MODEL, CALIBRATION, AND PARAMETER RISK

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Aarhus Quant Factory – Symposium

Aarhus Quant Day

17 January, 2014
• What is Calibration ?
• Model Risk : A Perfect Calibration ! Now What ?
• Calibration Risk
• New Calibration Methods : Moment Matching
• Conic Finance Perspective

• Joint work with: Florence Guillaume, Jurgen Tistaert, Erwin Simons, Dilip Madan.
What is Calibration?

“A derivative pricing model is said to be calibrated to a set of benchmark instruments if the value of these instruments, computed in the model, correspond to their market prices.” (Encyclopedia of Quantitative Finance)
What is Calibration?

- Calibration in a nutshell:

  Derivatives market prices
  
  Pricing model
  
  Model parameters
  
  Derivatives model prices

  Find: model parameters that match model prices as best as possible with market prices

  Optimization problem: Finding minimum “distance”
What is Calibration?

- Standard calibration problem: minimize distance $f$ between market $\{ P_i, \ i = 1, \ldots, M \}$ and model prices $\{ \hat{P}_i, \ i = 1, \ldots, M \}$ of liquid derivatives.
- Typically perfect match not possible.
- **Optimal match:**

$$p^* : f(\{ P_i \}, \{ \hat{P}_i \}, p^*) \leq f(\{ P_i \}, \{ \hat{P}_i \}, p'), \quad p^*, p' \in p; \quad (1)$$

where $p = \text{model parameter set}$.
- Common choice of $f$:

$$f(\{ P_i \}, \{ \hat{P}_i \}, p) = \text{RMSE}(\{ P_i \}, \{ \hat{P}_i \}, p) = \sqrt{\frac{1}{M} \sum_{i=1}^{M} (P_i - \hat{P}_i(p))^2}$$
# Calibration in Equity Land

## TABLE 4: IMPLIED VOLATILITY SURFACE EUROSTOXX 50, OCT 7TH 2003

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Calibration in E2C land

\[
\begin{array}{cccccccccc}
T & 1 & 2 & 3 & 4 & 5 & 7 & 10 \\
CDS (bp) & 50.7 & 73.7 & 97.6 & 109.2 & 116.3 & 122.4 & 127.1 \\
r (%) & 2.12 & 1.62 & 1.75 & 1.93 & 2.13 & 2.47 & 2.77 \\
Digital Put Price (%) & 1.00 & 2.63 & 4.97 & 7.13 & 9.17 & 12.66 & 17.19 \\
\sigma (%) & 92.35 & 77.26 & 71.22 & 67.03 & 64.17 & 60.47 & 57.89
\end{array}
\]
Calibration in CDO Land

Gaussian vs Lévy Base Correlation
19th of March 2008
iTraxx Europe Series 8 5Y

Base correlation vs tranche

KU LEUVEN
Model and Calibration Risk

- **Model Risk**

  One can calibrate different advanced models to a given market setting.

  Suppose the calibration fits are good.

  One can then price exotics under the calibrated model.

  Different models can give rise to significant different exotic prices.

<table>
<thead>
<tr>
<th>Model</th>
<th>rmse</th>
<th>ape</th>
<th>aae</th>
<th>arpe</th>
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Table 3: Lookback Option Prices

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<td>730.84</td>
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<td>NIG-OUΓ</td>
<td>722.34</td>
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</table>

Figure 9: Cliquet Prices: \( f_{loc} = -0.03, c_{loc} = 0.05, c_{glo} = +\infty, \ T = 3, N = 6, t_i = i/2 \)
Model and Calibration Risk

• Calibration Risk

In the calibration procedure one has to make different choices:

• Which function to minimize (rmse, ape, aae, arpe, …) ?
• Which vanilla to calibrated on (ATM, OTM, …) ?
• Use a weighting and if so how to weight ?
• Calibrate on prices or on implied volatilities ?
• Which starting value to take (yesterday’s optimum, random, historical,…)?
• Which search algorithm to use (Nelder-Nead, Gauss, Newton Gradient decent,….) ?
• Pre-fix some parameters and how ?

Suppose the calibration under different choices are good, but lead to different optimal parameters.
Different parameter settings can give rise to significant different exotic prices.
Model and Calibration Risk

• Example: The Heston Model

  Stock price process:

  \[
  \frac{dS_t}{S_t} = (r - q)dt + \sqrt{v_t}dW_t, \quad S_0 \geq 0
  \]

  Squared volatility process:

  \[
  dv_t = \kappa (\eta - v_t) dt + \lambda \sqrt{v_t} d\tilde{W}_t, \quad v_0 = \sigma_0^2 \geq 0, \]

  where \( W = \{W_t, t \geq 0\} \) and \( \tilde{W} = \{\tilde{W}_t, t \geq 0\} \) are correlated standard BM such that \( \text{Cov}(dW_t, d\tilde{W}_t) = \rho dt \).

  Parameters involved:

  • \( \nu_0 > 0 \): initial variance
  • \( \kappa > 0 \): mean reversion rate
  • \( \eta > 0 \): long run variance
  • \( \lambda > 0 \): volatility of variance
  • \( \rho \): correlation
Model and Calibration Risk

- **Calibration Risk**: The choice of the objective function

Evolution of the Cliquet price through time (local floor = -0.03, global floor = 0, local cap = 0.05, global cap = +∞, N = 8, T = 2, t_i=i/4) – market implied reduced calibration

\[
\text{RMSE} = \sqrt{\frac{\sum_{j=1}^{N} (P_j - \hat{P}_j)^2}{N}}
\]
\[
\text{APE} = \frac{1}{\text{mean}_j \hat{P}_j} \sum_{j=1}^{N} \frac{|P_j - \hat{P}_j|}{N}
\]
\[
\text{ARPE} = \frac{1}{N} \sum_{j=1}^{N} \frac{|P_j - \hat{P}_j|}{\hat{P}_j}
\]
Model and Calibration Risk

- **Calibration Risk: pre-fixing**
  - $\nu_0 = \text{square of the spot price of the VIX index expressed in units:}$
    \[ \nu_0 = \left( \frac{\text{VIX}(t_0)}{100} \right)^2 \]
  - **moving window (MW) estimate:**
    \[ \eta_{\text{MW}} = \frac{1}{T_{\text{VIX}}} \int_{-T_{\text{VIX}}}^{0} \left( \frac{\text{VIX}(t)}{100} \right)^2 \, dt = \text{mean}_{-T_{\text{VIX}} \leq t \leq 0} \left( \frac{\text{VIX}(t)}{100} \right)^2. \tag{1} \]
  - **exponentially weighted moving average (EWMA) estimate:**
    \[ \eta_{\text{EWMA}} = (1 - \alpha) \sum_{i=1}^{N} \alpha^{N-i} \left( \frac{\text{VIX}(t_i)}{100} \right)^2 \tag{2} \]
  - **mark-to-market (MTM) estimate:**
    \[ P(K, T) - C(K, T) = \exp(-rT)(K - \text{VIX}_T) \]
    \[ \eta_{\text{MTM}} = \left( \frac{K_{\text{ATM}}}{100} \right)^2 = \left( \frac{L\text{VIX}(t_0)}{100} \right)^2. \tag{3} \]
Model and Calibration Risk

• Calibration Risk
Model and Calibration Risk

- Calibration Risk
Model and Calibration Risk
Model and Calibration Risk

- Calibration Risk
Model and Calibration Risk

- Calibration Risk

Evolution of the Cliquet price through time (local floor = -0.03, global floor = 0, local cap = 0.05, global cap = +∞, N = 8, T = 2, t=i/4)
Moment Matching Calibration

• Using the implied moment formula (cfr. VIX calculation)

\[
\mathbb{E} \left[ \left( \log \left( \frac{S_T}{S_0} \right) \right)^N \right] = \left( (r - q)T \right)^N + \exp(\mu T) \left( \int_0^F \frac{N}{K^2} \left( (N - 1) \left( \log \left( \frac{K}{S_0} \right) \right)^{N-2} - \left( \log \left( \frac{K}{S_0} \right) \right)^{N-1} \right) P(K, T) dK \right. \\
+ \left. \int_F^\infty \frac{N}{K^2} \left( (N - 1) \left( \log \left( \frac{K}{S_0} \right) \right)^{N-2} - \left( \log \left( \frac{K}{S_0} \right) \right)^{N-1} \right) C(K, T) dK \right).
\]

• One can back out of the option price surface the implied variance, skewness, kurtosis and higher moments of the risk-neutral return distribution.

• Since the above moments are also known in close form for many popular models (Heston, VG, …), one tries to fit these moments as best as possible.
Moment Matching Calibration

- Moment matching Calibration

\[
E[g(S_T)] = g(\nu) + g'(\nu) \left( \exp((r - \delta)T)S_0 - \nu \right) + \exp(\nu T) \left( \int_0^K g''(K) P(K,T) dK + \int_\nu^\infty g''(K) C(K,T) dK \right)
\]

\{ variance, skewness, kurtosis \}

\[ C(K,T) \]

\[ P(K,T) \]
Moment Matching Calibration

- For VG, we have

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<th>$\text{VG}(\sigma, \nu, \theta)$</th>
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<tr>
<td>skewness</td>
<td>$\frac{\theta \nu \left(3 \sigma^2 + 2 \nu \theta^2\right)}{\left(\sigma^2 + \nu \theta^2\right)^{3/2}}$</td>
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<tr>
<td>kurtosis</td>
<td>$3 \left(1 + 2 \nu - \frac{\nu \sigma^4}{\left(\sigma^2 + \nu \theta^2\right)^2}\right)$</td>
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- Knowing the statistics and solving for the parameters:

$$
\begin{aligned}
    c_1 \sigma^6 + c_2 \sigma^4 + c_3 & = 0 \\
    \nu & = \frac{s^2 \nu^3}{T^2 \left(\sigma^2 + 2 \frac{\nu}{T}\right)^2 \left(\frac{\nu}{T} - \sigma^2\right)} \\
    \theta & = \text{sign}(s) \sqrt{\frac{\frac{\nu}{T} - \sigma^2}{\nu}} \\
\end{aligned}
\begin{aligned}
    c_1 & = \frac{(k^{3/2} - 1)T}{k^{3/2} \nu s^2} \\
    c_2 & = 3 \frac{k^{3/2} - 1}{s^2} - 1 \\
    c_3 & = 2 \frac{\nu^2}{T^2} \left(1 - 2 \frac{k^{3/2} - 1}{s^2}\right)
\end{aligned}
$$

where $\nu$, $s$ and $k$ denotes the market implied variance, skewness and kurtosis of the log asset return for the time horizon $T$. 
Moment Matching Calibration

• The system admits a solution iff

\[ 6 + 3s^2 - 2k < 0. \]

• If no perfect matching solution exists one can look for the “closest” one in the existence domain.

• MAIN ADVANTAGES
  o No integration is needed
  o Extremely fast
  o No initial parameter needed
  o No search-algorithm involved
# Moment Matching Calibration

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<td>0.91781</td>
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<td>0.2470</td>
<td>-0.4671</td>
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<td>1.13425</td>
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<td>0.7491</td>
<td>-0.3128</td>
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<tr>
<td>1.63288</td>
<td>0.1730</td>
<td>1.0591</td>
<td>-0.2676</td>
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<tr>
<td>2.13151</td>
<td>0.3146</td>
<td>1.9280</td>
<td>-0.9170</td>
</tr>
</tbody>
</table>

KU LEUVEN
Moment Matching Calibration

- **VG-Term structure model average results**

<table>
<thead>
<tr>
<th></th>
<th>Carr-Madan</th>
<th>moment matching (1)</th>
<th>moment matching (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMSE</td>
<td>0.4416</td>
<td>4.0043</td>
<td>2.4203</td>
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<tr>
<td>variance</td>
<td>0.0531</td>
<td>0</td>
<td>0</td>
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<tr>
<td>skewness</td>
<td>0.8514</td>
<td>0.1876</td>
<td>0</td>
</tr>
<tr>
<td>kurtosis</td>
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<td>0.0563</td>
<td>2.3616</td>
</tr>
<tr>
<td>cpu (sec)</td>
<td>59.0324</td>
<td>0.000608</td>
<td>0.000413</td>
</tr>
</tbody>
</table>

**Evolution of the RMSE (VG)**

- Carr-Madan
- Moment matching (1)
- Moment matching (2)
Moment Matching Calibration

- **Heston – VS calibration**

\[
\mathbb{E} \left[ \frac{1}{T} \int_0^T \sigma_t^2 dt \right] = \frac{2}{T} \exp(rT) \left( \int_0^{F_0} \frac{1}{K^2} P(K, T) dK + \int_{F_0}^{\infty} \frac{1}{K^2} C(K, T) dK \right).
\]

\[
\mathbb{E} \left[ \int_0^T \nu_t dt \right] = \eta T + \frac{\nu_0 - \eta}{\kappa} (1 - e^{-\kappa T})
\]

Legend:
- Market price
- Adjusted market price (parabolic)
- Adjusted market price (reduced set)
- Adjusted market price (interpolation)
- Model reduce fit
- Model interpolation fit
Conic Finance

- **Bid-Ask Pricing**
  - We will make use of the \( \text{minmaxvar} \) distortion function:
    \[
    \Phi(u; \lambda) = 1 - \left( 1 - u^{\frac{1}{1+\lambda}} \right)^{1+\lambda}
    \]
  - We use non-linear expectation to calculate (bid and ask) prices.
  - The distorted expectation of a random variable with distribution function \( F(x) \) is defined
    \[
    dE(X; \lambda) = E^\lambda[X] = \int_{-\infty}^{+\infty} x d\Phi(G(x); \lambda).
    \]
Conic Finance

• Bid-Ask Pricing

The ask price of payoff $X$ is determined as

$$\text{ask}(X) = -\exp(-rT)E^\lambda[-X].$$

The bid price of payoff $X$ is determined as

$$\text{bid}(X) = \exp(-rT)E^\lambda[X].$$

Hence for the BID price we have put more weight on the down-side. For the ASK the upside has been receiving more weighted.
Conic Finance

MC:

Option Price = \( \exp(-rT)E_Q[\text{Payoff}] \)

\[ \approx \exp(-rT)\frac{1}{N} \sum_{i=1}^{N} \text{ith simulated payoff} \]

\[ = \exp(-rT)\frac{1}{N} \sum_{i=1}^{N} \text{ith sorted simulated payoff} \]

DISTORTED UNIFORM (1/N) WEIGHTS:

\[ \text{Bid Price} = \exp(-rT)\sum_{i=1}^{N} w_i \text{ith sorted simulated payoff} \]
Conic Finance

• Bid-Ask Pricing

These formulas are derived by noting that the cash-flow of selling at its ask price and buying at its bid price is acceptable in the relevant market.

We say that a risk/cash-flow $X$ is acceptable if

$$E_Q[X] \geq 0 \text{ for all measures } Q \text{ in a convex set } \mathcal{M}.$$ 

$\mathcal{M}$ is a set of test-measures under which cash-flows need to have positive expectation.

Operational cones were defined by Cherney and Madan and depend solely on the distribution function $G(x)$ of $X$ and a distortion function $\phi$. One can show that we need to have that the distorted expectation is positive:

$$de(X; \lambda) = E^\lambda[X] = \int_{-\infty}^{+\infty} x d\Phi(G(x); \lambda).$$
Conic Finance

• How does everything relate to each other?

Model Risk: different models \(-\) different Q measures \(-\) different exotics
Calibration Risk: same model \(-\) different parameters/Q \(-\) different exotics.

Buying X at its bid price is acceptable:
\[ X - \text{bid}(X) \in A \]

Selling X at its ask price is acceptable:
\[ \text{ask}(X) - X \in A \]

Ask price is supremum of test-measure prices:
\[ \text{ask}(X) = \sup_{Q \in \mathcal{M}} E_Q[X] \]

Bid price is infimum of test-measure prices:
\[ \text{bid}(X) = \inf_{Q \in \mathcal{M}} E_Q[X] \]
Conclusion

- Calibration is an important procedure in derivatives pricing.
- Model risk and calibration risks are omnipresent and can lead to wide ranges in exotic pricing.
- Alternative calibration methods are currently developed.
- Moment matching calibration is not using a search algorithm and hence also not a starting value. It relies on closed-form expressions of the underlying moments, which can be readily estimated from market quotes. It is extremely fast.
- Conic finance puts model and calibration risk into a bid-ask perspective. The more uncertainty there is on the model/calibration to be used the wider the bid-ask spread.

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Thank you for your attention!