Optimal Taxation of Foreign Assets and Production Factors in a Small Open Economy*

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Abstract
This paper studies the dynamic Ramsey taxation of foreign assets in a small open economy model with international mobility of capital. The benevolent government of the small open economy chooses taxes on factors of local production and net foreign assets to finance an exogenous stochastic stream of government expenditures. The government can fully commits to future policies and balanced budgets, and can observe net foreign assets at the aggregate level but not the resident household’s individual accounts. The paper finds that the optimal tax scheme replicates a flat tax rate on households total assets, no matter whether they are held at home or abroad. Furthermore, numerical simulations show that it is optimal to tax foreign assets and physical capital rents to insure against fiscal shocks, while the expected burden of fiscal expenditures is mostly borne by labor income. The welfare gains of introducing the tax on foreign assets according to the Ramsey policy are quantified between 2.3 percent and 0.4 percent of annual consumption. The results can be related to the bilateral withholding tax agreements that Austria and the United Kingdom signed with Switzerland in the ongoing European debt crises.

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1 Introduction

Recently, proposals for more effective taxation of foreign assets have returned to the political agenda. A number of these proposals have their roots in bilateral and multilateral tax agreements. For example, the OECD records a peak in the number of bilateral Tax Information Exchange Agreements (TIEA) signed after the financial crises. The stock of 47 TIEAs by the end of 2008 has increased by a flow of 197 agreements in 2009, 200 in 2010, and 67 in 2011.\footnote{http://www.oecd.org/ctp/exchangeofinformation/taxinformationexchangeagreements.htm.} At the same time, in the course of the ongoing European debt crises, Austria and the United Kingdom made progress in enforcing taxation of foreign assets by signing bilateral withholding tax agreements with Switzerland.\footnote{These agreements allow to tax foreign assets at an aggregate level without breaking the Swiss bank secrecy, and serve as a role model for further agreements with Greece and Italy.}

While the taxation of foreign assets is heatedly debated in politics since budgets are tight, the macroeconomic effects of doing so are hardly studied in a dynamic open economy context. The normative dynamic taxation literature\footnote{With reference to the seminal contribution of Ramsey (1927), this branch of the optimal taxation literature is often labeled Ramsey taxation. A typical setting of the so called Ramsey problem is a benevolent government that commits in period zero to a sequence of distortionary taxes to finance an exogenous stream of nonproductive government expenditures. In contrast to the Mirrleesian approach to optimal taxation (Diamond and Mirrlees, 1971), the set of tax instruments available to the government is taken as given. The optimal policy that solves the Ramsey problem is called the Ramsey plan, or the Ramsey policy.} in economies with capital, initiated by Judd (1985) and Chamley (1986), has primarily focused on the closed economy. Razin and Sadka (1991b), Correia (1996a) and Atkeson, Chari, and Kehoe (1999) have extended the dynamic analysis to the small open economy, but abstracted from fiscal shocks and the feasibility of bilateral tax agreements. This paper aims to shed light into this gap of the Ramsey taxation literature and provides an analysis of the optimal taxation of foreign assets and production factors in a model with fiscal shocks, international mobility of capital and imperfect enforceability of personal income taxation.
The setup implies that taxing household’s asset income on the *residence principle* is not feasible, but with the bilateral tax agreement net foreign assets can be taxed at the aggregate level. This paper finds that most of the closed economy results on optimal capital income taxation found in Zhu (1992) and Chari, Christiano, and Kehoe (1994) carry over to the small open economy if the government has at hand the tax instrument on foreign assets.

First, the paper shows analytically that the tax on foreign assets is an effective tax instrument to replicate a tax on household’s total assets which cannot be observed by the government. Thus, it is not optimal for the government to discriminate between asset held abroad or at home. Second, the quantitative analysis shows that in a stochastic steady-state, governments should optimally use a state-contingent tax on foreign asset income in combination with a tax on physical capital rents to insure against fiscal shocks. So, in the model environment, taxing foreign assets at a fixed rate is not optimal when the economy is exposed to fiscal shocks. In particular, it is not optimal to exempt foreign assets from taxation. Third, the fiscal burden is mostly borne by labor income. As the mobility of assets increases, the economy approaches a mean tax rate on foreign assets close to zero which is consistent with the existing results on Ramsey taxation in the deterministic open economy (Correia, 1996a). Finally, depending on the mobility of assets, the welfare gains of introducing a tax on foreign assets are measured between 2.3 percent and 0.4 percent of annual consumption. Most of these welfare gains are realized in the early periods.

To derive the presented results the paper relies on three major assumptions. (i) The government fully commits to policies announced in the initial period, (ii) the considered economy and its government is atomic compared to the rest of the world, (iii) the government runs a balanced budget in every period. These will be discussed in the following along with two minor technical assumptions that concern the stationarity and the timing of the model.

While there is a consensus in the literature that no government has direct access to a commitment technology, several mechanisms have been
proposed in the literature that can replicate it. Lucas and Stokey (1983) use
the maturity structure of public debt to mimic optimal fiscal policy with
commitment. Persson, Persson, and Svensson (1987) focus on nominal
debt to make monetary policy time-consistent. Finally, Chari and Kehoe
(1990) propose reputational mechanisms to substitute for the commitment
technology. I follow the reduced form approach of the Ramsey taxation
literature assuming that the government has access to a perfect commit-
m ent technology. In particular, I model the full commitment to future
government policies with a recursive contract between the government
and the private agents along the lines of Marcet and Marimon (2011). This
assumption allows me to characterize the optimal policies in a simple re-
cursive form and to use standard function approximation to solve numer-
ically for the Ramsey plan.

An alternative would be to assume that the government has no access
to any form of commitment and mechanisms that replicate it are inoper-
ative. In such a setting, the government can condition its policies only
on fundamental states. Krusell, Quadrini, and Ríos-Rull (1996, 1997) and
Krusell and Ríos-Rull (1999) were among the first to study such Markov-
perfect policies in the context of optimal dynamic taxation. The formal def-
inition of the Markov-perfect equilibrium is given in Maskin and Tirole
(2001), and Klein, Krusell, and Ríos-Rull (2008) provide a general frame-
work in the context of public policy. Debortoli and Nunes (2010) consider
policies under loose commitment where the optimal plans can be periodi-
cally revised by future government with some probability. It is well docu-
mented in the closed economy literature that in a Markov-perfect equilib-
rium capital income taxes will be higher than in a Ramsey equilibrium (see
Klein and Ríos-Rull (2003), for example). Allowing for Markov-perfect
policies would also affect the welfare analysis, because such policies re-
fect the cost of commitment that arise from time-inconsistency problem
in the Ramsey policies.4

4The detailed analysis of a Markov-perfect equilibrium in the context of my model is
part of my future research. In Appendix B of this paper I characterize some deterministic
long-run properties of the Markov-perfect policy.
A small open economy is considered because the focus is on tax instruments that are supported with a bilateral, or possibly a multilateral tax agreement which excludes strategic motives that result in tax competition. Two-country models of dynamic tax competition with international mobility of capital have been studied in Klein, Quadrini, and Rios-Rull (2005) and Luthi (2009). The former paper provides a positive theory that matches the United States’ heavy reliance on capital income taxation compared to Europe. While Klein, Quadrini, and Rios-Rull (2005) focus on Markov-perfect policies, Luthi (2009) follows the Ramsey taxation literature assuming full commitment to government policies. In the paper she quantifies the welfare cost of tax competition in comparison with a fiscal union. However, with a balanced budget, the long-run results for the two regimes coincide. So, I expect that introducing tax competition in my model would not fundamentally change the steady-state analysis, but only affect the transition.

I abstract from government debt because it considerably streamlines the analysis. Allowing for government debt requires an extensive analysis on its own and is beyond the scope of this paper. The Ramsey taxation literature for the closed economy has found that allowing for state-contingent government debt leads to the indeterminacy of optimal capital income taxes (Zhu (1992), Chari, Christiano, and Kehoe (1994)). With state in-contingent debt it can be optimal to build up a buffer stock of assets (Aiyagari et al., 2002). Moreover, I want to concentrate on the question how political leaders should trade off different tax instruments in times of tight budgets. In the context of Ramsey taxation, balanced budget rules have first been studied in Stockman (2001) for the closed economy. Klein and Rios-Rull (2003) have extended the analysis to time-consistent Markov-perfect policies.

Exposing the standard small open economy model to a stochastic environment leads to non-stationarity of consumption and net foreign assets. Schmitt-Grohé and Uribe (2003) compare several proposals that have been brought up to impose stationarity on the stochastic small open economy model. They conclude that all approaches lead to very similar business
cycle dynamics. In the considered environment, I follow their approach of assuming a portfolio adjustment cost for households. This will be enough to impose stationarity, and I will interpret these cost as a measure of the mobility of assets. In the numerical analysis, I will report results for a broad range of these cost, including the case where the cost are negligible.

Finally, a trivial solution to most Ramsey problems with capital accumulation is that the government optimally taxes initial period assets lump-sum up to the present-value of future government expenditures. This avoids distortionary taxation in all other periods. To make the problem interesting, and arguably also more realistic, the literature restricts the Ramsey problem in two different ways. Correia (1996a) and Luthi (2009), for example, restrict the tax rate on capital income to be smaller than some upper bound. The result is that the non-depreciated capital of the initial period is simply taxed at the maximum rate in the following periods and declines thereafter. Chari, Christiano and Kehoe (1994), on the other hand, proposed that the government inherits initial taxes on assets from the past which rules out excessive initial period taxation with a simple timing assumption. I follow the latter approach to avoid arbitrary constraints on excessive taxation in initial periods.

Closely related to my work is the paper of Klein and Ríos-Rull (2003) who study the optimal dynamic taxation of labor and capital income in a stochastic closed economy with a balanced budget constraint. However, these authors focus on the differences between the government’s optimal Ramsey plan and the Markov-perfect policy, while I concentrate on Ramsey taxation in the open economy with imperfect enforceability of personal income taxation. For the Ramsey plan, Klein and Ríos-Rull (2003) find that the tax burden of government expenditures is borne almost completely by labor income whereas capital income taxation is used to finance fiscal expenditure surprises. I show that most of their results have an analog in the open economy if the government has at hand the tax on foreign assets as a policy instrument.

This paper is also related to the literature on optimal capital controls. In a two-period context, Razin and Sadka (1991a) study capital controls
in the presence of capital flight, and Jeanne and Korinek (2010) show that restricting capital inflows in boom times increase welfare in the presence financial market imperfections. Costinot, Lorenzoni, and Werning (2012) study in a two-country model how governments can use a tax on foreign assets to strategically manipulate the dynamic terms-of-trade. And Jeanne (2011) studies how capital controls can be used to permanently undervalue the real exchange rate. Finally, in the international trade context, the paper also relates to the international taxation literature reviewed in Dixit (1985) and Gordon and Hines (2002).

The remainder of this paper is organized as follows. Section 2 presents the benchmark environment of the small open economy without the tax on foreign assets. The associated Ramsey problem and the optimal policies are characterized in Section 3. In Section 4, the tax instrument on foreign assets is introduced. The numerical simulation of the stochastic steady-state moments and the welfare analysis is provided in Section 5, and Section 6 concludes.

2 Benchmark Environment

This section describes a benchmark small open economy where only factors of local production, namely physical capital and labor, can be taxed. It is assumed that initially the government lacks an international tax agreement on foreign assets such that resident household’s unobservable assets cannot be taxed. In Section 4, I will study the macroeconomic effects of introducing such a bilateral tax agreement and compare it to the benchmark economy laid out in this section.

The considered stochastic small open economy model features households, firms and a benevolent government. Firms produce a homogeneous good which is internationally tradable and can be used for private and public consumption as well as savings. Private agents have access to an international capital market which transforms household’s assets into physical capital at no cost. Households are not allowed to change their
residence, so labor is not internationally mobile. In this benchmark econ-
omy, the government taxes labor income of households and physical cap-
ital rents paid by firms at rates $\tau^n(s^t)$ and $\tau^k(s^t)$, respectively. Uncertainty is modeled in a standard way. In each period $t$, an exogenous and stochastic state $s_t$ is realized. The state $s_t$ is driven by a Markov process with finite support. Finally, let $s^t = (s_0, \ldots, s_t)$ denote the history of realized states up to period $t$.

2.1 Households

Aggregate household behavior can be described by a representative household choosing state-contingent consumption, $c(s^t)$, leisure time, $\ell(s^t)$, and the level of assets $a(s^t)$, to

$$\max \sum_{t,s^t} E_{s^t|s_0} \beta^t u(c(s^t), \ell(s^t)),$$

subject to the period-by-period private budget constraint

$$c(s^t) + a(s^t) + q(a(s^{t-1}))$$

$$\leq (1 - \tau^n(s^t)) (L - \ell(s^t)) \frac{w(s^t)}{p(s^t)} + a(s^{t-1}) \left[ 1 + \frac{r(s^t)}{p(s^t)} - \delta \right],$$

with the initial states $a_0$ and $s_0$ given. The domestic demand for final goods is served at price $p(s^t)$, and the asset $a(s^t)$ represents a claim to one unit of the world capital stock in the next period yielding the return $r(s^t)$. Capital is depreciating at the constant rate $\delta$ and asset holdings are subject to the borrowing constraint\(^5\)

$$a(s^t) \geq 0.$$

\(^5\)Imposing the borrowing constraint is a convenient way to avoid a transversality condition on assets. In equilibrium, the borrowing constraint will never be binding. For a similar formulation with transversality conditions see Stockman (2001).
Finally, I introduce a convex portfolio adjustment cost, $q(a)$, for the household along the lines of Schmitt-Grohé and Uribe (2003) to make the model stationary.\footnote{As proposed in Schmitt-Grohé and Uribe (2003), the cost function, $q(a)$ is such that deviations from a long-run asset level are punished at an increasing rate. This imposes stationarity on the foreign asset level. In the numerical analysis, this adjustment cost will be very small.}

\section*{2.2 Production}

The production sector can be summarized by a representative firm who produces the final good, $y(s^t)$, using capital, $k(s^t)$, and labor, $n(s^t)$, with the constant returns to scale technology

$$y(s^t) = f(k(s^t), n(s^t)).$$

The production sector is competitive. The final good is the same across countries and sold at local price $p(s^t)$. The government levies a tax $\tau^k(s^t)$ on capital rents paid by local firms. Profit maximization then implies that the tax-adjusted rental rate of capital, $r(s^t)(1 + \tau^k(s^t))$, and the domestic wage, $w(s^t)$, are given by their value marginal product

\begin{align*}
    r(s^t)(1 + \tau^k(s^t)) &= f_1(k(s^t), n(s^t)) p(s^t) \\
    w(s^t) &= f_2(k(s^t), n(s^t)) p(s^t).
\end{align*}

\section*{2.3 Government}

The government chooses the state contingent tax instruments on the production factors, $\tau^n(s^t)$ and $\tau^k(s^t)$, to finance exogenous government expenditures, $g(s^t)$, in every state of the history $s^t$, such that

$$g(s^t) \leq \tau^n(s^t)(L - \ell(s^t)) \frac{w(s^t)}{p(s^t)} + \tau^k(s^t)k(s^t) \frac{r(s^t)}{p(s^t)},$$
where government expenditures are nonproductive\(^7\) and driven by the exogenous stochastic Markov process

\[
\Gamma(g(s^{t+1})|g(s^t)). \tag{2}
\]

As in Stockman (2001) and Klein and Rios-Rull (2003), I assume that the government cannot issue debt or accumulate assets. This allows me to focus on a state where the government has to insure against fiscal surprises by trading off the available tax instruments.

### 2.4 Markets

Because agents are forbidden to change their residence, the labor market is only domestic. Domestic market clearing requires

\[
n(s^t) = L - \ell(s^t), \tag{3}
\]

where \(L\) denotes the total time endowment of households. The small open economy can run a non-balanced capital and trade account, but is subject to international price equalization

\[
p^* = p(s^t), \quad r^* = r(s^t),
\]

where \(p^*\) and \(r^*\) denote the price of the final good and the rental rate in the international markets, respectively. Henceforth, I will chose the international price of the final good as the numéraire, i.e., \(p^* = 1\).

\(^7\)This paper focuses on the optimal tax policy in response to fiscal shocks and not on the optimal level of public expenditures. Following the tradition of the Ramsey taxation literature in assuming that expenditures are nonproductive is not crucial as long as public consumption is separable from private consumption in household’s preferences. However, the long-run analysis would change if government expenditures enter the production function as an untaxed factor (Correia, 1996a).
2.5 Decentralized Equilibrium

Given the sequence of state-contingent tax policies

\[ \{ \tau^k(s^t), \tau^n(s^t) \} \]

and a world rental rate, \( r^* \), a decentralized equilibrium is a state-contingent sequence of choices

\[ \{ c(s^t), \ell(s^t), k(s^t), a(s^t) \} \]

which is consistent with the borrowing constraint on assets (1), rational expectations about the transition of government expenditures (2), labor market clearing (3), the intratemporal labor-leisure trade-off,

\[ 0 = u_2(c(s^t), \ell(s^t)) - u_1(c(s^t), \ell(s^t)) \left( 1 - \tau^n(s^t) \right) f_2(k(s^t), n(s^t)) , \] (4)

the intertemporal Euler equation

\[ 0 = \beta E_{s^t+1|s^t} u_1(c(s^{t+1}), \ell(s^{t+1})) \left[ 1 + r^* - \delta - q_1(a(s^t)) \right] \]
\[ - u_1(c(s^t), \ell(s^t)) a(s^t) , \] (5)

the international no arbitrage condition

\[ 0 = \frac{f_1(k(s^t), n(s^t))}{1 + \tau^k(s^t)} - r^* \] (6)

and the private budget constraint

\[ 0 = (1 - \tau^n(s^t)) \left( L - \ell(s^t) \right) f_2(k(s^t), n(s^t)) + a(s^{t-1})(1 + r^* - \delta) \]
\[ - \left[ c(s^t) + a(s^t) + q(a(s^{t-1})) \right] , \] (7)
where the initial conditions $s_0$ and $a_0$ are taken as given. The formulation of the decentralized equilibrium for the benchmark environment shows that the government’s fiscal instruments will distort the intratemporal allocation of consumption and leisure, and possibly the international allocation of capital. The intertemporal saving decision of households is not directly affected by the tax policy. This will no longer be true when the government also has at hand the tax on foreign assets, a case that will be discussed in Section 4.

3 Ramsey Problem

The benevolent government of the small open economy chooses its tax policy in period zero to maximize the equilibrium welfare of domestic agents subject to the government budget constraint and the decentralized behavior of agents. More formally, it chooses allocations, $c(s^t), \ell(s^t), k(s^t), a(s^t)$, and tax policy $\tau^k(s^t), \tau^n(s^t)$, to

$$\max \sum_{t,s^t} E_{s^t|s_0} \beta^t u\left(c(s^t), \ell(s^t)\right),$$

subject to Equations (1)-(7) and the government budget constraint

$$g(s^t) \leq \tau^n(s^t) (L - \ell(s^t)) f_2(k(s^t), L - \ell(s^t)) + \tau^k(s^t) k(s^t) r^*, \quad (8)$$
given $s_0$ and $a_0$. A key property of the Ramsey problem is that the one-period forward looking constraint in the private Euler equation (5) introduces a potential for time-inconsistency in the Ramsey plan as described in Kydland and Prescott (1977). Namely, in period zero the government might announce state-contingent tax policies that it will want to reoptimize after entering that state. As is common on the literature, I will simply assume that the government can fully commit to policies announced in the initial period.\(^8\) However, because of the forward-looking constraint

\(^8\)See Barro and Gordon (1983), Lucas and Stokey (1983), Persson, Persson, and Svensson (1987), and Chari and Kehoe (1990) for mechanisms that can substitute for such a
standard dynamic programming cannot be applied. I will rely instead on the more general approach of Marcet and Marimon (2011) to address this issue.

3.1 Primal Formulation

I follow the primal approach to Ramsey taxation as applied in Lucas and Stokey (1983) and Stockman (2001) to substitute out the tax rates from the constraints of the Ramsey problem. This allows me to solve for the optimal allocation directly, and the optimal linear tax system then follows from calculating the taxes rates given the optimal allocation. In particular, the open economy model under consideration has two non-redundant tax instruments that allow me to substitute out two constraints of the Ramsey problem. First, the efficiency condition (4) can be used to reduce the labor income tax, $\tau_n(s^t)$, in the equilibrium private budget constraint (7) to yield

$$0 = u_2(c(s^t), \ell(s^t))(L - \ell(s^t)) + u_1(c(s^t), \ell(s^t))\left[a(s^{t-1})(1 + r^* - \delta) - [c(s^t) + a(s^t) + q(a(s^{t-1}))]\right]. \tag{9}$$

Second, adding private and public budgets from Equations (7) and (8), respectively, using the constant returns to scale property of the domestic production technology, and substituting out the tax on capital rents, $\tau_k(s^t)$, with the international no arbitrage condition (6) results in the domestic resource constraint

$$0 \leq f(k(s^t), L - \ell(s^t)) + (a(s^{t-1}) - k(s^t))r^* - [c(s^t) + a(s^t) - (1 - \delta)a(s^{t-1}) + g(s^t) + q(a(s^{t-1}))], \tag{10}$$

In Appendix B, I also provide some long-run properties of the optimal policies in a Markov-perfect equilibrium where the government cannot commit to future policies.

In the case of time-inconsistency, the Ramsey plan is a time-variant function of the state space $(a, g)$. 

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Compared to the resource constraint in a closed economy, the factor income from abroad, \( (a(s^t-1) - k(s^t)) r^* \), enters the resource constraint as an additional term. As I have reduced labor supply and the two tax instruments with the constraints (3), (4), and (6), and turned the government budget constraint into the resource constraint, the primal formulation of the government’s decision problem reads

\[
\max_{c(s^t), \ell(s^t), k(s^t), a(s^t)} \sum_{t=0}^{\infty} \beta^t u \left( c(s^t), \ell(s^t) \right),
\]

subject to (1), (2), (5), (9) and (10), given \( s_0 \) and \( a_0 \).

### 3.2 Recursive Formulation

The primal formulation of the period zero Ramsey problem in (11) can be formulated along the lines of a recursive contract between the government and agents as proposed by Marcet and Marimon (2011). The recursive form of the associated augmented Lagrangian reads

\[
W(a, \mu, g) = \min_{\gamma, \mu', c, \ell, k, a'} \max_{c, \ell} \left[ u(c, \ell) + \beta E_{s^t|s_0} W(a', \mu', g) - \gamma u_1(c, \ell) a' + \mu a u_1(c, \ell) \left[ 1 + r^* - \delta - q_1(a) \right] \right]
\]

subject to the resource constraint

\[
0 \leq f(k, L - \ell) + (a - k)r^* - \left[ c + a' - (1 - \delta)a + g + q(a) \right],
\]

the private budget constraint

\[
0 = u_2(c, \ell)(L - \ell) + u_1(c, \ell) \left[ a(1 + r^* - \delta) - [c + a' + q(a)] \right],
\]

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10A general treatment of recursive methods for dynamic incentive problems is provided in Messner, Pavoni, and Sleet (2012).
and the transition of the so called pseudo state,

\[ \mu' = \gamma, \]

with \(a_0, \mu_0 = 0, s_0,\) and \(\Gamma(g' | g)\) given. In the recursive formulation, the additional state variable \(\mu\) keeps track of past policy promises of the government and enters as a time varying weight to the reward function of the recursive formulation. Because of the forward-looking constraint, the Ramsey problem in (11) is not recursive on the state space \((a, g)\), but the introduction of the pseudo state variable, \(\mu\), makes the Ramsey problem recursive on the augmented state space, \((a, \mu, s)\). Based on this augmented recursive form, standard recursive methods can be applied to characterize the Ramsey plan. Given the optimal allocation, the optimal tax scheme then follows from the tax wedges implied by the decentralized equilibrium,

\[
\tau^n = 1 - \frac{u_2(c, \ell)}{u_1(c, \ell)f_2(k, L - \ell)}, \\
\tau^k = \frac{f_1(k, L - \ell)}{r^*} - 1.
\]

### 3.3 Production Efficiency

Physical capital, \(k\), only affects the resource constraint of the Ramsey problem’s recursive formulation. Thus, capital is optimally allocated to maximize resources in every period given the remaining variables. The corresponding equilibrium condition reads

\[ 0 = f_1(k, L - \ell) - r^*. \]

Comparing this allocation with the equilibrium no arbitrage condition (13) implies an optimal zero tax on physical capital rents, \(\tau^k = 0\), in all periods. The intuition for this result is similar to the one provided in Gordon (1983). Because the supply of physical capital is perfectly elastic in the small open economy, the entire burden of either a tax on capital rents or labor income
is borne by labor. Therefore, it is efficient to tax labor income directly, because the tax on capital rents induces an additional distortion on the allocation of capital. As a direct consequence of this result, the full burden of fiscal expenditures is borne by labor income which is taxed at the rate,

$$\tau^n = \frac{g}{(L - \ell) f_2 (k, L - \ell)}.$$

Since the labor income tax, $\tau^n$, is fully determined by the government’s budget constraint the government would never deviate from the announced tax policy ex post even if it were allowed to break the recursive contract signed in period zero. This implies that the Ramsey plan is time-consistent and the pseudo state is no longer needed to make the Ramsey policy recursive, $\mu = 0$. Given the optimal state-contingent tax policies described above, the optimal allocation follows immediately from the decentralized equilibrium stated in equations (1)-(6).

## 4 Taxation of Foreign Assets

This section introduces a tax on foreign assets to the benchmark environment presented in Sections 2 and 3. To avoid the rather implausible case of lump-sum taxation in the initial period, I follow the timing assumption proposed by Chari, Christiano, and Kehoe (1994) also applied in Klein and Ríos-Rull (2003), assuming that the tax on foreign assets income, $\tau^x(s^{t-1})$, is predetermined. As is the case for the bilateral withholding tax agreements between Austria, Germany and the United Kingdom with Switzerland, I choose foreign asset income as the relevant tax base.$^{11}$ The government budget constraint then reads

$$g(s^t) \leq \tau^n(s^t) (L - \ell(s^t)) f_2 (k(s^t), L - \ell(s^t)) + \tau^x(s^t)(s^t) r(s^t),$$

$$\quad + \tau^*(s^{t-1}) (a(s^{t-1}) - k(s^{t-1})) r^*.$$

$^{11}$An alternative would be to tax the volume of foreign assets, or apply the domestic rental rate $r(s^t)$ instead of the world rate, $r^*$. This would only result in a level effect for the optimal tax, but not affect the optimal tax scheme in other respects.
Depending on the net foreign asset position, the tax $\tau(x^{s-1})$ can also turn into a subsidy if the capital account is negative.

### 4.1 International No Arbitrage

The introduction of the tax on foreign assets affects the international no arbitrage condition for the capital market,

$$\left(1 - \tau(x^{s-1})\right)r^* = r(s^t). \tag{15}$$

The after-tax rental rate of increasing foreign assets has to be equal to the return of allocation capital to the domestic economy. As a consequence, the private Euler equation of the decentralized equilibrium becomes

$$\begin{align*}
0 &= \left[ \beta E_{s^{t+1}|s^t} u_1 \left(c(s^{t+1}), \ell(s^{t+1})\right) \left[1 + (1 - \tau(x^{s^t}))r^* - \delta - q_1(a(s^t))\right] \\
&\quad - u_1 \left(c(s^t), \ell(s^t)\right) \right] a(s^t), \tag{16}
\end{align*}$$

and the international no arbitrage condition now reads

$$\begin{align*}
0 &= \frac{f_1(k(s^t), n(s^t))}{(1 + \tau^x(s^t))(1 - \tau(x^{s-1}))} - r^*. \tag{17}
\end{align*}$$

Thus, a positive value of $\tau(x^{s-1})$ has the same effect as tax on resident household’s individual asset income and a subsidy on physical capital rents. Most importantly, it is no longer the case that foreign assets can simply escape taxation of the domestic government.

### 4.2 Ramsey Problem

Having the tax on foreign asset as an additional policy instrument allows for a recursive formulation of the augmented Lagrangian on the state space $(a, \mu, \tau^x, g)$. However, the private budget constraint can be used to reduce the tax on foreign assets from the primal formulation of the Ram-
sey problem for all $t \geq 1$. From period one onwards, the resulting Lagrangian is then recursive on the lower dimensional state space, $(a, \mu, g)$ which allows for a direct comparison with the recursive formulation of the benchmark economy. To focus on the stochastic long-run properties, I will abstract from the optimal period zero allocation in the remainder of this section which is not crucial for the analysis. In particular, given the optimal choices from the initial period, $a(s_0)$ and $\mu(s_0)$, the recursive form of the augmented Lagrangian from period one onwards reads

$$W(a, \mu, g) = \min_{\gamma', \mu'} \max_{c, \ell, k, a'} W(a', \mu', g') - \gamma u_1(c, \ell) a'$$

$$+ \mu \left[ u_1(c, \ell) \left( c + a' + q(a) - a q_1(a) \right) - u_2(c, \ell) \left( L - \ell \right) \right]$$

subject to the resource constraint

$$0 \leq f(k, L - \ell) + (a - k)r^* - \left[ c + a' - (1 - \delta)a + g + q(a) \right],$$

and pseudo state transition,

$$\mu' = \gamma,$$

with $a(s_0), \mu(s_0), g(s^1)$, and $\Gamma(g' | g)$ given. Since having the tax on foreign assets as a policy instrument is the same as removing constraints from the benchmark Ramsey problem, welfare must be higher compared to the benchmark economy. Given the optimal allocation, the optimal linear tax scheme for all $t \geq 1$ follows from equation (12), and

$$\tau^k = \frac{f_1(k, L - \ell)}{(1 - \tau^x)r^*} - 1,$$

$$\tau^x = 1 - \frac{c + a' - (1 - \delta)a + q(a) - (1 - \tau^x)(L - \ell)f_2(k, L - \ell)}{ar^*}.$$

---

12The derivation of the primal formulation, as well as the augmented Lagrangian from period zero onwards is delegated to Appendix A. It is a straightforward application of the proposition stated in Stockman (2001).
In period 0, $\tau_0^x$ is inherited from the past.

### 4.3 Optimal Tax on Capital Rents

The equilibrium condition with respect to the optimal allocation of physical capital remains unchanged compared to the benchmark economy and is still given by equation (14). The supply of physical capital is still perfectly elastic, thus it is still inefficient to distort the allocation of capital. However, since the direct tax on foreign assets implicitly subsidizes capital rents, the tax on capital rents should be used to exactly offset the distortion that is created by the tax on foreign asset income,

$$\tau^k = \frac{\tau^x}{1 - \tau^x}.$$

The total tax revenue from foreign assets and physical capital,

$$\tau^k r + \tau^x (a - k) r^* = \tau^k a r,$$

then reveals that total assets, $a$, no matter whether located at home or abroad, are effectively taxed at the same rate, $\tau^k r$.

### 4.4 Optimal Labor Income Tax

The optimality conditions with respect to consumption, $c$, and leisure, $\ell$, can be combined with (12) to derive the optimal labor income tax rate as a function of the optimal allocation and the pseudo state, $\mu$,

$$\tau^n = \frac{u_{11}(c, \ell) f_2(k, L - \ell) - u_{12}(c, \ell) [\gamma a' - \mu[c + a' + qa(a) - aq_1(a)]]}{(1 + \mu) u_1(c, \ell) f_2(k, L - \ell)} + \frac{\mu [u_{21}(c, \ell) f_2(k, L - \ell) - u_{22}(c, \ell)] (L - \ell)}{(1 + \mu) u_1(c, \ell) f_2(k, L - \ell)}.$$
Suppose for a moment that the cross derivative of the utility function is close to zero, then the second term of this expression will be positive. The term

$$\gamma a' - \mu[c + a' + q(a) - aq_1(a)],$$

then determines whether the tax rate on labor income is smaller or bigger than zero. In the stochastic steady-state, $\gamma$ and $\mu$ will take similar values, moreover the portfolio adjustment cost of assets will be small compared to consumption expenditures. This then implies a positive labor income tax in the stochastic steady-state which is in line with the existing literature on state-contingent Ramsey taxation in the closed economy (see Chari, Christiano, and Kehoe (1994), and Klein and Ríos-Rull (2003), for example).

### 4.5 Optimal Tax on Foreign Assets

The optimality condition with respect to the future asset level pins down the government’s Euler equation,

$$0 = \beta E_{g'} \lambda' \left[ 1 + r^* - \delta - q_1(a') \right] - \lambda + (\mu - \gamma) u_1(c, \ell)$$

$$- \beta E_{g'} \mu'a'q_{11}(a')u_1(c', \ell') \right] \lambda'. \tag{19}$$

The comparison with the private Euler equation (16) reveals that the convexity of the cost function, $q_{11}(a)$, influences the long-run level of the optimal tax on foreign assets. Combining the private Euler equation (16) with the government Euler equation (19), the optimal tax on foreign assets in an interior deterministic steady-state can be written as

$$\tau^x = \frac{\mu}{\lambda r^*} u_1(c, \ell) aq_{11}(a).$$

$$^{13}$$For utility functions additively separable in consumption and leisure the argument applies exactly.
This implies that a convex portfolio adjustment cost will result in a positive deterministic long-run tax rate on foreign asset income. The intuition for this result is related to the one provided in Correia (1996b). According to the private Euler equation (5), assets’ long-run elasticity of substitution with respect to the rental rate, \((1 - \tau^*)r^* = r\), is given by,

\[
\varepsilon_{a,r} \equiv \frac{1/\beta - [1 - \delta - q_1(a)]}{aq_{11}(a)}.
\]

Thus, the convex adjustment cost makes assets a less than perfect elastic tax base, and therefore, a more efficient source of tax revenue in the long-run. However, as the portfolio adjustment cost goes to zero, assets become perfectly elastic and the entire burden of either a tax on asset income, capital rents, or labor income is borne by labor. Because the indirect taxation introduces additional distortions, it is efficient to tax labor income directly which implies that the deterministic steady-state tax on foreign assets will go to zero. However, as is similarly shown in Zhu (1992), Chari, Christiano, and Kehoe (1994) and Klein and Rios-Rull (2003) for the capital income tax in the closed economy, the stochastic steady-state tax on foreign assets will fluctuate around this long-run value, because predetermined assets are not perfectly elastic even absent the portfolio adjustment cost.

5 Numerical Analysis

This section provides a quantitative analysis of the two tax regimes studied in the previous sections. Henceforth, I label the benchmark economy without the tax on foreign assets the \(NT\)-Economy, and the economy with the tax on foreign assets the \(T\)-Economy. In a first step, the moments of the stochastic steady-state tax rates will be simulated. In particular, I want to focus on the question how the government of the \(T\)-Economy should optimally implement state-contingent taxes on foreign asset income to insure against surprises in fiscal expenditures. Moreover, I will also derive implications for the optimal labor income tax policy. The calibration of
the model along the lines of Klein and Ríos-Rull (2003) will allow for a comparison with their results derived for the closed economy. In a further step, I run the policy experiment of an unexpected introduction of the tax on foreign assets in the NT-Economy. This experiment will allow to quantify the welfare gains of such a policy change, as well as to study the transitional dynamics.

5.1 Functional Forms and Calibration

The utility function is of the standard constant relative risk aversion (CRRA), and the production function of the standard Cobb-Douglas form,

\[
u(c, \ell) = \left(\frac{c^\eta \ell^{1-\eta}}{1-\sigma}\right)^{1-\sigma} - 1, \quad f(k, n) = Ak^\alpha n^{1-\alpha},
\]

which I take from the benchmark simulation of Klein and Ríos-Rull (2003). The preference parameters \(\eta\) and \(\sigma\) capture the relative taste for leisure and the inverse of the intertemporal elasticity of substitution, respectively. As in Correia (1996a), I assume that the world economy is symmetric to the small open economy and in a deterministic steady-state which implies a world rental rate,

\[r^* = \frac{1}{\beta} - (1 - \delta).\]

Following Schmitt-Grohé and Uribe (2003), the portfolio adjustment cost are of the convex form

\[q(a) = \phi \frac{2}{2}(a - \bar{a})^2,
\]

where \(\bar{a}\) denotes an exogenous long-run target level of assets. The parameter \(\phi\) controls the magnitude and the convexity of these portfolio adjustment cost as well as the stationarity properties of the model. The free total productivity parameter \(A\) is calibrated to normalize the output of

\[\text{For simplicity, I assume that there are no portfolio adjustment cost in the world economy.}\]
the deterministic steady-state of the $NT$-economy with a portfolio adjustment cost parameter $\phi = 0.05$ to unity. The long-run level of assets $\bar{a}$ is calibrated to the deterministic steady-state capital stock of the same economy. As proposed by Klein and Ríos-Rull (2003, Table 2), government expenditures evolve according to a two-state Markov chain with elements, $g \in \{0.184, 0.216\}$, and a symmetric probability state transition matrix with persistence parameter $\rho = 0.835$. The mean government expenditures are then 20 percent of output. With the exception of the parameter $\phi$ which I let vary over a range of small values, the remaining parameters are chosen as listed in Table 1 and taken from Klein and Ríos-Rull (2003).\textsuperscript{15}

The numerical simulation shows that the ergodic set of the stationary equilibrium distributions gets very large for values of $\phi$ below 0.005, so I will limit the numerical analysis to values of $\phi$ above this threshold value. This choice is roughly in line with the value calibrated in Schmitt-Grohé and Uribe (2003, Table 4).

\textbf{5.2 $NT$-Economy Results}

Starting with the benchmark economy where the tax on foreign assets is not available to the government, Table 2 reports the simulated steady-state moments of the tax rate on labor income for the $NT$-Economy. A range of values for the portfolio adjustment cost parameter, $\phi \in \{.05, .01, .005\}$, are considered. The numerical simulation shows that the government optimally sets an average steady-state labor income tax rate of around 31 percent to finance government expenditures. Because the labor tax is mainly

\begin{table}[h]
\centering
\begin{tabular}{lcccccccc}
\hline
Parameter & $A$ & $a$ & $b$ & $\delta$ & $\eta$ & $L$ & $r^*$ & $\sigma$ \\
\hline
Value      & 2.04 & 0.36 & 0.97 & 0.08 & 0.20 & 1.00 & 0.11 & 1.00 \\
\hline
\end{tabular}
\caption{Calibration of parameters}
\end{table}

\textsuperscript{15}The taste for leisure, $\eta$, takes a relatively high value resulting in leisure choices around four fifths of the available time, $L$. Klein and Ríos-Rull (2003) motivate this by the observation that the total time spent on working out of the working age population is around a fifth of the available time.
driven by the government budget constraint, it is not surprising that the correlation with innovations in the government expenditures is positive and close to unity. Finally, because the government has no access to foreign asset taxation and government debt, the only way to finance fiscal surprises is to use the labor income tax. This results in a standard deviation for the simulated labor income tax rate that is slightly above the one of government expenditures.

5.3 \( T \)-Economy Results

Moving on to the case of foreign asset taxation, Table 3 reports the moments of the stochastic steady-state tax rates for the \( T \)-Economy. The table shows that the long-run burden of government expenditures is mostly borne by labor income. However, if the portfolio adjustment cost are relatively high a significant share of expenditures is also financed through taxing foreign asset income and capital rents. Remember that this combination of policies replicates a tax on resident household’s total asset income without distorting the capital allocation in the economy. Not very
surprisingly, the magnitude of foreign asset taxation becomes smaller as the mobility of assets increases. As the portfolio adjustment cost vanish, in the parameterization of the third column of Table 3, the average fiscal burden is almost borne only by household’s labor income.

A further observation is that the standard deviation of the tax on foreign assets is around for times that of government expenditures. At the same time the foreign asset tax is highly positively correlated to government expenditures, thus, unexpected government expenditures are optimally financed through the taxation of foreign asset income and capital rents. In short, this means that the the role of capital income taxation in the closed economy derived in Chari, Christiano, and Kehoe (1994) carries over to the small open economy if the government has at hand the tax on foreign assets.

The labor income tax rate is negatively correlated to the tax on foreign assets as well as innovations in government expenditures. Thus, it is optimal to finance a labor income tax cut with an increases in the tax on foreign assets if a expenditure shock hits the economy. This pattern is more pronounced as the international mobility of assets increases and the economy is more distorted by the sizable labor income tax. In general, these observations are also consistent with the closed economy results reported in Klein and Rios-Rull (2003, Table 7).

Table 4 provides the comparison between the mean steady-state variables for the T-Economy and the NT-Economy. A fist observation is that the government of the T-Economy runs a much higher capital account deficit than that of the NT-Economy. The simple reason being that the positive stochastic steady-state tax on foreign assets distorts resident’s savings in assets downwards compared to the benchmark economy, while the level of physical capital is higher because agents supply more labor. As a direct consequence of the higher capital and labor supply, the T-Economy also produces more final goods. Finally, the mean stochastic steady-state income of resident household’s in the T-Economy falls below the income of the NT-Economy as the labor income tax rate gets higher. This maps also into the asset level and consumption which are both falling with the
5.4 Welfare Analysis

A key property of the Ramsey plan is that the introduction of the tax on foreign asset income increases welfare viewed from the initial period. The informal reason being that the government is provided with an additional tax instrument that it could simply commit to leave aside if it generated welfare losses. Because Table 4 reports that the foreign asset tax depresses steady-state income there must be gains of the tax in the initial periods to make up for losses in steady-state income. This is in line with the results of Chari, Christiano, and Kehoe (1994) and Stockman (2001), who find that the gains from switching to the Ramsey policy are mostly realized in initial periods.

To quantify the value of having the additional tax instrument on foreign assets, I consider the amount \( x \) by which consumption in the \( NT \)-Economy needs to be scaled up in each state of the history \( s^t \) to equate the period zero welfare of the \( T \)-Economy,

\[
U^T(a_0, g_0) = u(c^{NT}(a_0, g_0)(1 + x(a_0, g_0)), e^{NT}(a_0, g_0)) \\
+ \beta E_{g'|g} U^{NT}(a_{1NT}, g_1),
\]

Table 4: Stochastic steady-state comparison.

<table>
<thead>
<tr>
<th>Adjustment cost, ( \phi )</th>
<th>(.05)</th>
<th>(.01)</th>
<th>(.005)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type-Economy</td>
<td>( NT)</td>
<td>( T)</td>
<td>( NT)</td>
</tr>
<tr>
<td>Assets, ( a )</td>
<td>3.25</td>
<td>2.78</td>
<td>3.25</td>
</tr>
<tr>
<td>Capital, ( k )</td>
<td>3.25</td>
<td>3.76</td>
<td>3.25</td>
</tr>
<tr>
<td>Foreign assets, ( (a - k) / y )</td>
<td>0.00</td>
<td>-0.84</td>
<td>0.00</td>
</tr>
<tr>
<td>Labor, ( n )</td>
<td>0.17</td>
<td>0.20</td>
<td>0.17</td>
</tr>
<tr>
<td>Output, ( y )</td>
<td>1.00</td>
<td>1.16</td>
<td>1.00</td>
</tr>
<tr>
<td>Income, ( y + (a - k)r^* )</td>
<td>1.00</td>
<td>1.05</td>
<td>1.00</td>
</tr>
<tr>
<td>Consumption, ( c )</td>
<td>0.54</td>
<td>0.62</td>
<td>0.54</td>
</tr>
</tbody>
</table>
where $\tilde{U}^{NT}(a,g)$ is defined recursively,

$$
\tilde{U}^{NT}(a,g) = u(c^{NT}(a,g)(1 + x(a_0, g_0)), \ell^{NT}(a,g)) + \beta E_{g'} \tilde{U}^{NT}((a^{NT})', g'),
$$

and the superscript denotes the Ramsey plan of the $T$-Economy and the $NT$-Economy, respectively. The same welfare measure has been proposed in Stockman (2001) to compare the welfare losses of different budget rules. For convenience, I choose the initial assets level, $a_0$, to be the mean of the stochastic steady-state asset level in the $NT$-Economy. The amount $x$ can then be interpreted as the welfare gain of unexpectedly introducing the tax on foreign assets to an $NT$-Economy in the stochastic steady-state.

Table 5 reports the welfare gains in consumption equivalents, $x(a_0, g_0)$, for both a high and a low realization of initial government expenditures, $g_0$. Not very surprisingly, the long-run welfare effects of introducing the tax on foreign assets depend on the convexity of the portfolio adjustment cost which are an inverse measure of the steady-state price elasticity of assets. The reported welfare gains of having the additional tax instruments vary between 2.3 percent and 0.4 percent of annual consumption in the $NT$-Economy.

<table>
<thead>
<tr>
<th>Adjustment cost, $\phi$</th>
<th>.05</th>
<th>.01</th>
<th>.005</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial state, $g_0$</td>
<td>low</td>
<td>high</td>
<td>low</td>
</tr>
<tr>
<td>Welfare gain (in %)</td>
<td>2.27</td>
<td>2.04</td>
<td>0.94</td>
</tr>
</tbody>
</table>

Table 5: Welfare gains in consumption equivalents, $x$.

5.5 Transitional Dynamics

To disentangle the short-run from the long-run effects of the Ramsey policies, I illustrate the transitional dynamics with a policy experiment in the form of an unexpected introduction of the tax on foreign assets to the average $NT$-Economy in the stochastic steady-state. To make this exercise
interesting, I assume that the government is allowed to reoptimize in the period of the policy change.\footnote{This policy experiment is equivalent to comparing an NT-Economy to a T-Economy starting from the same initial asset and expenditure level, $a_0$ and $g_0$, respectively, and setting $\tau^x_0 = 0$.}

Figure 1 illustrates the transitional dynamics of the average labor income tax, $\tau^n$, the average labor supply, $n$, the average tax on foreign assets, $\tau^x$, and the average stock of assets, $a$, before and after the policy change. The bottom left panel shows that the policy change allows the government to provide households with a higher level of leisure for several periods after the policy change. At the same time, also a higher level of consumption can be afforded by reducing the stock of assets in the transition to the new steady-state, as shown in the bottom right panel. In the steady-state, however, momentary utility $u(c, \ell)$ is almost unchanged compared to the NT-Economy, thus the welfare gains are mostly realized in initial periods.

In the period of the policy change the zero tax on foreign assets is inherited from the former policy regime, so there is not much potential for welfare gains. However, with a peak in period one after the policy change, the government optimally taxes foreign assets at a rate between 15 percent and 8 percent. The reason for this is that the price elasticity of the assets is lowest in the first period because of the non-depreciated stock of assets inherited from the past. After the peak there is a reduction in the foreign asset tax followed by a smooth transition to the steady-state where the mean level of the tax on foreign assets is mainly driven by the convexity of the portfolio adjustment cost and not by the low elasticity of the non-depreciated stock of assets. The labor income tax also peaks in period one after the policy change because its tax base drops substantially. The peak is followed by a smooth transition to the steady-state tax rate.

In summary, the welfare gains of introducing a tax on foreign assets can be substantial. In particular, most of the gains can be realized in periods just after policy change.
Figure 1: Transitional dynamics.
6 Conclusion

This paper analyzed the dynamic Ramsey taxation of foreign assets in a model with international mobility of capital and fiscal shocks. The benevolent government of a small open economy chooses taxes on factors of local production and net foreign assets to finance an exogenous stochastic stream of government expenditures. The government fully commits to future policies and balanced budgets, and can observe net foreign assets at the aggregate level but not the resident household’s individual accounts. The paper finds that the optimal tax scheme replicates an individual asset income tax on the residence principle. Furthermore, numerical simulations show that such a government should tax foreign assets and physical capital rents to insure against fiscal shocks, while the expected burden of fiscal expenditures is mostly borne by labor income. The welfare gains of introducing the tax on foreign assets according to the Ramsey policy are quantified between 2.3 percent and 0.4 percent of annual consumption.

The results of this study can be related to the bilateral tax agreements that have regained momentum in the political discussion of the ongoing European debt crises. Prominent examples are the bilateral withholding tax agreements that Austria and the United Kingdom have signed with Switzerland. Switzerland is asked to collect taxes on asset income of foreign citizens on behalf of their home country. This study suggests that the tax on foreign assets should be state-contingent, while moderate in the expected level. Moreover, the tax on foreign assets should be peaking in periods after the introduction where most of the welfare gains can be realized.

An important direction for future research is the deviation from the assumption that the government can fully commit to future policies. Contrary to the Ramsey policy, the consideration of Markov-perfect policies, for example, will reflect the cost of commitment arising from the time-inconsistency problem introduced by the tax instrument on foreign assets. This would be a crucial step in going from the normative analysis in this paper to a positive one.
7 References


A Taxation of Foreign Assets

A.1 Decentralized Equilibrium

Given the sequence of state-contingent tax policies
\[ \{ \tau^k(s^t), \tau^n(s^t), \tau^x(s^t) \}, \]
and a world rental rate, \( r^* \), a decentralized equilibrium is a state-contingent sequence of choices
\[ \{ c(s^t), \ell(s^t), k(s^t), a(s^t) \}_{t,s^t} \]
which is consistent with the borrowing constraint on assets (1), rational expectations about the transition of government expenditures (2), labor market clearing (3), the intratemporal labor-leisure trade-off (4), the intertemporal Euler equation (16), the international no arbitrage condition (17), and the private budget constraint
\[
0 = (1 - \tau^n(s^t)) (L - \ell(s^t)) f_2 (k(s^t), n(s^t))
+ a(s^{t-1}) \left[ 1 + (1 - \tau^x(s^{t-1})) r^* - \delta \right] - \left[ c(s^t) + a(s^t) + q(a(s^{t-1})) \right],
\]
where the initial conditions \( s_0, a_0, \) and \( \tau_0^x \) are taken as given.

A.2 Primal Formulation

The tax on foreign assets can be reduced from the constraint set of the Ramsey problem for all \( t \geq 1 \) by combining (20) and (16) to the yield a set
of implementability constraints,

\[
0 = \left[ \beta E_{s^{t+1}|s^t} \left\{ u_1(c(s^{t+1}), \ell(s^{t+1})) \left[ c(s^{t+1}) + a(s^{t+1}) + q(a(s^t)) \right] \\
- a(s^t)q_1(a(s^t)) \right\} - u_2(c(s^{t+1}), \ell(s^{t+1})) (L - \ell(s^{t+1})) \right] \\
- u_1(c(s^t), \ell(s^t)) \right] a(s^t). \tag{21}
\]

as proposed in the proposition of Stockman (2001). Because the resource constraint (10) remains unaffected by the additional tax instrument, the primal formulation of the Ramsey problem then reads

\[
\max_{c(s^t), \ell(s^t), k(s^t), a(s^t)} \sum_{s^t} E_{s^{t+1}|s^t} \beta^{t+1} u\left(c(s^t), \ell(s^t)\right)
\]

subject to (1), (2), (10), and (21), and the private budget constraint from the initial period

\[
0 = u_2(c(s_0), \ell(s_0)) (L - \ell(s_0)) \\
+ u_1(c(s_0), \ell(s_0)) \left[ a_0 \left[ 1 + (1 - \tau_0^x) r^* - \delta \right] - \left[ c(s_0) + a(s_0) + q(a_0) \right] \right], \tag{22}
\]

given \( s_0, a_0 \) and \( \tau_0^x \). Following again Marcet and Marimon (2011), the associated formulation of the augmented Lagrangian can be written as

\[
W_0(a_0, \tau_0^x, \mu(s_0)) = \min_{\gamma(s_0) c(s_0), \ell(s_0), k(s_0), a(s_0)} \max_{u(c(s_0), \ell(s_0))} \left[ c(s_0) + a(s_0) + q(a_0) \right] \\
+ \beta E_{s^{1}|s_0} W(a(s_0), \mu(s_0), g(s^1)) \\
- \gamma(s_0) u_1(c(s_0), \ell(s_0)) a(s_0),
\]
subject to the initial period resource constraint,

\[
0 \leq f(k(s_0), L - \ell(s_0)) + (a_0 - k(s_0)) r^* - [c(s_0) + a(s_0) - (1 - \delta)a_0 + g(s_0) + q(a_0)],
\]

the initial period private budget constraint \(22\), state transition \(\mu(s_0) = \gamma(s_0)\), the recursive form of \(W(a(s_0), \mu(s_0), g(s^1))\) defined in \(18\), and given \(s_0, a_0, \tau^x_0\), and \(\Gamma(g(s^{t+1})|g(s^t))\). Thus, the only difference of the period zero allocation to the allocations for \(t \geq 1\), is the initial period constraint of the private budget imposed by the predetermined initial tax rate on foreign assets, \(\tau^x_0\).

**B Markov-perfect Policies**

**B.1 Recursive Formulation**

As an alternative to the assumption of full commitment, I will consider here the other extreme that the government can only announce policies conditional on the payoff relevant state variables, \(a, \tau^x,\) and \(g\). This assumption is in line with the time-consistent public policy approach discussed in Klein, Krusell, and Rios-Rull (2008). To simplify the notation, I henceforth refer with \(\tau\) to the tax on foreign assets, \(\tau^x\). The recursive formulation of the government’s decision problem can then be written as

\[
\varphi(a, \tau, g) = \arg \max_{c, f, k, a', \tau'} u(c, \ell) + \beta E g'|g V(a', \tau', g')
\]

subject to the resource constraint

\[
0 \leq f(k, L - \ell) + (a - k)r^* - [c + a' - (1 - \delta)a + g + q(a)],
\]
the private Euler equation

\[
0 = \left[ \beta E_{g'} | g \right] u_1 (c(a', \tau', g'), \ell(a', \tau', g')) \left[ 1 + (1 - \tau') r^* - \delta - q_1(a') \right] \\
- u_1(c, \ell) a',
\]

and the private budget constraint

\[
0 = u_2(c, \ell)(L - \ell) + u_1(c, \ell) \left[ [1 + (1 - \tau) r^* - \delta] a - [c + a' + q(a)] \right],
\]

where

\[
V(a, \tau, g) = u(c(a, \tau, g), \ell(a, \tau, g)) + \beta E_{g'} | g \ V(a', \tau', g'),
\]

\[
\varphi(a, \tau, g) = \langle c(a, \tau, g), \ell(a, \tau, g), k(a, \tau, g), h(a, \tau, g), h^\tau(a, \tau, g) \rangle,
\]

\[
a' = h(a, \tau, g),
\]

\[
\tau' = h^\tau(a, \tau, g),
\]

with \( a_0, \tau_0, s_0 \) and \( \Gamma(g'|g) \) given. In the Markov-perfect formulation of the government problem, time-inconsistency induced by the forward-looking constraint is taken into account, because the future variables \( \varphi(a', \tau', g') \) can only be affected by the government through the manipulation of the endogenous state variables, \( a' \) and \( \tau' \). Once entered a state, \( (a, \tau, g) \), the government has no incentive to deviate from the announced policy, even if it were allowed to do so.

### B.2 Production Efficiency

Independent of the assumption on the government’s commitment technology physical capital, \( k \), only shows up in the resource constraint of the decision problem. Thus, physical capital is optimally chosen to maximize resources in every period given the other variables and the corresponding equilibrium condition is still given by (14). No matter whether the gov-
ernment can commit to future policies or not, it is never optimal to distort the allocation of capital.

B.3 Optimal Labor Income Tax

Combining the optimality conditions with respect to \( c \) and \( \ell \) with (12) yields the optimal time-consistent labor income tax,

\[
\tau^* = \psi u_{11}(c, \ell) f_2(k, L - \ell) - u_{12}(c, \ell) \left(1 + \theta\right) u_1(c, \ell) f_2(k, L - \ell)
\]

\[
+ \theta \left[ u_{22}(c, \ell) - u_{12}(c, \ell) + [u_{11}(c, \ell) - u_{21}(c, \ell)] \frac{u_2(c, \ell)}{u_1(c, \ell)} f_2(k, L - \ell) \right] (L - \ell),
\]

where \( \psi \) denotes the multiplier on the forward-looking constraint, and \( \theta \) the multiplier on the private budget constraint. A direct comparison to the Ramsey policy is not possible, but the two multipliers \( \psi \) and \( \theta \) will mainly determine the size of the optimal labor income tax.

B.4 Optimal Tax on Foreign Assets

The government’s Euler equation can be written as

\[
0 = \beta \left[ 1 + r^* - \delta - q_1(a') \right] E_{g'} \lambda' - \lambda
\]

\[
+ \beta \left[ 1 + (1 - \tau') r^* - \delta - q_1(a') \right] E_{g'} \theta' u_1(c', \ell') - \theta u_1(c, \ell)
\]

\[
+ \beta \phi \left[ 1 + (1 - \tau') r^* - \delta - q_1(a') \right] E_{g'} \frac{du_1(c', \ell')}{da'}
\]

\[
- q_{11}(a') E_{g'} u_1(c', \ell') \right] a'.
\]

In the jargon of Klein, Krusell, and Ríos-Rull (2008), this is called a generalized Euler equation, because not only the unknown functions enter the equilibrium conditions, but also the derivatives with respect to the endogenous states. So, by the very definition of the Markov-perfect equilibrium,
it is only well defined if indeed the equilibrium functions are differentiable with respect to the states. In an interior deterministic steady-state the private Euler equation implies that

\[ 0 = \beta \left[ 1 + (1 - \tau')r^* - \delta - q_1(a) \right] - 1, \]

therefore, the optimal long-run tax rate on capital flows reads

\[ \tau^* = \frac{H}{A} \left[ u_1(c, \ell)q_{11}(a) - \frac{d u_1(c, \ell)}{da} \right]. \]

This shows that in the Markov-perfect equilibrium, the long-run optimal tax on foreign assets must be positive even without the convex cost of adjusting the portfolio. The optimal time-consistent tax on foreign assets cannot be zero, because the government cannot commit not to tax predetermined assets once entered a state.