Trade liberalization and markup divergence: a general equilibrium approach.*

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Abstract

We propose a general equilibrium trade model which allows to capture intersectoral markup divergence under trade liberalization without assuming collusion in the non-traded sector. The main driving forces of our results are general equilibrium effects and variable markups. Using a new class of preferences, which we refer to as implicitly additive, we show how trade liberalization can lead to falling (rising) markups in traded (non-traded) sector.

1 Introduction

General equilibrium models which capture both product differentiation and intersectoral reallocation effects (Behrens and Murata, 2012; Bernard et al., 2007; Hsieh and Klenow, 2009) are a hot topic in modern international trade literature. Among other things, this models potentially allow to study specific features of markups’ behavior in different sectors under trade liberalization. However, despite growing interest to variable markups in trade from both empirical (Syverson, 2007; Martin, 2012) and theoretical (Feenstra and Weinstein, 2010; Ottaviano et al., 2002; Kichko et al., 2014) perspectives, surprisingly little has been done to bring this two strands of literature together.

There is widespread belief that trade liberalization in manufacturing leads to a drop in the markups for manufacturing goods, and a hike in the markups for services. In this paper, we address this issue using multi-sectoral monopolistic competition. More specifically, we develop a two-sector

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trade model based on monopolistic competition with variable elasticity of substitution. Both sectors produce differentiated goods, one of them being tradable (manufacturing) and the other one is non-tradable (services). The model yields clear-cut results and captures the aforementioned markup divergence.

We work with a fairly natural and rich sub-class of homothetic preferences, which we call *implicitly additive preferences*. These preferences combine two features which render the model highly tractable: (i) simple behavior of elasticity of substitution reminiscent of the one under additive preferences (see Zhelobodko et al., 2012), and (ii) well-defined sectoral price indices. Standard CES preferences belong to implicitly additive preferences as a special case. However, working with CES is inappropriate for our purpose, the reason being that in this case markups are well-known to remain invariant to trade liberalization.

Our main findings may be summarized as follows. First, we show that changes in the manufacturing price index are the only channel through which trade liberalization affects non-traded sector. Furthermore, we find a simple conditions on the demand side under which a drop in manufacturing prices leads to exit of some firms in the non-traded sector and relaxes toughness of competition among the remaining firms.

Second, we show that when the number of firms in the manufacturing sector is given, both domestic and foreign markups shrink in response to additional entry. Trade liberalization drives domestic (foreign) markup downwards (upwards). In other words, the entry effect and the direct trade liberalization effect on domestic markups work in the same direction, whereas for foreign markups these two effects are opposing.

Third, we provide comparative statics results of stable free-entry equilibria with respect to trade cost. We show that when iceberg-cost effect dominates the price-index effect, trade liberalization invites more entry to the manufacturing sector. More importantly, *a reduction in trade costs leads to a drop (hike) in domestic markups for manufacturing goods (services)*. A standard explanation of markup divergence between manufacturing and services, which has far-fetched policy implications, is that trade liberalization somehow fosters collusion between service-producing firms. In this paper, we argue that this phenomenon may arise due to solely general equilibrium effects.

In addition, we explore the properties of implicitly additive preferences and find them to be a flexible tool of incorporating variable markups into international trade models.

The paper is organized as follows. Section 2 describes the model. Section 3 focuses on the equilibrium in non-traded sector, while Section 4 studies free-entry equilibrium in traded sector and explores the consequences of trade liberalization. Section 5 concludes.

### 2 The model

We develop a general equilibrium model of trade between two countries, $H$ and $F$. Countries are assumed to be symmetric in population.
The economy of each country involves two sectors. One sector produces a traded good, while the other one produces a non-traded good. Both goods are differentiated. To be precise, each of the two goods is presented by a continuum of varieties.

2.1 Preferences

In this paper, we work with homothetic preferences. The reason for this is that in this case price indices are well-defined. As shown in Section 3, this makes the analysis of interaction between traded and non-traded sectors quite intuitive and straightforward.

Furthermore, since we seek to reach a prudent balance between generality and tractability, we choose to work with a special, but still fairly rich subclass of homothetic preferences. To define this class, we use the concept of elasticity of substitution.

It is shown in Zhelobodko et al. (2012) that under symmetric additive preferences elasticity of substitution $\bar{\sigma}(x_i, x_j, x)$ between varieties $i$ and $j$ is independent of the remaining consumption pattern $x$, given that both varieties are consumed in equal volumes. Thus, if $x_i = x_j = x$, we have

$$\bar{\sigma}(x, x, x) = \sigma(x),$$

which shows that additive preferences exhibit very simple behavior of elasticity of substitution. Since the nature of substitutability across varieties is a key demand-side feature in imperfect competition models, the aforementioned property explains, at least in part, why additive preferences are so popular (Krugman, 1979; Behrens and Murata, 2007; Zhelobodko et al., 2012, Simonovska, 2010). However, non-CES additive preferences are well-known to be non-homothetic, hence, they do not induce a well-defined price index.

In order to combine both desirable properties mentioned above, we restrict ourselves to such homothetic preferences for which there exist a function $s(x, u)$, such that

$$\bar{\sigma}(x, x, x) = s(x, u(x)).$$

In (1), $u(x)$ is the consumption index satisfying the standard properties: it is increasing, strictly quasi-concave and positive homogeneous of degree 1 in $x$.

Moreover, observe that at a symmetric consumption pattern given by $x = x1_{[0,N]}$, where $N$ is the mass of available varieties, we have

$$u(x) = x\nu(N), \quad s(x, u(x)) = s(x, x\nu(N)).$$

On the other hand, as shown by Parenti, Ushchev and Thisse (2014), when preferences are homothetic, $\sigma$ depends solely on $N$ at a symmetric consumption pattern. Combining this with (2)
yields that \( s(x, u) \) is positive homogeneous of degree 0. Hence, (1) boils down to
\[
\bar{\sigma}(x, x, x) = \sigma(x/u(x)). \tag{3}
\]

The intuition behind (3) is as follows. If \( \sigma(z) \) is a decreasing (increasing) function, then a higher consumption level of both varieties makes them worse (better) substitutes, while an increase in the overall level of consumption captured by \( u(x) \) does the opposite. However, under a proportional change in consumption of all varieties the degree of substitutability between any two of them remains the same.

The following proposition yields an alternative characterization of preferences satisfying (3).

**Proposition 1.** A symmetric homothetic preference relationship satisfies (3) if and only if the indifference curve with utility level \( u \) is described by
\[
\int_0^N \theta \left( \frac{x_i}{u} \right) \mathrm{d}i = 1, \tag{4}
\]
where \( \theta(z) \) is some non-negative, increasing, concave and twice continuously differentiable function.

**Proof.** Given \( \theta(z) \), the proof of the “if” part essentially boils down to choosing such \( \sigma(z) \) which satisfies
\[
\frac{1}{\sigma(z)} = r_\theta(z) \equiv -\frac{z\theta''(z)}{\theta'(z)}. \tag{5}
\]

Conversely, to prove the “only if” part, we choose \( \theta(z) \) satisfying (5) for a given \( \sigma(z) \). See Appendix 1 for technical details. Q.E.D.

In what follows, we refer to preferences for which (1) holds as *implicitly additive*, the reason being that they are equivalently defined by (4). Kimball (1995) uses (4) for a production function representation.

### 2.2 Consumers

Each country is endowed with a mass \( L \) of consumers, who share identical homothetic preferences given by
\[
U^k = U(u(x^k, x^{lk}), u_n(x^k_n)), \quad k, l \in \{H, F\},
\]
where \( U \) is the upper-tier utility, \( u \) and \( u_n \) are lower-tier utilities of consuming, respectively, traded and non-traded good, while .

We assume that \( U, u \) and \( u_n \) are strictly increasing, strictly quasi-concave and positive homogeneous of degree 1. We also assume that lower-tier utilities are implicitly additive (see Section
2.1. i.e. there exist increasing and concave functions $\theta$ and $\theta_n$, such that for any $x^{kk}$, $x^{lk}$ and $x^k_n$, $u$ and $u_n$ satisfy

$$\int_0^{N^k} \theta\left(\frac{x_{i,k}^k}{u}\right) \, di + \int_0^{N^l} \theta\left(\frac{x_{j,l}^k}{u}\right) \, dj = 1, \quad \int_0^{N^l} \theta_n\left(\frac{x_{i,n}^k}{u_n}\right) \, di = 1, \quad k, l \in \{H, F\}. \quad (6)$$

As shown in Appendix 2, the inverse demands are given by

$$\frac{\theta'\left(\frac{x_{i,k}^k}{u}\right)}{\mu} = p_{i,k}, \quad \frac{\theta'\left(\frac{x_{j,l}^k}{u}\right)}{\mu} = p_{j,l}, \quad \frac{\theta'\left(\frac{x_{i,n}^k}{u_n}\right)}{\mu_n} = p_n, \quad (7)$$

where $\mu$ and $\mu_n$ are sectoral market aggregates which involve the Lagrange multiplier of the budget constraint, as well as marginal utilities of consuming tradable and non-tradable goods.

2.3 Firms

Because countries are symmetric, we focus on a symmetric outcome. In what follows, we use the following notation:

$$x^H \equiv x^{HH} = x^{FF}, \quad x^F \equiv x^{FH} = x^{HF},$$

$$p^H \equiv p^{HH} = p^{FF}, \quad p^F \equiv p^{FH} = p^{HF},$$

$$N \equiv N^H = N^F.$$

Since we work with a monopolistically competitive setting, no firm can strategically manipulate $\mu$ and $\mu_n$. Thus, at a symmetric outcome profit-maximizing prices are given by

$$p^H = \frac{c}{1 - r_{\theta}\left(\frac{x^H}{u}\right)}, \quad p^F = \frac{c\tau}{1 - r_{\theta}\left(\frac{x^F}{u}\right)}, \quad p_n = \frac{c}{1 - r_{\theta}\left(\frac{x_n}{u_n}\right)}. \quad (8)$$

Combining (7) with (8), we obtain

$$\frac{\theta'\left(\frac{x^H}{u}\right)}{\mu} = \frac{c}{1 - r_{\theta}\left(\frac{x^H}{u}\right)}, \quad \frac{\theta'\left(\frac{x^F}{u}\right)}{\mu} = \frac{c\tau}{1 - r_{\theta}\left(\frac{x^F}{u}\right)}, \quad \frac{\theta'\left(\frac{x_n}{u_n}\right)}{\mu_n} = \frac{c}{1 - r_{\theta}\left(\frac{x_n}{u_n}\right)}.$$

3 Equilibrium in the non-traded sector

In this section, we first study how intrasectoral equilibrium in the non-traded sector behaves in response to shocks in prices on traded goods.
We start with the case when the number of firms in the non-traded sector is given. This allows us to study how entry affects the key sectoral variables.

Using (6) and (8) for a given number of non-traded firms \( N_n \), symmetric equilibrium conditions are given by

\[
P_n u_n = N_n \cdot \frac{c x_n}{1 - r_\theta \left( \frac{x_n}{u_n} \right)} = 1 - \alpha \left( \frac{P}{P_n} \right),
\]

\[
N_n \theta \left( \frac{x_n}{u_n} \right) = 1.
\]

Here \( \alpha(P/P_n) \) is the share of consumers’ expenditure on traded goods, which depends on the relative price. The functional form of \( \alpha(\cdot) \) is determined by the upper-tier utility.

Solving (10) for \( x_n/u_n \) yields

\[
\frac{x_n}{u_n} = \theta^{-1} \left( \frac{1}{N_n} \right).
\]

Since \( \theta \) is an increasing function, it is implied by (11) that relative consumption of each variety of services decreases with growing product variety in the service sector.

Furthermore, combining (8) and (11) we find that the equilibrium markup for the non-traded goods \( m_n \) is given by

\[
m_n \equiv \frac{P_n - c}{p_n} = r_\theta \left[ \theta^{-1} \left( \frac{1}{N_n} \right) \right].
\]

It is implied by (5) and (12) that \( m_n \) decreases in response to additional entry if and only if \( r_\theta(z) > 0 \) or, equivalently, if the elasticity of substitution \( \sigma(z) \) is a decreasing function. The latter condition may be interpreted as follows: an increase in consumption index \( u(x) \) makes varieties more differentiated.

We now come the question how price index for non-traded goods varies with entry. Dividing (9) by \( u_n \) and using (11) yields

\[
P_n = \frac{c N_n \theta^{-1}(1/N_n)}{1 - r_\theta[\theta^{-1}(1/N_n)]}.
\]

As seen from (13) if \( r_\theta(z) \) is increasing or moderately decreasing, additional entry leads to a price drop. Another words, more firms means tougher competition.

Total expenditure for non-traded goods is given by

\[
E(N_n, P) = L \left[ 1 - \alpha(P/P_n) \right].
\]

Combining (13) and (14) we find that \( E \) increases in response to product variety expansion. This result is in the line with the idea of love for variety. Furthermore, \( E \) decreases when traded
goods become cheaper due to substitution effect between tradables and non-tradables.

Finally, (12) and (14) imply that the equilibrium firm size \( q_n \) and operating profit \( \pi_n \) are given, respectively, by

\[
q_n(N_n, P) = \frac{cL}{N_n} \cdot \frac{1 - \alpha(P/P_n)}{1 - r_\theta(1/N_n)},
\]

(15)

\[
\pi_n(N_n, P) = \frac{L}{N_n} \cdot (1 - \alpha(P/P_n)) \cdot r_\theta(1/N_n).
\]

(16)

**Free-entry equilibrium.** Under free-entry the zero-profit condition

\[
\pi_n(N_n, P) = F
\]

(17)

must hold.

Equation (17) has a far-fetched implication: trade liberalization impacts the mass of firms \( N_n \) in the non-traded sector only through variations in price index for traded goods. Moreover, as implied by (12) – (16), all the key variables of the non-traded sector are pinned down by \( N_n \) and \( P \). As a consequence, the whole impact of a reduction in trade cost on the non-traded sector is fully captured by changes in \( P \).

As seen from (16), the following conditions are sufficient for \( \pi_n \) to be a decreasing function in \( N_n \): (A) \( r_\theta'(z) > 0 \), and (B) \( \alpha(P/P_n) \) decreases not too fast. In this case, (17) has a unique solution \( N_n^* \). Plugging \( N_n^* \) into (12) – (15), we determine a unique symmetric free-entry equilibrium.

Observe also that when traded goods get cheaper, \( \pi_n \) is shifted downwards because of tougher competition from the side of the traded sector. As a consequence, if (A) and (B) hold, a decrease in \( P \) results in falling \( N_n^* \). This, in turn, leads to an increase in both \( P_n^* \) and \( m_n^* \), as seen from (12) – (13).

The following proposition is a summary.

**Proposition 2.** Assume that \( r_\theta'(z) > 0 \), as well as that \( \alpha(P/P_n) \) decreases not too fast. Then, a unique symmetric free-entry equilibrium in the non-traded sector exists. Moreover, a reduction in prices on traded goods leads to (i) a decrease in the number of firms in the non-traded sector, (ii) more expensive non-traded goods, and (iii) higher markups for non-traded goods.

Several comments are in order. First, trade liberalization is likely to foster toughness of competition and drives prices for tradables downwards. Intuitively, we should expect that trade liberalization leads to a reduction of \( P \) in our model. Though we postpone a rigorous analysis of this question until section 3.2, we can gain here some intuition on the nature of interaction between the two sectors. To do so, we assume for the moment that \( P \) is exogenous. Proposition 2 then yields a clear-cut prediction of how the non-traded sector responds to a decrease in \( P \): the non-traded goods become more expensive, some firms exit, while the remaining firms enjoy relaxed competition and charge higher markups. In particular, *prices in traded and non-traded sectors move in*
the opposite directions.

Second, what do the assumptions of Proposition 2 mean? As discussed above, \( r'^\theta(z) > 0 \) is a necessary and sufficient condition for entry to generate a pro-competitive effect, which is intuitively plausible. Moreover, this condition has an alternative interpretation solely in terms of the demand-side: higher consumption index fosters product differentiation. That \( \alpha(P/P_n) \) is a moderately decreasing function means low substitutability between traded and non-traded goods. To illustrate, consider the CES upper-tier utility:

\[
U = \left( \beta U^{(\sigma-1)/\sigma} + (1 - \beta) U_n^{(\sigma-1)/\sigma} \right)^{\sigma/(\sigma-1)},
\]

\[0 < \beta < 1 < \sigma.\]

In this case, we have

\[
\alpha \left( \frac{P}{P_n} \right) = \frac{(P/P_n)^{1-\sigma}}{(1 - \beta)/\beta + (P/P_n)^{1-\sigma}}.
\]

Equation (18) implies that, when \( \sigma \) gets closer and closer to 1, the slope of \( \alpha(\cdot) \) is less and less steep. As a consequence, the operating profit (16) is more likely to decrease in \( N_n \) when tradables and non-tradables are poor substitutes. This assumption makes sense, for most non-traded goods – especially services – are likely to be poor substitutes to, say, manufacturing goods, which are typically traded. Thus, in what follows we consider the situation when (A) and (B) hold as a benchmark case.

Third, when (A) and (B) do not hold, multiplicity of free-entry equilibria may arise. In this case, we focus on stable equilibria, i.e. those where

\[
\frac{\partial \pi_n}{\partial N_n} \bigg|_{N_n = N_n^*} < 0.
\]

How \( P \) affects \( \alpha \). Finally, we discuss how changes in \( P \) affect sectoral shares of consumers’ expenditure. Clearly, two effects are at work: (i) the substitution effect between traded and non-traded goods, which drives \( \alpha \) downwards when tradables become more expensive; (ii) the market expansion effect, which arises because an increase in \( 1 - \alpha \) leads to a hike in demand for non-traded goods, inviting more firms to enter the sector of non-tradables. This makes competition tougher and results in a decrease in \( P_n \). Consequently, \( \alpha \) shrinks further. Thus, both effects work in the same direction.

Mathematically, the first effect is captured by direct impact of \( P \) on \( \alpha \) given \( P_n \), while the second effect occurs because a shock in \( P \) triggers a change in \( P_n \), which may be captured as follows: solving (17) for \( N_n \) and plugging the result into (13), we obtain a decreasing function \( P_n = P_n(P) \).
In order to capture the total impact of $P$ on $\alpha$, we define

$$a(P) \equiv \alpha(P/P_n(P)).$$

Because both effects discussed above work in the same direction, it is clear that $a(P)$ decreases with $P$.

4 Equilibrium in the traded sector and trade liberalization

We are now equipped to study the effect of trade liberalization on the whole economy. Sectoral equilibrium conditions in the traded sector are as follows:

$$x^H \frac{r_{\theta}(x^H/u)}{1 - r_{\theta}(x^H/u)} + \tau x^F \frac{r_{\theta}(x^F/u)}{1 - r_{\theta}(x^F/u)} = \frac{F}{cL},$$

(19)

$$N \left( \theta(\frac{x^H}{u}) + \theta(\frac{x^F}{u}) \right) = 1,$$

(20)

$$\frac{\theta'(\frac{z^H}{u}) (1 - r_{\theta}(\frac{z^H}{u}))}{\theta'(\frac{z^F}{u}) (1 - r_{\theta}(\frac{z^F}{u}))} = \frac{1}{\tau},$$

(21)

$$P_u = N \cdot \left( \frac{cH}{1 - r_{\theta}(\frac{z^H}{u})} + \frac{cF}{1 - r_{\theta}(\frac{z^F}{u})} \right) = a(P).$$

(22)

We start with solving (20) - (21) for relative consumption levels $x^H/u$ and $x^F/u$. To simplify notation, we set

$$z^H \equiv \frac{x^H}{u}, \quad z^F \equiv \frac{x^F}{u}.$$

Given the number of firms $N$ in each country, the (20)-locus is a downward-sloping curve on the $(z^H, z^F)$-plane, since $\theta(\cdot)$ is an increasing function. On the other hand, the slope of (21)-locus is positive, which is implied by the second order condition of profit maximization. As a consequence, the two curves have a unique intersection point, which we denote by $(\bar{z}^H(N, \tau), \bar{z}^F(N, \tau))$.

We now study how $\bar{z}^H(N, \tau)$ and $\bar{z}^F(N, \tau)$ vary with number of firms $N$ and trade cost $\tau$. When $N$ increases, the (20)-locus is shifted downwards, while the (21)-locus remains unchanged. Thus, both $\bar{z}^H$ and $\bar{z}^F$ decrease in the number of firms. An intuitive explanation of this result is love for variety: additional entry leads to splitting consumers’ budget over a broader range of varieties. Finally, a higher trade cost leads to a downward shift of the (21)-locus, while the (20)-locus remains unchanged. Thus, $\bar{z}^H$ increases with trade cost, while $\bar{z}^F$ decreases with trade cost.
Observe that markups charged by firms on domestic markets are given by \( r_\theta \left( \bar{z}^H(N, \tau) \right) \), while those on foreign market equal \( r_\theta \left( \bar{z}^F(N, \tau) \right) \). It is implied by the above analysis that additional entry drives both markups downward if and only if \( r_\theta(z) \) is an increasing function. In other words, whether the effect of entry is pro- or anti-competitive is ruled totally by the nature of consumers’ variety-loving behavior. Furthermore, it is worth pointing out that under a given number of firms trade liberalization always shifts domestic and foreign markups in the opposite directions. More precisely, a decrease in \( \tau \) drives domestic (foreign) markups downwards (upwards) if and only if \( r_\theta(z) \) is an increasing function. Otherwise, the result is reversed.

Next, we use (22) to show how price index \( P \) and consumption index \( u \) vary with the mass of firms \( N \) and trade cost \( \tau \):

\[
P = \bar{P}(N, \tau) \equiv cN \cdot \left( \frac{\bar{z}^H(N, \tau)}{1 - r_\theta(\bar{z}^H(N, \tau))} + \frac{\tau \bar{z}^F(N, \tau)}{1 - r_\theta(\bar{z}^F(N, \tau))} \right),
\]

\[
u = \bar{u}(N, \tau) \equiv \frac{a \left( \bar{P}(N, \tau) \right)}{\bar{P}(N, \tau)}.
\]

If the lower-tier utility of consuming tradable goods is CES, then it is readily verified that \( \bar{P}(N, \tau) \) decreases in \( N \) and increases in \( \tau \), while \( \bar{u}(N, \tau) \) displays a mirror image behavior.

It remains to pin down the number of firms \( N \), which is endogenously determined by free entry mechanism. To do so, we use the zero profit condition (19), which now takes the form

\[
\bar{u}(N, \tau) \left[ \bar{z}^H(N, \tau) \frac{r_\theta(\bar{z}^H(N, \tau))}{1 - r_\theta(\bar{z}^H(N, \tau))} + \tau \bar{z}^F(N, \tau) \frac{r_\theta(\bar{z}^F(N, \tau))}{1 - r_\theta(\bar{z}^F(N, \tau))} \right] = \frac{F}{cL}.
\] (23)

How the left-hand side of (23) varies with \( N \) is a priori ambiguous. In particular, multiplicity of equilibria may arise. However, since the left-hand side of (23) is the traded good producer’s operating profit \( \pi(N, \tau) \), it makes sense to focus on “stable” equilibria, i.e. those where \( \partial \pi / \partial N < 0 \) at \( N = N^* \), where by \( N^* \) we denote a solution of (23). As for variations in \( \tau \), their impact on firms’ profits has a twofold nature: trade liberalization (i) reduces costs, which leads to rising profits, and (ii) shifts the price index \( \bar{P}(N, \tau) \), which may result in tougher competition, hence in lower profits. In any case, it is implied by (23) that at any stable equilibrium trade liberalization leads to an increase in the equilibrium mass of firms \( N^* \) if and only if the former effect suppresses the latter, i.e. when \( \partial \pi / \partial \tau < 0 \). If, in addition, \( \bar{P}(N, \tau) \) decreases in \( N \) and increases in \( \tau \), then trade liberalization also drives down the equilibrium value of price index \( P^* \equiv \bar{P}(N^*, \tau) \).

5 Concluding remarks

To be completed.
References


Appendix

Appendix 1. Assume that (4) holds. Then, since the inverse demands are given by (7) (see Appendix 2 for the proof), the elasticity $\eta_i$ of the inverse demand for variety $i$ is given by

$$\eta_i = r_\theta \left( \frac{x_i}{u(x)} \right),$$

where $r_\theta$ is defined by (5). On the other hand, as shown by Parenti, Ushchev and Thisse (2014), for any symmetric preference defined over a continuum of goods the following equality holds:

$$\bar{\sigma}(x_i, x_j, x)|_{x_i=x_j} = \frac{1}{\eta_i}.$$  \hspace{1cm} (25)

Combining (24) with (25) and setting $\sigma(z) \equiv 1/r_\theta(z)$, we obtain (3). This proves the “if” part of Proposition 1.

Proof of Proposition 1.

To prove the “only if” part, assume that (3) holds for a given $\sigma(z)$. It is straightforward to check that an implicitly additive preference with $\theta$ defined by

$$\theta(z) \equiv \int_0^z \exp \left( - \int_1^\zeta \frac{d\zeta}{\xi \sigma(\xi)} \right) d\zeta$$

also satisfies (3) for the same $\sigma(z)$. Referring to the fact that a preference relationship is uniquely determined by the elasticity of substitution completes the proof. Q.E.D.

Appendix 2: deriving inverse demands.

Then consumer’s problem may be written as follows:

$$\max_{u, \bar{x}} u \quad \text{s.t.} \quad \int \limits_0^N p_i x_i \, d\bar{i} = \int \limits_0^N \theta \left( \frac{x_i}{u} \right) \, d\bar{i} = 1.$$  \hspace{1cm} (26)
Setting $z_i \equiv x_i/u$, we reformulate the constraints as

$$
\int_0^N p_iz_i\,di = \frac{1}{u}, \quad \int_0^N \theta(z_i)\,di = 1.
$$

Since maximizing $u$ is equivalent to minimizing $1/u$, we come to the following equivalent reformulation of problem (26):

$$
\min_{z_i} \int_0^N p_i z_i\,di \quad \text{s.t.} \quad \int_0^N \theta(z_i)\,di = 1. \tag{27}
$$

The first-order conditions for (27) are given by $p_i = \lambda \theta'(z_i)$, which immediately implies (7). Q.E.D.