1,t}(x, y)dx}{\Delta} dy - (1-u) \int_{\underline{p}}^p \frac{1\left[\frac{da(y)}{dt} > 0\right] \int_{\tilde{a}}^a g_{1,t}(x, y)dx}{\Delta} dy \\
& \quad \Downarrow
\end{aligned}$$

Taking limits.

$$\begin{aligned}
& \lim_{\Delta \rightarrow 0} \frac{G_{1,t+\Delta}(a, p) - G_{1,t}(a, p)}{\Delta} = 0 = \\
& u\lambda(\theta)\Gamma(p) \int_{\underline{a}}^a s(x, b)g_{0,t}(x)dx - (1-u)\delta \int_{\underline{p}}^p \int_{\underline{a}}^a g_{1,t}(x, y)dx dy \\
& - (1-u)\lambda(\theta)\bar{\Gamma}(p) \int_{\underline{p}}^p \int_{\underline{a}}^a s(x, y)g_{1,t}(x, y)dx dy \\
& + (1-u) \int_{\underline{p}}^p 1\left[\frac{da(a, y)}{dt} < 0\right] \lim_{\Delta \rightarrow 0} \frac{\int_{a_t - \frac{da(a_t, y)}{dt}\Delta}^a g_{1,t}(x, y)dx}{\frac{da(a_t, y)}{dt}\Delta} \frac{da(a_t, y)}{dt} dy \\
& - (1-u) \int_{\underline{p}}^p 1\left[\frac{da(a, y)}{dt} > 0\right] \lim_{\Delta \rightarrow 0} \frac{\int_{a_t}^a g_{1,t}(x, y)dx}{\frac{da(a_t, y)}{dt}\Delta} \frac{da(a_t, y)}{dt} dy
\end{aligned}$$

Dividing by $\frac{da(a_t, y)}{dt}$ is potentially troublesome since there exists a level \hat{a} for all values of a where $\frac{da(\hat{a}, y)}{dt} = 0$. This is circumvented only by defining the equation for $\frac{da(a, y)}{dt} \neq 0$.

$$\begin{aligned}
& \lim_{\Delta \rightarrow 0} \frac{G_{1,t+\Delta}(a, p) - G_{1,t}(a, p)}{\Delta} = 0 = \\
& u\lambda(\theta)\Gamma(p) \int_{\underline{a}}^a s(x, b)g_{0,t}(x)dx - (1-u)\delta \int_{\underline{p}}^p \int_{\underline{a}}^a g_{1,t}(x, y)dx dy \\
& - (1-u)\lambda(\theta)\bar{\Gamma}(p) \int_{\underline{p}}^p \int_{\underline{a}}^a s(x, y)g_{1,t}(x, y)dx dy \\
& - (1-u) \int_{\underline{p}}^p 1\left[\frac{da(a, y)}{dt} < 0\right] \lim_{\Delta \rightarrow 0} \frac{\widehat{G}_{1,t}(a - \frac{da(a_t, y)}{dt}\Delta, y) - \widehat{G}_{1,t}(a, y)}{-\frac{da(a_t, y)}{dt}\Delta} \frac{da(a_t, y)}{dt} dy \\
& - (1-u) \int_{\underline{p}}^p 1\left[\frac{da(a, y)}{dt} > 0\right] \lim_{\Delta \rightarrow 0} \frac{\widehat{G}_{1,t}(a, y) - \widehat{G}_{1,t}(a - \frac{da(a_t, y)}{dt}\Delta, y)}{\frac{da(a_t, y)}{dt}\Delta} \frac{da(a_t, y)}{dt} dy
\end{aligned}$$

where $\widehat{G}_{1,t}(a, y) = \int_a^a g_{1,t}(x, y)dx$. Finally we get the integral equation tying down the distribution of productivities and assets.

$$\begin{aligned}
& u\lambda(\theta)\Gamma(p) \int_a^a s(x, b)g_{0,t}(x)dx - (1-u) \int_p^p \left[1\left[\frac{da(a, y)}{dt} < 0\right] \frac{dG_{1,t}(a, y)}{da} \right] \frac{da(a, y)}{dt} dy = \\
& (1-u)\delta \int_p^p \int_a^a g_{1,t}(x, y)dx dy + (1-u)\lambda(\theta)\bar{\Gamma}(p) \int_p^p \int_a^a s(x, y)g_{1,t}(x, y)dx dy \\
& + (1-u) \int_p^p 1\left[\frac{da(a, y)}{dt} > 0\right] \frac{dG_{1,t}(a, y)}{da} g_1(a, y) \frac{da(a, y)}{dt} dy
\end{aligned}$$

■

C Figures

Figure 1: Lorenz Curve of Wealth in Data

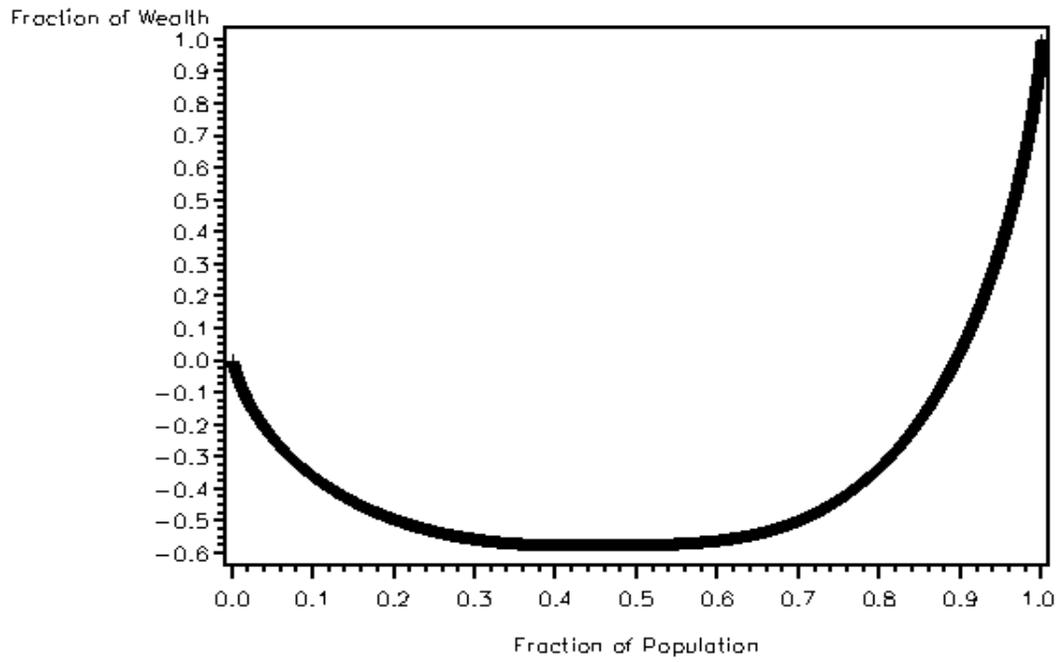


Figure 2: Non-Parametric Regression of Wage on Wealth

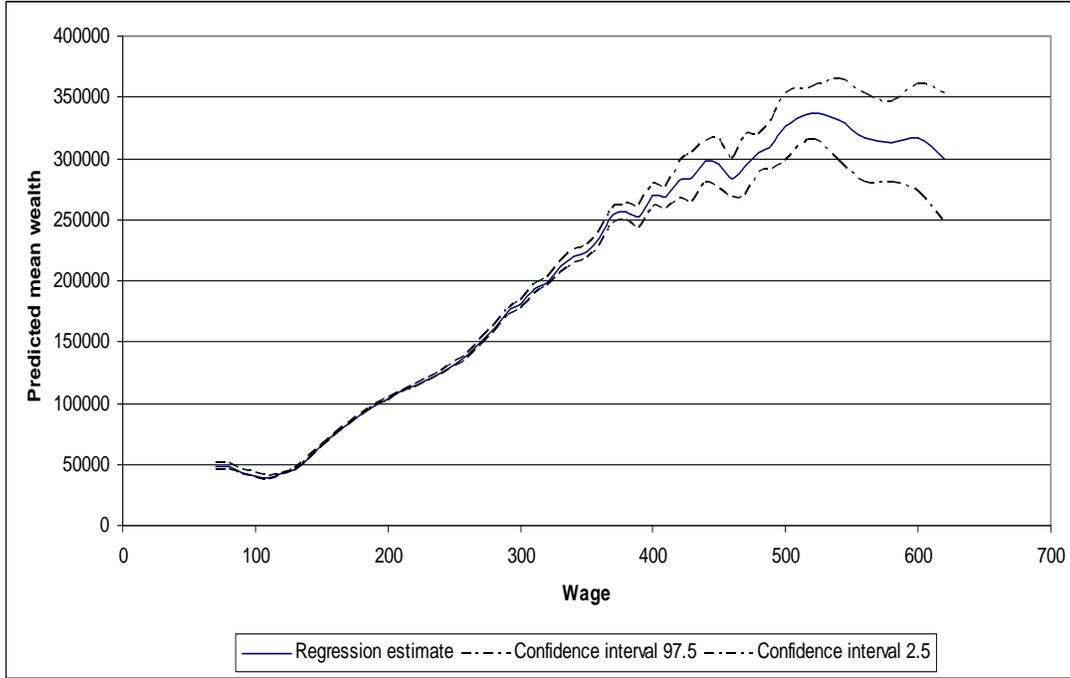


Figure 3: Lorenz Curve of Wealth in Simulated Model

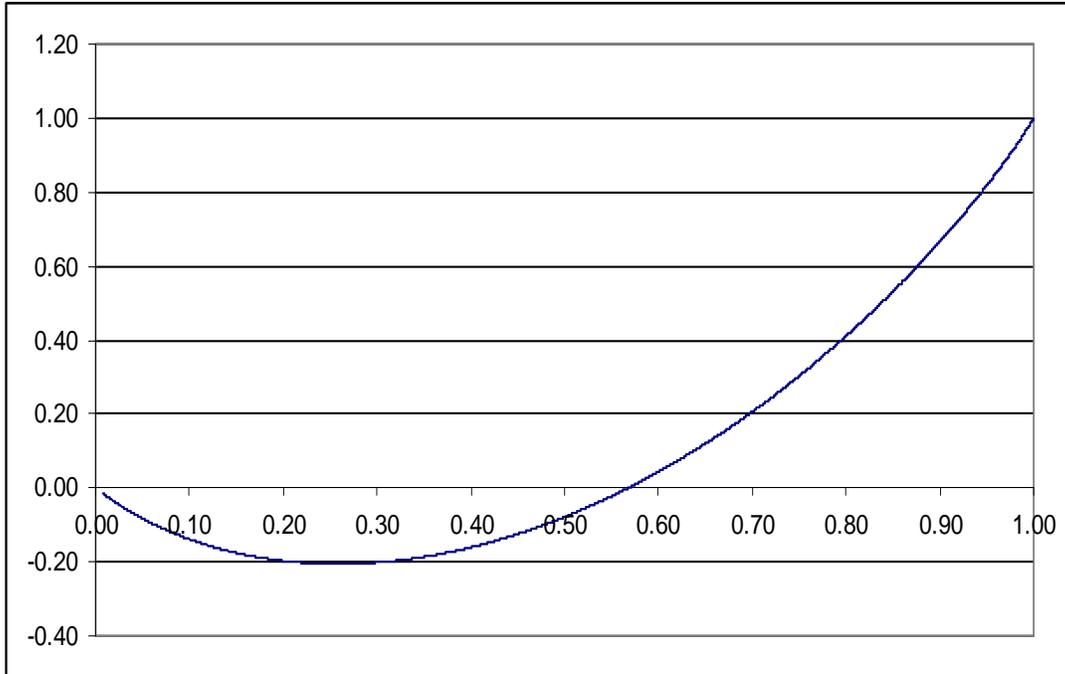


Figure 4: Welfare implications of different UI levels in an equilibrium model

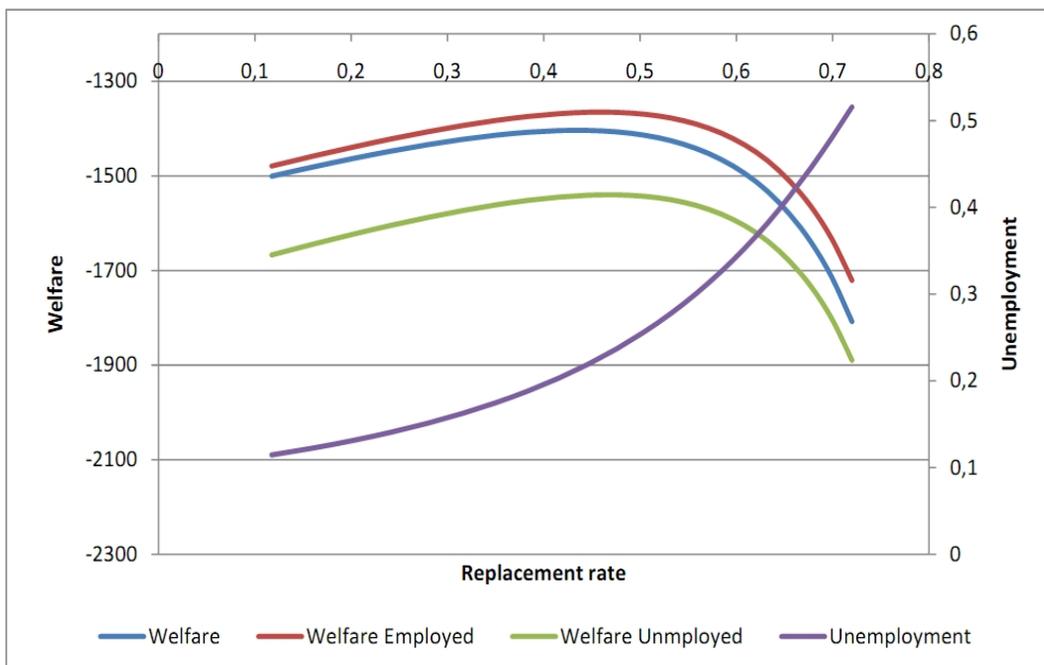


Figure 5: Welfare implications of different UI levels in a partial equilibrium model

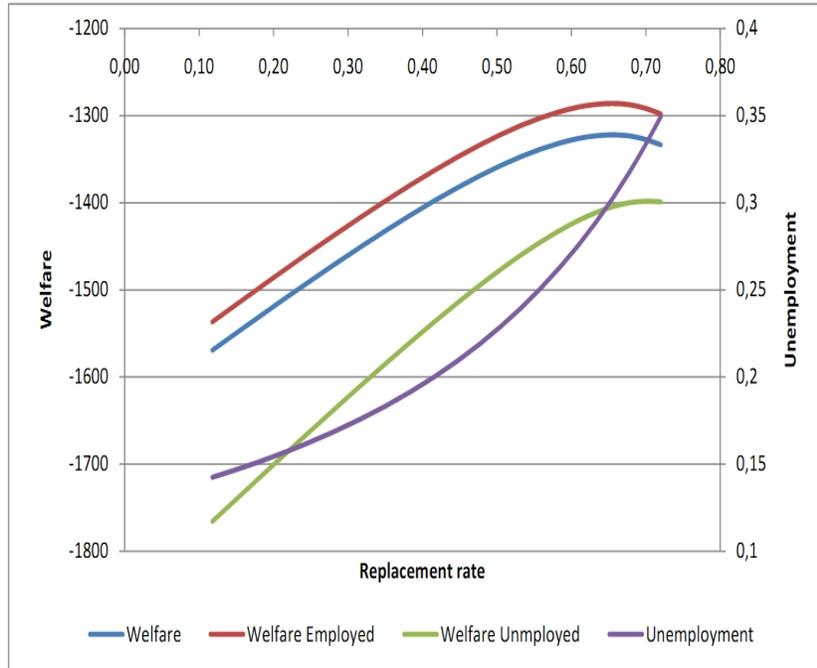


Figure 6: Aggregate Welfare as a function of productivity

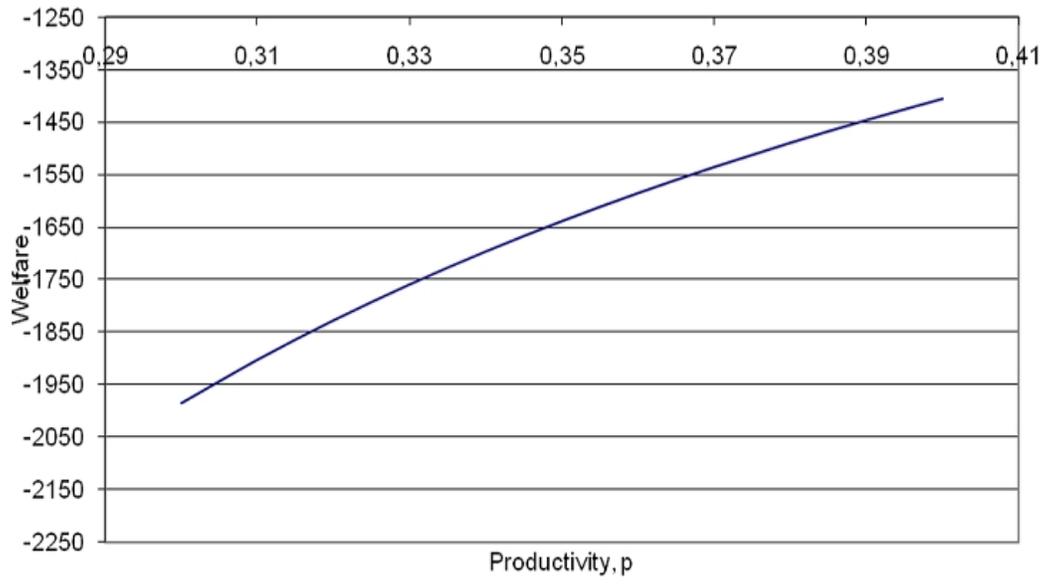
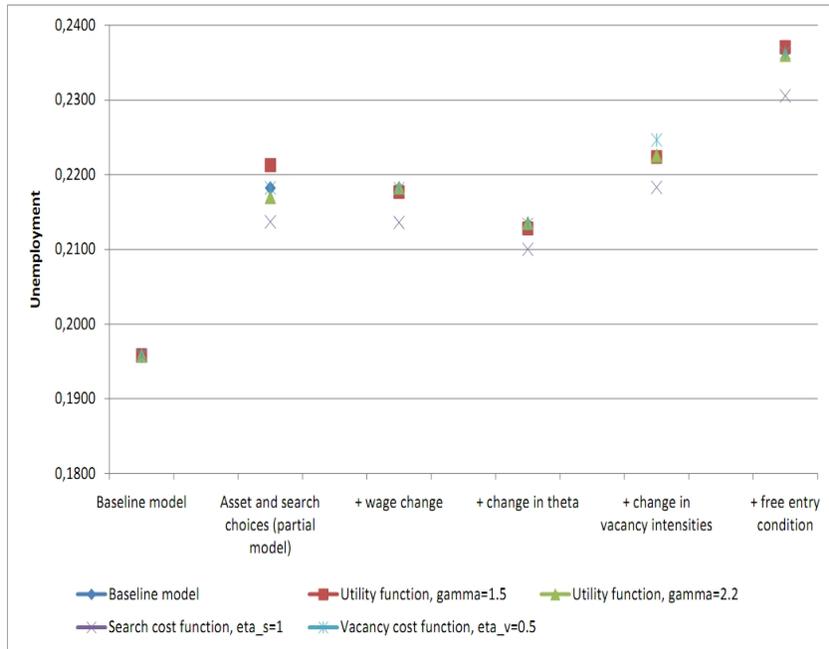


Figure 7: Unemployment decomposition using calibrated parameters from robustness



D Tables

Table 1: DESCRIPTIVE STATISTICS ON HOURLY WAGE AND WEALTH

	Mean	Std. Dev.	Skewness	Min. Value	Max. Value
Hourly wage	196.31	65.68	1.63	55.96	679.13
Wealth (in 10.000 DKK)	8.63	32.11	2.13	-95.85	258.94
- Employed	10.05	33.91	1.94	-95.85	258.94
- Non-employed	2.40	21.53	3.69	-95.84	258.69

Table 2: PH ESTIMATES OF WEALTH

	Estimate	Std. Err.	Estimate	Std. Err.
Employment duration				
Wealth (in 100.000 DKK)	-0.05930*	0.00020	-0.03692*	0.00021
+ Covariates	NO		YES	
Unemployment duration				
Wealth (in 100.000 DKK)	0.03556*	0.00076	-0.01380*	0.00077
+ Covariates	NO		YES	
Non-employment duration				
Wealth (in 100.000 DKK)	-0.00558*	0.00035	-0.01949*	0.00036
+ Covariates	NO		YES	

Notes:

a) Covariates include: Dummy for homeowner, age, education length, dummy for education level, experience, and dummy for females.

b) '*', and '**' indicate significance at one and five percent level, respectively.

Table 3: Fixed Parameters

Parameter	Definition	Value
φ	Worker discount rate	$1.05^{1/12} - 1$
r	Interest rate	$1.03^{1/12} - 1$
β	Bargaining power	0.5
b	Unemployment benefits	0.1
p	Productivity level	0.40
γ	Relative risk aversion	2
α_s	Search cost scale	0.1
η_s	Search cost curvature	1.5
α_v	Vacancy cost scale	0.15
η_v	Vacancy cost curvature	1
χ	Matching function elasticity	0.5

Table 4: Calibrated Parameters

	Definition	Moment	Target	Model	Calibrated parameter
i	Entrycost	M	0.0967	0.0967	135.73
\underline{a}	Lower level of assets	$E(a unemployed)$	0.2040	0.2040	-2.68
δ	Job destruction rate	$Pr(U E \text{ last period})$	0.0113	0.0113	0.0113
δ_f	Firm destruction rate	$Pr(U E \text{ last period} firm \text{ closure})$	0.0013	0.0013	0.0013
δ_j	Job specific destruction rate	$Pr(U E \text{ last period} no \text{ firm closure})$	0.0100	0.0100	0.0100
π	Matching function scale	$1 - u$	0.8042	0.8042	0.0111

Table 5: Equilibrium Quantities

Quantity	Definition	Model Value	Data value
Average asset of employed	$E(a employed)$	1.62	0.77
Variance of assets	$Var(a)$	3.92	5.98
Min. asset value	\underline{a}	-2.68	-7.31
Max. asset value	\bar{a}	6.46	19.7

Table 6: DECOMPOSITION OF THE EFFECT OF HIGHER UI
Unemployment rate

Baseline model - $b=0.1$	0.1958
High UI - $b=0.125$	
Change in search and asset choices	0.2189
+ wage change	0.2178
+ change in theta	0.2139
+ change in vacancy intensities	0.2233
+ free entry condition	0.2373

Table 7: EFFECT OF DIFFERENT TAXATION SCHEMES

	Income tax	VA tax	Tax on capital gains	Wealth tax
Unemployment	0.1958	0.1966	0.1887	0.1776
Welfare	-1,405.606	-1,400.212	-1,417.925	-1,439.384
- Unemployed	-1,547.988	-1,541.921	-1,563.399	-1,588.976
- Employed	-1,370.940	-1,365.526	-1,384.093	-1,407.086
Avg. Assets	1.347	1.743	0.562	-0.505

Table 8: NEW CALIBRATED PARAMETERS FOR ROBUSTNESS

	\underline{a}	i	ϕ
Baseline model	-2.68	135.73	0.0111
Utility function, $\gamma = 1.5$	-1.79	135.73	0.0143
Utility function, $\gamma = 2.2$	-3.02	135.73	0.0101
Search cost function, $\eta_s = 1$	-2.68	135.73	0.0140
Vacancy cost function, $\eta_v = 0.5$	-2.68	110.75	0.0101

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