Production Flexibility and Capacity Investment under Demand Uncertainty

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Motivation

- Many firms nowadays face high demand volatility:
  - influences desirability to invest in production capacity
  - influences choice of capacity level
  - raises the value of being able to adapt production decision

→ Capacity Size

→ Flexibility
Production Flexibility is crucial for Companies:

- **Car Industry**: Credit crunch recession $\Rightarrow$ significant drop in demand $\Rightarrow$ companies had to downscale production $\Rightarrow$ low utilization rates

**Figure 1: Articles, the Guardian, 2009**

- **LCD industry**: During initial stage producing at full capacity. Later on competition on the supply side led to overcapacity.
Research Question

Optimal Investment Strategy of Firm

- Optimal investment timing
- Capacity size
- Optimal output rate

→ Production Flexibility

⇓

Impact on Investment decision
Outline

- Introduction
- Literature
- Model
- Solution
- Results
  - Utilization Rate
  - Capital vs. Labor
  - Value of Flexibility
- Summary/Conclusion
Model Setup

- Price setting monopolist
- Demand uncertainty
- Continuous time setting
- **Production Flexibility**:
  - production can fluctuate over time
  - between zero and capacity level
  - no adjustment costs
- One time investment $\rightarrow$ capacity size
- Solution method:
  - Dynamic programming approach
Main Aspects

→ Optimize the

● Timing and
● Size of investment
● Output rate

for investment in flexible and inflexible capacity, respectively.

→ Analyze investment in flexible capacity:
  ○ Utilization rate
  ○ Capital vs. labor intensive industries

→ Compare the two investment strategies (flexible vs. inflexible) regarding timing and size.
Operations Strategy literature:

irreversible investment
uncertain future rewards
leeway about timing

⇒ Real Options theory

Assumed either flexibility in timing or flexible technology:
  - Different types of flexibility: He & Pindyck (1992), Chronopoulos et al. (2011)
  - Closest related: Dangl (1999)
**Literature (2)**

- **Production Economics:**
  - Bengtsson (2001): relates RO literature to manufacturing flexibility
Model - Production Quantity

- Inverse demand function (linear)
  \[ p(q_t, t) = \theta_t - \gamma q_t \]

- \( \theta_t \) follows the geometric Brownian motion
  \[ d\theta_t = \alpha \theta_t dt + \sigma \theta_t dW_t \]

- Production quantity at time \( t \), \( q_t \), price \( p \), \( \gamma > 0 \)
- Variable unit production cost \( c \)
- In case of unlimited capacity \( \Rightarrow \) optimal production quantity:
  \[ q_t^* = \arg\max_{q(t)} [(p - c)q_t] = \max\left(0, \frac{\theta_t}{2\gamma} - \frac{c}{2\gamma}\right) \]
Model - Flexible Capacity

- Capacity \( K \)
- Production cannot exceed capacity: \( 0 \leq q_t \leq K \)
  
  \[
  q_t' = \min(q_t^*, K)
  \]
  
  depending on \( \theta_t \).

- Capacity holding cost \( c_hK \)

- Profit flow at time \( t \) equals

\[
\pi(\theta_t, K) = \begin{cases} 
-c_hK & \text{for } 0 \leq \theta_t < c \\
\frac{(\theta_t-c)^2}{4\gamma} - c_hK & \text{for } c \leq \theta_t < 2\gamma K + c \\
(\theta_t - \gamma K - c - c_h)K & \text{for } \theta_t \geq 2\gamma K + c
\end{cases}
\]

- Firm chooses capacity size \( K \) at moment of investment.
- Investment cost: \( I(K) = \delta K^\lambda, \lambda < 1 \) (robustness check: \( \lambda > 1 \))
Solution Method

- Project value $V(\theta, K)$ satisfies

\[
V(\theta_t, K) = \Pi(\theta_t, K)dt + E \left[ V(\theta_t + d\theta_t, K)e^{-rdt} \right]
\]

with constant discount rate $r$.

$$\Rightarrow$$

\[
V_{\text{flex}}(\theta, K) = \begin{cases} 
L_1(K) \theta^{\beta_1} - \frac{c_h K}{r} & \text{for } 0 \leq \theta < c \\
M_1(K) \theta^{\beta_1} + M_2 \theta^{\beta_2} \quad & \\
\frac{1}{4\gamma} \left[ \frac{\theta^2}{r-2\alpha-\sigma^2} - \frac{2c\theta}{r-\alpha} + \frac{c^2}{r} \right] - \frac{c_h K}{r} & \text{for } c \leq \theta < 2\gamma K + c \\
N_2(K) \theta^{\beta_2} + \frac{K}{r-\alpha} \theta - \frac{K(K\gamma+c+c_h)}{r} & \text{for } \theta \geq 2\gamma K + c
\end{cases}
\]

$\beta_1$ ($\beta_2$) is the positive (negative) root of the quadratic polynomial

\[
\frac{1}{2}\sigma^2 \beta^2 + \left( \alpha - \frac{1}{2}\sigma^2 \right) \beta - r = 0.
\]
Solution Method (2)

(1) Optimal capacity choice $K^*(\theta)$:
$$\max_{K(\theta)} V(\theta, K(\theta)) - cK(\theta) \text{ for every } \theta$$

(2) Optimal investment threshold $\theta^*$
Value of Waiting $F(\theta) = \text{Value of Investing } V(\theta) - cK(\theta)$
- waiting region $\theta < \theta^*$
- stopping region $\theta > \theta^*$
Results (Optimal Investment in Region II)

Figure 2: Investment Strategy (Optimal Capacity $K^*(\theta)$ and Production Quantity $q^*(\theta)$). Parameter values: $\sigma = 0.15$, $\alpha = 0.02$, $r = 0.1$, $\gamma = 1$, $c = 100$, $c_h = 100$, $\delta = 1000$ and $\lambda = 0.7$. 
Results (Optimal Investment in Region II)

Figure 2: Investment Strategy (Optimal Capacity $K^*($θ$)$ and Production Quantity $q^*($θ$)$). Parameter values: $\sigma = 0.15$, $\alpha = 0.02$, $r = 0.1$, $\gamma = 1$, $c = 100$, $c_h = 100$, $\delta = 1000$ and $\lambda = 0.7$. 
Results (Optimal Investment in Region III)

Figure 3: Investment Strategy (Optimal Capacity $K^*(\theta)$ and Production Quantity $q^*(\theta)$). Parameter values: $\sigma = 0.05$, $\alpha = 0.002$, $r = 0.1$, $\gamma = 1$, $c = 100$, $c_h = 100$, $\delta = 1000$ and $\lambda = 0.7$. 
Results (Optimal Investment in Region III)

Figure 3: Investment Strategy (Optimal Capacity $K^*(\theta)$ and Production Quantity $q^*(\theta)$). Parameter values: $\sigma = 0.05$, $\alpha = 0.002$, $r = 0.1$, $\gamma = 1$, $c = 100$, $c_h = 100$, $\delta = 1000$ and $\lambda = 0.7$. 
Results (Capacity Utilization)

Effect of increasing uncertainty on the utilization at moment of investment:

\[ u := \frac{q^*(\theta^*)}{K^*(\theta^*)} \]

Table 1: Investment strategy with the occupation rate (Parameter values: \(\alpha = 0.02, r = 0.1, \gamma = 1, c = 100, c_h = 100, \delta = 1000\) and \(\lambda = 0.7\))

<table>
<thead>
<tr>
<th>(\sigma)</th>
<th>(\theta^*)</th>
<th>(K^<em>(\theta^</em>))</th>
<th>(q^<em>(\theta^</em>))</th>
<th>(u)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>489.78</td>
<td>223.66</td>
<td>194.89</td>
<td>87.14%</td>
</tr>
<tr>
<td>0.15</td>
<td>857.15</td>
<td>747.19</td>
<td>378.57</td>
<td>50.67%</td>
</tr>
<tr>
<td>0.2</td>
<td>3726.02</td>
<td>16175.4</td>
<td>1813.01</td>
<td>11.21%</td>
</tr>
</tbody>
</table>
Robustness (Convex Investment Cost)

Table 2: Comparing Investment Strategies with Concave ($\lambda < 1$) and Convex ($\lambda > 1$) Investment Cost Structure (Parameter values: $\alpha = 0.02$, $r = 0.1$, $\gamma = 1$, $c = 100$, $c_h = 100$ and $\delta = 1000$)

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$\sigma$</th>
<th>$\theta^*$</th>
<th>$K^<em>(\theta^</em>)$</th>
<th>$q^<em>(\theta^</em>)$</th>
<th>$u$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7</td>
<td>0.1</td>
<td>489.78</td>
<td>223.66</td>
<td>194.89</td>
<td>87.14%</td>
</tr>
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<td>0.7</td>
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<td>3726.02</td>
<td>16175.4</td>
<td>1813.01</td>
<td>11.21%</td>
</tr>
<tr>
<td>1.1</td>
<td>0.1</td>
<td>763.308</td>
<td>287.631</td>
<td>331.654</td>
<td>100%</td>
</tr>
<tr>
<td>1.1</td>
<td>0.15</td>
<td>1401.02</td>
<td>866.889</td>
<td>650.508</td>
<td>75.04%</td>
</tr>
<tr>
<td>1.1</td>
<td>0.2</td>
<td>7502.41</td>
<td>19461.3</td>
<td>3701.21</td>
<td>19.02%</td>
</tr>
</tbody>
</table>
## Capital vs. Labor intensive

<table>
<thead>
<tr>
<th></th>
<th>Flexible</th>
<th>Numerical example</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Labor intensive</strong></td>
<td>Capacity Utilization</td>
<td>low</td>
</tr>
<tr>
<td>$(c_h &lt; c)$</td>
<td></td>
<td>$c_h = 0, c = 100$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\theta^* = 274.7$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$K^<em>(\theta^</em>) = 263.0$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$q^<em>(\theta^</em>) = 87.4$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$ur = 33%$</td>
</tr>
<tr>
<td><strong>Capital intensive</strong></td>
<td></td>
<td>high</td>
</tr>
<tr>
<td>$(c_h &gt; c)$</td>
<td></td>
<td>$c_h = 100, c = 0$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\theta^* = 279.5$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$K^<em>(\theta^</em>) = 118.3$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$q^<em>(\theta^</em>) = 139.8$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$ur = 100%$</td>
</tr>
</tbody>
</table>

Figure 4: Capacity utilization
Results (Impact of Capacity Holding Cost)

Figure 5: Effect of increasing capacity holding cost $c_h$ on optimal capacity size $K^*(\theta^*)$ and investment threshold $\theta^*$. ($c = 100$)
Results (Impact of Capacity Holding Cost) (2)

Figure 6: Effect of increasing capacity holding cost $c_h$ and capacity size $K$ on the investment threshold $\theta^*$ when capacity size is a constant parameter. ($c = 100$)
Results (Impact of Production Flexibility)

Figure 7: Impact of Flexibility and Uncertainty on the Investment Strategy.
Parameter values: $\alpha = 0.02$, $r = 0.1$, $\gamma = 1$, $c = 100$, $c_h = 100$, $\delta = 1000$ and $\lambda = 0.7$. 
Results (Impact of Production Flexibility) (2)

Figure 8: Impact of Flexibility and Uncertainty on the Investment Strategy. Parameter values: $\alpha = 0.02$, $r = 0.1$, $\gamma = 1$, $c = 200$, $c_h = 0$, $\delta = 1000$ and $\lambda = 0.7$. 
Robustness (Impact of Production Flexibility)

Investment cost structure $I(K) = \delta K^\lambda$ with

$\lambda < 1 \Rightarrow$ concave or

$\lambda > 1 \Rightarrow$ convex

Figure 9: Impact of Flexibility and Uncertainty on the Investment Strategy for Convex Investment Cost.
Conclusions

- Our paper extends Real Option theory by considering, besides timing decision, flexibility in production and capacity choice.

- In contrast to earlier literature we show that two investment cases need to be taken into account.

- Main Results:
  - Utilization is decreasing strikingly in demand uncertainty.
  - Capital vs. labor intensive industry
  - Impact of capacity holding cost on investment strategy.
  - Comparison of investment strategy for flexible and inflexible firm.
    - Two contrary effects as to the timing of the investment.
    - Flexible firm invests in higher capacity. Capacity difference (flexible vs. inflexible firm) increases in uncertainty.
Extensions

- Competitive setting
- Different sources of uncertainty
- Case Study
- ...