Random Assignment with Non-Random Peers: A Structural Approach to Counterfactual Treatment Assessment

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Abstract

Recent years have witnessed an explosion of applied economics research on peer effects. However, attempts to leverage peer effects by creative group assignment have often failed to account for endogenous network formation, which has the potential to lead to biased estimates and flawed policy prescriptions. To account for this, I model network endogeneity as an omitted variable problem within the context of a non-linear peer effects model, providing conditions in which the parameters of this model are identified even in the presence of network endogeneity. I then model network formation as a means of identifying the omitted parameters. In a departure from prior literature, I model network formation as a static game in which actors make continuous linking decisions subject to a total effort constraint, showing that this model has a unique, interior, stable equilibrium. I provide conditions under which parameters of the network formation model are identified. I then estimate the model using data from a girls’ empowerment program in rural Rajasthan, India. Finally, using the estimated parameters, I compare simulated outcomes to those from a realized out-of-sample policy before evaluating outcomes under a counterfactual assignment rule.

1I thank my dissertation committee of Dan Ackerberg, Tanya Rosenblat, Jeff Smith, and Rebecca Thornton for their support throughout this process. I acknowledge the International Food Policy Research Institute for funding the Bal Sabha data collection. This paper has benefited from extensive conversations with Lawrence Blume, Vincent Boucher, Yann Bramoulé, Sarah Johnston, Benjamin Lipsius, David Miller, Bryan Stuart, Chris Sullivan, Adam Szeidl, Evan Taylor, and Andrew Usher, as well as participants at numerous seminars at the University of Michigan. All errors remain my own.
1 Introduction

A large literature documents how peers affect many of economic outcomes and choices. However, in many situations, peers are also choices. Failure to account for this crucial fact may lead to crucially flawed estimates of peer effects and also policy prescriptions with unintended consequences (see Carrell, Sacerdote, and West 2013; Angrist 2014). This paper fills the gap by positing a structural model of peer effects that accounts for network endogeneity and shows how to estimate the model using detailed data on outcomes and network connections.

In the context of the linear-in-means model (see, e.g., Manski 1993), I posit network endogeneity as an omitted-variable problem. I then provide a network-formation model in which actors make simultaneous, continuous linking decisions subject to a total effort constraint. The continuous nature of the model greatly simplifies equilibrium characterization as well as computation. I provide identification results showing that estimation of the network-formation model solves the omitted-variables problem that confounds peer effects estimation. I then estimate the model using data from a randomized experiment in rural Rajasthan and show that the model performs reasonably well in predicting the results of out-of-sample alternative policies.

The data for this paper comes from a randomized trial of a girls’ empowerment program in rural Rajasthan, India. As part of the program, 13 girls in each school are chosen to form a Bal Sabha (Girls’ Parliament). A study team conducted a randomized trial of the program, in which we have a Control group as well as two treatment groups: one with participants assigned by election, another with participation randomly assigned. As part of this effort, we collected detailed longitudinal data on network connections along a number of dimensions, as well as outcome data. Concurrent work by Delavallade, Griffith, and Thornton (2016) assesses the reduced-form effects of the program under both selection regimes as implemented. This paper aims to assess the effect of the program under alternative assignment regimes unobserved in the data. This is accomplished by a model that accounts for both peer effects and endogenous network formation as part of a structural model.

Recognizing that peers are often choices, economists have exploited random assignment to peer groups as a solution to network endogeneity. For example, prominent studies have exploited random assignment to dorms (Sacerdote 2001, Stinebrickner and Stinebrickner 2006), university class sections (De Giorgi, Pellizzari, and Radaelli 2010), and squadrons at the Air Force Academy
While in static settings this approach can identify the mean effect of peers on outcomes, care must be taken when extrapolating out-of-sample.

Estimates of the effects of alternative policies may be further confounded when the relevant peer group is a subset of the assigned larger group. For example, within a classroom, students may choose their friends by ability, gender, caste, or any number of other observed or unobserved characteristics. That is, even when group assignment is random, peer groups may be endogenous conditional on group assignment. Ignoring this latter source of endogeneity may lead to crucially biased estimates of the effects of counterfactual policies. This is especially true when proposed alternative assignments have substantially different available peers (see Carrell, Sacerdote, and West 2013).

To perform counterfactual policy analysis in such a context requires a structural model that accounts for peer group formation conditional on group assignment. Given the sobering findings in Carrell, Sacerdote, and West (2013), prominent recent papers have expressed doubt at the very idea of out-of-sample prediction in the presence of network endogeneity (Angrist 2014; Sacerdote 2014). My approach, on the other hand, is to explicitly model peer selection and outcomes conditional on peers as two parts of an overarching model. First, peer groups are formed according to a simultaneous-move game. Then, conditional on these peer groups, outcomes are determined by a process that accounts for peer influence.

To model peer effects, I start with the linear-in-means model as posited by Manski (1993), slightly transformed as in Carrell, Sacerdote, and West (2013). In this setting, network endogeneity arises from the presence of unobserved factors that affect both outcomes and the decision to form network links. To account for this, I augment the model with individual-specific unobserved variables (see, e.g., Goldsmith-Pinkham and Imbens 2013). This effectively treats network endogeneity as an omitted variable issue, similar in spirit to “latent space” models of network formation (see Jackson 2014). Conditional on observation of the omitted individual-specific parameters, I show that the parameters of the peer effects model are identified under much weaker assumptions than typically employed. Specifically, conditional on controlling for the unobserved confounders, parameters of the peer effects model are identified even in the presence of network endogeneity.

I then demonstrate that these unobserved variables can be recovered by estimating a structural
model of network formation. I model the network formation process as a static, simultaneous-move game in which players make continuous linking decisions subject to a total effort constraint. This necessarily builds in tradeoffs between linking decisions. Further, the model accounts for homophily, complementarity between linking decisions, and a number of other patterns prevalent in the data. With sufficient within-network variation in exogenous characteristics, parameters of the network formation model are identified. Crucially, identification includes scalar unobservables that can be used to control for network endogeneity in the peer-effects model. Importantly, I show that the model has a unique interior equilibrium. This allows for estimation of the network formation game without reference to partial identification (Sheng 2012) or complicated equilibrium-selection procedures (Christakis et al. 2011; Badev 2013; Mele 2010).

Armed with these identification results, I estimate the parameters of the network-formation and peer effects models. I then show that these estimated parameters perform reasonably well in predicting out-of-sample outcomes. Finally, I construct a counterfactual assignment rule that assigns students with the lowest predicted outcomes to participate in the program. With simulations using the estimated parameters, I show that this policy has no effect on Educational Aspirations but substantially lower Gender Roles attitudes.

This paper’s primary contribution lies in providing a method for assessing the effects of counterfactual policies in the presence of endogenous network formation. The need for this is particularly acute given recent papers documenting randomized interventions that affect networks themselves (see Comola and Prina 2014; Delavallade, Griffith, and Thornton 2016; Vasilaky and Leonard 2015). In these settings, changing network structure can be an important channel through which interventions affect outcomes, and failure to account for changing network structure may bias results—especially counterfactual exercises. My method provides a method for estimating a structural model using experimental variation (see, e.g. Todd and Wolpin 2006), as well as adding to a body of work comparing randomized to non-randomized assignments, as demonstrated in, for example, Shadish, Clark, and Steiner (2008).

As a necessary part of this paper’s main contribution, I also advance the literature on the identification and estimation of network-formation games. By positing the game as one of continuous linking decisions subject to a total effort constraint, I show that the game has a unique interior equilibrium. This allows the econometrician to escape problems of equilibrium selection (see, e.g., Badev
2013; Christakis et al. 2011; Mele 2010) and partial identification (see Sheng 2012) that tend to complicate identification and estimation in the context of discrete games. This vastly simplifies identification and estimation, and allows for avoidance of equilibrium-selection specification when performing counterfactual simulations.

The paper proceeds as follows. Section 2 describes the program under study as well as deriving a number of key reduced-form facts to motivate features of the structural model. Section 3 provides the peer effects model that posits network endogeneity in the peer effects context as an omitted-variable problem. Section 4 then develops the network-formation model as a means of controlling for these structural unobservables. Applying the identification results, Section 5 presents structural estimation results, showing that the model’s predictions mesh well with realized out-of-sample outcomes. Using these parameter estimates, Section 6 compares predicted outcomes to those of a realized out-of-sample group of schools. Section 7 then simulates outcomes under an alternative assignment regime whereby students with the lowest predicted outcomes are assigned to participate in the program. Section 8 concludes.

2 Empirical Setting and Data Description

This section describes the empirical setting and the experimental design. Through this exercise, we learn that the program has negative but insignificant effects on outcomes but strongly significant effects on network structure. We further learn crucial features of the observed networks, including that the constructed continuous measure of networks is consistent with binary measures that are typically used in peer-effects models.

2.1 Bal Sabha Program and Experimental Design

Here, I describe the Bal Sabha program, as well as the experimental design. The program, operated by the NGO Educate Girls, takes place in government schools in rural Rajasthan, India. As part of the program, 13 girls in grades 6 through 8 are chosen to form a Bal Sabha (Girls’ Parliament). Under the NGO’s preferred assignment rule, these girls are elected to participate by all students in those grades, including boys. Accordingly, these elections lead to non-random selection, a fact that is documented in Delavallade, Griffith, and Thornton (2016).
The program focuses on developing so-called “soft skills.” These skills include leadership and self-confidence as well as attitudes and aspirations about education, age at marriage, and gender roles. The larger goal of the program is for girls to employ these skills as a means of overcoming barriers to their own education, such as early marriage.

The intervention consists of a series of five “games” during which village volunteers work through increasingly difficult scenarios. Through activities such as role playing, girls are taught to develop their own voice in difficult situations such as, for example, their parents desiring to have them marry young or end their schooling. The parliaments meet biweekly over a span of approximately six months during the academic year. The total intervention time is approximately 25-50 hours.

2.2 Study Design and Data Collection

As part of the rollout of the program to a new district during the 2013-14 academic year, a study team designed and implemented a randomized trial. A sample of thirty schools was chosen in two administrative blocks. None of these schools had ever had a Bal Sabha prior to the start of the study.

Prior to treatment assignment, we performed three data collection activities in each school. First, Bal Sabha elections were held in all schools, including those that would later serve as controls. Second, girls in each school filled out an extensive questionnaire that gathered background demographic information as well as data on attitudes, aspirations, and expectations along a number of dimensions. Third, prior to treatment assignment, we also conducted a pairwise network census among all girls in each of the 30 schools.

After baseline data collection, schools were assigned to one of three treatment groups. T1 schools conducted the Bal Sabha with elected participants, as is customary for the program as implemented by Educate Girls. Schools assigned to T2 received the Bal Sabha program, but the 13 participants were chosen randomly. Finally, Control schools did not receive the program at all.

The program was implemented over a period of approximately six months. During this time, village volunteers led the 13 participating girls through the games-based curriculum. Girls chosen to participate were encouraged to share their learning and experiences with their classmates who were not participants, and effects spilling over to non-participants is a key feature of the implementing organization’s theory of change.
At the conclusion of program implementation, enumerators returned to each school to conduct an endline survey that measured outcomes similar to those measured at baseline. Further, in order to assess the effect of the program on networks, we conducted an additional pairwise network census. Accordingly, this data allows us to measure the program’s effects on both endline outcomes—as measured by aspirations and attitudes—and endline networks.

2.3 Demographics and Outcomes at Baseline

Table 1 provides descriptive statistics for the full sample of 1319 girls at baseline.\(^2\) Note that approximately 28 percent of the girls were elected to participate, suggesting an average of approximately 44 girls in each school. Enrollment seems to be skewed toward students in Standard 6 (the omitted category). Finally, note substantial variation in caste, as 37% of the sample are members of Scheduled Castes/Scheduled Tribes, while 44.5% are in Other Backwards Castes. The omitted caste category, General or upper castes, comprises 18.5% of the sample. From these means, we see that there are fewer girls in higher grades, suggestive of attrition, as well as large lower-caste populations.

This paper focuses on two outcomes: educational aspirations/expectations, and attitudes about gender roles. These outcome measures are constructed as the normalized first principal component of all relevant survey questions. Girls have higher educational aspirations if, for example, they indicate they would like to complete university, as compared to stating they would like to complete only eighth grade. Girls have higher Gender Roles attitudes if, for example, they say it is okay for a wife to disagree with her husband in public.

Baseline outcomes are summarized in Table 1, Panel B. Since the mean of the outcome variables is 0 by construction in the data, these means indicate that girls have below average Educational Aspirations and above average Gender Roles attitudes. This conforms to our priors that girls have lower Educational Aspirations than boys but higher Gender Roles attitudes.\(^3\)

Table 2 presents regression results of baseline outcomes on the baseline covariates in Panel A of Table 3. The results show that these outcomes vary substantially by baseline characteristics. For

\(^2\)The sample consists of all girls who have non-missing data on the covariates in Panel A. This rule excludes less than 1% of eligible girls.

\(^3\)Baseline balance is presented in Appendix B. Table A.1 shows balance across treatment arms, while Table A.2 shows within-school balance between girls (randomly) selected to participate in T2 and those not selected.
### Table 1: Baseline Variable Descriptives

<table>
<thead>
<tr>
<th>Panel A: Baseline Covariates</th>
<th>Mean</th>
<th>S.D.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elected</td>
<td>0.281</td>
<td>0.449</td>
</tr>
<tr>
<td>Standard 7</td>
<td>0.318</td>
<td>0.466</td>
</tr>
<tr>
<td>Standard 8</td>
<td>0.306</td>
<td>0.461</td>
</tr>
<tr>
<td>Scheduled Caste</td>
<td>0.252</td>
<td>0.435</td>
</tr>
<tr>
<td>Scheduled Tribe</td>
<td>0.118</td>
<td>0.323</td>
</tr>
<tr>
<td>Other Backward Caste</td>
<td>0.445</td>
<td>0.497</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: Baseline Outcomes</th>
<th>Mean</th>
<th>S.D.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Educational Aspirations</td>
<td>-0.188</td>
<td>1.028</td>
</tr>
<tr>
<td>Gender Roles</td>
<td>0.125</td>
<td>0.992</td>
</tr>
</tbody>
</table>

Notes: Robust standard errors in parentheses, clustered by school. Sample is all girls in all schools. N = 1319 in 30 schools.

example, elected girls have higher educational aspirations, while lower caste girls have substantially lower levels of both baseline outcomes. Accordingly, if the program is to be targeted at those most “at need,” it may make sense to target lower caste girls for participation in the program rather than the more popular girls who are chosen by election.

### 2.4 Baseline Network Descriptives, and a Continuous Measure of Connectedness

As mentioned above, we collected extensive network data at both baseline and endline. This data collection consisted of each girl in the study sample answering a series of binary questions about every other girl in her school in grades 6 through 8. I categorize these variables by whether they are choices, such as being friends, or static variables, such as living in close proximity. These are described in Table 3 in Panels A and B, respectively. I have highlighted the “She is a friend” measure in gray, as that is the link definition commonly reported in the literature. Note that the friendship networks are quite dense: on average, girls indicate that 55.9 percent of other girls in their school are friends, as shown in the shaded row of Table 3. Other measures of connectedness, on the other hand, suggest less dense networks. For example, only 20.6% of girls said that they had visited the other girl’s home within the past week.

While the bulk of the economics literature on networks treats links as binary, the additional
Table 2: Baseline Outcome Heterogeneity

<table>
<thead>
<tr>
<th></th>
<th>Educational Aspirations</th>
<th>Gender Roles</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>Elected</td>
<td>0.203**</td>
<td>-0.043</td>
</tr>
<tr>
<td></td>
<td>(0.077)</td>
<td>(0.074)</td>
</tr>
<tr>
<td>Standard 7</td>
<td>-0.022</td>
<td>0.116</td>
</tr>
<tr>
<td></td>
<td>(0.076)</td>
<td>(0.069)</td>
</tr>
<tr>
<td>Standard 8</td>
<td>0.060</td>
<td>0.172**</td>
</tr>
<tr>
<td></td>
<td>(0.092)</td>
<td>(0.084)</td>
</tr>
<tr>
<td>Scheduled Caste</td>
<td>-0.265**</td>
<td>-0.338**</td>
</tr>
<tr>
<td></td>
<td>(0.116)</td>
<td>(0.126)</td>
</tr>
<tr>
<td>Scheduled Tribe</td>
<td>-0.322***</td>
<td>-0.391***</td>
</tr>
<tr>
<td></td>
<td>(0.103)</td>
<td>(0.133)</td>
</tr>
<tr>
<td>Other Backwards Caste</td>
<td>-0.312***</td>
<td>-0.069</td>
</tr>
<tr>
<td></td>
<td>(0.087)</td>
<td>(0.102)</td>
</tr>
<tr>
<td>Constant</td>
<td>-0.013</td>
<td>0.210**</td>
</tr>
<tr>
<td></td>
<td>(0.087)</td>
<td>(0.089)</td>
</tr>
<tr>
<td>Observations</td>
<td>1,319</td>
<td>1,319</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.024</td>
<td>0.029</td>
</tr>
</tbody>
</table>

Notes: Robust standard errors in parentheses, clustered by school. *** p<0.01, ** p<0.05, * p<0.1.

measures of connectedness allow for capture of more variation in link intensity.\(^4\)

In order to exploit this additional information—and as an input into the structural estimation process described in later sections of this paper—I construct a “continuous” measure of connectedness by collapsing the measures in Table 3, Panel A into a single index. While this provides for more exploitable variation, these variables are highly correlated with each other. To account for the covariance structure, I take the first principal component, then scaling such that the scalar term has unit variance.

In Table 4, I compare the continuous measure of connectedness to the binary one. For the latter, I follow the bulk of the literature in using the student’s response to “She is a friend” as a binary link measure. Panel A shows the probability that a student in the group identified on the y-axis indicates an individual on the x-axis is a friend. For example, the likelihood that a member of a Scheduled Caste names another Scheduled Caste member as a friend is 62.2%, while the likelihood of her naming a member of General Castes is 49.6%. Comparison of (shaded) elements

\(^4\)If connectedness is indeed a latent continuous measure, an additional motivation for use of the first principal component is to reduce measurement error (see Black and Smith 2006).
Table 3: Baseline Network Variable Descriptives

<table>
<thead>
<tr>
<th>Panel A: Choice Network Variables</th>
<th>Mean</th>
<th>S.D.</th>
</tr>
</thead>
<tbody>
<tr>
<td>I speak with her regularly</td>
<td>0.603</td>
<td>0.489</td>
</tr>
<tr>
<td>She is a friend.</td>
<td>0.559</td>
<td>0.497</td>
</tr>
<tr>
<td>I know her as well as my parents, brothers, sisters.</td>
<td>0.317</td>
<td>0.465</td>
</tr>
<tr>
<td>In past week, we studied together outside school</td>
<td>0.201</td>
<td>0.401</td>
</tr>
<tr>
<td>In past week, I visited her home.</td>
<td>0.206</td>
<td>0.405</td>
</tr>
<tr>
<td>In past week, she visited my home.</td>
<td>0.187</td>
<td>0.390</td>
</tr>
<tr>
<td>She often tries/wants to do same things as me.</td>
<td>0.198</td>
<td>0.399</td>
</tr>
<tr>
<td>I like to talk to her when I have a problem.</td>
<td>0.232</td>
<td>0.422</td>
</tr>
<tr>
<td>She likes to talk to me when I have a problem.</td>
<td>0.220</td>
<td>0.414</td>
</tr>
<tr>
<td>I can trust her to keep my secrets.</td>
<td>0.220</td>
<td>0.414</td>
</tr>
<tr>
<td>I listen to her when she says something.</td>
<td>0.367</td>
<td>0.482</td>
</tr>
<tr>
<td>She listens to me when I say something.</td>
<td>0.359</td>
<td>0.480</td>
</tr>
<tr>
<td>I wish I could be like her.</td>
<td>0.162</td>
<td>0.368</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: Static Network Variables</th>
<th>Mean</th>
<th>S.D.</th>
</tr>
</thead>
<tbody>
<tr>
<td>She is a relative.</td>
<td>0.210</td>
<td>0.407</td>
</tr>
<tr>
<td>We are in the same caste.</td>
<td>0.271</td>
<td>0.444</td>
</tr>
<tr>
<td>I can walk to her house in less than 5 minutes.</td>
<td>0.220</td>
<td>0.414</td>
</tr>
</tbody>
</table>

Notes: Sample is all pairs of students. N = 78,238 in 30 schools. Missing data imputed via iterative EM algorithm with logit specification (see Appendix D).
along the diagonal with others in the same row suggests individuals are much more likely to claim as friends others of their own population grouping. The final column provides the p-value of a test of the equality between the probability of an individual in that row indicating a same-type other individual is a friend with the probability of her indicating an individual in a different category is a friend. Note that this test suggests strong homophily among members of Scheduled Castes and General Castes, but less evidence for Scheduled Tribes and Other Backwards Castes under the binary link definition.

Table 4: Homophily by Population Group

<table>
<thead>
<tr>
<th></th>
<th>SC</th>
<th>ST</th>
<th>OBC</th>
<th>General</th>
<th>P-value of Test for Homophily</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Binary Network Definition</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SC</td>
<td>0.622</td>
<td>0.533</td>
<td>0.532</td>
<td>0.496</td>
<td>0.004</td>
</tr>
<tr>
<td></td>
<td>(0.027)</td>
<td>(0.037)</td>
<td>(0.030)</td>
<td>(0.037)</td>
<td></td>
</tr>
<tr>
<td>ST</td>
<td>0.526</td>
<td>0.544</td>
<td>0.483</td>
<td>0.487</td>
<td>0.393</td>
</tr>
<tr>
<td></td>
<td>(0.041)</td>
<td>(0.035)</td>
<td>(0.060)</td>
<td>(0.058)</td>
<td></td>
</tr>
<tr>
<td>OBC</td>
<td>0.538</td>
<td>0.514</td>
<td>0.534</td>
<td>0.540</td>
<td>0.995</td>
</tr>
<tr>
<td></td>
<td>(0.040)</td>
<td>(0.048)</td>
<td>(0.043)</td>
<td>(0.041)</td>
<td></td>
</tr>
<tr>
<td>Gen</td>
<td>0.487</td>
<td>0.449</td>
<td>0.511</td>
<td>0.607</td>
<td>0.002</td>
</tr>
<tr>
<td></td>
<td>(0.016)</td>
<td>(0.026)</td>
<td>(0.023)</td>
<td>(0.029)</td>
<td></td>
</tr>
<tr>
<td><strong>Panel B: Continuous Link Intensity Definition</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SC</td>
<td>1.225</td>
<td>0.923</td>
<td>0.888</td>
<td>0.835</td>
<td>0.006</td>
</tr>
<tr>
<td></td>
<td>(0.107)</td>
<td>(0.110)</td>
<td>(0.084)</td>
<td>(0.072)</td>
<td></td>
</tr>
<tr>
<td>ST</td>
<td>0.959</td>
<td>1.090</td>
<td>0.888</td>
<td>0.917</td>
<td>0.123</td>
</tr>
<tr>
<td></td>
<td>(0.071)</td>
<td>(0.087)</td>
<td>(0.115)</td>
<td>(0.121)</td>
<td></td>
</tr>
<tr>
<td>OBC</td>
<td>0.927</td>
<td>0.880</td>
<td>0.921</td>
<td>0.876</td>
<td>0.344</td>
</tr>
<tr>
<td></td>
<td>(0.085)</td>
<td>(0.092)</td>
<td>(0.080)</td>
<td>(0.061)</td>
<td></td>
</tr>
<tr>
<td>Gen</td>
<td>0.810</td>
<td>0.784</td>
<td>0.833</td>
<td>1.131</td>
<td>0.003</td>
</tr>
<tr>
<td></td>
<td>(0.044)</td>
<td>(0.050)</td>
<td>(0.039)</td>
<td>(0.098)</td>
<td></td>
</tr>
</tbody>
</table>

Notes: N = 78,238 in 30 schools. Values indicate mean value of link for individual in group on the y-axis with respect to individual in group on the x-axis. Robust standard errors in parentheses, clustered by school. Final column presents p-value of test that mean value of link is equal for same type and other types. That is, it tests the equality of the mean on the diagonal to the pooled mean of off-diagonal elements within the same row. SC = Scheduled Caste, ST = Scheduled Tribe, OBC = Other Backwards Caste.

Panel B performs the same exercise as Panel A, except with means of the continuous measure of connectedness. Therefore, in Panel B the means in the table represent the mean connectedness value that an individual in the group on the y-axis assigns to an individual on the x-axis. From this, we see that the same general patterns of homophily hold: Scheduled Castes and General castes
show substantial homophily, with less evidence for Scheduled Tribes and Other Backward Castes. This provides suggestive evidence that the continuous measure of connectedness reflects similar network patterns as the binary “She is a friend” measure that has received the bulk of attention in the literature.

2.5 Evidence for Stylized Facts about Networks

Table 5 presents additional facts about networks. These regression results are presented for descriptive purposes, making no claims as to causation. They are presented here to motivate features of the network formation model in the following sections.

First, Panel A shows that the sum of each individual’s links is increasing in the size of her school, as defined by the number of girls in our sample within a given school. For Column (1), this is simply the number of friends she claims to have. For Column (2), the dependent variable is the sum of her scalar link measures with reference to all other girls in her school. This shows that the number (or sum) of links is increasing with school size.

Second, Panel B shows that average link value is decreasing in school size. This is indicated by the negative and highly significant coefficient on the “School Size” variable. This suggests that there may be tradeoffs in linking strategies, and that each individual’s linking decisions with others within her school are not independent of each other. This casts doubt on the simple dyadic model of link formation as employed in, for example, Goldsmith-Pinkham and Imbens (2013) and Comola and Prina (2014). In a dyadic model, the number of other individuals in your potential network is irrelevant, and thus the coefficient would be zero.

Third, Panel C shows that linking decisions are complementary but not symmetric. That is, individual $i$’s decision to link to $j$ depends upon $j$’s decision to link to $i$, as shown by the highly significant positive coefficients in Panel C. While linking strategies are clearly complementary, neither of the two measures (binary and continuous) is symmetric, as would be indicated by a coefficient of unity.$^5$ That is, while in- and out-links are correlated, they are not symmetric.

$^5$Even if links are indeed symmetric, measurement error in the network measure would tend to attenuate the estimated coefficient away from 1.
Table 5: Network Size and Complementarity

<table>
<thead>
<tr>
<th></th>
<th>Network Definition</th>
<th>Binary (1)</th>
<th>Continuous (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Relationship between School Size and Link Count</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>School Size</td>
<td>0.575***</td>
<td>0.391***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.029)</td>
<td>(0.026)</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>20.852***</td>
<td>8.284***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.963)</td>
<td>(1.418)</td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>1,319</td>
<td>1,319</td>
<td></td>
</tr>
<tr>
<td>R-squared</td>
<td>0.490</td>
<td>0.638</td>
<td></td>
</tr>
<tr>
<td><strong>Panel B: Relationship between School Size and Link Value</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>School Size</td>
<td>-0.002***</td>
<td>-0.004***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.001)</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>0.678***</td>
<td>1.283***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.021)</td>
<td>(0.084)</td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>78,238</td>
<td>78,238</td>
<td></td>
</tr>
<tr>
<td>R-squared</td>
<td>0.023</td>
<td>0.035</td>
<td></td>
</tr>
<tr>
<td><strong>Panel C: Relationship between In- and Out-Link Values</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>In-Link Value</td>
<td>0.194***</td>
<td>0.216***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.012)</td>
<td>(0.039)</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>0.431***</td>
<td>0.734***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.028)</td>
<td>(0.042)</td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>78,238</td>
<td>78,238</td>
<td></td>
</tr>
<tr>
<td>R-squared</td>
<td>0.038</td>
<td>0.047</td>
<td></td>
</tr>
</tbody>
</table>

Notes: Robust standard errors in parentheses, clustered by school. *** p<0.01, ** p<0.05, * p<0.1. Dependent variable for Panel A is sum (or count) of links under appropriate definition. Dependent variable for Panels B and C is value of out-link under appropriate definition. Unit of observation is individual student in Panel A, dyad (pair of students) in Panels B and C.
2.6 Reduced-From Treatment Effects

Before proceeding to structural modeling, here I present reduced-form treatment effects. Because of non-random assignment within T1 schools, I restrict this exercise to T2 and Control schools. Accordingly, we can interpret differences between those chosen for participation in T2 and those not chosen as causal, which is not the case in T1 schools. The analysis shows that the program has negative effects on endline outcomes, especially for participants, and the program also affects networks within T2 schools.

2.6.1 On Outcomes

First, I estimate reduced-form treatment effects with specifications as in Equation (1). The omitted category in these regressions is all students in Control schools.

\[ y_{is} = \beta_0 + \beta_1 T_{2s} \times Participant_{is} + \beta_2 T_{2s} \times NonParticipant_{is} + \epsilon_{is} \]  

(1)

Table 6 shows reduced-form treatment effects. While noting possible lack of power, first note that the point estimates of the program’s effects are negative in all specifications. Further, the point-estimated effects of approximately -.2 standard deviations are substantively meaningful, at least among participants. Additionally, the effect on both outcomes is more negative for participants than non-participants.

Next, I investigate treatment effect heterogeneity. Heterogeneity may occur along many dimensions, such as those defined by the variables in Table 1. In order to reduce dimensionality, I use the Control schools to predict endline outcomes conditional on baseline outcomes and individual-level covariates. The regression results for this prediction are given in Table 7. In a sense, this uses the Control group as a counterfactual to predict what would have occurred in treatment schools in the absence of treatment, conditional on variables observed at baseline. We can see from these results that being a member of a lower caste predicts lower Educational Aspirations and Gender Roles outcomes. Using the predicted outcomes from this regression, I then group students into predicted outcome terciles. Low, Middle, and High predicted terciles are denoted by \( \hat{L} \), \( \hat{M} \), and \( \hat{H} \), respectively.\(^6\)

\(^6\)While the table presents the coefficient estimates used to predict \( \hat{L} \), \( \hat{M} \), and \( \hat{H} \) in T1 and T2, I use a leave-one-out
Table 6: Reduced-From Treatment Effects

<table>
<thead>
<tr>
<th></th>
<th>Education</th>
<th>Gender Roles</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>Participant in T2</td>
<td>-0.182</td>
<td>-0.220</td>
</tr>
<tr>
<td></td>
<td>(0.205)</td>
<td>(0.166)</td>
</tr>
<tr>
<td>Non-Participant in T2</td>
<td>-0.044</td>
<td>0.006</td>
</tr>
<tr>
<td></td>
<td>(0.134)</td>
<td>(0.103)</td>
</tr>
<tr>
<td>Baseline Outcome</td>
<td>0.326***</td>
<td>0.092*</td>
</tr>
<tr>
<td></td>
<td>(0.045)</td>
<td>(0.051)</td>
</tr>
<tr>
<td>Constant</td>
<td>-0.016</td>
<td>0.049</td>
</tr>
<tr>
<td></td>
<td>(0.103)</td>
<td>(0.082)</td>
</tr>
<tr>
<td>Observations</td>
<td>920</td>
<td>920</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.004</td>
<td>0.120</td>
</tr>
</tbody>
</table>

Notes: Robust standard errors in parentheses, clustered by school. *** p<0.01, ** p<0.05, * p<0.1. Regressions restricted to T2 and C. Omitted category is all girls in Control.

Table 7: Defining Predicted Outcome Terciles

<table>
<thead>
<tr>
<th></th>
<th>Educational Aspirations</th>
<th>Gender Roles</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>Elected</td>
<td>0.032</td>
<td>-0.010</td>
</tr>
<tr>
<td></td>
<td>(0.116)</td>
<td>(0.060)</td>
</tr>
<tr>
<td>Standard 7</td>
<td>0.258</td>
<td>0.095</td>
</tr>
<tr>
<td></td>
<td>(0.211)</td>
<td>(0.200)</td>
</tr>
<tr>
<td>Standard 8</td>
<td>0.163</td>
<td>0.175</td>
</tr>
<tr>
<td></td>
<td>(0.204)</td>
<td>(0.167)</td>
</tr>
<tr>
<td>Scheduled Caste</td>
<td>-0.510***</td>
<td>-0.170</td>
</tr>
<tr>
<td></td>
<td>(0.150)</td>
<td>(0.162)</td>
</tr>
<tr>
<td>Scheduled Tribe</td>
<td>-0.430*</td>
<td>-0.816</td>
</tr>
<tr>
<td></td>
<td>(0.192)</td>
<td>(0.494)</td>
</tr>
<tr>
<td>Other Backwards Caste</td>
<td>-0.342</td>
<td>-0.273***</td>
</tr>
<tr>
<td></td>
<td>(0.201)</td>
<td>(0.059)</td>
</tr>
<tr>
<td>Baseline Outcome</td>
<td>0.286***</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>(0.054)</td>
<td>(0.058)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.226</td>
<td>0.224</td>
</tr>
<tr>
<td></td>
<td>(0.161)</td>
<td>(0.173)</td>
</tr>
<tr>
<td>Observations</td>
<td>393</td>
<td>395</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.162</td>
<td>0.051</td>
</tr>
</tbody>
</table>

Robust standard errors in parentheses, clustered by school. *** p<0.01, ** p<0.05, * p<0.1. Estimation restricted to students in Control schools.
Next, I estimate heterogeneous treatment effects with regressions of the form in Equation (2).

\[ y_{is} = \sum_{k=1}^{3} I_k (\beta_{0k} + \beta_{1k} T2_s \times Participant_{is} + \beta_{2k} T2_s \times NonParticipant_{is}) + \epsilon_{is} \]  

(2)

In this specification, \( I_k \) is an indicator for being in each predicted tercile.\(^7\) Results for this specification are in Table 8. Note that there are strongly negative effects for Gender Roles among Participants in T2, but only for those in the middle predicted tercile. This presents suggestive evidence of heterogeneous treatment effects for Gender Roles in T2 schools, with heterogeneity defined by predicted outcome tercile. Further, while noting lack of power to detect small differences, I note that there are no significant effects on non-participants for any predicted outcome tercile.

2.6.2 On Networks

While I find suggestive evidence for negative treatment effects on outcomes, there is much stronger evidence for treatment effects on networks. Since we have random within-school variation in T2 schools, I present reduced-form treatment effect estimates broken down by whether each node involved in the link is chosen for participation. To do this, I estimate Equation (3).

\[ L_{ijs} = \gamma_0 + \gamma_1 Participant_{is} + \gamma_2 Participant_{js} + \gamma_3 Participant_{is} \times Participant_{js} + u_{ijs} \]  

(3)

These results are presented in Table 9, where Columns (2) and (4) additionally control for baseline link values.

Results for the binary link definition show significant effects for the interaction term \( \gamma_3 \). That is, if both students are chosen to participate, the probability of a link is much larger than if only one is chosen. Similar patterns hold for the continuous definition in Columns (3) and (4), where we see much larger average link value if both are chosen. For the continuous link definition, we see evidence for substitution of links: if only one is chosen, the average link value decreases.

These results contain powerful implications for evaluation of counterfactual assignment policies. Participation in the program has a substantial effect on the identity of others with whom individuals

---

\(^7\)That is, \( I_1 = \hat{L}, I_2 = \hat{M}, I_3 = \hat{H} \).
<table>
<thead>
<tr>
<th></th>
<th>Education (1)</th>
<th>Gender Roles (2)</th>
<th>Gender Roles (3)</th>
<th>Gender Roles (4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{L} )</td>
<td>(-0.363^{**})</td>
<td>0.080</td>
<td>(-0.009)</td>
<td>(-0.021)</td>
</tr>
<tr>
<td></td>
<td>(0.163)</td>
<td>(0.193)</td>
<td>(0.116)</td>
<td>(0.123)</td>
</tr>
<tr>
<td>( \hat{M} )</td>
<td>(-0.088)</td>
<td>(-0.077)</td>
<td>(0.127^{**})</td>
<td>0.117**</td>
</tr>
<tr>
<td></td>
<td>(0.088)</td>
<td>(0.083)</td>
<td>(0.045)</td>
<td>(0.048)</td>
</tr>
<tr>
<td>( \hat{H} )</td>
<td>0.373***</td>
<td>0.329***</td>
<td>0.094</td>
<td>0.066</td>
</tr>
<tr>
<td></td>
<td>(0.069)</td>
<td>(0.098)</td>
<td>(0.124)</td>
<td>(0.125)</td>
</tr>
<tr>
<td>Participant in T2 * ( \hat{L} )</td>
<td>(-0.288)</td>
<td>(-0.366)</td>
<td>(-0.059)</td>
<td>(-0.044)</td>
</tr>
<tr>
<td></td>
<td>(0.274)</td>
<td>(0.267)</td>
<td>(0.246)</td>
<td>(0.249)</td>
</tr>
<tr>
<td>Participant in T2 * ( \hat{M} )</td>
<td>(-0.065)</td>
<td>(-0.091)</td>
<td>(-0.532^{***})</td>
<td>(-0.534^{***})</td>
</tr>
<tr>
<td></td>
<td>(0.160)</td>
<td>(0.150)</td>
<td>(0.177)</td>
<td>(0.167)</td>
</tr>
<tr>
<td>Participant in T2 * ( \hat{H} )</td>
<td>(-0.060)</td>
<td>(-0.073)</td>
<td>(-0.126)</td>
<td>(-0.083)</td>
</tr>
<tr>
<td></td>
<td>(0.219)</td>
<td>(0.220)</td>
<td>(0.354)</td>
<td>(0.332)</td>
</tr>
<tr>
<td>Non-Participant in T2 * ( \hat{L} )</td>
<td>(-0.116)</td>
<td>(-0.176)</td>
<td>0.104</td>
<td>0.119</td>
</tr>
<tr>
<td></td>
<td>(0.192)</td>
<td>(0.184)</td>
<td>(0.174)</td>
<td>(0.177)</td>
</tr>
<tr>
<td>Non-Participant in T2 * ( \hat{M} )</td>
<td>0.024</td>
<td>0.008</td>
<td>0.035</td>
<td>0.029</td>
</tr>
<tr>
<td></td>
<td>(0.113)</td>
<td>(0.110)</td>
<td>(0.196)</td>
<td>(0.176)</td>
</tr>
<tr>
<td>Non-Participant in T2 * ( \hat{H} )</td>
<td>0.052</td>
<td>0.057</td>
<td>(-0.160)</td>
<td>(-0.143)</td>
</tr>
<tr>
<td></td>
<td>(0.155)</td>
<td>(0.153)</td>
<td>(0.218)</td>
<td>(0.208)</td>
</tr>
<tr>
<td>Baseline outcome * ( \hat{L} )</td>
<td>0.338***</td>
<td>0.059</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.097)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Baseline outcome * ( \hat{M} )</td>
<td>0.155**</td>
<td>0.100</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.072)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Baseline outcome * ( \hat{H} )</td>
<td>0.059</td>
<td>0.122**</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.098)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>920</td>
<td>920</td>
<td>920</td>
<td>920</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.118</td>
<td>0.145</td>
<td>0.020</td>
<td>0.032</td>
</tr>
<tr>
<td>Test1 P-value</td>
<td>0.704</td>
<td>0.578</td>
<td>0.269</td>
<td>0.246</td>
</tr>
<tr>
<td>Test2 P-value</td>
<td>0.678</td>
<td>0.490</td>
<td>0.414</td>
<td>0.442</td>
</tr>
</tbody>
</table>

Notes: Robust standard errors in parentheses, clustered by school. Regressions restricted to T2 and C. Omitted category is C, \( \hat{L} \), \( \hat{M} \), and \( \hat{H} \) predicted from baseline variables (see Table 7). Test1 is a test of equality of the interactions of \( \hat{L} \), \( \hat{M} \), and \( \hat{H} \) with Participant in T2. Test2 is a test of equality of the interactions of \( \hat{L} \), \( \hat{M} \), and \( \hat{H} \) with Non-Participant in T2.
interact. If we believe that effects of programs diffuse through networks, then failing to account for the effect of the program on the structure of the network itself may lead to erroneous predictions. Accordingly, any attempt to predict outcomes under counterfactual assignments needs to account for the effect of the program on networks.

### Table 9: Reduced-Form Treatment Effects on Networks

<table>
<thead>
<tr>
<th>Network Definition</th>
<th>Binary</th>
<th>Continuous</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>Participant (Self)</td>
<td>-0.007</td>
<td>-0.010</td>
</tr>
<tr>
<td></td>
<td>(0.018)</td>
<td>(0.014)</td>
</tr>
<tr>
<td>Participant (Alter)</td>
<td>0.002</td>
<td>-0.007</td>
</tr>
<tr>
<td></td>
<td>(0.030)</td>
<td>(0.026)</td>
</tr>
<tr>
<td>Participant (Both)</td>
<td>0.068*</td>
<td>0.051*</td>
</tr>
<tr>
<td></td>
<td>(0.033)</td>
<td>(0.024)</td>
</tr>
<tr>
<td>Baseline Measure (Self)</td>
<td>0.153***</td>
<td>0.243***</td>
</tr>
<tr>
<td></td>
<td>(0.014)</td>
<td>(0.015)</td>
</tr>
<tr>
<td>Baseline Measure (Alter)</td>
<td>0.218***</td>
<td>0.082***</td>
</tr>
<tr>
<td></td>
<td>(0.005)</td>
<td>(0.019)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.535***</td>
<td>0.313***</td>
</tr>
<tr>
<td></td>
<td>(0.016)</td>
<td>(0.011)</td>
</tr>
<tr>
<td>Observations</td>
<td>19,430</td>
<td>19,430</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.001</td>
<td>0.080</td>
</tr>
<tr>
<td>P-value of Test</td>
<td>0.186</td>
<td>0.372</td>
</tr>
</tbody>
</table>

Notes: Sample restricted to girls in T2 schools. Robust standard errors in parentheses, clustered by school. *** p<0.01, ** p<0.05, * p<0.1. Dependent variable is existence/intensity of link between i (self) and j (alter), as indicated by i at endline. Test is a test of significance of sum of coefficients for Participant (self), Participant (Alter), and Participant (Both). Missing network data imputed via algorithm described in Appendix D.

#### 2.7 Reduced-Form Stylized Facts

The descriptive analysis as well as reduced-form treatment effects provide a number of stylized facts that a predictive model must rationalize. At baseline, we see that outcomes vary substantially with observed characteristics, especially caste groupings. We further see the following five stylized facts about networks.

1. Networks are not independent of observed characteristics but rather exhibit substantial homophily.

2. Links are not symmetric, but are complementary. That is, the existence/intensity of individ-
ual $i$’s link to individual $j$ is positively correlated with the existence/intensity of individual $j$’s link to $i$.

3. The network exhibits substantial degree heterogeneity. That is, there is substantial variation in the number (binary definition) or sum (continuous) of links.

4. Average number of links (for binary) and sum of link values (for continuous) is growing in school size.

5. Average link value is decreasing in school size, suggesting that links are not independent. Rather, this suggests that there are tradeoffs to an individual in making link decisions.

Further, from the endline regressions presented above, we see suggestive evidence that participation in the Bal Sabha program affects endline outcomes, and that these effects may vary by background characteristics. Finally, we see substantial reduced-form effects on the existence and intensity of network links, even conditional upon baseline networks. The structural model that follows provides features that account for all of these reduced-form patterns.

3 Peer Effects Model

Here, I describe the model of peer effects. This provides the key identification result for the peer effects model, which posits network endogeneity as an omitted-variable problem (see, e.g., Goldsmith-Pinkham and Imbens 2013), while also allowing for non-linear peer effects (Carrell, Sacerdote, and West 2013). Conditional on these unobserved variables, the parameters of the peer effects model are identified.

3.1 The Problem of Endogenous Networks

The peer effects model begins with the standard linear-in-means model (see, e.g., Manski 1993).\footnote{Blume et al. (2015) provide micro-foundations for this model as well as a generalization of various identification results derived since Manski (1993).} As discussed in Carrell, Sacerdote, and West (2013), with some assumptions the canonical model
can be rewritten as Equation (4).\(^9\)

$$y_{is1} = \alpha_0 + \alpha_1 P_{is} + \alpha_2 \bar{P}_{is1} + u_{is1}$$ \hspace{1cm} (4)

where \(y_{is1}\) is some outcome at endline (time 1), \(P_{is}\) is an indicator for individual \(i\) in school \(s\) being chosen to participate in the program, and \(\bar{P}_{is1}\) is individual \(i\)’s peer group mean participation. The variable \(u_{is1}\) is unobserved. Therefore, from Equation (4), we see that expected outcome \(y_{is1}\) is a linear function of a student’s participation status and the participation status of her peers, along with an additional additive unobserved component.

Equation (4) requires a definition of peer group mean variable \(\bar{P}_{is1}\). This in turn requires the choice of how to weight peers. That is,

$$\bar{P}_{is} = \frac{\sum_{j \neq i} w_{ij}s}{\sum_{k \neq i} w_{iks}} P_{js}$$ \hspace{1cm} (5)

where \(w_{ij}s\) is some weight for the intensity of the link between individuals \(i\) and \(j\). For example, when links are binary, then \(w_{ij}s \in \{0, 1\}\) and thus \(\bar{P}_{is1}\) is merely the fraction of an individual’s peers who are also chosen to participate. If link values are continuous, then \(\bar{P}_{is1}\) may weight “closer” or “stronger” links more. For purposes here, with directional links \(g_{ij}s\) and \(g_{jis}\) defining the links between individuals \(i\) and \(j\), I take as given the weighting function \(w_{ij}s = g_{ij}s + g_{jis}\). See Appendix C for a fuller discussion of the issue of weighting.

I augment this model by decomposing the error term \(u_{is1}\) in a manner similar to Goldsmith-Pinkham and Imbens (2013), who effectively include the unobserved variable \(a_{is}\). In contrast to that paper, however, I follow the suggestion by Bramoullé (2013) to include a peer effect in the unobserved variable, which amounts to including \(\bar{a}_{is1}\) as an additional regressor in the structural peer effects model. Let \(u_{is1} = \alpha_3 a_{is} + \alpha_4 \bar{a}_{is1} + v_{is1}\). Accordingly, Equation (4) becomes Equation (6).

$$y_{is1} = \alpha_0 + \alpha_1 P_{is} + \alpha_2 \bar{P}_{is1} + \alpha_3 a_{is} + \alpha_4 \bar{a}_{is1} + v_{is1}$$ \hspace{1cm} (6)

\(^9\)Note that the bulk of the literature focuses on estimating an endogenous peer effect, which is not the focus of the model here.
With this formulation, network endogeneity biases peer effects estimates whenever $\bar{P}_{i,s1}$ is correlated with either $a_{is}$ or $\bar{a}_{is1}$. That is, if one estimates Equation (4) without controlling for $a_{is}$, estimates of $\alpha_2$ will be biased due to correlation between $(P_{i,s1}, \bar{P}_{i,s1})$ and $u_{i,s1}$ (which includes $(a_{is}, \bar{a}_{is1})$).

As an example for when $\text{cov}(\bar{P}_{i,s1}, a_{is}) \neq 0$, suppose that $a_{is}$ is unobserved academic ability, and that this unobserved ability is positively associated with outcome $y_{i,s1}$. Endogeneity arises when $a_{is}$ also plays a part in the network formation process, whereby those with higher ability also are more likely to link with participants. Therefore, those with higher $a_{is}$ will tend to have more of their links be with participants, bringing about positive correlation between $\bar{P}_{i,s1}$ and $a_{is}$. Note that this endogeneity may arise even when $P_{is}$ is exogenous, such as the case when participation is assigned randomly. That is, even with random assignment, estimation that does not account for unobserved $a_{is}$ may be biased in the presence of endogenous network formation.

In addition to accounting for network endogeneity, I further allow for the possibility of nonlinear peer effects. As in Carrell, Sacerdote, and West (2013), these non-linear peer effects account for the fact that peer means $\bar{P}_{i,s1}$ and $\bar{a}_{is}$ may affect different types of individuals differently. This is accounted for by the variables $I_{isk}(z_{is}), k = 1, ..., K$, which define a set of $K$ indicators for being in different categories of the population. The partition could be defined by grade level, gender, race, baseline outcome, or any other function of exogenous characteristics. Note that the elements of $z_{is}$ are assumed to be exogenous variables, where $E[v_{i,s1}|z_{is}] = 0$. With this additional step, Equation (6) becomes Equation (7).

$$y_{i,s1} = \sum_{k=1}^{K} I_{isk} \left( \alpha_{0k} + \alpha_{1k} P_{is} + \alpha_{2k} \bar{P}_{i,s1} + \alpha_{3k} a_{is} + \alpha_{4k} \bar{a}_{is1} \right) + v_{i,s1}$$

(7)

Non-linear peer effects are identified by the different coefficients $\alpha_{2k}$ and $\alpha_{4k}$ across different values of $k$.$^{11}$

Finally, in order to increase power, I also estimate a version of Equation (7) that includes

---

$^{10}$It is not sufficient that those with higher ability link more (or less) with all students. Endogeneity arises because ability leads to differential valuation of network links based on ability.

$^{11}$This model allows for non-linear direct effects (identified by $\alpha_{1k}$ and $\alpha_{3k}$) as well as non-linear peer effects (identified by $\alpha_{2k}$ and $\alpha_{4k}$).
baseline outcomes. This specification is defined by Equation (8).

\[
y_{is1} = \sum_{k=1}^{K} I_{isk} \left( \alpha_{0k} + \alpha_{1k} P_{is} + \alpha_{2k} \bar{P}_{is1} + \alpha_{3k} a_{is} + \alpha_{4k} \bar{a}_{is1} + \alpha_{5k} y_{is0} + \alpha_{6k} \bar{y}_{is0} \right) + v_{is1}
\]  

(8)

### 3.2 Identification Results for the Peer Effects Model

With the outcome equation formulated as in Equation (7), identification is straightforward. Define the parameter vector \( \alpha = (\alpha_{01}, ..., \alpha_{41}, \alpha_{02}, ..., \alpha_{4k}) \). Conditional on observed \( P \), the vector of unobserved \( A \), and the network \( G \), the parameters of the model \( \alpha \) are identified, a result given in Proposition 1. Note here that \( P_s \) is the vector of \( P \) for school \( s \), and similar for \( A_s \) and \( G_s \).

**Proposition 1.** Suppose that

1. \((P_{is}', \bar{P}_{is1}, a_{is}, \bar{a}_{is1}) \perp \perp (P_{jt}, \bar{P}_{jt1}, a_{jt}, \bar{a}_{jt1}) \forall s \neq t.\)

2. \( \alpha \in \Theta_\alpha \subset \mathbb{R}^{15} \), where \( \Theta_\alpha \) is compact.

3. \((P_{is}', \bar{P}_{is1}, a_{is}, \bar{a}_{is1}) \in X \subset \mathbb{R}^4 \), where \( X \) is compact.

4. \( E[v_{is}|P_s, G_s, A_s] = 0 \forall j \)

Then the parameter vector \( \alpha \) of Equation (7) is identified as \( s \to \infty \).

**Proof.** This is a standard OLS result other than the fact that it relies upon consistent estimates of \( a_{is} \). \( \square \)

This model embeds the standard linear-in-means model as well as the more general model used by Carrell, Sacerdote, and West (2013). The authors of that paper essentially assumed that \( \alpha_{3k} = \alpha_{4k} = 0 \) for all \( k \). The standard linear-in-means model typically further assumes that \( K = 1 \) and thus \( I_{isk} = 1 \forall i, s \), implying are no non-linear effects. Accordingly, the identification result in Equation (1) states weaker conditions than those previously used in the literature on peer effects.

Finally, it bears repeating that identification and thus consistent estimation in the presence of network endogeneity depends crucially on an initial estimate of unobserved \( a_{is} \). This estimate is obtained from estimation of the network-formation process, which is described in the next section.
Conditional on $a_{is}$ and given the assumptions of Proposition 1, we can recover the true parameters of the peer effects model.\footnote{Average school size in the data is 40 girls, and thus we have 40 data points with which to estimate $a_{is}$ for each $i$. If, despite this, the prior estimation returns noisy estimates of $a_{is}$, this should induce attenuation in estimates of $\alpha$.}

### 3.3 Relation to Other Models

The peer effects model here combines two approaches that have received substantial attention in the statistics and econometrics literature. First, I specify arbitrary latent characteristics $a$ that must be accounted for. Second, conditional on these latent characteristics, the model posits a parametric control function approach. These twin approaches allow for identification of the parameters of the peer effects model in the presence of certain types of network endogeneity.

First, I posit $a$ as an unobserved, “latent” characteristic, making this model similar to the “latent space” models described in Jackson (2014). The primary example of such an approach to peer effects identification is Goldsmith-Pinkham and Imbens (2013). Such models posit that unobserved “latent” characteristics play a part in the process being modeled. As Jackson describes, a key feature of such models is that the latent characteristic may be any unobserved—and possibly difficult-to-measure—characteristic, such as “ability” or “ambition.” \footnote{“Latent space” models have also been heavily used in industrial organization, for example in Berry, Levinsohn, and Pakes (1995), in their pathbreaking methodology for demand estimation.}

Further, conditional on these latent characteristics $a$, identification of the model’s parameters is achieved via a parametric control function approach. Rearrangement of Equation (7) shows this.

\[
y_{is1} = \sum_{k=1}^{K} I_{isk} (\alpha_{0k} + \alpha_{1k} P_{is} + \alpha_{2k} \bar{P}_{is1}) + \sum_{k=1}^{K} I_{isk} (\alpha_{3k} a_{is} + \alpha_{4k} \bar{a}_{is1}) + v_{is1}
\]

\[
y_{is1} = \sum_{k=1}^{K} I_{isk} (\alpha_{0k} + \alpha_{1k} P_{is} + \alpha_{2k} \bar{P}_{is1}) + f(a_{is}, \bar{a}_{is}, z_{is}) + v_{is1}
\] (9)

\[
y_{is1} = \sum_{k=1}^{K} I_{isk} (\alpha_{0k} + \alpha_{1k} P_{is} + \alpha_{2k} \bar{P}_{is1}) + u_{is1}
\] (10)

The control function is $f(a_{is}, \bar{a}_{is}, Z_{is})$. Endogeneity arises because $P_{is}$ and $\bar{P}_{is1}$ may depend on $a_{is}$ and $\bar{a}_{is1}$. This implies correlation between these regressors and $u_{is1}$, leading to biased estimates of $\alpha_{1k}$ and $\alpha_{2k}$ in Equation (10). Estimating the control function $f$ allows for identification in the presence of this endogeneity.
Identification in the presence of endogeneity via control functions has found wide application in applied econometrics, and the model here follows in this tradition. As pointed out by Bramoullé (2013), the model in Goldsmith-Pinkham and Imbens (2013) is similar in flavor to the canonical Heckman selection model, which itself uses a parametric control function approach to identification. Employment of control functions to account for unobserved heterogeneity has also found widespread application in industrial organization, particularly in the estimation of production functions (Ackerberg, Caves, and Frazer 2015; Levinson and Petrin 2003; Olley and Pakes 1996).

4 A Structural Model of Network Formation

The prior section showed that, conditional on the observed network and unobserved variables A, the parameters of the peer-effects model are identified. This section demonstrates that the unobserved vector A is identified through observation of the network-formation process. Therefore, after A is recovered through the network-formation estimation procedure, these variables can be plugged in to provide for consistent estimation of the peer-effects parameters.

4.1 Simple Model

To fix ideas and intuition, I develop a simple version of the network-formation model. This simple model sets aside the unobserved variables $a_{is}$ that will be added into the model later. Through this, we develop intuition behind equilibrium results, the instrumentation strategy, and identification.

4.1.1 Players, Strategy Space, and Utility

For a given school s, there are $N_s$ players in the network formation game. $N_s$ is assumed to be determined exogenously. In the context here, $N_s$ is the number of girls in a given school s in grades 6-8.

Each player i in school s chooses whether to be linked to each of the other $N_s - 1$ players. More formally, each player i in school s chooses a vector of actions $g_{is} \in \mathbb{R}_{+}^{N_s-1}$. Importantly, in a deviation from the bulk of the theoretical network-formation literature, link intensity is continuous:
\( g_{ijs} \in [0, \infty) \). Further, individuals make these choices subject to a total effort constraint as spelled out in Assumption 1. Each individual’s objective is to maximize utility subject to this constraint.

**Assumption 1.** For each \( i = 1, \ldots, N \), \( \sum_{j \neq i} c_{ijs} g_{ijs} \leq M_{is} \), where \( c_{ijs} \) is the cost to individual \( i \) of forming a link with \( j \) and \( M_{is} \) is individual \( i \)'s endowment. Further, \( M_{is} \in [\underline{M}, \bar{M}] \subset \mathbb{R}_{++} \) and \( c_{ijs} \in [c, \bar{c}] \subset \mathbb{R}_{++} \).

The budget constraint serves two purposes in the model. First, it imposes a structured way in which individuals trade off the costs and benefits of different linking strategies. If individual \( i \) chooses to increase \( g_{ijs} \) (her link to \( j \)), then she must decrease some \( g_{iks} \) (her link to \( k \)). Second, \( M_{is} \) may vary across students and may depend on observed or unobserved characteristics. Accordingly, \( M_{is} \) allows for out-degree heterogeneity: individuals with higher \( M_{is} \) have a higher effort endowment and thus will tend to have more out-links in equilibrium, conditional on other variables in the model.

Finally, note that the lower bounds on cost implicitly imposes the restriction that network size is bounded above for each individual: even as the size of a person’s school grows infinitely, the number of total links \( \sum_{j \neq i} g_{ijs} \leq \frac{M_{is}}{c} \). Compact support of \( M_{is} \) implies further that network size is bounded above across individuals.

Utility is a function of the realized network \( G_s \) as well as exogenous characteristics of all students in school \( s \), \( X_s \). Following prior models (e.g., Badev 2013; Mele 2010), I assume that the utility of links is additive. Similar to these models, I assume that individuals derive different utilities depending upon how “mutual” their links are. The utility to individual \( i \) of a network \( G_s \) is given as follows:

\[
U_{is}(G_s, X_s) = \sum_{j \neq i} u_{ijs} = \sum_{j \neq i} g_{ijs}^{\alpha} g_{jis}^{\beta} e_{ij}(X_is, X_js)
\]

The utility to individual \( i \) from his link to \( j \) depends upon both his linking strategy \( g_{ijs} \) and on \( j \)'s linking strategy via \( g_{jis} \). Additionally, utility of links depends upon the two individuals’ observable

---

14Exceptions are found in Baumann (2016), Bloch and Dutta (2009), and Rogers (2006). Jackson (2008) briefly mentions models of this type in a section titled “Weighted Network Formation.”

15Budget constraints are rare in models of network formation, but are also employed in, for example, Baumann (2016), Bloch and Dutta (2009), and Boucher (2015).
characteristics \( X_{is} \) and \( X_{js} \).

**Assumption 2.** The following restrictions hold:

1. \( X_{is} \) and \( f() \) are bounded in \( \mathbb{R}^k \) and \( \mathbb{R} \), respectively.

2. \( 0 < \beta < (1 - \alpha) < 1 \)

Assumption 2 imposes additional structure on the utility function. The functional form implies that all links are marginally valuable, except when \( g_{jis} = 0 \). Hence, in the absence of a budget constraint, all individuals would choose to be maximally linked to all others.

This model is limited in two important ways. First, utility from given links depends *only* on the link between those two individuals as well as their characteristics. Importantly, the utility to \( i \) of linking to \( j \) does *not* depend upon \( j \)'s links, other than his link to \( i \). Thus, this model does not allow for utility from linking to popular individuals. Conversely, it does not allow for congestion externalities, whereby a link to a given individual is less valuable when that individual has more links. Through the budget constraint, however, the model *does* allow for tradeoffs between links.

Second, and in contrast to the structure discussed in Bramoullé (2013) and Blume et al. (2015), individuals do not consider final outcomes \( y_{is} \) in making their linking decisions. This assumption is made more plausible in educational contexts by the findings in Carrell, Sacerdote, and West (2013), who show that Air Force members tend to choose peers by homophily. In such a context, at least for parts of the population, a homophilic linking strategy would tend to lead to lower academic outcomes.

### 4.1.2 Equilibrium

Here, I provide results showing equilibrium existence and uniqueness. In contrast to models with discrete action spaces, the continuity of the model allows for the use of Nash equilibrium rather than pairwise stability.

As spelled out above, each individual chooses a vector of links \( g_{is} \) to maximize his utility subject to others’ linking decisions. Proposition 2 provides the paper’s primary existence result. Existence is guaranteed by the concavity of the game, a result that dates back at least to Rosen (1965). However, the Nash Equilibrium is not unique. It may be possible to have an equilibrium with any combination of \( g_{ijs} = g_{jis} = 0 \).
For example, there exists an equilibrium in which each person is connected only to one other person, on whom he exhausts his entire endowment of effort. Further, an empty network is an equilibrium.

Accordingly, to refine the set of equilibria, I define a strictly positive equilibrium as a Nash equilibrium in which each person’s strategy profile exhibits strictly positive links. That is, for a strategy profile to be a strictly positive equilibrium, it must be a Nash equilibrium and $g_{ijs} > 0$ for every $i, j \neq i$.

**Proposition 2.** There exists a Nash equilibrium for the network-formation game. Further, there exists a strictly positive equilibrium.

*Proof.* See Appendix A.

A necessary condition for a strictly positive equilibrium requires that the following first-order conditions hold:

\[
\frac{\partial U_{is}}{\partial g_{ijs}} = \alpha g_{ijs}^{\alpha-1} (\beta g_{ijs} - c_{ijs} e^{f(X_{is}, X_{js})}) - c_{ijs} \lambda_{is} = 0 \quad \forall \; i, j \neq i \tag{12}
\]

\[
\frac{\partial U_{is}}{\partial \lambda_{is}} = M_{is} - \sum_{j \neq i} c_{ijs} g_{ijs} = 0 \quad \forall \; i \tag{13}
\]

Importantly, however, there is only one interior equilibrium, as stated in Proposition 3. Intuitively, uniqueness derives from the concavity of the network-formation game. This uniqueness result is quite important, and is extremely useful for estimation and simulation.

**Proposition 3.** The strictly positive equilibrium of the game is unique.

*Proof.* See Appendix A.

### 4.1.3 Relation to Potential Games

As further justification for focusing attention on the strictly positive equilibrium, here I relate the game as developed to the theory of potential games. Relationships between network formation and potential games has been previously discussed in the literature, most prominently in Badev (2013). I show that a special case of the game developed here is a potential game. I further demonstrate that, in this special case, the potential function allows for refinement of the set of
equilibrium strategy profiles to include only the strictly positive equilibrium. Relation of network formation games

In general, the network formation game is not a potential game. To see this fact, note that

\[
\frac{\partial^2 U_{is}}{\partial g_{ijs} \partial g_{jis}} = \alpha \beta g_{ijs}^{\alpha-1} g_{jis}^{\beta-1} e^{f(X_{is}, X_{js})}
\]

In a simpler setting, Theorem 4.5 in Monderer and Shapley (1996) state that a sufficient and necessary condition for a continuous game to be a potential game is that the partial derivatives in Equations (14) and (15) are equal, which in general does not hold here.\(^{16}\) However, there exists a special case in which Monderer and Shapley’s symmetry condition does hold. In this special case, the network formation game is a potential game with the potential function defined in Proposition 4.

**Proposition 4.** If \(\beta = \alpha\) and \(f(X_{is}, X_{js}) = f(X_{js}, X_{is})\), then the network formation game is an exact potential game with potential function defined as \(P(G_s, X_s) = \frac{1}{2} \sum_{i=1}^{N_s} \sum_{j \neq i} (g_{ijs} g_{jis})^{\alpha \beta} e^{f(X_{is}, X_{js})}\).

**Proof.** See Appendix A.

In this special case of the game, equilibrium results follow directly from known results of potential games. Importantly, Monderer and Shapley (1996) note that the potential can be used for equilibrium refinement, in a way that is useful for the task at hand. They show that the set of potential maximizers is a subset of the set of Nash equilibria.

**Proposition 5.** If \(\beta = \alpha\) and \(f(X_{is}, X_{js}) = f(X_{js}, X_{is})\), then the unique strictly positive equilibrium is the unique maximizer of the potential function \(P(G_s, X_s)\).

**Proof.** See Appendix A.

**Corollary 1.** The unique strictly positive equilibrium is efficient in that it maximizes total utility.

\(^{16}\)I note that they proved this fact when strategy sets are intervals of the real line. In my setting, strategy sets are compact subsets of \(\mathbb{R}^{N_s-1}\). Accordingly, I do not rely on their theorem to prove Proposition 4.
Proof. From the definition of the potential function, \( P(G_s, X_s) = \frac{1}{2} \sum_{i=1}^{N_s} U_{is}(G_s, X_s) \). Therefore, the set of values that maximizes the potential function also maximizes the sum of utilities, and the result follows from Proposition 5.

Proposition 5 provides an important result for the game at hand. While the set of Nash equilibria is quite large, the set of strategy profiles that maximize the potential function is a singleton, containing only the strictly positive equilibrium strategy profile. This, together with the efficiency result in Corollary 1, provides additional support for the focus on the strictly positive equilibrium in the empirical analysis that follows.

4.2 Identification Results for the Simple Model

The prior subsection showed that there exists a unique strictly positive Nash equilibrium. Identification proceeds by assuming that we observe \( S \) networks in this equilibrium state. In this subsection, I provide conditions under which parameters of the network formation game are identified. In particular, Assumption 3 states what is observed. I note that this assumption allows for observations to be arbitrarily dependent within schools.

Assumption 3. For each \( s = 1, ..., S, i = 1, ..., N_s \), we observe a vector of characteristics and links \((X'_{is}, g'_{is}) \in X \times G\), where \( X \subset \mathbb{R}^m \) and \( G \subset \mathbb{R}^{N_s - 1} \) are compact, \( m = \text{dim}(X_{is}) \), and \( N_s \) is the number of agents in school \( s \). Further, \((X'_{is}, g'_{is}) \perp \perp (X'_{jt}, g'_{jt}) \forall s \neq t\).

4.2.1 Identification Arguments for Networks

Before proceeding to identification results, we require a slight detour to discuss identification in the context of networks. Dependence upon network links complicates asymptotics. In the model developed here, dependence has two sources. First, since utility depends upon the mutual-ness of links, individual \( i \)'s link choice to \( j \) depends on \( j \)'s choice to \( i \). Second, the budget constraint imposes dependence among all of an individual’s links. Accordingly, we require identification arguments that account for these cross-sectional dependencies.

Accounting for these dependencies, identification results in network-formation models have taken two different strategies, both of which I employ here. Leung (2015) refers to these two strategies as “many market” and “large market” asymptotics. First, many market asymptotics
depend upon observation of a number of different networks. That is, in our context, identification is achieved as $S$, the number of schools, approaches infinity. Such arguments can be employed to identify parameters that are common across networks.

In contrast, identification of parameters that are only observed within a single market requires observation of arbitrarily large networks. As discussed in Graham (2014), for a network with $N$ agents, the econometrician observes $N - 1$ linking decisions per agent. Importantly in our context, we need to identify individual-specific parameters $a_{is}$ that are only observed within a single school. Identification of these parameters leverages such large market asymptotics, where parameters are identified as the size of the network $s$–which contains individual $i$–grows. Such asymptotics are non-standard, since the dimension of the parameter vector is also increasing.

### 4.2.2 Instrumentation Strategy and Identification

The network formation model as spelled out above has two sources of endogeneity, for which I employ two distinct strategies. First, I difference out endogenous variables that depend only on $i$. Second, to control for endogeneity of individual $j$’s network choice, I employ a budget set instrument, whereby exogenous variation is obtained via variation in the utility of potential links. I then show that, conditional on appropriate exogeneity assumptions and rank conditions, crucial parameters of the network-formation model are identified.

Before proceeding to results, I rearrange the first-order conditions and redefine some variables. First, Equation (12) becomes Equation (16) and then Equation (17).\(^{17}\)

\[
\log g_{ij} = \frac{\log \alpha}{1 - \alpha} + \frac{\beta}{1 - \alpha} \log g_{jis} + \frac{f(X_{is}, X_{js})}{1 - \alpha} - \frac{\log \lambda_{is}}{1 - \alpha} - \frac{\log c_{ij}}{1 - \alpha} \tag{16}
\]

\[
\tilde{g}_{ij} = \tilde{\alpha} + \tilde{\beta} \tilde{g}_{jis} + \tilde{f}(X_{is}, X_{js}) - \tilde{\lambda}_{is} - \tilde{c}_{ij} \tag{17}
\]

Importantly, the parameters $\alpha$ and $\beta$ are subsumed into a composite parameter $\frac{\beta}{1 - \alpha}$, identified hereafter as $\tilde{\beta}$. Additionally, assume the following functional form:

\[
\tilde{f}(X_{is}, X_{js}) = \gamma_1 X_{is} + \delta_1 X_{is} X_{js} + \gamma_3 X_{js} \tag{18}
\]

---

\(^{17}\)To get from Equation (16) to Equation (17), define and substitute $\tilde{g}_{ij} = \log g_{ij}$, $\tilde{\alpha} = \frac{\log \alpha}{1 - \alpha}$, $f(X_{is}, X_{js}) = \frac{f(X_{is}, X_{js})}{1 - \alpha}$, $\tilde{\lambda}_{is} = \frac{\log \lambda_{is}}{1 - \alpha}$, and $\tilde{c}_{ij} = \frac{\log c_{ij}}{1 - \alpha}$. 

30
In the data as described above, all $X_{is}$ are binary variables. Accordingly, homophily is identified by the coefficient $\delta_1$ being positive (and possibly $\gamma_1$ and $\gamma_3$ being negative). Substitution and rearrangement of terms yields the following:

$$\tilde{g}_{ijs} = \tilde{\beta} \tilde{g}_{jis} + (\tilde{\alpha} + \gamma_1 X_{is} - \tilde{\lambda}_{is}) + \delta_1 X_{is} X_{js} + \gamma_3 X_{js} - \tilde{c}_{ijs} \quad (19)$$

The econometric issue is to identify and estimate the parameters of Equation (19).

Identification is complicated due to two sources of endogeneity in Equation (19). First, $\tilde{\lambda}_{is}$, which identifies the (log) shadow value of additional effort endowment, necessarily depends upon $\tilde{c}_{ijs}$, the cost of linking. Second, whenever $\tilde{\beta} > 0$, $\tilde{g}_{jis}$ depends upon $\tilde{g}_{ijs}$, which depends on $\tilde{c}_{ijs}$. I solve these issues by using two different strategies.

The first strategy leverages the “panel” nature of the data by applying a standard differencing-out method. However, instead the standard two dimensions of individuals $i$ and time $t$, here we have two dimensions “out” $i$ and “in” $j$. For all variables in Equation (19), perform a “within $i$” transformation. That is, define $\bar{g}_{iis}^i = \frac{1}{N_s - 1} \sum_{k \neq i} \tilde{g}_{iks}$ and $\dot{g}_{iis}^i = \tilde{g}_{iis} - \bar{g}_{iis}^i$. Other variables are defined similarly, leading to Equation (20).

$$\dot{g}_{ijs}^i = \tilde{\beta} \dot{g}_{jis}^i + \delta_1 X_{is} \dot{X}_{js}^i + \gamma_3 \dot{X}_{js}^i - \dot{c}_{ijs}^i \quad (20)$$

This transformation eliminates all terms that vary only with $i$, including the necessarily endogenous term $\tilde{\lambda}_{is}$.

Second, I employ novel instruments for the necessarily endogenous $\dot{g}_{jis}^i$ terms. The instrument relies upon tradeoffs between different linking strategies, which in turn relies upon the non-dyadic structure of the network formation model. Intuitively, due to the budget constraint, individual $j$’s linking decision to $i$ depends upon his alternative options for links. That is, it depends upon the utility he derives from linking to other individuals $k$, where $k \neq i$, which in turn depends upon $k$’s characteristics. Crucially, the instrument works through the budget constraint and thus the shadow value of effort.

Simple algebra shows how these instruments are relevant. First, take the mirror image of
Equation (19), replacing \( i \) with \( j \) and \( j \) with \( i \), leading to Equation (21).

\[
\tilde{g}_{jis} = \tilde{\beta}\tilde{g}_{ij}s + (\tilde{\alpha} + \gamma_1 X_{js} - \tilde{\lambda}_{js}) + \delta_1 X_{js} X_{is} + \gamma_3 X_{is} - \tilde{c}_{jis}
\]  

(21)

Next, perform the “within \( i \)” transformation, leading to Equation (22).

\[
\dot{g}_{jis} = \tilde{\beta}\dot{g}_{ij}s + \gamma_1 \dot{X}_{js} - \dot{\lambda}_{js} + \delta_1 X_{is} \dot{X}_{js} - \dot{c}_{jis}
\]  

(22)

The terms on the right-hand side of Equation (22) suggest instrument candidates. However, \( \dot{g}_{ij}s \) is the dependent variable in Equation (20) and thus necessarily depends on \( \tilde{c}_{ij}s \). Further, \( \dot{X}_{js} \) and \( X_{is} \dot{X}_{js} \) are on the right-hand side of that same equation and thus not excludable. Accordingly, instruments must come through the term \( \dot{\lambda}_{js} \).

Relevant instruments are revealed by decomposing the term \( \dot{\lambda}_{js} \). This shows that

\[
\dot{\lambda}_{js} = \tilde{\lambda}_{js} - \frac{1}{N_s - 1} \sum_{k \neq i} \lambda_{ks} = \tilde{\lambda}_{js} - \frac{1}{N_s - 1} \sum_k \lambda_{ks} + \frac{1}{N_s - 1}\tilde{\lambda}_{is}
\]  

(23)

The middle term is constant for all \( i \) and \( j \) within the school \( s \). However,

\[
\tilde{\lambda}_{js} = \frac{1}{N_s - 2} \sum_{k \neq i, k \neq j} \left( -\tilde{g}_{jk} + \tilde{g}_{jk} + X_{ks} \gamma_1 + X_{js} X_{ks} \delta_1 - c_{kj} \right)
\]  

(24)

\[
\tilde{\lambda}_{is} = \frac{1}{N_s - 2} \sum_{k \neq i, k \neq j} \left( -\tilde{g}_{ik} + \tilde{g}_{ik} + X_{ks} \gamma_1 + X_{is} X_{ks} \delta_1 - c_{kj} \right)
\]  

(25)

Equations (24) and (25) motivate the use of the following instruments:

1. \( \frac{1}{N_s - 2} \sum_{k \neq i, k \neq j} X_{ks} \)
2. \( \frac{1}{N_s - 2} \sum_{k \neq i, k \neq j} X_{js} X_{ks} \)
3. \( \frac{1}{N_s - 2} \sum_{k \neq i, k \neq j} X_{is} X_{ks} \)

These instruments are the mean characteristics of individuals other than \( i \) and \( j \) within school \( s \), as well as those characteristics interacted with \( i \) and \( j \)’s characteristics.

To provide intuition for these instruments, I employ a brief example. Suppose there are three individuals in a given school: \( i \), \( j \), and \( k \). Students come in two types, Blue and Green, and \( X \) is
an indicator for being type Green. Green type students exhibit strong homophily ($\delta_1 > 0$), and they are also more desirable as links to Blue types ($\gamma_1 > 0$). Suppose $i$ and $j$ are both type Blue. Variation in $k$’s type clearly affects $i$ and $j$’s link decisions to each other: if $k$ is Green, then both $i$ and $j$ will link more to $k$ than if $k$ is Blue. Due to the budget constraint, linking more to $k$ necessitates that they link less to each other.

Similarly, suppose that $i$ and $j$ are both Green. They will tend to link more to $k$ if $k$ is Green than if $k$ is Blue. Linking more to $k$ necessitates linking less to each other. Similar intuition shows that their linking strategy toward each other is similarly affected by $k$’s type if $i$ and $j$ are of different types (one Blue and one Green). Accordingly, variation in characteristics of other students serves as a relevant instrument in determining $j$’s linking strategy toward $i$.

Now that relevance has been established, Assumption 4 provides the primary excludability assumption. This assumes mean independence of unobserved costs from all covariates, both those of the two individuals involved with the specific link and others. Independence of unobserved costs from all covariates is necessary for the instruments discussed above to be valid.

**Assumption 4.** $E[\log c_{ijs}|X_{ks}] = 0 \forall k$.

**Assumption 5.** $(\tilde{\beta}, \delta_1', \gamma_3') \in \Theta$, a compact subset of $\mathbb{R}^{2m+1}$, where $m = \dim(X_{is})$.

**Proposition 6.** Define $b_{ijs} = [\hat{g}_{ijs}', X_{is} \hat{X}_{jss}, \hat{X}_{ijs}]$ and $z_{ijs} = [X_{is} \hat{X}_{jss}, \hat{X}_{ijs}, X_{js} \hat{X}_{iis}, \hat{X}_{iis}]$. Given Assumptions 3, 4, and 5, $(\tilde{\beta}, \delta_1', \gamma_3')$ is identified if $E[z_{ijs}' b_{ijs}]$ is of rank $2m + 1$.

**Proof.** See Appendix A. □

The model’s main identification argument is stated in Proposition 6. I note that, due to the “within $i$” transformation, parameters for terms that vary only with $i$ are not identified. Importantly, $\lambda_{is}$, $\bar{\alpha}$, and $\gamma_1$ are not identified, but this amounts to non-identification of the scale of each individual’s utility. In contrast, parameters that identify the utility tradeoffs that $i$ makes in her linking decisions are identified. The parameters $\tilde{\beta}$, $\delta_1$, and $\gamma_3$ identify these relative tradeoffs.

There are at least two situations in which identification fails the hypotheses of Proposition 6. First, if $\delta_1$ and $\gamma_3$ are both zero, then the constructed instruments are irrelevant, since then the $X$ characteristics are irrelevant to the link-formation process. Second, the instruments may be collinear with the exogenous regressors. Importantly, if $X$ is an indicator for treatment that
is assigned by school in a randomized trial, the instruments will be collinear and thus the rank condition fails. In sum, identification requires that exogenous characteristics vary within schools and that these exogenous characteristics are relevant for network formation.

4.3 Adding in Scalar Unobservables

Recall that the purpose of the network formation model is to recover the parameter vector $A$ to control for network endogeneity in the peer effects model. Having derived results for the simple model, I now add these into the model. These must be estimated in order to control for the endogeneity of $P_{is}$ and $\bar{P}_{is}$ in the peer-effects outcome equation from the prior section.

4.3.1 Equilibrium and Functional Form

Scalar unobservables $a_{is}$ and $a_{js}$ are included in the model as part of the function $f$. That is, these scalar unobservables change the relative utilities of the various linking strategies. I make the following assumption on the functional form of $f$:

$$
\tilde{f}(X_{is}, X_{js}, a_{is}, a_{js}) = \gamma_1 X_{is} + \gamma_2 a_{is} + \delta_1 X_{is} X_{js} + \delta_2 X_{is} a_{js} + \delta_3 a_{is} X_{js} + \delta_4 a_{is} a_{js} + \gamma_3 X_{js} + \gamma_4 a_{js}
$$

In all specifications, the vector of observed variables $X_{is}$ contains a participation indicator $P_{is}$. In order for omitted $a$ to bias estimates of the peer effects model, it must change the relative utilities derived from links conditional on $X$ (and thus $P$). Note the centrality of the interactions between $a$ and $X$ here. For example, if $\delta_3$ is positive, then individuals with higher $a_{is}$ derive more utility from linking with participant individuals (for whom $P_{js} = 1$) than those without such characteristic (for whom $P_{js} = 0$). This leads them to having higher $\bar{P}_{is}$ in the outcome equation, which is clearly correlated positively with $a_{is}$.

With the additional assumption that $a_{is}$ is scalar, the equilibrium results for the simple case extend to the case with scalar unobservables. That is, the results in Propositions 2 through 5 hold. Equilibria exist, the strictly positive equilibrium is unique, and the results for potential games follow as well.

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4.3.2 Identification Results with Scalar Unobservables

The simple model effectively assumes \( a_{is} = 0 \) for every individual. This rules out the primary source of endogenous network formation that leads to bias in the peer effects estimates. Adding these back into the model, Equation (20) becomes Equation (27).

\[
\dot{g}_{ij} = \tilde{\beta} g_{ij} + \delta_1 X_{is} \dot{X}_{is} + \delta_2 a_{is} \dot{a}_{is} + \delta_3 a_{is} \dot{X}_{js} + \delta_4 a_{is} \dot{a}_{js} + \gamma_3 \dot{X}_{is} + \gamma_4 a_{is} \dot{a}_{js} - \dot{c}_{ij}(27)
\]

Again, the mirror image of Equation (27) provides instruments for endogenous \( \dot{g}_{j} \).

\[
\dot{g}_{j} = \tilde{\beta} \dot{g}_{j} + \delta_1 X_{js} \dot{X}_{js} + \delta_2 a_{js} \dot{a}_{js} + \delta_3 a_{js} \dot{X}_{is} + \delta_4 a_{js} \dot{a}_{is} + \gamma_3 \dot{X}_{js} + \gamma_4 a_{js} \dot{a}_{is} - \dot{c}_{j}(28)
\]

Assumption 6 serves to identify some of the parameters of the model.

**Assumption 6.** The following exogeneity conditions hold:

1. \( \mathbb{E}[\log c_{ij}|X_{ks}, a_{ks}] = 0 \quad \forall \, k \)

2. \( \mathbb{E}[a_{is}|X_{ks}] = 0 \quad \forall \, k \)

3. \( \mathbb{E}[a_{is}|a_{js}] = 0 \quad \forall \, j \neq i \)

The first part of the assumption is similar to Assumption 4 and implies that unobserved costs are (mean) independent of individual-level observed and unobserved variables. The second part serves to separate the composite term \( (\gamma_3 X_{js} + \gamma_4 a_{js}) \), while the third part of Assumption 6 rules out correlation among these unobserved variables.

**Assumption 7.** \( (\tilde{\beta}, \delta_1', \delta_2', \delta_3', \delta_4', \gamma_3', \gamma_4') \in \Theta, \) a compact set in \( \mathbb{R}^{4m+3} \), where \( m = \text{dim}(X_{is}) \).

Further, \( a_{js} \in \Omega \quad \forall \, j, s \), where \( \Omega \) is a compact set in \( \mathbb{R} \).

Identification results are analogous to those of the simpler model. Proposition 7 states the first result.

**Proposition 7.** Define \( b_{ij} = [\dot{g}_{ij}, X_{is} \dot{X}_{is}, \dot{X}_{is}] \) and \( z_{ij} = [X_{is} \dot{X}_{js}, \dot{X}_{is}, X_{js} \dot{X}_{is}, \dot{X}_{is}] \). Given Assumptions 3, 6, and 7, the parameters \( \tilde{\beta}, \gamma_1, \) and \( \delta_1 \) are identified if \( \mathbb{E}[z_{ij}\, b_{ij}] \) is of rank \( l \geq 2m + 1 \), where \( m = \text{dim}(X_{is}) \).
Additional assumptions are necessary to identify the remaining network-formation parameters, stated in Assumption 8. Proposition 8 provides conditions under which the parameters are identified, but only to scale. These parameters are only identified to scale due to the fact that we can re-scale them by correspondingly re-scaling the latent variable $a$. For a given normalization, such at $\sigma_a^2 = 1$, these parameters are identified absolutely.

Assumption 8. The following conditional variance restrictions hold:

1. $\mathbb{E}[a_{is}^2 | X_{ks}] = \sigma_a^2 \forall k$

2. $\mathbb{E}[a_{is}^2 | a_{js}] = \sigma_a^2 \forall j \neq i$

Proposition 8. Given Assumptions 6 and 8, the parameters $\gamma_2$, $\delta_2$, $\delta_3$, and $\delta_4$ are identified to scale if the following rank conditions hold:

1. $\text{rank}(\mathbb{E}[z'_{ij}s b_{ij}s]) = 2m + 1$, where $b_{ij}s = [\dot{g}_{jis}, X_{is}\dot{X}_{js}, \dot{X}_{js}]

2. $\text{rank}(\mathbb{E}[z'_{ij}s b_{ij}s^2]) = m + 1$, where $b_{ij}s^2 = [1, X_{js}]

3. $\text{rank}(\mathbb{E}[z'_{ij}s X_{is}]) = k

4. $\text{rank}(\mathbb{E}[z'_{ij}s]) = 1$

Proof. See Appendix A.

Assumption 9. For a given $s$, $\text{cov}(X_{is}, X_{js}) = 0 \forall i \neq j.$

Proposition 9 provides the main identification result for $A$. Recall that the peer effects model relies upon controlling for latent variables $a_{is}$, and thus we need to recover these variables in order to identify its parameters. While results to this point have relied upon "many market" asymptotics, identification of $A$ relies upon "large market" asymptotics. For an individual in a school of size $N_s$, we observe $N_s - 1$ links. Note that, as in Proposition 8, the parameter $A$ is only identified to scale, a scale that can be fixed with a convenient normalization.
Proposition 9. For a given $s$, $\gamma_4 + \delta_2 \mathbb{E}_{i \neq j} [X_{is}] \neq 0 \Rightarrow a_{js}$ is identified to scale for all $j$.

Proof. See Appendix A.

These three propositions provide the primary identification results for the model with scalar unobservables. Observation of many networks provides identification of parameters common to all networks, as given in Propositions 7 and 8. Observation of a large number of linkages within each network provides identification of the vector of individual-specific parameter vector $A$, as given in Proposition 9. Note again that, similar to the simple case in the prior subsection, parameters that only involve variables that vary with $i$ are not identified. As in the simpler case, this non-identification result essentially amounts to the inability to make welfare claims about different network configurations, which is beyond the scope of this paper. Rather, with the identified parameters, we can make claims about expected outcomes under counterfactual policies.

5 Structural Estimation Results

Structural estimation consists of two steps. First, I estimate the parameters of the network formation game. Next, conditional on these parameters—particularly the structural unobservables $A$—I estimate the parameters of the outcome equation which accounts for peer effects.

5.1 Network Formation Estimation

The section estimates the network formation model using the identification results in the prior section. First, I discuss how I handle missing network data. Then I estimate the parameters of the network formation model. As expected, we see that the process exhibits substantial homophily, and we further find that the latent variables $a$ play an important part in network formation.

5.1.1 Missing Network Data

As described in Section 2 above, network data was collected via school visits after the conclusion of the Bal Sabha program. Accordingly, we have missing network data for two reasons. First, some students were not present in school on the date of the survey. Second, students may not have properly answered the survey questions.
Missing network data has the potential to confound estimation for a number of reasons. If data is missing non-randomly, listwise deletion leads to biased estimates of even network-level descriptive statistics (see, e.g., Chandrasekhar and Lewis 2011). In the specific model outlined here, missing network data means that we do not observe an individual’s entire vector of network choices. If certain types of students, defined by observed or unobserved characteristics, are more likely to be absent on the day of the network survey, then we need a way of accounting for these students.

Accordingly, a method of reconstructing the missing network data is needed. Chandrasekhar and Jackson (2014) provide a method that constructs networks based upon the probability of observing given dyad and triad relationships in the data. Williams (2016) recently extended this method to allow for missingness to vary by observed characteristics. He shows that the method does a reasonable job in reconstructing simulated missing data in AddHealth, then applies the method to simulate networks at the Air Force Academy. The method employed in Chandrasekhar and Jackson (2014) arises out of the exponential random graphs model of network formation, and does not model tradeoffs between linking strategies.

Fortunately here, the network formation model can be pressed into service to fill in missing data. In his application at the Air Force Academy, Williams (2016) does not model network formation; rather, he models only outcomes conditional on the observed network. In contrast, I have posited a specific model of network formation that can be used to reconstruct missing data.

Accordingly, I employ an iterative EM algorithm that uses the network model itself to simulate missing data. Details are in Appendix D, but the basic structure is as follows. First, fill in the missing data arbitrarily. Second, estimate the network formation model with this dataset. Third, using these estimated parameters and implied distributions of unobserved data, simulate values for missing network data. Repeat the second and third steps for sufficient iterations to converge to the distribution of both the simulated networks and the estimated parameters. This generates a Markov Chain of simulated networks and estimated parameters. After sufficient burn-in period, I take draws from this chain as the simulated parameters and networks.

5.1.2 Network Formation Parameters

Estimation of the network formation parameters is performed via GMM. Essentially, estimation consists of finding parameter values of the structural parameter \((\hat{\beta}, \gamma, \delta)\) and the scalar unobserv-
ables $a$ such that the sample analogues of the assumed moment conditions are empirically true. I estimate this via GMM, with moments motivated by the identification results.\footnote{An alternative and somewhat less computationally burdensome procedure estimates the structural parameters $\theta = (\hat{\beta}, \gamma, \delta)$ and the scalar unobservable vector $A$ iteratively. However, this procedure has the limitation that it does not allow for closed-form standard errors. Therefore, while point estimation is sped up, the need to bootstrap the procedure leads this procedure to be substantially slower overall due to increased burden of variance estimation.} As discussed in the prior subsection, in order to correct for missing data, the GMM routine is the minimization step of the iterative EM algorithm.

Estimated parameters of the network-formation game are given in Table 10. In Panel A, we see that $\hat{\beta}$ is positive and large, indicating that effort levels of the two actors forming a link are strongly complementary, consistent with the reduced-form facts. Importantly, this is true even when controlling for a large set of observed and unobserved characteristics. Further $\hat{\beta}$ is significantly less than one, as required for the network-formation process to have a unique interior equilibrium. The point estimate of $\gamma_2$ is large and highly significant, suggesting that scalar unobservables play a large part in network formation. Similarly, $\delta_4$ are similarly positive and significant, suggesting that individuals who both have higher $a$ are likely to have stronger links.

Panel B presents parameter estimates that show how observed and unobserved variables interact in determining the utility of network links. The first column shows that, for example, members of Scheduled Castes and Scheduled Tribes are less valuable as network links than are Generals (the omitted category), and they are particularly less likely to form links if these individuals are chosen to participate in the Bal Sabha program.

The second column shows substantial homophily along a number of dimensions, as indicated by the $\delta_1$ coefficients. Those in Standard 7 and 8 are much more likely to be linked, and homophily is enhanced if both are chosen as Bal Sabha participants. We further see substantial homophily among participants in the Bal Sabha if both are members of Scheduled Castes, Scheduled Tribes, or Other Backwards Castes.

The final two columns show estimates of the effects of interactions between observed characteristics and unobserved $a$. Many of these estimated coefficients are highly significant and large in magnitude. This suggests that these interactions are quite important in individuals’ decisions about network formation. Accordingly, failure to account for these interactions has the potential to crucially biased estimates of the parameters of the peer effects model.
Table 10: Structural Network Formation Parameter Estimates

**Panel A: Parameters Not involving Covariates**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Std. Error</th>
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</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.150**</td>
<td>(0.073)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.620***</td>
<td>(0.026)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.057***</td>
<td>(0.012)</td>
</tr>
</tbody>
</table>

**Panel B: Parameters involving Covariates**

<table>
<thead>
<tr>
<th>X Variable</th>
<th>$\gamma_1$</th>
<th>$\delta_1$</th>
<th>$\delta_2$</th>
<th>$\delta_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elected</td>
<td>0.369***</td>
<td>0.116**</td>
<td>0.039</td>
<td>0.125***</td>
</tr>
<tr>
<td>Standard 7</td>
<td>-0.080***</td>
<td>0.674***</td>
<td>-0.021</td>
<td>0.037</td>
</tr>
<tr>
<td>Standard 8</td>
<td>-0.079***</td>
<td>0.718***</td>
<td>-0.096***</td>
<td>0.066**</td>
</tr>
<tr>
<td>SC</td>
<td>-0.564***</td>
<td>0.614***</td>
<td>-0.230***</td>
<td>-0.070*</td>
</tr>
<tr>
<td>ST</td>
<td>-0.504***</td>
<td>0.624***</td>
<td>-0.078*</td>
<td>-0.020</td>
</tr>
<tr>
<td>OBC</td>
<td>-0.251***</td>
<td>0.194***</td>
<td>-0.084***</td>
<td>0.005</td>
</tr>
<tr>
<td>Participant</td>
<td>0.026</td>
<td>0.036</td>
<td>-0.130</td>
<td>0.043</td>
</tr>
<tr>
<td>Participant \times Elected</td>
<td>-0.004</td>
<td>-0.105</td>
<td>0.040</td>
<td>-0.019</td>
</tr>
<tr>
<td>Participant \times Standard 7</td>
<td>-0.206***</td>
<td>-0.116</td>
<td>0.072</td>
<td>0.036</td>
</tr>
<tr>
<td>Participant \times Standard 8</td>
<td>-0.006</td>
<td>-0.119</td>
<td>0.040</td>
<td>-0.038</td>
</tr>
<tr>
<td>Participant \times SC</td>
<td>0.280*</td>
<td>0.083</td>
<td>0.161</td>
<td>0.049</td>
</tr>
<tr>
<td>Participant \times ST</td>
<td>0.232*</td>
<td>-0.123</td>
<td>0.061</td>
<td>-0.001</td>
</tr>
<tr>
<td>Participant \times OBC</td>
<td>-0.043</td>
<td>0.095</td>
<td>-0.035</td>
<td>-0.018</td>
</tr>
</tbody>
</table>

Notes: N = 58,530 in 20 schools. Robust standard errors in parentheses, clustered by school. *** p<0.01, ** p<0.05, * p<0.1. Parameters estimated via GMM. Missing data imputed and estimates adjusted via algorithm described in Appendix D. SC = Scheduled Caste, ST = Scheduled Tribe, OBC = Other Backwards Caste. Omitted Categories are Not Elected, Standard 6, and General.
5.2 Peer Effects Estimates

This section presents estimates of the peer effects model, as specified by Equation (7). Similar to the network formation issue, I first describe how I treat missing outcome data. Then I present the estimated parameters, from which we see that structural unobservables \( a \) are important in determining outcomes. We further see substantial evidence of heterogeneous peer effects.

5.2.1 Missing Outcome Data

Similar to the network link variables, I encounter missing data for two reasons. First, some girls were not present on the day that the endline questionnaires were administered. Second, even if they were present, some students did not answer the relevant questions.

To account for this possibly non-random missing data, I employ an iterative EM algorithm. Estimation is done by OLS, which then imputes outcomes according to the estimated distribution of unobserved variables.

Importantly, the parameters of the peer effects model are estimated conditional on a realized network and unobserved parameters \( A \). Accordingly, to account for variance in imputing the network data, I take draws from the imputed distribution of networks and unobserved vector \( A \), as these were calculated as part of the network formation estimation process. Conditional on each draw of the network at \( A \), I iterate the algorithm 500 times to minimize sensitivity to starting values.

5.2.2 Peer Effects Parameter Estimates

Tables 11 and 12 present estimated parameters of the peer effects model. These estimates are calculated via OLS conditional on the realized network and estimated \( A \). From these, I construct Participant and \( \bar{A} \). From this we see that latent variable \( A \) plays a large part in determining outcomes, particularly Gender Roles attitudes.

First, Table 11 gives results for Educational Aspirations. Column (1) estimates the simple model that does not account for baseline outcomes or \( A \). Column (2) adds \( A \) and peer effects on \( A \), while Column (3) includes baseline outcomes as well as peer effects for baseline outcomes. Finally, Column (4) includes all regressors and is thus the most general model.
From Columns (2) and (4), we see that the latent variable $A$ positively impacts Educational Aspirations for those in the lowest predicted outcome tercile. We further see that students’ peers’ $A$ affects Educational Aspirations for those in the highest predicted outcome tercile in Column (4). These results suggest that the unobserved variable $A$ is an important determinant of endline Educational Aspirations.

Results for Gender Roles attitudes are presented in Table 12, which is structured similarly to Table 11. In Column (4), we see that those in the middle predicted outcome tercile are significantly negatively affected by $A$, while those in the highest predicted tercile are significantly positively affected. We further see negative peer effects of baseline outcomes for those in the highest predicted tercile.

Column (1) is a naive estimate of the effect of the program without accounting for either baseline outcomes or scalar unobservables $a_{is}$. Column (2) adds in scalar unobservables, Column (3) adds in baseline outcomes, and Column (4) includes all of these regressors. Accordingly, Column (4) is the most general specification, and I focus the discussion on those results.

None of the interactions with $a_{is}$ have significant coefficients, suggesting that these scalar unobservables do not play a substantial direct role in bringing about educational aspirations.\footnote{Alternatively, the insignificant coefficients may be due to attenuation. That is, $A$ is estimated with variance thus observed with measurement error.} However, there is a large negative coefficient of -.725 on the peer effect of $a_{is}$ for those in the lowest predicted outcome tercile. That is, students who have a lower projected outcome do worse due to interaction of those with higher unobservables.

Analogous results for Gender Roles outcomes are presented in Table 12. Note that most of the action appears to be among those in the middle of the predicted outcome distribution, as shown by coefficients involving $\hat{M}$. Those in this group are negatively affected by participation and are also negatively affected by their peers’ participation, although this latter effect is not significant. Conversely, these same students are positively affected by their peers’ baseline outcomes as well as their peers’ baseline scalar unobservables, as measured by $\overline{X}$. Further, the results present some evidence of heterogeneous peer effects.
<table>
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<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
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<td>( \hat{L} )</td>
<td>-0.310***</td>
<td>-0.358***</td>
<td>0.188***</td>
<td>0.148*</td>
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<td>(0.018)</td>
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<tr>
<td>( \hat{M} )</td>
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<td>-0.028</td>
<td>-0.009</td>
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<td></td>
<td>(0.011)</td>
<td>(0.041)</td>
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<td>(0.045)</td>
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<td>( \hat{H} )</td>
<td>0.423***</td>
<td>0.463***</td>
<td>0.448***</td>
<td>0.517***</td>
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<tr>
<td></td>
<td>(0.010)</td>
<td>(0.028)</td>
<td>(0.014)</td>
<td>(0.041)</td>
</tr>
<tr>
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<td>-0.013</td>
</tr>
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<td>(0.048)</td>
<td>(0.029)</td>
<td>(0.037)</td>
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<td>0.039</td>
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<td>(0.032)</td>
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<td>(0.208)</td>
<td>(0.056)</td>
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<td>Participant ( \times \hat{M} )</td>
<td>-0.112</td>
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<td>(0.083)</td>
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<td>(0.078)</td>
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<td>-0.396***</td>
<td>-0.425***</td>
<td>-0.357***</td>
<td>-0.406***</td>
</tr>
<tr>
<td></td>
<td>(0.093)</td>
<td>(0.103)</td>
<td>(0.078)</td>
<td>(0.096)</td>
</tr>
<tr>
<td>A ( \times \hat{L} )</td>
<td>0.124***</td>
<td>0.121***</td>
<td>0.121***</td>
<td>0.121***</td>
</tr>
<tr>
<td></td>
<td>(0.046)</td>
<td>(0.044)</td>
<td>(0.044)</td>
<td>(0.044)</td>
</tr>
<tr>
<td>A ( \times \hat{M} )</td>
<td>0.090***</td>
<td>0.086***</td>
<td>0.086***</td>
<td>0.086***</td>
</tr>
<tr>
<td></td>
<td>(0.033)</td>
<td>(0.032)</td>
<td>(0.032)</td>
<td>(0.032)</td>
</tr>
<tr>
<td>A ( \times \hat{H} )</td>
<td>0.119***</td>
<td>0.108***</td>
<td>0.108***</td>
<td>0.108***</td>
</tr>
<tr>
<td></td>
<td>(0.041)</td>
<td>(0.036)</td>
<td>(0.036)</td>
<td>(0.036)</td>
</tr>
<tr>
<td>( \bar{A} \times \hat{L} )</td>
<td>0.600</td>
<td>0.332</td>
<td>0.332</td>
<td>0.332</td>
</tr>
<tr>
<td></td>
<td>(0.956)</td>
<td>(0.542)</td>
<td>(0.542)</td>
<td>(0.542)</td>
</tr>
<tr>
<td>( \bar{A} \times \hat{M} )</td>
<td>-0.238</td>
<td>-0.245</td>
<td>-0.245</td>
<td>-0.245</td>
</tr>
<tr>
<td></td>
<td>(0.273)</td>
<td>(0.264)</td>
<td>(0.264)</td>
<td>(0.264)</td>
</tr>
<tr>
<td>( \bar{A} \times \hat{H} )</td>
<td>0.312**</td>
<td>-0.428***</td>
<td>-0.428***</td>
<td>-0.428***</td>
</tr>
<tr>
<td></td>
<td>(0.149)</td>
<td>(0.164)</td>
<td>(0.164)</td>
<td>(0.164)</td>
</tr>
<tr>
<td>Baseline Outcome ( \times \hat{L} )</td>
<td>0.292***</td>
<td>0.284***</td>
<td>0.284***</td>
<td>0.284***</td>
</tr>
<tr>
<td></td>
<td>(0.012)</td>
<td>(0.018)</td>
<td>(0.018)</td>
<td>(0.018)</td>
</tr>
<tr>
<td>Baseline Outcome ( \times \hat{M} )</td>
<td>0.136***</td>
<td>0.134***</td>
<td>0.134***</td>
<td>0.134***</td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
<td>(0.016)</td>
<td>(0.016)</td>
<td>(0.016)</td>
</tr>
<tr>
<td>Baseline Outcome ( \times \hat{H} )</td>
<td>0.046***</td>
<td>0.036**</td>
<td>0.036**</td>
<td>0.036**</td>
</tr>
<tr>
<td></td>
<td>(0.012)</td>
<td>(0.015)</td>
<td>(0.015)</td>
<td>(0.015)</td>
</tr>
<tr>
<td>Baseline Outcome ( \times \hat{L} )</td>
<td>0.606***</td>
<td>0.574***</td>
<td>0.574***</td>
<td>0.574***</td>
</tr>
<tr>
<td></td>
<td>(0.033)</td>
<td>(0.149)</td>
<td>(0.149)</td>
<td>(0.149)</td>
</tr>
<tr>
<td>Baseline Outcome ( \times \hat{M} )</td>
<td>0.348***</td>
<td>0.342***</td>
<td>0.342***</td>
<td>0.342***</td>
</tr>
<tr>
<td></td>
<td>(0.033)</td>
<td>(0.055)</td>
<td>(0.055)</td>
<td>(0.055)</td>
</tr>
<tr>
<td>Baseline Outcome ( \times \hat{H} )</td>
<td>0.714***</td>
<td>0.724***</td>
<td>0.724***</td>
<td>0.724***</td>
</tr>
<tr>
<td></td>
<td>(0.028)</td>
<td>(0.056)</td>
<td>(0.056)</td>
<td>(0.056)</td>
</tr>
</tbody>
</table>

Robust standard errors in parentheses, clustered by school. *** \( p<0.01 \), ** \( p<0.05 \), * \( p<0.1 \). \( N = 1319 \) in 30 schools in all specifications. Missing data imputed and estimates adjusted via algorithm described in Appendix D. Standard errors calculations account for variance in estimating generated regressors \( \bar{A} \).
<table>
<thead>
<tr>
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<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{L} )</td>
<td>0.079***</td>
<td>0.084</td>
<td>0.085***</td>
<td>0.085</td>
</tr>
<tr>
<td></td>
<td>(0.011)</td>
<td>(0.067)</td>
<td>(0.013)</td>
<td>(0.064)</td>
</tr>
<tr>
<td>( \hat{M} )</td>
<td>0.190***</td>
<td>0.254***</td>
<td>0.043***</td>
<td>0.101**</td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td>(0.045)</td>
<td>(0.010)</td>
<td>(0.043)</td>
</tr>
<tr>
<td>( \hat{H} )</td>
<td>0.091***</td>
<td>0.104***</td>
<td>0.000</td>
<td>0.020</td>
</tr>
<tr>
<td></td>
<td>(0.010)</td>
<td>(0.035)</td>
<td>(0.013)</td>
<td>(0.035)</td>
</tr>
<tr>
<td>Participant ( \times \hat{L} )</td>
<td>-0.038</td>
<td>-0.039</td>
<td>-0.045</td>
<td>-0.045</td>
</tr>
<tr>
<td></td>
<td>(0.027)</td>
<td>(0.028)</td>
<td>(0.028)</td>
<td>(0.028)</td>
</tr>
<tr>
<td>Participant ( \times \hat{M} )</td>
<td>-0.455***</td>
<td>-0.468***</td>
<td>-0.512***</td>
<td>-0.522***</td>
</tr>
<tr>
<td></td>
<td>(0.025)</td>
<td>(0.031)</td>
<td>(0.023)</td>
<td>(0.026)</td>
</tr>
<tr>
<td>Participant ( \times \hat{H} )</td>
<td>0.105**</td>
<td>0.120**</td>
<td>0.121***</td>
<td>0.140***</td>
</tr>
<tr>
<td></td>
<td>(0.042)</td>
<td>(0.047)</td>
<td>(0.041)</td>
<td>(0.047)</td>
</tr>
<tr>
<td>Participant ( \times \hat{L} )</td>
<td>-0.343***</td>
<td>-0.348***</td>
<td>-0.347***</td>
<td>-0.338***</td>
</tr>
<tr>
<td></td>
<td>(0.058)</td>
<td>(0.085)</td>
<td>(0.059)</td>
<td>(0.078)</td>
</tr>
<tr>
<td>Participant ( \times \hat{M} )</td>
<td>-0.400***</td>
<td>-0.421***</td>
<td>0.186***</td>
<td>0.160**</td>
</tr>
<tr>
<td></td>
<td>(0.047)</td>
<td>(0.084)</td>
<td>(0.046)</td>
<td>(0.067)</td>
</tr>
<tr>
<td>Participant ( \times \hat{H} )</td>
<td>-0.525***</td>
<td>-0.523***</td>
<td>-0.240***</td>
<td>-0.238***</td>
</tr>
<tr>
<td></td>
<td>(0.055)</td>
<td>(0.058)</td>
<td>(0.058)</td>
<td>(0.062)</td>
</tr>
<tr>
<td>( A \times \hat{L} )</td>
<td>-0.032</td>
<td>-0.029</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.037)</td>
<td>(0.036)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( A \times \hat{M} )</td>
<td>0.004</td>
<td>0.015</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.033)</td>
<td>(0.034)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( A \times \hat{H} )</td>
<td>0.055</td>
<td>0.041</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.037)</td>
<td>(0.037)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \bar{X} \times \hat{L} )</td>
<td>-0.030</td>
<td>-0.030</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.407)</td>
<td>(0.407)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \bar{X} \times \hat{M} )</td>
<td>-0.452</td>
<td>-0.395</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.349)</td>
<td>(0.283)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \bar{X} \times \hat{H} )</td>
<td>-0.131</td>
<td>-0.208</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.309)</td>
<td>(0.315)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Baseline Outcome ( \times \hat{L} )</td>
<td>0.050***</td>
<td>0.049***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td>(0.008)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Baseline Outcome ( \times \hat{M} )</td>
<td>0.032***</td>
<td>0.024**</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.007)</td>
<td>(0.009)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Baseline Outcome ( \times \hat{H} )</td>
<td>0.077***</td>
<td>0.07***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td>(0.011)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Baseline Outcome ( \times \hat{L} )</td>
<td>-0.059**</td>
<td>-0.042</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.025)</td>
<td>(0.031)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Baseline Outcome ( \times \hat{M} )</td>
<td>0.499***</td>
<td>0.499***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.018)</td>
<td>(0.043)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Baseline Outcome ( \times \hat{H} )</td>
<td>0.248***</td>
<td>0.257***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.023)</td>
<td>(0.031)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Robust standard errors in parentheses, clustered by school. *** \( p<0.01 \), ** \( p<0.05 \), * \( p<0.1 \). \( N = 1319 \) in 30 schools in all specifications. Missing data imputed and estimates adjusted via algorithm described in Appendix D. Standard errors calculations account for variance in estimating generated regressors \( \hat{A} \).
6 Out-of-Sample Validation

6.1 Simulation Method

While a structural model can allow for out-of-sample prediction, our confidence in the model can be bolstered significantly by comparison of the model’s predictions to realized out-of-sample outcomes. Fortunately here, I have an out-of-sample treatment group that can be used as a holdout sample, as suggested by Todd and Wolpin (2006). In T1 schools, participation in the program was assigned by election rather than randomly, as was done in T2 schools. Therefore, having used T2 and Control to estimate the model, I can use the estimated parameters to predict outcomes conditional on all participants being chosen by election. Comparing these predictions to the actual realized outcomes in T1 schools provides a check on the model’s predictive power.

Counterfactual simulation relies upon simulation of unobservables. The network-formation model includes three unobservables: $c_{ijs}$, $M_{is}$, and $a_{is}$. The cost variables are by construction independent of all observables and $a_{is}$. Accordingly, they are drawn from a random normal distribution with mean zero and variance $\hat{\sigma}_c^2$, where $\hat{\sigma}_c^2$ is the empirical variance of these residuals. Scalar variable $a_{is}$ is similarly drawn from a random normal with mean zero and variance 1 (recall that this mean and variance are imposed as moment conditions). Finally, $M_{is}$ is drawn from a log-normal distribution allowing for some dependence on observable characteristics. I take the distribution of observed characteristics in T1 as given in all simulations in order to avoid any possible composition issues.

After the network-formation process is simulated, I move on to simulating outcomes. The parameters in Tables 11 and 12 are used to predict outcomes conditional on the simulated network and simulated $A$. Again, to avoid any composition bias, all simulations are done on 10 schools with the exact distribution of observed covariates as found in T1 schools.

6.2 Comparison to T1

Simulation results are presented in Table 13. Simulations for Educational Aspirations are presented in Panel A. Note that both simulation specifications are overly optimistic about mean Educational Aspirations. This could be due to a number of issues, including a possible discouragement effect of choice by election for those not selected. That is, it may be the case that the program
carried out with elected participants serves to reinforce marginalization for girls not elected to the program.

While the model’s predicted Educational Aspirations may be biased upwards, it does a good job of getting at treatment effect heterogeneity. Under both specifications, we see that the model predicts that non-elected girls have lower Educational Aspirations at endline as do those in lower castes.

The model does a better job of predicting the overall mean for Gender Roles attitudes, as shown in Panel B. While the model predicts that elected girls have lower endline Gender Roles attitudes than their non-elected counterparts, it does not match the pattern of heterogeneity across caste groupings as well. For example, both specifications predict much higher mean Gender Roles attitudes than are observed among OBC girls. In sum, while not performing perfect prediction, the model does a credible job of matching many features of the out-of-sample realized outcomes.

7 Counterfactual Policy Evaluation

7.1 Counterfactual Assignments

From the descriptive regression results in Table 2, we see that those in lower castes have substantially lower baseline outcomes, and those who were elected to participate have higher outcomes. Table 7, which was used to predict endline outcomes in the absence of treatment, shows similar patterns. Optimizing outcomes via creative assignment essentially amounts to choosing a set of individuals who will benefit the most from treatment, while accounting for the indirect effects of the program through network change and peer effects. Accordingly, an obvious place to start in thinking about alternative policies is to treat those who are in the most need. Accordingly, I have designed an assignment rule whereby girls with the lowest predicted outcomes, as calculated by the estimates in Table 7, are assigned to participate in the program.

When comparing program outcomes for counterfactual assignment policies, we need a relevant comparison. As a benchmark, girls in the randomly-assigned T2 schools seem like a natural point of departure. In order to eliminate potential composition effects, I simulate the counterfactual assignment using the same distribution of background covariates as found in T2.
Table 13: Comparison of Realized to Predicted in T1 Schools (Educational Aspirations)

### Panel A: Educational Aspirations

<table>
<thead>
<tr>
<th></th>
<th>All</th>
<th>Elected</th>
<th>Not Elected</th>
<th>SC</th>
<th>ST</th>
<th>OBC</th>
<th>General</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Observed in T1</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>-0.279</td>
<td>-0.053</td>
<td>-0.398</td>
<td>-0.364</td>
<td>-0.489</td>
<td>-0.392</td>
<td>0.085</td>
</tr>
<tr>
<td>Standard Error of Mean</td>
<td>0.151</td>
<td>0.180</td>
<td>0.163</td>
<td>0.233</td>
<td>0.179</td>
<td>0.189</td>
<td>0.234</td>
</tr>
<tr>
<td>N</td>
<td>330</td>
<td>114</td>
<td>216</td>
<td>64</td>
<td>32</td>
<td>153</td>
<td>81</td>
</tr>
</tbody>
</table>

**Simulated with Same Covariate Distribution as T1**

**Without Baseline Outcomes**

| Mean of Simulated Means | -0.140 | -0.091 | -0.166      | -0.318 | -0.242 | -0.188 | 0.132   |

**With Baseline Outcomes**

| Mean of Simulated Means | -0.170 | -0.150 | -0.180      | -0.269 | -0.257 | -0.236 | 0.068   |

### Panel B: Gender Roles Attitudes

<table>
<thead>
<tr>
<th></th>
<th>All</th>
<th>Elected</th>
<th>Not Elected</th>
<th>SC</th>
<th>ST</th>
<th>OBC</th>
<th>General</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Observed in T1</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>-0.022</td>
<td>-0.085</td>
<td>0.012</td>
<td>0.066</td>
<td>0.085</td>
<td>-0.339</td>
<td>0.462</td>
</tr>
<tr>
<td>Standard Error of Mean</td>
<td>0.147</td>
<td>0.210</td>
<td>0.149</td>
<td>0.240</td>
<td>0.239</td>
<td>0.121</td>
<td>0.100</td>
</tr>
<tr>
<td>N</td>
<td>332</td>
<td>116</td>
<td>216</td>
<td>65</td>
<td>33</td>
<td>153</td>
<td>81</td>
</tr>
</tbody>
</table>

**Simulated with Same Covariate Distribution as T1**

**Without Baseline Outcomes**

| Mean of Simulated Means | -0.083 | -0.186 | -0.027      | -0.130 | -0.076 | -0.074 | -0.065 |

**With Baseline Outcomes**

| Mean of Simulated Means | -0.005 | -0.101 | 0.046       | -0.078 | -0.084 | 0.012  | 0.053  |

Notes: Standard errors clustered by school. Simulation results in Panel A correspond to Columns (2) (without baseline outcomes) and (4) (with baseline outcomes) of Table 11. Simulation results in Panel B correspond to Columns (2) (without baseline outcomes) and (4) (with baseline outcomes) of Table 12. Simulations based upon 1000 repetitions, with residuals drawn from random normal. SC = Scheduled Caste, ST = Scheduled Tribe, OBC = Other Backwards Caste.
7.2 Simulated Outcomes

Table 14 demonstrates the results of this simulation. Note that the simulations were performed separately for the two outcomes, as the assignment rule does not in general assign the same girls under both outcomes. The results of this simulation are quite troublesome, and suggest that assigning those most “in need” may not a good strategy. Further, while noting lack of power, girls are predicted to perform worse on average for every group with the exception of those not elected.

We see a similar negative effect of this assignment rule for Gender Roles attitudes. As shown in Panel B of Table 14, girls perform on average slightly worse than under random assignment in T2. Further, the alternative assignment policy seems to have the largest negative impact on elected girls.

In contrast, this assignment rule appears to do worse than random assignment for Gender Roles attitudes. On average, girls Gender Roles attitudes are 0.08 to 0.10 standard deviations lower than under random assignment. In particular, elected girls and those in higher castes (General and OBC) perform much worse, while members of Scheduled Castes and Scheduled Tribes are expected to perform similarly.

7.3 Optimal Treatment Assignment

In a sense, assessment of the effects of counterfactual assignment policies is an exercise in statistical treatment assignment (see Smith and Staghoj 2009). That is, we search for statistical rules that enhance average outcomes, either for the entire population or some subgroup. This is the exact exercise that Carrell, Sacerdote, and West (2013) pursue in their analysis of Air Force Academy squadron assignment. The model here can be employed in a similar exercise that accounts for network dynamics conditional on assignment. However, as discussed by Carrell, Sacerdote, and West (2013), this is a complicated integer programming problem which, in the case of my model, is further compounded by the need to simulate network dynamics as part of the optimization routine. Accordingly, such an exercise has an extreme computational burden and thus beyond the scope of the current project.
Table 14: Counterfactual Policy Assigning Girls with Lowest Predicted Outcomes

### Panel A: Educational Aspirations

<table>
<thead>
<tr>
<th></th>
<th>All</th>
<th>Elected</th>
<th>Not Elected</th>
<th>SC</th>
<th>ST</th>
<th>OBC</th>
<th>General</th>
</tr>
</thead>
<tbody>
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<td><strong>Observed in T2</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>-0.092</td>
<td>0.125</td>
<td>-0.184</td>
<td>-0.207</td>
<td>-0.230</td>
<td>-0.098</td>
<td>0.539</td>
</tr>
<tr>
<td>Standard Error of Mean</td>
<td>0.130</td>
<td>0.165</td>
<td>0.153</td>
<td>0.194</td>
<td>0.233</td>
<td>0.157</td>
<td>0.182</td>
</tr>
<tr>
<td>N</td>
<td>335</td>
<td>99</td>
<td>236</td>
<td>93</td>
<td>53</td>
<td>159</td>
<td>30</td>
</tr>
</tbody>
</table>

**Simulated with Same Covariate Distribution as T2**

**Without Baseline Outcomes**

| Mean of Simulated Means | -0.155 | -0.080 | -0.187 | -0.324 | -0.267 | -0.099 | 0.267 |

**With Baseline Outcomes**

| Mean of Simulated Means | -0.133 | -0.105 | -0.145 | -0.285 | -0.240 | -0.084 | 0.264 |

### Panel B: Gender Roles Attitudes

<table>
<thead>
<tr>
<th></th>
<th>All</th>
<th>Elected</th>
<th>Not Elected</th>
<th>SC</th>
<th>ST</th>
<th>OBC</th>
<th>General</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Observed in T2</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>-0.009</td>
<td>0.056</td>
<td>-0.036</td>
<td>-0.221</td>
<td>0.038</td>
<td>0.082</td>
<td>0.078</td>
</tr>
<tr>
<td>Standard Error of Mean</td>
<td>0.169</td>
<td>0.247</td>
<td>0.158</td>
<td>0.224</td>
<td>0.214</td>
<td>0.182</td>
<td>0.293</td>
</tr>
<tr>
<td>N</td>
<td>335</td>
<td>99</td>
<td>236</td>
<td>93</td>
<td>52</td>
<td>159</td>
<td>31</td>
</tr>
</tbody>
</table>

**Simulated with Same Covariate Distribution as T2**

**Without Baseline Outcomes**

| Mean of Simulated Means | -0.073 | -0.124 | -0.052      | -0.101 | -0.050 | -0.073 | -0.033 |

**With Baseline Outcomes**

| Mean of Simulated Means | -0.046 | -0.096 | -0.025      | -0.109 | -0.059 | -0.020 | 0.035  |

Notes: Standard errors clustered by school. Simulation results in Panel A correspond to Columns (2) (without baseline outcomes) and (4) (with baseline outcomes) of Table 11. Simulation results in Panel B correspond to Columns (2) (without baseline outcomes) and (4) (with baseline outcomes) of Table 12. Simulations based upon 500 repetitions, with residuals drawn from random normal. SC = Scheduled Caste, ST = Scheduled Tribe, OBC = Other Backwards Caste.
8 Conclusion

Out-of-sample prediction from random assignment is fraught with difficulties, a fact starkly demonstrated by Carrell, Sacerdote, and West (2013). This is especially true when the assignment process affects peer groups as well as outcomes of interest. This paper’s primary contribution lies in providing a method to account for network endogeneity. I accomplish this by pairing a non-linear model of peer effects with a novel model of network formation.

The peer effects model posits network endogeneity as an omitted-variable problem. I show that, conditional on observation of these latent variables, parameters of the peer effects model are identified even in the presence of network endogeneity. This formulation motivates the need to estimate these latent variables as part of the network formation process.

I then model network formation as a game of continuous choice in which individuals choose link values subject to a total effort constraint. I show that there exists a unique interior equilibrium and that this equilibrium is characterized by a set of first-order conditions. I then provide conditions under which parameters of the network formation process are identified, including the latent variables that cause network endogeneity.

With these identification results, I estimate the model using detailed data on outcomes and network links collected as part of a randomized evaluation of a girls’ empowerment program in Rajasthan, India. I then compare the results to realized out-of-sample outcomes. Finally, I simulate outcomes under an assignment rule that preferentially chooses participants with the lowest predicted outcomes.

Future work will further explore different counterfactual assignment rules. An extension would use the estimated parameters solve for “optimal” assignment in a similar manner to Carrell, Sacerdote, and West (2013). Such an approach holds promise as it will control for network endogeneity. However, given the need to simulate networks repeatedly, such an exercise is computationally intensive and thus beyond the scope of this paper.
References


Sacerdote, Bruce. 2001. “Peer Effects with Random Assignment: Results for Dartmouth Room-


Appendix A: Proof of Propositions.

Proposition 2

Proof. Existence of equilibrium follows directly from Rosen (1965). Given each other player’s strategies, each player’s utility function is concave in his own strategy $g_{is}$. Therefore, existence of equilibrium follows from Theorem 1 of Rosen (1965).

I show existence of a strictly positive equilibrium in three steps. First, I show existence of equilibrium in a version of the game in which players’ strategy sets are bounded below by $g > 0$. Second, I show that, for sufficiently small $g$, the lower bound is non-binding. Finally, I demonstrate that the equilibrium of the bounded game is an equilibrium when players are allowed to link zero with other players (that is, when $g = 0$).

**Step 1: Existence with Strictly Positive Strategy Sets**

Define a network-formation game in which individuals maximize utility as defined by Equation (11). Different than the game defined in the text, however, they must form strictly positive links with each individual. That is, for each $i, j \neq i$, $g_{ijs} \geq g$, where $g > 0$ (strictly). Set $g$ sufficiently small that each player’s strategy set is non-void: $g \in (0, \frac{M}{(N-1)c})$.

As defined by Rosen (1965) and Ui (2008), for each $i$, $U_{is}(g_{is}, g_{-is})$ is concave for every $g_{-is}$. Accordingly, the game is a smooth concave game on a compact strategy set. Thus, by Lemma 1 in Ui (2008) and the notes afterward, a Nash Equilibrium of this game exists.

**Step 2: Lower Bound is Non-Binding for Sufficiently Small $g$**

The result in Step 1 applies for any $g > 0$ such that strategy sets are non-void. Suppose $g \in (0, \frac{M}{(f_{\max} f_{\min}^\beta) \frac{1}{c}}) \cap (0, \frac{M}{(N-1)c})$, where $f_{\min} \leq e^{f(X_{is},X_{js})} \leq f_{\max} \forall i, j \neq i$. Define $\lambda_{is}$ as the Lagrange Multiplier for the budget constraint, and $\mu_{ij}$ as the Lagrange Multiplier for the lower-bound constraint $g_{ijs} - g \geq 0$ for $i, j \neq i$. Therefore, the following Kuhn-Tucker conditions hold for individual $i$ and all $j, k \neq i$:

\[
\alpha g_{ijs}^{\alpha-1} g_{jis}^\beta e^{f(X_{is},X_{js})} - \lambda_{is} c_{ij} + \mu_{ijs} = 0 \tag{A.1}
\]

\[
\alpha g_{iks}^{\alpha-1} g_{kis}^\beta e^{f(X_{is},X_{ks})} - \lambda_{is} c_{iks} + \mu_{iks} = 0 \tag{A.2}
\]

\(^{20} f_{\min} \text{ and } f_{\max} \text{ are well-defined and finite due to compactness of the range of the function } f \text{ and continuity of the exponential function.} \)
Suppose the constraint binds for some \( g_{ijs} \), and thus \( \mu_{ijs} > 0 \). Since \( g < \frac{M}{(N-1)^{1/2}} \), the constraint must not bind for some \( k \neq j, i \). Therefore, \( \mu_{iks} = 0 \). Combine Equations (A.1) and (A.2) through \( \lambda_{is} \) as follows:

\[
\alpha g_{ijs}^{\alpha - 1} g_{jis}^{\beta} \frac{e^f(X_{is}, X_{js})}{c_{ij}} + \mu_{ijs} = \alpha g_{iks}^{\alpha - 1} g_{kis}^{\beta} \frac{e^f(X_{is}, X_{ks})}{c_{ik}} \quad (A.3)
\]

On the right-hand side of Equation (A.4), \( g_{ijs} = g \), \( g_{jis} \geq g \), and \( e^f(X_{is}, X_{js}) \geq f_{\min} \). On the right-hand side, \( g_{iks}^{\alpha - 1} < g^{\alpha - 1} \) and \( g_{kis} \leq \frac{M}{c} \). After substitution, we see that

\[
g^{\alpha + \beta - 1} \frac{f_{\min}}{c} < g^{\alpha - 1} \left( \frac{M}{c} \right) ^{\beta} \frac{f_{\max}}{c} \quad (A.5)
\]

Rearrangement reveals

\[
g < \frac{M}{c} \left( \frac{f_{\max}}{f_{\min}} \frac{c}{c} \right)^{\beta} \quad (A.6)
\]

This implies a contradiction since we assumed that \( g \in (0, \frac{M}{c} \left( \frac{f_{\max}}{f_{\min}} \frac{c}{c} \right)^{\beta}) \). Accordingly, for sufficiently small \( g \), the constraint \( g_{ijs} \geq g \) does not hold for any pair \( i, j \neq i \). Therefore, with this restriction, there exists an equilibrium in which \( g_{ijs} > g \forall i, j \neq i \) (strictly).

Step 3: Equilibrium of the bounded game is still an equilibrium when players are allowed to choose links of 0.

Step 2 above shows that, for arbitrarily small \( g \), an equilibrium exists in which the lower-bound condition is non-binding for every pair \( i, j \neq i \). Therefore, for all \( i \), any deviation in which \( g'_{ijs} > 0 \forall i, j \neq i \) cannot lead to higher utility to \( i \) than this equilibrium allocation. I now show that this fact remains true if we allow players to choose \( g'_{ijs} = 0 \).

Assume players play the strategies played in the equilibrium described in Step 2. For every \( i, j \neq i \), define this strategy as \( g_{ijs} \). At this point, \( \mu_{ijs} = 0 \forall i, j \neq i \), and Equation (A.1) becomes

\[
\alpha g_{ijs}^{\alpha - 1} g_{jis}^{\beta} e^f(X_{is}, X_{js}) = \lambda_{is} c_{ij} \quad (A.7)
\]
From this, we see that

\[
U_i(g_{is}, g_{\neq is}) = \sum_{j \neq i} g_{ij}^{\alpha} g_{jis}^{\beta} e^{f(X_{is}, X_{js})} = \frac{1}{\alpha} \sum_{j \neq i} \lambda_{is} c_{ij} g_{is} \\
= \lambda_{is} \frac{M_{is}}{\alpha} \tag{A.8}
\]

Suppose that this is not an equilibrium of the game in which \( g = 0 \). Therefore, for some \( i \), there exists an alternative strategy \( g'_{is} \) in which \( g'_{ij} = 0 \) for some \( j \neq i \) where \( U_i(g'_{is}, g_{\neq is}) > U_i(g_{is}, g_{\neq is}). \)

The utility from links where \( g''_{ij} > 0 \) is bounded above by the utility derived from solving the First Order Conditions in Equation (A.7), restricted to positive links. Define \( g''_{ij} \) as the hypothetical set of links in which these FOCs hold whenever \( g''_{ij} > 0 \). Note that \( g''_{ij} > \iff g'_{ij} > 0 \), and \( \lambda''_{is} \) as the Lagrange Multiplier corresponding to this constrained utility-maximizing strategy. So,

\[
\alpha (g''_{ij})^{\alpha-1} g_{jis}^{\beta} e^{f(X_{is}, X_{js})} = \lambda''_{is} c_{ij} \forall j \neq i \vert g'_{ij} > 0 \tag{A.10}
\]

Therefore,

\[
\frac{g''_{ij}}{g_{ij}} = \left( \frac{\lambda''_{is}}{\lambda_{is}} \right)^{\frac{1}{\alpha-1}} \tag{A.11}
\]

From this, we see that \( \frac{g''_{ij}}{g_{ij}} \) is constant across all \( j \) for whom \( g''_{ij} > 0 \). Clearly, \( g''_{ij} > g_{ij} \) whenever \( g''_{ij} > 0 \). Further, \( 0 < \alpha < 1 \Rightarrow \lambda''_{is} < \lambda_{is} \). From this, we see that

\[
U_{ij}(g'_{is}, g_{\neq is}) = \sum_{j \neq i} (g'_{ij})^{\alpha} g_{jis}^{\beta} e^{f(X_{is}, X_{js})} \tag{A.12}
\]

\[
= \sum_{j \neq i} 1\{g'_{ij} > 0\} (g'_{ij})^{\alpha} g_{jis}^{\beta} e^{f(X_{is}, X_{js})} \tag{A.13}
\]

\[
\leq \frac{1}{\alpha} \sum_{j \neq i} 1\{g'_{ij} > 0\} \lambda''_{is} g''_{ij} c_{ij} \tag{A.14}
\]

\[
= \lambda''_{is} \frac{M_{is}}{\alpha} \tag{A.15}
\]

\[
< \lambda_{is} \frac{M_{is}}{\alpha} = U_i(g_{is}, g_{\neq is}) \tag{A.16}
\]

This shows that any deviation in which \( g_{ij} = 0 \) for some \( j \) makes agent \( i \) strictly worse off.
Therefore, the strictly positive equilibrium is also an equilibrium of the game when $g = 0$.

\[ \square \]

**Proposition 3**

Proof. Suppose there are two equilibria \((g, \lambda)\) and \((g', \lambda')\), where \(g = (g_{12s}, g_{13s}, \ldots, g_{NN-1s})\) and \(\lambda = (\lambda_1s, \ldots, \lambda_Ns)\). Equations (12) and (13), the First Order necessary conditions for strictly positive equilibrium, imply

\[
(\alpha - 1)(\log g_{ijs} - \log g'_{ijs}) + \beta(\log g_{jis} - \log g'_{jis}) - (\log \lambda_{is} - \log \lambda'_{is}) = 0 \quad \forall \, i, j \neq i \quad (A.17)
\]

\[
\sum_{j \neq i} c_{ijs}(g_{ijs} - g'_{ijs}) = 0 \quad \forall \, i \quad (A.18)
\]

Define \(\tilde{\beta} = \frac{\beta}{1-\alpha}\) and \(\tilde{\lambda}_{is} = \log \lambda_{is} - \alpha\). After substitution and rearrangement, Equation (A.17) becomes

\[
(\log g_{ijs} - \log g'_{ijs}) = \tilde{\beta}(\log g_{jis} - \log g'_{jis}) - (\tilde{\lambda}_{is} - \tilde{\lambda}'_{is}) \quad \forall \, i, j \neq i
\]

(A.19)

By symmetry,

\[
(\log g_{jis} - \log g'_{jis}) = \tilde{\beta}(\log g_{ijs} - \log g'_{ijs}) - (\tilde{\lambda}_{js} - \tilde{\lambda}'_{js}) \quad \forall \, i, j \neq i
\]

(A.20)

Substitute Equation (A.20) into Equation (A.19) and rearrange, yielding

\[
(\log g_{ijs} - \log g'_{ijs}) = -\frac{1}{1 - \tilde{\beta}^2} \left(\tilde{\beta}(\tilde{\lambda}_{js} - \tilde{\lambda}'_{js}) + (\tilde{\lambda}_{is} - \tilde{\lambda}'_{is})\right) \quad \forall \, i, j \neq i
\]

(A.21)

Since the log function is continuously differentiable for all positive values, the Mean Value Theorem $\Rightarrow \exists \, g_{ijs}' \in [g_{ijs}, g'_{ijs}]$, where $\log g_{ijs} - \log g'_{ijs} = \frac{1}{g_{ijs}}(g_{ijs} - g'_{ijs})$ and $g_{ijs}' > 0$. Make this substitution and multiply by $-(1 - \tilde{\beta}^2)g_{ijs}'\cdot c_{ijs}$:

\[
-(1 - \tilde{\beta}^2)c_{ijs}(g_{ijs} - g'_{ijs}) = c_{ijs}g_{ijs}' \left(\tilde{\beta}(\tilde{\lambda}_{js} - \tilde{\lambda}'_{js}) + (\tilde{\lambda}_{is} - \tilde{\lambda}'_{is})\right) \quad \forall \, i, j \neq i
\]

(A.22)
Next, sum across $j \neq i$, substitute and rearrange:

$$-(1 - \tilde{\beta})^2 \sum_{j \neq i} c_{ijs}(g_{ijs} - g'_{ijs}) = \sum_{j \neq i} c_{ijs}g^*_{ijs} \left(\tilde{\beta}(\tilde{\lambda}_{js} - \tilde{\lambda}'_{js}) + (\tilde{\lambda}_{is} - \tilde{\lambda}'_{is})\right) \forall i$$  \hspace{1cm} (A.23)

$$0 = \left(\sum_{j \neq i} c_{ijs}g^*_{ijs}\right) (\tilde{\lambda}_{is} - \tilde{\lambda}'_{is}) + \tilde{\beta} \sum_{j \neq i} c_{ijs}g^*_{ijs} (\tilde{\lambda}_{js} - \tilde{\lambda}'_{js}) \forall i$$  \hspace{1cm} (A.24)

This defines a linear system of $N$ equations and $N$ unknowns, as defined by $Ab = 0$ in Equation (A.25):

$$\begin{bmatrix}
\sum_{j \neq 1} c_{1js}g^*_{1js} & \tilde{\beta}c_{12s}g^*_{12s} & \ldots & \tilde{\beta}c_{1Ns}g^*_{1Ns} \\
\tilde{\beta}c_{21s}g^*_{21s} & (\sum_{j \neq 2} c_{2js}g^*_{2js}) & \ldots & \tilde{\beta}c_{2Ns}g^*_{2Ns} \\
\ldots & \ldots & \ldots & \ldots \\
\tilde{\beta}c_{N1s}g^*_{N1s} & \ldots & (\sum_{j \neq 1} c_{Njs}g^*_{Njs}) & \ldots
\end{bmatrix}
\begin{bmatrix}
\tilde{\lambda}_{1s} - \tilde{\lambda}'_{1s} \\
\tilde{\lambda}_{2s} - \tilde{\lambda}'_{2s} \\
\ldots \\
\tilde{\lambda}_{Ns} - \tilde{\lambda}'_{Ns}
\end{bmatrix} = 0$$  \hspace{1cm} (A.25)

Clearly, $A$ being invertible will guarantee $\tilde{\lambda}_{is} - \tilde{\lambda}'_{is} = 0 \forall i$.

Suppose $A$ is not invertible. Therefore, 0 is an eigenvalue of $A$ with an associated eigenvector $v$. Let $v_m$ be the largest element of $v$ and, w.l.o.g., $v_m > 0$. So, $v_m \geq v_j \geq -v_m \forall j \neq m$. Now,

$$v_m\left(\sum_{j \neq m} c_{mjs}g^*_{mjs}\right) + \tilde{\beta} \sum_{j \neq m} v_j c_{mjs}g^*_{mjs} \geq v_m\left(\sum_{j \neq m} c_{mjs}g^*_{mjs}\right) - \tilde{\beta}v_m \sum_{j \neq m} c_{mjs}g^*_{mjs}$$  \hspace{1cm} (A.26)

$$> v_m\left(\sum_{j \neq m} c_{mjs}g^*_{mjs}\right)(1 - \tilde{\beta}) > 0$$  \hspace{1cm} (A.27)

This contradicts that 0 is an eigenvalue. Therefore, $A$ is invertible, and $\tilde{\lambda}_{is} = \tilde{\lambda}'_{is} \forall i$.

Finally, from Equation (A.21), we see that $\tilde{\lambda}_{is} - \tilde{\lambda}'_{is} = 0 \forall i, j \neq i \Rightarrow (\log g_{ijs} - \log g'_{ijs}) = 0 \forall i, j \neq i$. Therefore, $(g, \lambda) = (g', \lambda')$ and the equilibrium is unique.

\[\square\]
Proposition 4

Proof. Let \( g_{ks} = (g_{k1s}, ..., g_{kNs}) \) be agent \( i's \) strategy vector, and \( g_{-is} \) be the strategy vectors of the other \( N_s - 1 \) players. The definition of a potential game requires that, for every \( k, g_{ks}, g_{-ks}, \)

\[
P(\mathbf{g}_{ks}, \mathbf{g}_{-ks}, \mathbf{X}_s) - P(\mathbf{g}_{ks}', \mathbf{g}_{-ks}, \mathbf{X}_s) = U_{is}(\mathbf{g}_{ks}, \mathbf{g}_{-ks}, \mathbf{X}_s) - U_{is}(\mathbf{g}_{ks}', \mathbf{g}_{-ks}, \mathbf{X}_s)
\]

Simple substitution and the assumption \( f(\mathbf{f}(\mathbf{X}_{is}, \mathbf{X}_{ks})) = \mathbf{f}(\mathbf{X}_{ks}, \mathbf{X}_{is}) \) shows this to be the case. Define \( u_{ij}(\mathbf{g}_{ks}, \mathbf{g}_{-ks}) = (g_{ij}g_{jis})^\alpha e^f(X_{is}, X_{js}) \) and \( u_{ij}(\mathbf{g}_{ks}', \mathbf{g}_{-ks}) \) similarly. Note that \( u_{ij}(\mathbf{g}_{ks}, \mathbf{g}_{-ks}) = u_{ij}(\mathbf{g}_{ks}', \mathbf{g}_{-ks}) \) whenever \( i, j \neq k \). So,

\[
P(\mathbf{g}_{ks}, \mathbf{g}_{-ks}, \mathbf{X}_s) - P(\mathbf{g}_{ks}', \mathbf{g}_{-ks}, \mathbf{X}_s) = \frac{1}{2} \sum_{i=1}^{N_s} \sum_{j \neq i} (u_{ijs}(\mathbf{g}_{ks}, \mathbf{g}_{-ks}) - u_{ijs}(\mathbf{g}_{ks}', \mathbf{g}_{-ks}))
\]

\[
= \frac{1}{2} \sum_{j \neq k} (u_{kjs}(\mathbf{g}_{ks}, \mathbf{g}_{-ks}) - u_{kjs}(\mathbf{g}_{ks}', \mathbf{g}_{-ks}))
\]

\[
+ \frac{1}{2} \sum_{i \neq k} (u_{iks}(\mathbf{g}_{ks}, \mathbf{g}_{-ks}) - u_{iks}(\mathbf{g}_{ks}', \mathbf{g}_{-ks}))
\]

\[
= \sum_{j \neq k} (u_{jks}(\mathbf{g}_{ks}, \mathbf{g}_{-ks}) - u_{jks}(\mathbf{g}_{ks}', \mathbf{g}_{-ks}))
\]

\[
= \sum_{j \neq k} (g_{kjs}g_{jks})^\alpha e^f(X_{ks}, X_{js}) - \sum_{j \neq k} (g'_{kjs}g_{jks})^\alpha e^f(X_{ks}, X_{js})
\]

\[
= U_{is}(\mathbf{g}_{ks}, \mathbf{g}_{-ks}, \mathbf{X}_s) - U_{is}(\mathbf{g}_{ks}', \mathbf{g}_{-ks}, \mathbf{X}_s)
\]

\[\square\]

Proposition 5

Proof. First, it is clear that the potential function \( P(G_s, X_s) \) is a continuous function in \( G_s \) on a compact set. Therefore, there exists some \( G_s^* \) that maximizes the potential function.

Monderer and Shapley (1996) showed that the set of strategy profiles that maximize \( P \) is a subset of the set of equilibrium profiles. Since the game as a potential game is a special case of the broader game, Proposition 2 provides existence results, and Proposition 3 provides that the strictly positive equilibrium is unique. Therefore, by showing that any other equilibrium is not a potential function maximizer, by necessity the strictly positive one must be.
I prove this by demonstrating that any equilibrium strategy profile that is not the strictly positive equilibrium cannot maximize $P$.

Take any equilibrium network $G_s$ other than the strictly positive one. Therefore, $g_{ijs} = g_{jis} = 0$ for some $i, j \neq i$. For all $d \in [0, \frac{M_{is}}{c_{ijs}}] \cap [0, \frac{M_{js}}{c_{jis}}]$, define a deviation profile as follows:

\[
\begin{align*}
g'_{ijs} &= g'_{jis} = d \\
g'_{iks} &= \frac{M_{is} - dc_{ij}}{M_{is}} g_{iks} \forall k \neq i, j \\
g'_{jks} &= \frac{M_{js} - dc_{jis}}{M_{js}} g_{jks} \forall k \neq i, j \\
g'_{kls} &= g_{kls} \forall k \neq i, j, \forall l
\end{align*}
\]

That is, $g_{iks}$ and $g_{jks}$ adjust proportionally. Note that $g_{iks} = 0 \Rightarrow g'_{iks} = 0$. Such a deviation is feasible (within the constraint set) for all $d$ as restricted above.

Now, define a function $F(d)$ as follows:

\[
F(d) = P(G_s', X_s) - P(G_s, X_s) = d^{2\alpha} e^{f(X_is, X_js)} + ((\frac{M_{is} - dc_{ijs}}{M_{is}})^{\alpha} - 1)U_{is}(G_s, X_s) + ((\frac{M_{js} - dc_{jis}}{M_{js}})^{\alpha} - 1)U_{js}(G_s, X_s)
\]

(A.28)

For notational convenience, let $U_{is} = U_{is}(G_s, X_s)$ and $U_{js} = U_{js}(G_s, X_s)$. The function $F(d)$ is continuous within its domain and differentiable for all $d > 0$. Further,

\[
F'(d) = 2\alpha d^{2\alpha - 1} e^{f(X_is, X_js)} - \frac{\alpha c_{ijs} U_{is}}{M_{is}} (\frac{M_{is} - dc_{ijs}}{M_{is}})^{\alpha - 1} - \frac{\alpha c_{jis} U_{js}}{M_{js}} (\frac{M_{js} - dc_{jis}}{M_{js}})^{\alpha - 1}
\]

(A.29)

\[
\geq 2\alpha d^{2\alpha - 1} e^{f(X_is, X_js)} - \frac{\alpha c}{M} \left( U_{is}(\frac{M_{is}}{M_{is} - dc_{ijs}})^{1-\alpha} + U_{js}(\frac{M_{js}}{M_{js} - dc_{jis}})^{1-\alpha} \right)
\]

(A.30)

Next, I show that there exists $d$ such that $F'(d)$ is strictly positive. There are two distinct cases:

(1) $U_{is} = U_{js} = 0$, and (2) $U_{is} + U_{js} > 0$.

Case 1: $U_{is} = U_{js} = 0$
In this case, Equation (A.30) becomes

\[ F'(d) = 2\alpha d^{2\alpha-1} e^{f(X_{is}, X_{js})} \]  \hspace{1cm} (A.31)

which is strictly positive for all \( d > 0 \). Therefore, by the Mean Value Theorem, there exists some \( d^* \in (0, d) \) such that \( F(d) - F(0) = F'(d^*)(d - 0) \). So, \( F(d) > 0 \).

**Case 2: \( U_{is} + U_{js} > 0 \)**

When \( U_{is} + U_{js} > 0 \), further restrict \( d \) such that \( d \in (0, \frac{M_{is}}{2c_{ij}}) \cap (0, \frac{M_{js}}{2c_{ij}}) \cap (0, \frac{(\bar{\tau}(U_{is} + U_{js})}{Me^{f(X_{is}, X_{js})}}\frac{1}{1 - \alpha})^{21} \). This set is non-void since the supremum of each interval is a strictly positive number. Within this restricted set,

\[ F'(d) > 2\alpha \left( \frac{\bar{\tau}(U_{is} + U_{js})}{Me^{f(X_{is}, X_{js})}} \right) e^{f(X_{is}, X_{js})} - \frac{\alpha \bar{\tau}}{M} 2^{1-\alpha} (U_{is} + U_{js}) \]

\[ = \frac{\alpha \bar{\tau}(U_{is} + U_{js})}{M} (2 - 2^{1-\alpha}) > 0 \]  \hspace{1cm} (A.32)

Therefore, for any \( d \in (0, \frac{M_{is}}{2c_{ij}}) \cap (0, \frac{M_{js}}{2c_{ij}}) \cap (0, \frac{(\bar{\tau}(U_{is} + U_{js})}{Me^{f(X_{is}, X_{js})}}\frac{1}{1 - \alpha})^{21} \), by the Mean Value Theorem, there exists some \( d^* \in (0, d) \) such that \( F(d) - F(0) = F'(d^*)(d - 0) \). So, \( F(d) > 0 \).

Bringing it all together, for any equilibrium \( G_s \) where \( g_{ij} = g_{ji} = 0 \) for some \( i, j \neq i \), there exists some feasible deviation \( G'_s \) in which \( F(d) > 0 \) and thus \( P(G'_s, X_s) > P(G_s, X_s) \). Therefore, each such \( G_s \) is not a maximizer of the potential function. Accordingly, the unique strictly positive strategy profile, for which \( g_{ij} > 0 \forall i, j \neq i \), maximizes the potential function.

\[ \square \]

**Proposition 6**

**Proof.** This result is a typical panel IV result, allowing for arbitrary correlation of variables within clusters. Let \( S \) be the number of schools (potential networks) observed and \( N \) be the number of actors per school. Starting with Equation (20) and the instrument set \( z_{ij} \), we see that

\[ z'_{ij}g_{ij} = z'_{ij}g_{ij} + z'_{ij}X_{is} + z'_{ij}X_{js} + z'_{ij}\tilde{X}_{is} + z'_{ij}\tilde{X}_{js} \]

\[ = z'_{ij}b_{ij} + z'_{ij}c_{ij} \]  \hspace{1cm} (A.33)

\[ 21 \text{ In this case where the game is a potential game, } \beta = \alpha. \text{ The assumption 2 therefore implies that } 1 - 2\alpha > 0. \]
where \( \theta = (\tilde{\beta}, \delta', \gamma')' \). Next, sum across all schools and pairs of students:

\[
\frac{1}{SN(N-1)} \sum_{s=1}^{S} \sum_{i=1}^{N} \sum_{j \neq i} b'_{ij} z_{ij} z'_{ij} \tilde{g}_{ij} = \sum_{s=1}^{S} \sum_{i=1}^{N} \sum_{j \neq i} \frac{1}{SN(N-1)} b'_{ij} z_{ij} z'_{ij} b_{ij} \theta
\]

\[
- \frac{1}{Sn(n-1)} \sum_{s=1}^{S} \sum_{i=1}^{N} \sum_{j \neq i} b'_{ij} z_{ij} z'_{ij} \tilde{c}_{ij} \tag{A.34}
\]

Since \( \tilde{c}_{ij} \) is a linear combination of terms that are assumed to be independent across \( s \), \( \tilde{c}_{ij} \) is also independent across \( s \). Further, all terms are bounded and thus have finite variance. Let \( w_{ij} \) be an element of one of the matrices in Equation (A.34). For any such variable, \( C[w_{ij}, w_{kl}] = 0 \) whenever \( s \neq t \). Further, since each \( w_{ij} \) is identically distributed within a school, \( \mathbb{V}[w_{ij}] = \mathbb{V}[w_{kl}] \forall i, j, k, l \). Further,

\[
\mathbb{V}[\bar{w}_{ij}] = \mathbb{V}\left[\frac{1}{SN(N-1)} \sum_{s=1}^{S} \sum_{i=1}^{N} \sum_{j \neq i} w_{ij}\right] = \frac{1}{S^2 N^2 (N-1)^2} \left( SN(N-1) \mathbb{V}[w_{ij}] + S \left( \sum_{i=1}^{N} \sum_{j \neq i} \sum_{k=1}^{N} \sum_{l \neq j} C[w_{ij}, w_{kl}] \right) \right)
\]

\[
\leq \frac{1}{S} \left( \frac{1}{N(N-1)} + 1 \right) \mathbb{V}[w_{ij}] \tag{A.35}
\]

where the final line applies the Cauchy Schwarz Inequality (\( C[w_{ij}, w_{kl}] \leq \mathbb{V}[w_{ij}] \)). Therefore, \( \lim_{S \to \infty} \mathbb{V}[\bar{w}_{ij}] = 0 \) and by Chebyshev’s Inequality,

\[
\text{plim}_{S \to \infty} \bar{w}_{ij} = \mathbb{E}[w_{ij}] \tag{A.36}
\]

I here note that all terms in Equation (A.34) are sample averages. Therefore, we can apply Equation (A.36) to each element of these matrices. Thus, \( \frac{1}{SN(N-1)} \sum_{s=1}^{S} \sum_{i=1}^{N} \sum_{j \neq i} b'_{ij} z_{ij} z'_{ij} \tilde{g}_{ij} \to_p \mathbb{E}[b'_{ij} z_{ij} z'_{ij} \tilde{g}_{ij}] \), etc. Now, replacing the terms in Equation (A.34) with probability limits,

\[
\mathbb{E}[b'_{ij} z_{ij} z'_{ij} \tilde{g}_{ij}] = \mathbb{E}[b'_{ij} z_{ij} z'_{ij} b_{ij}] \theta - \mathbb{E}[b'_{ij} z_{ij} z'_{ij} \tilde{c}_{ij}] \tag{A.37}
\]

Assumption 4 implies that the final term in Equation (A.37) is zero, while the rank condition of
Proposition 6 implies invertibility of \( \mathbb{E}[b'_{ij}z_{ij}z'_{ij}s_{ij}b_{ij}] \). Therefore,

\[
\theta = (\mathbb{E}[b'_{ij}z_{ij}s_{ij}z'_{ij}b_{ij}])^{-1}\mathbb{E}[b'_{ij}z_{ij}z'_{ij}s_{ij}] \tag{A.38}
\]

and thus the parameters \( \tilde{\beta}, \delta_1, \) and \( \gamma_3 \) are identified.

\[\Box\]

**Proposition 7**

**Proof.** This proof is very similar to Proposition 6 but relies upon the additional exogeneity conditions in Assumption 6. Starting with Equation (27) and the instrument set \( z_{ij} \), we see

\[
z'_{ij}g'_{ij} = z'_{ij}s_{ij}z_{ij} \tilde{\beta} + z'_{ij}s_{ij}Xa_{is}a_{js}\delta_1 + z'_{ij}s_{ij}a_{is}A_{js}\delta_2 + z'_{ij}s_{ij}a_{is}A_{ja}\delta_3 + z'_{ij}s_{ij}a_{is}A_{ja}\delta_4 + z'_{ij}s_{ij}\gamma_4 - z'_{ij}c'_{ij} \tag{A.39}
\]

Rearrangement of terms shows that

\[
z'_{ij}g'_{ij} = z'_{ij}s_{ij}b_{ij}\theta + z'_{ij}s_{ij}Xa_{is}A_{ja}\delta_2 + z'_{ij}s_{ij}a_{is}A_{ja}\delta_3 + z'_{ij}s_{ij}a_{is}A_{ja}\delta_4 + z'_{ij}s_{ij}a_{is}A_{ja}\gamma_4 - z'_{ij}c'_{ij} \tag{A.40}
\]

where \( \theta = (\tilde{\beta}, \delta_1', \gamma_3') \). Next, sum across all schools and all pairs of students. So,

\[
\frac{1}{SN(N-1)} \sum_{s=1}^{S} \sum_{i=1}^{N} \sum_{j \neq i} b'_{ij}s_{ij}z_{ij}s_{ij}g_{ij} = \frac{1}{SN(N-1)} \sum_{s=1}^{S} \sum_{i=1}^{N} \sum_{j \neq i} (b'_{ij}s_{ij}z_{ij}s_{ij}g_{ij} + b'_{ij}s_{ij}z_{ij}s_{ij}Xa_{is}A_{ja}\delta_2
\]

\[
+ b'_{ij}s_{ij}z_{ij}s_{ij}a_{is}A_{ja}\delta_3 + b'_{ij}s_{ij}z_{ij}s_{ij}a_{is}A_{ja}\delta_4 + b'_{ij}s_{ij}z_{ij}s_{ij}a_{is}A_{ja}\gamma_4
\]

\[
+ b'_{ij}s_{ij}z_{ij}s_{ij}c_{ij} \tag{A.41}
\]

Since \( c'_{ij} \) is a linear combination of terms that are assumed to be independent across \( s, \) \( c'_{ij} \) is also independent across \( s. \) Further, all terms are bounded and thus have finite variance. Let \( w_{ij} \) be an element of one of the matrices in Equation (A.41). For any such variable, \( \mathbb{C}[w_{ij}, w_{kl}] = 0 \)
whenever \( s \neq t \). Further,

\[
\mathbb{V}[\bar{w}_{ijs}] = \mathbb{V}\left[ \frac{1}{SN(N-1)} \sum_{s=1}^{S} \sum_{i=1}^{N} \sum_{j \neq i} w_{ijs} \right] = \frac{1}{S^2 N^2 (N-1)^2} \left( SN(N-1) \mathbb{V}[w_{ijs}] + S \left( \sum_{s=1}^{S} \sum_{i=1}^{N} \sum_{j \neq i} C[w_{ijs}, w_{kls}] \right) \right) \leq \frac{1}{S} \left( \frac{1}{N(N-1)} + 1 \right) \mathbb{V}[w_{ijs}] \tag{A.42}
\]

where the final line applies the Cauchy-Schwarz Inequality \((C[w_{ijs}, w_{kls}] \leq \mathbb{V}[w_{ijs}])\). Therefore, \( \lim_{S \to \infty} \mathbb{V}[\bar{w}_{ijs}] = 0 \) and by Chebyshev’s Inequality,

\[
\text{plim}_{S \to \infty} \bar{w}_{ijs} = \mathbb{E}[w_{ijs}] \tag{A.43}
\]

I here note that all terms in Equation (A.41) are sample averages. Therefore, we can apply Equation (A.43) to each element of these matrices. Thus, \( \frac{1}{SN(N-1)} \sum_{s=1}^{S} \sum_{i=1}^{N} \sum_{j \neq i} \mathbb{E}[b'_{ijs} z_{ijs} z'_{ijs} \bar{g}_{ijs}] \to_p \mathbb{E}[b'_{ijs} z_{ijs} z'_{ijs} \bar{g}_{ijs}] \), etc. Now, replacing the terms in Equation (A.41) with probability limits,

\[
\mathbb{E}[b'_{ijs} z_{ijs} z'_{ijs} \bar{g}_{ijs}] = \mathbb{E}[b'_{ijs} z_{ijs} z'_{ijs} b_{ijs}] \theta + \mathbb{E}[b'_{ijs} z_{ijs} z'_{ijs} X_{is} \hat{a}^i_{js}] \delta_2 + \mathbb{E}[b'_{ijs} z_{ijs} z'_{ijs} a_{is} \hat{X}^i_{js}] \delta_3 \\
+ \mathbb{E}[b'_{ijs} z_{ijs} z'_{ijs} a_{is} \hat{a}^i_{js}] \delta_4 + \mathbb{E}[b'_{ijs} z_{ijs} z'_{ijs} \hat{a}^i_{js}] \gamma_4 - \mathbb{E}[b'_{ijs} z_{ijs} z'_{ijs} \hat{c}^i_{ijs}] \tag{A.44}
\]

The first part of Assumption 6 implies that the final term in Equation (A.44) is zero. Note that \( z_{ijs} \) and \( b_{ijs} \) are simply functions of \( x_{ks} \). Therefore, application of L.I.E. implies that \( \mathbb{E}[b'_{ijs} z_{ijs} z'_{ijs} X_{is} \hat{a}^i_{js}] = \mathbb{E}[b'_{ijs} z_{ijs} z'_{ijs} X_{is}] \mathbb{E}[\hat{a}^i_{js} | b_{ijs}, z_{ijs}] = 0 \). By similar argument, \( \mathbb{E}[b'_{ijs} z_{ijs} z'_{ijs} a_{is} \hat{X}^i_{js}] = 0 \) and \( \mathbb{E}[b'_{ijs} z_{ijs} z'_{ijs} \hat{a}^i_{js}] = 0 \). Further, the third part of Assumption 6 implies

\[
\mathbb{E}[b'_{ijs} z_{ijs} z'_{ijs} a_{is} \hat{a}^i_{js}] = \mathbb{E}[b'_{ijs} z_{ijs} z'_{ijs} a_{is} \hat{a}^i_{js}] = \mathbb{E}[b'_{ijs} z_{ijs} z'_{ijs} a_{is} \hat{a}^i_{js}] = 0 \tag{A.45}
\]
where we condition on all \( k \) and \( l \neq i \). Substituting these results into Equation (A.44) shows that

\[
E[b'_{ijas}z_{ijas}\tilde{g}_{ijas}] = E[b'_{ijas}z_{ijas}b_{ijas}]\theta
\]  

(A.46)

The rank condition guarantees the existence of \((E[b'_{ijas}z_{ijas}b_{ijas}])^{-1}\) and thus

\[
\theta = (E[b'_{ijas}z_{ijas}b_{ijas}])^{-1}E[b'_{ijas}z_{ijas}\tilde{g}_{ijas}]
\]  

(A.47)

So, \( \theta = (\tilde{\beta}, \delta_1', \gamma_3') \) is identified.

**Proposition 8**

The first rank condition, together with Assumption 6 and Proposition 7, imply that \( \tilde{\beta} \) is identified. I prove the rest of the proposition in three steps: (1) Scale identification of \( \delta_2 \) and \( \gamma_2 \), (2) Scale identification of \( \delta_3 \), and (3) Scale identification of \( \delta_4 \).

**Step 1: Scale identification of \( \delta_2 \) and \( \gamma_2 \)**

Proof. To begin, multiply Equation (27) by \( z_{ijas}'a_{js} \) and sum across \( SN(N-1) \) observations. So,

\[
1 \cdot \frac{1}{SN(N-1)} \sum_{s=1}^{S} \sum_{i=1}^{N} \sum_{j \neq i} z_{ijas}'a_{js}\tilde{g}_{ijas} = \frac{1}{SN(N-1)} \sum_{s=1}^{S} \sum_{i=1}^{N} \sum_{j \neq i} (z_{ijas}'a_{js}\tilde{g}_{ijas}\tilde{\beta} + z_{ijas}'a_{js}a_{is}\tilde{X}_{is}\delta_1 \\
+ z_{ijas}'a_{js}X_{is}\beta_1 a_{js}'\tilde{\beta}_2 + z_{ijas}'a_{js}a_{is}\tilde{X}_{js}\delta_3 + z_{ijas}'a_{js}a_{is}\tilde{a}_{js}'\delta_4 \\
+ z_{ijas}'a_{js}\tilde{X}_{ia}\gamma_3 + z_{ijas}'a_{js}a_{is}\tilde{a}_{js}'\gamma_4 - z_{ijas}'a_{js}\tilde{c}_{ijas})
\]  

(A.48)

Since \( \tilde{c}_{ijas} \) is a linear combination of terms that are assumed to be independent across \( s \), \( \tilde{c}_{ijas} \) is also independent across \( s \). Further, all terms are bounded and thus have finite variance. Let \( w_{ijas} \) be an element of one of the matrices in Equation (A.48). For any such variable, \( C[w_{ijas}, w_{klt}] = 0 \).
whenever \( s \neq t \). Further,

\[
\mathbb{V}[\bar{w}_{ij}^s] = \mathbb{V}[\frac{1}{SN(N-1)} \sum_{s=1}^{S} \sum_{i=1}^{N} \sum_{j \neq i} w_{ij}s] \\
= \frac{1}{SN^2(N-1)^2} \left( SN(N-1)\mathbb{V}[w_{ij}s] + S \left( \sum_{i=1}^{N} \sum_{j \neq i} \sum_{k=1}^{N} \sum_{l \neq j} C[w_{ij}s, w_{kl}s] \right) \right) \\
\leq \frac{1}{S} \left( \frac{1}{N(N-1)} + 1 \right) \mathbb{V}[w_{ij}s]
\]

(A.49)

where the final line applies the Cauchy-Schwarz Inequality (\( C[w_{ij}s, w_{kl}s] \leq \mathbb{V}[w_{ij}s] \)). Therefore,

\[
\lim_{S \to \infty} \mathbb{V}[\bar{w}_{ij}^s] = 0
\]

and by Chebyshev’s Inequality,

\[
\text{plim}_{S \to \infty} \bar{w}_{ij}^s = \mathbb{E}[w_{ij}s]
\]

(A.50)

I here note that all terms in Equation (A.41) are sample averages. Therefore, we can apply Equation (A.50) to each element of these matrices. Thus, \( \frac{1}{SN(N-1)} \sum_{s=1}^{S} \sum_{i=1}^{N} \sum_{j \neq i} z'_{ijs}a_{js}\tilde{g}^t_{ij}s \to_p \mathbb{E}[z'_{ijs}a_{js}\tilde{g}^t_{ij}s] \), etc. Now, replacing the terms in Equation (A.41) with probability limits,

\[
\mathbb{E}[z'_{ijs}a_{js}\tilde{g}^t_{ij}s] = \mathbb{E}[z'_{ijs}a_{js}\tilde{g}^t_{ij}s] + \mathbb{E}[z'_{ijs}a_{js}X_is\tilde{X}_is^t] \delta_1 + \mathbb{E}[z'_{ijs}a_{js}X_is\tilde{a}^t_{ijs}] \delta_2 + \mathbb{E}[z'_{ijs}a_{js}a_{is}\tilde{X}_ijs^t] \delta_3 \\
+ \mathbb{E}[z'_{ijs}a_{js}a_{is}\tilde{a}^t_{ijs}] \delta_4 + \mathbb{E}[z'_{ijs}a_{js}X_is\tilde{X}_is] \gamma_3 + \mathbb{E}[z'_{ijs}a_{js}a_{is}\tilde{a}^t_{ijs} \gamma_4] - \mathbb{E}[z'_{ijs}a_{js}\delta_{ijs}^t] \quad \text{(A.51)}
\]

Assumption 6 implies that \( \mathbb{E}[z'_{ijs}a_{js}\delta_{ijs}^t] = 0 \).

Assumption 6 and L.I.E. imply that \( \mathbb{E}[z'_{ijs}a_{js}X_is\tilde{X}_is^t] = \mathbb{E}[z'_{ijs}X_is\tilde{X}_is^t] \mathbb{E}[a_{js}|z_{ijs}, X_is, \tilde{X}_is] = 0 \).

Similarly, \( \mathbb{E}[z'_{ijs}a_{js}X_is^t] = \mathbb{E}[z'_{ijs}X_is^t \mathbb{E}[a_{js}|z_{ijs}, \tilde{X}_is] = 0 \). Independence of \( a_{js} \) and \( a_{ks} \) when \( k \neq j \).
implies \( \mathbb{E}[z'_{ij} a_{js} a_{is} \dot{X}'^i_{js}] = \mathbb{E}[z'_{ij} \dot{X}'^i_{js} \mathbb{E}[a_{js} a_{is} | z_{ij}] \dot{X}'^i_{js}] = 0 \), and similarly \( \mathbb{E}[z'_{ij} a_{js} a_{is} \dot{a}'_{js}] = 0 \). Next,

\[
\mathbb{E}[z_{ij} a_{js} \dot{X}_{is} \dot{a}'_{js}] = \mathbb{E}[z'_{ij} a^2_{js} X_{is}] - \sum_{k \neq i} \mathbb{E}[z'_{ij} a_{ks} a_{js} X_{is}]
\]

\[
= \frac{N - 2}{N - 1} \mathbb{E}[z'_{ij} a^2_{js} X_{is}] - \sum_{k \neq i, k \neq j} \mathbb{E}[z'_{ij} a_{ks} a_{js} X_{is}]
\]

\[
= \frac{N - 2}{N - 1} \mathbb{E}[z'_{ij} a^2_{js} X_{is}]
\]

\[
= \frac{N - 2}{N - 1} \mathbb{E}[z'_{ij} X_{is}] \sigma_a
\]

(A.52)

where \( \sigma_a \) is the variance of the scalar unobservable \( a \). Similarly,

\[
[z_{ij} a_{js} \dot{a}'_{js}] = \mathbb{E}[z'_{ij} a^2_{js}] - \sum_{k \neq i} \mathbb{E}[z'_{ij} a_{ks}]
\]

\[
= \frac{N - 2}{N - 1} \mathbb{E}[z'_{ij} a^2_{js}] - \sum_{k \neq i, k \neq j} \mathbb{E}[z'_{ij} a_{ks} a_{js}]
\]

\[
= \frac{N - 2}{N - 1} \mathbb{E}[z'_{ij} a^2_{js}]
\]

\[
= \frac{N - 2}{N - 1} \mathbb{E}[z'_{ij}] \sigma_a
\]

(A.53)

Combining these results and substituting into Equation (A.51), now

\[
\mathbb{E}[z'_{ij} a_{is} \dot{g}'_{jis}] = \mathbb{E}[z'_{ij} a_{is} \dot{g}'_{jis}] \tilde{\beta} + \frac{N - 2}{N - 1} \sigma_a (\mathbb{E}[z'_{ij} X_{is}] \delta_2 + \mathbb{E}[z'_{ij}] \gamma_2)
\]

\[
= \mathbb{E}[z'_{ij} a_{is} \dot{g}'_{jis}] \tilde{\beta} + \frac{N - 2}{N - 1} \sigma_a \mathbb{E}[z'_{ij} b^1_{jis}] \begin{bmatrix}
\delta_2 \\
\gamma_2
\end{bmatrix}
\]

(A.54)

Next, assume there exists \( \theta_1 = (\tilde{\beta}, \delta_2, \gamma_2) \) and \( \theta_1' = (\tilde{\beta}', \delta'_2, \gamma'_2) \). Further, let \( \sigma_a^2 \) and \( (\sigma_a')^2 \) both be finite.

From Equation (A.54), it must be true that

\[
0 = \mathbb{E}[z'_{ij} a_{is} \dot{g}'_{jis}] (\tilde{\beta} - \tilde{\beta}') + \frac{N - 2}{N - 1} \left( \sigma_a^2 \mathbb{E}[z'_{ij} b^1_{jis}] \begin{bmatrix}
\delta_2 \\
\gamma_2
\end{bmatrix} - (\sigma_a')^2 \mathbb{E}[z'_{ij} b^1_{jis}] \begin{bmatrix}
\delta'_2 \\
\gamma'_2
\end{bmatrix} \right)
\]

(A.55)
From above, $\tilde{\beta}$ is identified, and thus $(\tilde{\beta} - \tilde{\beta}') = 0$. Therefore,

$$0 = \mathbb{E}[z_{ijs}b_{ijs}^2] \left( \sigma_a^2 \begin{bmatrix} \delta_2 \\ \gamma_2 \end{bmatrix} - (\sigma_a)^2 \begin{bmatrix} \delta'_2 \\ \gamma'_2 \end{bmatrix} \right)$$

(A.56)

The second rank condition implies that there exists some $(m + 1) \times l$ matrix $\mathbf{A}_1$ such that $\mathbf{A}_1 \mathbb{E}[z_{ijs}'b_{ijs}^2]$ is of rank 2$m$. Therefore, $(\mathbf{A}_1 \mathbb{E}[z_{ijs}'b_{ijs}^2])^{-1}$ exists and

$$0 = \left( \sigma_a^2 \begin{bmatrix} \delta_2 \\ \gamma_2 \end{bmatrix} - (\sigma_a)^2 \begin{bmatrix} \delta'_2 \\ \gamma'_2 \end{bmatrix} \right)$$

(A.57)

Accordingly, $\delta_2$ and $\gamma_2$ are identified up to the scale factor $\sigma_a^2$.

**Step 2:** Scale identification of $\delta_3$ Multiply Equation (27) by $z'_{ijs}a_{is}$. So,

$$z'_{ijs}a_{is}\hat{g}'_{ijs} = z'_{ijs}a_{is}g_{ij}^{ijs}\tilde{\beta} + z'_{ijs}a_{is}X_{is}\hat{X}_{js}^{i}\delta_1 + z'_{ijs}a_{is}X_{is}\hat{a}_{js}^{i}\delta_2 + z'_{ijs}a_{is}^2\hat{X}_{js}^{i}\delta_3 + z'_{ijs}a_{is}^2\hat{a}_{js}^{i}\delta_4 + z'_{ijs}a_{is}\hat{X}_{is}^{i}\gamma_3 + z'_{ijs}a_{is}\hat{a}_{js}^{i}\gamma_4 - z'_{ijs}a_{is}c_{ijs}^{i}$$

(A.58)

Next, take the mean over all $SN/(N - 1)$ observations. So,

$$\frac{1}{SN(N - 1)} \sum_{s=1}^{S} \sum_{i=1}^{N} \sum_{j \neq i} z'_{ijs}a_{is}\hat{g}'_{ij} = \frac{1}{SN(N - 1)} \sum_{s=1}^{S} \sum_{i=1}^{N} \sum_{j \neq i} (z'_{ijs}a_{is}g_{ij}^{ijs}\tilde{\beta} + z'_{ijs}a_{is}X_{is}\hat{X}_{js}^{i}\delta_1 + z'_{ijs}a_{is}^2\hat{X}_{js}^{i}\delta_3 + z'_{ijs}a_{is}^2\hat{a}_{js}^{i}\delta_4 + z'_{ijs}a_{is}\hat{X}_{is}^{i}\gamma_3 + z'_{ijs}a_{is}\hat{a}_{js}^{i}\gamma_4 - z'_{ijs}a_{is}c_{ijs}^{i})$$

(A.59)

By the same argument as in Step 1, $\frac{1}{SN(N-1)} \sum_{s=1}^{S} \sum_{i=1}^{N} \sum_{j \neq i} z'_{ijs}a_{is}g_{ij}^{ijs}$, etc. So, replace the matrices in Equation (A.59) with their probability limits.

$$\mathbb{E}[z'_{ijs}a_{is}g_{ij}^{ijs}] = \mathbb{E}[z'_{ijs}a_{is}g_{ij}^{ijs}\tilde{\beta}] + \mathbb{E}[z'_{ijs}a_{is}X_{is}\hat{X}_{js}^{i}\delta_1] + \mathbb{E}[z'_{ijs}a_{is}X_{is}\hat{a}_{js}^{i}\delta_2] + \mathbb{E}[z'_{ijs}a_{is}^2\hat{X}_{js}^{i}\delta_3] + \mathbb{E}[z'_{ijs}a_{is}^2\hat{a}_{js}^{i}\delta_4] + \mathbb{E}[z'_{ijs}a_{is}\hat{X}_{is}^{i}\gamma_3] + \mathbb{E}[z'_{ijs}a_{is}\hat{a}_{js}^{i}\gamma_4] - \mathbb{E}[z'_{ijs}a_{is}c_{ijs}^{i}]$$

(A.60)

Assumption 6 implies $\mathbb{E}[z'_{ijs}a_{is}c_{ijs}^{i}] = 0$. Further, $\mathbb{E}[z_{ijs}a_{is}X_{is}\hat{X}_{js}^{i}] = \mathbb{E}[z_{ijs}a_{is}X_{is}\hat{X}_{js}^{i}] \mathbb{E}[a_{is}|z_{ijs}, X_{is}, \hat{X}_{js}^{i}] = 0$ and similarly $\mathbb{E}[z_{ijs}a_{is}X_{is}^{i}] = 0$. Independence of $a_{is}$ and $a_{js}$ from each other and from $X$
implies $E[z_{ij}a_{is}X_{is}a'_{js}^i] = E[z_{ij}'X_{is}E[a_{is}|a'_{js},X_{is},z_{ij},X_{is}]] = 0$, and by similar logic $E[z_{ij}a_{is}a'_{js}] = 0$. Further,

$$
[z'_{ij}a_{is}^2a'_{js}] = E[z_{ij}'a_{is}^2a_{js}] - \sum_{k \neq i} E[z_{ij}^i a_{is}^2a_{ks}]
\quad = E[z_{ij}^i a_{is}^2E[a_{js}|a_{is}]] - \sum_{k \neq i} E[z_{ij}a_{is}^2E[a_{ks}|a_{is}]]
\quad = 0
$$

(A.61)

From Assumption 8, it follows that $E[z_{ij}'a_{is}^2\tilde{X}_{js}^i] = E[z_{ij}'\tilde{X}_{js}^i]\sigma_a^2$, where $\sigma_a^2$ is the variance of $a$. Now, substituion of these results into Equation (A.60) yields

$$
E[z_{ij}a_{is}g_{ij}^i] = E[z_{ij}'a_{is}g_{ij}^i] + \tilde{\beta} + E[z_{ij}'\tilde{X}_{js}^i]\sigma_a^2\delta_3
$$

(A.62)

Now, assume there exists some parameter vector $\theta_2 = (\tilde{\beta}, \gamma_3)$ and $\theta'_2 = (\tilde{\beta}', \gamma_3)$. These vectors are associated with finite $\sigma_a^2$ and $(\sigma_a)^2$. So,

$$
0 = E[z_{ij}a_{is}g_{ij}^i](\tilde{\beta} - \tilde{\beta}') + E[z_{ij}'\tilde{X}_{js}^i]\sigma_a^2\delta_3 - (\sigma_a^2)'\delta'_3
$$

(A.63)

Identification of $\tilde{\beta}$ implies $\tilde{\beta} = \tilde{\beta}'$. So,

$$
0 = E[z_{ij}'\tilde{X}_{js}^i]\sigma_a^2\delta_3 - (\sigma_a^2)'\delta'_3
$$

(A.64)

The third rank condition further implies that there exists some $mxl$ matrix $A_2$ such that $A_2E[z_{ij}'\tilde{X}_{js}^i]$ is of full rank $m$. Therefore, $(A_2E[z_{ij}'\tilde{X}_{js}^i])^{-1}$ exists. So,

$$
0 = \sigma_a^2\delta_3 - (\sigma_a^2)'\delta'_3
$$

(A.65)

Accordingly, the parameter vector $\delta_3$ is identified up to the scale factor $\sigma_a^2$.

**Step 3: Scale identification of $\delta_4$**
Finally, multiply Equation (27) by \( z'_{ij}s a_{is} a_{js} \). So,

\[
z'_{ij} s a_{is} a_{js} \hat{g}'_{ij} = z'_{ij}s a_{is} a_{js} \hat{g}'_{ij} + z'_{ij} s a_{is} a_{js} X_is \hat{X}^i_{js} \beta_3 + z'_{ij} s a_{is} a_{js} X_is \hat{a}'_{js} \delta_2 + z'_{ij} s a_{is} a_{js}^2 \hat{X}^i_{js} \delta_3 \tag{A.66}
\]

Next, take the mean over all \( SN(N-1) \) observations. So,

\[
\frac{1}{SN(N-1)} \sum_{s=1}^{S} \sum_{i=1}^{N} \sum_{j \neq i} z'_{ij} s a_{is} a_{js} \hat{g}'_{ij} = \frac{1}{SN(N-1)} \sum_{s=1}^{S} \sum_{i=1}^{N} \sum_{j \neq i} (z'_{ij} s a_{is} a_{js} \hat{g}'_{ij} \beta_3 + z'_{ij} s a_{is} a_{js} X_is \hat{X}^i_{js} \delta_1 + z'_{ij} s a_{is} a_{js}^2 \hat{X}^i_{js} \delta_3 + z'_{ij} s a_{is} a_{js} \hat{a}'_{js} \delta_4
\]

By the same argument as in Step 1, \( \frac{1}{SN(N-1)} \sum_{s=1}^{S} \sum_{i=1}^{N} \sum_{j \neq i} z'_{ij} s a_{is} a_{js} \hat{g}'_{ij} \to p \) \( \mathbb{E}[z'_{ij} s a_{is} a_{js} \hat{g}'_{ij}] \), etc. So, replace the matrices in Equation (A.67) with their probability limits, yielding

\[
\mathbb{E}[z'_{ij} s a_{is} a_{js} \hat{g}'_{ij}] = \mathbb{E}[z'_{ij} s a_{is} a_{js} \hat{g}'_{ij}] \beta_3 + \mathbb{E}[z'_{ij} s a_{is} a_{js} X_is \hat{X}^i_{js}] \delta_1 + \mathbb{E}[z'_{ij} s a_{is} a_{js} X_is \hat{a}'_{js}] \delta_2 + \mathbb{E}[z'_{ij} s a_{is} a_{js}^2 \hat{X}^i_{js}] \delta_3 + \mathbb{E}[z'_{ij} s a_{is} a_{js}^2 \hat{a}'_{js}] \delta_4 + \mathbb{E}[z'_{ij} s a_{is} a_{js} \hat{a}'_{js}] \gamma_3 + \mathbb{E}[z'_{ij} s a_{is} a_{js} \hat{a}'_{js} \gamma_4 - \mathbb{E}[z'_{ij} s a_{is} a_{js} c'_{ij} s] \tag{A.68}
\]

Assumption 6 implies \( \mathbb{E}[z'_{ij} s a_{is} a_{js} c'_{ij} s] = \mathbb{E}[z'_{ij} s a_{is} a_{js}] \mathbb{E}[c'_{ij} s | z_{ij}, a_{is}, a_{js}] = 0 \). Application of Assumption 6 and L.I.E. together imply \( \mathbb{E}[z'_{ij} s a_{is} a_{js} X_is \hat{X}^i_{js}], \mathbb{E}[z'_{ij} s a_{is} a_{js} X_is \hat{a}'_{js}], \mathbb{E}[z'_{ij} s a_{is} a_{js}^2 \hat{X}^i_{js}], \mathbb{E}[z'_{ij} s a_{is} a_{js}^2 \hat{a}'_{js}], \mathbb{E}[z'_{ij} s a_{is} a_{js} \hat{a}'_{js}], \text{and } \mathbb{E}[z'_{ij} s a_{is} a_{js} \hat{a}'_{js}] \) are also zero. Further,

\[
\mathbb{E}[z'_{ij} s a_{is}^2 a_{js} \hat{a}'_{js}] = \mathbb{E}[z'_{ij} s a_{is}^2 a_{js}] - \sum_{k \neq i} \mathbb{E}[z'_{ij} s a_{is} a_{js} a_{ks}]
\]

\[
= \frac{N-2}{N-1} \mathbb{E}[z'_{ij} s a_{is}^2 a_{js}] - \sum_{k \neq i, k \neq j} \mathbb{E}[z'_{ij} s a_{is} a_{js} a_{ks}]
\]

\[
= \frac{N-2}{N-1} \mathbb{E}[z'_{ij} s a_{is}^2 a_{js}]
\]

\[
= \frac{N-2}{N-1} (\sigma^2) \mathbb{E}[z'_{ij} s] \tag{A.69}
\]
Substitution into Equation (A.67) yields

\[ E[z'_{ijs}a_ia_ja_ks\hat{g}_{ijs}] = E[z'_{ijs}a_ia_ja_ks\hat{g}_{ijs}]\tilde{\beta} + \frac{n-2}{n-1}(\sigma_a^2)^2E[z'_{ijs}]\delta_4 \]  

(A.70)

Assume there exist parameter vectors \( \theta_3 = (\tilde{\beta}, \delta_4) \) and \( \theta'_3 = (\tilde{\beta}', \delta'_4) \), with associated \( \sigma_a^2 \) and \( (\sigma'_a)^2 \).

Equation (A.70) thus implies that

\[ 0 = E[z'_{ijs}a_ia_ja_ks\hat{g}'_{ijs}](\tilde{\beta} - \tilde{\beta}') + \frac{n-2}{n-1}(\sigma_a^2)^2E[z'_{ijs}])(\delta_4 - \delta'_4) \]  

(A.71)

Identification of \( \beta \) implies \( (\tilde{\beta} - \tilde{\beta}') = 0 \). Further, the fourth rank condition implies that there exists some \( 1 \times l \) matrix \( A_3 \) such that \( A_3E[z'_{ijs}] \) is of rank 1. Therefore, \( 0 = (\sigma_a^2)^2\delta_4 - ((\sigma'_a)^2)^2\delta'_4 \), and \( \delta_4 \) is identified to up to the scale factor \( \sigma_a^2 \).

\[ \square \]

**Proposition 9**

The prior propositions have provided conditions under which \( \tilde{\beta} \), \( \delta \), and \( \gamma \) are identified. So, I proceed under the assumption that these parameters are identified. I now proceed to show that, conditional on these parameters being identified, \( a_ks \) is identified for all \( j \) as \( s \to \infty \).

First, for any \( i, j, k \),

\[ \tilde{\beta}(\tilde{g}_{jjs} - \tilde{g}_{kis}) = \delta_1 X_is(X_{js} - X_{ks}) + \delta_2 X_is(A_{js} - A_{ks}) + \delta_3 A_is(X_{js} - X_{ks}) \\
\quad + \delta_4 A_is(A_{js} - A_{ks}) + \gamma_3(X_{js} - X_{ks}) + \gamma_4(A_{js} - A_{ks}) - (\tilde{c}_{ijs} - \tilde{c}_{jis}) \]  

(A.72)

Since every element on the right-hand side of Equation (A.72) is bounded, \( (\tilde{g}_{jjs} - \tilde{g}_{kis}) - \tilde{\beta}(\tilde{g}_{jjs} - \tilde{g}_{kis}) \) is also bounded. Therefore, it has finite variance. Note further that it does not depend on \( N \). Note that

\[ \frac{1}{N-1} \sum_{k \neq i} \left( (\tilde{g}_{jjs} - \tilde{g}_{kis}) - \tilde{\beta}(\tilde{g}_{jjs} - \tilde{g}_{kis}) \right) = \hat{g}'_{jjs} - \tilde{\beta}\hat{g}'_{jjs}. \]
Therefore, in the limit, Equation (A.73) becomes

\[
\frac{1}{(N-1)} \sum_{i \neq j} (\dot{g}_{ijs} - \beta \hat{g}_{ijs}) = \frac{1}{(N-1)} \sum_{i \neq j} \left( \delta_1 X_{is} \bar{X}_{js}^i + \delta_2 X_{js} \dot{X}_{is}^i + \delta_3 a_{is} \bar{X}_{js} + \delta_4 a_{is} \dot{a}_{js} + \gamma_3 \dot{X}_{is} + \gamma_4 \dot{a}_{js} - c_{ij} \right)
\]

(A.73)

Finite variance and independence implies that \( \frac{1}{(N-1)} \sum_{i \neq j} \left( \dot{g}_{ijs}^i - \beta \hat{g}_{ijs}^i \right) = \mathbb{E}_{i \neq j} \left[ \left( \dot{g}_{ijs} - \beta \hat{g}_{ijs} \right) \right] + o_p(1) \) for any \( j \). Similarly,

- \( \frac{1}{(N-1)} \sum_{i \neq j} X_{is} \bar{X}_{js} = X_{js} \mathbb{E} [X_{is}] - \mathbb{E} [X_{is}^2] + o_p(1) \)
- \( \frac{1}{(N-1)} \sum_{i \neq j} X_{is} \dot{X}_{js} = a_{js} \mathbb{E} [X_{is}] + o_p(1) \)
- \( \frac{1}{(N-1)} \sum_{i \neq j} a_{is} \bar{X}_{js} = o_p(1) \)
- \( \frac{1}{(N-1)} \sum_{i \neq j} a_{is} \dot{a}_{js} = o_p(1) \)
- \( \frac{1}{(N-1)} \sum_{i \neq j} \dot{X}_{is} = X_{js} - \mathbb{E} [X_{is}] + o_p(1) \)
- \( \frac{1}{(N-1)} \sum_{i \neq j} \dot{a}_{js} = a_{js} + o_p(1) \)
- \( \frac{1}{(N-1)} \sum_{i \neq j} \dot{c}_{ij} = o_p(1) \)

Therefore, in the limit, Equation (A.73) becomes

\[
\mathbb{E}_{i \neq j} \left[ \left( \dot{g}_{ijs} - \beta \hat{g}_{ijs} \right) \right] = \delta_1 (X_{js} \mathbb{E} [X_{is}] - \mathbb{E} [X_{is}^2]) + \delta_2 a_{js} \mathbb{E} [X_{is}] + \gamma_3 (X_{js} - \mathbb{E} [X_{is}]) + \gamma_4 a_{js} + o_p(1)
\]

(A.74)

Rearrangement yields

\[
a_{js} (\gamma_4 + \delta_2 \mathbb{E} [X_{is}]) = \mathbb{E}_{i \neq j} \left[ \left( \dot{g}_{ijs} - \beta \hat{g}_{ijs} \right) \right] - \delta_1 (X_{js} \mathbb{E} [X_{is}] - \mathbb{E} [X_{is}^2]) - \gamma_3 (X_{js} - \mathbb{E} [X_{is}]) + o_p(1)
\]

(A.75)

Now, suppose there exist \( a'_{js} \neq a_{js} \). From Equation (A.75), we see that \( (a'_{js} - a_{js}) (\gamma_4 + \delta_2 \mathbb{E} [X_{is}]) = o_p(1) \). Therefore, \( (\gamma_4 + \delta_2 \mathbb{E} [X_{is}]) \neq 0 \) \( \Rightarrow (a'_{js} - a_{js}) = o_p(1) \) and thus \( a_{js} \) is point identified.
## Appendix B: Supplementary Tables and Figures

### Table A.1: Baseline Balance Across Schools

<table>
<thead>
<tr>
<th></th>
<th>Panel A: Baseline Covariates</th>
<th></th>
<th>Panel B: Baseline Outcomes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>T1</td>
<td>T2</td>
<td>C</td>
</tr>
<tr>
<td>Elected</td>
<td>0.321 (0.056)</td>
<td>0.291 (0.037)</td>
<td>0.240 (0.061)</td>
</tr>
<tr>
<td>Standard 7</td>
<td>0.331 (0.016)</td>
<td>0.311 (0.041)</td>
<td>0.315 (0.029)</td>
</tr>
<tr>
<td>Standard 8</td>
<td>0.346 (0.030)</td>
<td>0.272 (0.034)</td>
<td>0.303 (0.016)</td>
</tr>
<tr>
<td>SC</td>
<td>0.195 (0.057)</td>
<td>0.267 (0.068)</td>
<td>0.285 (0.084)</td>
</tr>
<tr>
<td>ST</td>
<td>0.118 (0.033)</td>
<td>0.175 (0.082)</td>
<td>0.073 (0.028)</td>
</tr>
<tr>
<td>OBC</td>
<td>0.459 (0.063)</td>
<td>0.459 (0.069)</td>
<td>0.423 (0.092)</td>
</tr>
<tr>
<td>Education Aspirations</td>
<td>-0.217 (0.144)</td>
<td>-0.146 (0.105)</td>
<td>-0.201 (0.096)</td>
</tr>
<tr>
<td>Gender Roles</td>
<td>0.135 (0.171)</td>
<td>0.042 (0.183)</td>
<td>0.186 (0.086)</td>
</tr>
</tbody>
</table>

Notes: Robust standard errors in parentheses, clustered by school. Sample is 1319 students in 30 schools.
Table A.2: Baseline Balance Within T2 Schools

<table>
<thead>
<tr>
<th></th>
<th>Participant</th>
<th>Non-Participant</th>
<th>P-value of Balance Test</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Baseline Covariates</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Elected</td>
<td>0.362</td>
<td>0.260</td>
<td>0.135</td>
</tr>
<tr>
<td></td>
<td>(0.055)</td>
<td>(0.037)</td>
<td></td>
</tr>
<tr>
<td>Standard 7</td>
<td>0.244</td>
<td>0.340</td>
<td>0.060</td>
</tr>
<tr>
<td></td>
<td>(0.055)</td>
<td>(0.040)</td>
<td></td>
</tr>
<tr>
<td>Standard 8</td>
<td>0.339</td>
<td>0.242</td>
<td>0.204</td>
</tr>
<tr>
<td></td>
<td>(0.071)</td>
<td>(0.032)</td>
<td></td>
</tr>
<tr>
<td>SC</td>
<td>0.283</td>
<td>0.260</td>
<td>0.779</td>
</tr>
<tr>
<td></td>
<td>(0.084)</td>
<td>(0.074)</td>
<td></td>
</tr>
<tr>
<td>ST</td>
<td>0.205</td>
<td>0.161</td>
<td>0.335</td>
</tr>
<tr>
<td></td>
<td>(0.102)</td>
<td>(0.074)</td>
<td></td>
</tr>
<tr>
<td>OBC</td>
<td>0.433</td>
<td>0.470</td>
<td>0.620</td>
</tr>
<tr>
<td></td>
<td>(0.091)</td>
<td>(0.068)</td>
<td></td>
</tr>
<tr>
<td><strong>Panel B: Baseline Outcomes</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Educational Aspirations</td>
<td>-0.083</td>
<td>-0.174</td>
<td>0.489</td>
</tr>
<tr>
<td></td>
<td>(0.142)</td>
<td>(0.111)</td>
<td></td>
</tr>
<tr>
<td>Gender Roles</td>
<td>0.006</td>
<td>0.058</td>
<td>0.706</td>
</tr>
<tr>
<td></td>
<td>(0.169)</td>
<td>(0.202)</td>
<td></td>
</tr>
</tbody>
</table>

Notes: Robust standard errors in parentheses, clustered by school. Sample is 412 students in 10 T2 schools.
Appendix C: Weighting in the Construction of Peer Means

In the standard setting with binary directed link data, peer weighting is a near-trivial matter and thus construction of peer means is fairly straightforward. In a binary setting, there are four obvious link definitions between individuals $i$ and $j$:

1. An “OUT” link exists if individual $i$ indicates that $j$ is a friend.
2. An “IN” link exists if individual $j$ indicates that $i$ is a friend.
3. An “OR” link exists if either an “OUT” link or an “IN” link exists.
4. An “AND” link exists if both an “OUT” link and an “IN” link exist.

Note that the first two are necessarily directed, while the third and fourth are symmetric. For purposes of the reduced-form analysis in this paper, and to be consistent with the continuous results, I employ the “OUT” definition for binary network links.

Peer weighting is much more complicated when link intensities are continuous, as posited in the structural model developed in this paper. The following general assumptions on all weights that will be maintained throughout. While in principle these weights could be estimated, in order to preserve computational power, I assume that the function is known. Letting $g_{ijs}$ be the intensity of $i$’s link toward $j$, and $g_{jis}$ be the intensity of $j$’s link toward $i$, the following three definitions seem natural

1. “OUT” link weight is $g_{ijs}$
2. An “IN” link weight is $g_{jis}$
3. An “SUM” link weight is $g_{ijs} + g_{jis}$

Once these weight are constructed, they are normalized so that the sum of the weights for a given individual $i$ is one. For purposes of this paper, I employ the “SUM” weight definition for continuous link intensities. Future work will investigate the sensitivity of results to a choice of different weighting functions.
Appendix D: Graphical Reconstruction Algorithm

D.1: Baseline Network Data

To impute the baseline network data, I employ an iterative, single-imputation algorithm. The approach relies upon minimal modeling, and is used only for descriptive purposes and reduced-form results. The basic idea is to use the different network measures to impute the missing ones. Since the pattern is non-monotonic, I specify it via the following steps, as motivated by the discussion in Cameron and Trivedi Chapter 27.5 and 27.6. The algorithm proceeds as follows:

1. For each of the 16 (binary) network measures identified in Table 3, let $L_{ij0}^v$ be the link between $i$ and $j$ at baseline. Where data is missing impute as 0 or 1 with some arbitrary method.

2. For $v = 1, ..., 16$, estimate logit of $L_{ij0}^v$ on $L_{ij0}^w \forall w \neq v$ and $L_{ji0}^v \forall v$.

3. Given the estimated probabilities of a link from the logit estimates in Step 2, impute $L_{ij0}^v$ for each $v$.

4. Iterate Steps 2 and 3 sufficiently to achieve convergence to stationary distribution of networks.

5. Perform analysis using the final imputed values of $L_{ij0}^v$.

D.2: Endline Network Data

Data imputation requires a model. As briefly discussed in the main body of text, given that I have a model of network formation, it makes sense to use this model to impute missing network data. Accordingly, rather than using the logit method as for baseline, I impute network data by an iterative EM algorithm. The algorithm proceeds as follows:

1. For the continuous network measure $L_{ij1}$, impute missing data arbitrarily.

2. Using the imputed data, estimate the parameters of the network formation model. Recover moments of distributions of unobservables $a_{is}$, $M_{is}$ and $c_{is}$

3. Using the implied distributions of the unobserved variables $a_{is}$, $M_{is}$ and $c_{is}$, impute missing data. This step requires iteration of the network-formation process until an equilibrium consistent with the First-Order Conditions is reached.
4. Iterate Steps 2 and 3 sufficiently to reach convergence to a stable distribution of parameters and networks.

5. Take draws from this stable distribution. Construct point and variance estimates that properly adjust for imputation error.

**Endline Outcome Data**

Equation (7) provides the model’s equation whereby outcomes are determined conditional on networks, observed variables, and unobserved $A$. Data imputation here proceeds from the imputed full networks as follows:

1. Take $m$ draws from the converged distribution of networks and estimated parameters $A$.

2. For each draw

   2.1 Arbitrarily impute missing outcome data.

   2.2 Using imputed outcomes as well as the draw of networks and $A$, estimate the parameters of Equation (7).

   2.3 Using implied distribution of residuals from Step 2.2, impute outcome data where missing

   2.4 Iterate Steps 2.2 and 2.3 sufficient to reach convergence to stationary distribution. Take one draw from this distribution.

3. Given the final parameter values in Step 2.4, construct point and variance estimates that properly adjust for imputation error.