Listing Specs: The Effect of Attribute Orders on Choice*

Preliminary and Incomplete

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Abstract

Evidence from various choice domains shows that the presentation order of the attributes of the available options can have important effects on decisions. We introduce a decision-theoretic model that allows for such effects. We identify simple properties of observable behavior that characterize the model. We show how attribute-order effects can cause the choice of an alternative to depend on its position on a list. Finally, we apply our model to derive some new insights into strategic framing in advertising and phenomena like the endowment effect.

KEYWORDS: Attribute, Framing, Order, Muti-Attribute Choice, Primacy, Recency, Salience

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1 Introduction

Rich evidence shows that the order in which the attributes of choice alternatives are presented to individuals can affect their decisions, with potentially large welfare consequences. For example, choices of health plans can depend on how their attributes (like copay, deductibles, and premium) are presented (Ericson and Starc (2016)). People's willingness to pay for medical treatments can depend on the presentation position of their price (Kjær et al. (2006)). Doctors' diagnoses can depend on the order in which pieces of information are encountered (e.g., Bergus et al. (1995), Cunnington et al. (1997), Chapman and Elstein (2000)). Police investigations and jury decisions can depend on the presentation order of alibi and eyewitness evidence (Dahl et al. (2009)). Finally, consumers' evaluation of products can depend on the presentation order of their attributes (see, e.g., Levav et al. (2010), Kumar and Gaeth (1991) and references therein).[1,2]

Motivated by this evidence, in this paper we aim to introduce an explicit, decision-theoretic, model of framing of choice *alternatives* and its effects on decisions. Among the many forms that framing can take, we will focus on the presentation order of *attributes*. That is, we will view the attributes that physically describe an alternative as the given information to be framed; different presentation orders correspond to different frames, called attribute-order frames. We axiomatize a model of random choice where such frames affect the probability of choosing an alternative. We characterize when an individual is more susceptible to these frames. Given this, we investigate how attribute-order effects can spill over and determine how the position of an alternative on the choice list may influence its probability of being chosen—this may be called a list-order effect.[3] We consider applications to classic behavioral phenomena (like the endowment effect), advertisement, and rhetoric more generally.

Consider the following illustrative example. Health plans are often presented—say, to employees—in a table where each row is an attribute (copay, deductibles, premium) and each column is a plan. Mainstream choice theory assumes that the order in which rows are organized is irrelevant. By contrast, we allow for the possibility that this order affects the probability that each plan is eventually chosen (or the share of employees choosing each plan). For instance, this probability may chance if we move the premium from the first to the last row. The reasons and mechanisms behind these effects may be multiple and complex,

[1]Other papers in marketing and psychology report related evidence, but they are too many to list them here. See, e.g., Cornelissen and Werner (2014) and Auspurg and Jäckle (2017) for recent reviews as well as Chrzan (1994), Day and Prades (2010), Day et al. (2012)).

[2]Even the voters' support for political candidates may depend on the presentation order of their "attributes." For instance, in the 2016 U.S. Presidential race Hillary Clinton presented herself moving her maiden name (Rodham) to her middle name so as to emphasize her independence from her husband. See Shafer (2017) and the Guardian article (https://www.theguardian.com/media/mind-your-language/2016/nov/11/winning-words-the-language-that-got-donald-trumfp-elected).

[3]For evidence see, for example, Mantonakis et al. (2009) and references therein.

but are of second-order importance for us. Our goal is to develop a model which is consistent with choice data that exhibits those framing effects.

In this spirit, our analysis rests on the assumption that we can observe the choice frequencies of the alternatives in a menu as well as the specific ordering of attributes. Thus, we add a dimension—the attribute-order frame—to the usual dataset assumed by random-choice models. In our example, an health plan with n attributes can be represented as the vector $x_f = (x_{f(i)})_{i=1}^n$, where $x_{f(i)}$ is the value of attribute $f(i)$ (e.g., the premium) that appears in row i under frame f. For a different frame f', the same plan may feature the same attribute in a different row.

As a simple and first pass at studying attribute-order frames, we identify properties that the dataset has to satisfy for the choice frequencies to be consistent with an attribute-order Luce representation. To illustrate, consider a binary menu $\{x_f, y_f\}$. In our baseline model the probability that the decision maker chooses plan x_f takes the form

$$p(x_f, \{x_f, y_f\}) = \frac{e^{\sum_{i=1}^n \alpha(i) u_{f(i)}(x_{f(i)})}}{e^{\sum_{i=1}^n \alpha(i) u_{f(i)}(x_{f(i)})} + e^{\sum_{i=1}^n \alpha(i) u_{f(i)}(y_{f(i)})}}. \tag{1}$$

In this representation, α is a weighting function that depends on the attribute position and $u_{f(i)}$ is an attribute-specific utility function that captures the decision-maker (stable) underlying tastes for each attribute. Thus, we show how to identify such tastes, even though one might think that framing undermines the reliability of any preference elicitation.[4]

The weighting function α is the key difference between our framework and the standard framework. Depending on the shape of α, the model can capture several regularities in how attribute orders can influence choice. For instance, an increasing α can give rise to *recency* effects, whereby attributes presented later have a more prominent influence of the evaluation of alternatives. A decreasing α, instead, can give rise to *primacy* effects, whereby initial attributes have a more prominent influence.[5] An instance of this is the "leader-driven" effect: An alternative that starts ahead of the others in terms of the first attribute's value for the consumer is more likely to be chosen (Carlson et al. (2006)). More generally, some people may assign more weight to initial and final attributes, which can be accommodated by a U-shaped α. It is also possible that α is constant, which renders attribute orders irrelevant and so include the standard framework as a special case of ours.

Representation (1) is characterized by simple and intuitive axioms. Besides standard positivity and non-triviality axioms, the following are key. The first is a weakening of Luce's independence of irrelevant alternatives: The relative probability of choosing x_f over y_f should not depend on other alternatives in the menu, provided all alternatives have the same frame f.

[4]We are aware of the shortcomings of the Luce model (duplicates, etc.) and plan to address them in future versions of this paper.

[5]For evidence on the primacy and recency effects, see Kardes and Herr (1990), Haugtvedt and Wegener (1994), Payne et al. (2000), Bond et al. (2007), Ge et al. (2011).

This independence is allowed to fail if alternatives are framed differently within a menu (see Section 4). The second axiom requires that, again holding the frame fixed, the likelihood of choosing x_f over y_f should only depend on the attributes that differ between the two, not on the level of other attributes. This axiom delivers additive separability. The last key axiom compares choices across frames. Consider two alternatives, x_f and y_f, that differ only in the first attribute and re-frame them by only shifting this attribute to a later position i, obtaining $x_{f'}$ and $y_{f'}$. Then, the difference in the log-likelihood of choosing x_f over y_f should be proportional to the log-likelihood difference for $x_{f'}$ and $y_{f'}$, where the proportionality factor can depend only on the position i. This rules out an interaction between position and attribute type, which we however plan to consider in extensions of the model.

We extend our theory to cover choices from menus whose alternatives are presented in different frames (e.g., $\{x_f, y_{f'}\}$). For such menus, one possibility is that some alternative on the menu—for example, the first one—serves as a reference point in determining a frame which the decision maker uses for all alternatives.[6] For instance, if x_f features price first and quality second while $y_{f'}$ has the reverse order, the decision maker "reframes" the second alternative with the order f and then compares x_f and y_f. Our representation (1) allows us to express this response in terms of observables so as to test the model. In short, if a decision maker evaluates x_f and $y_{f'}$ adopting the specific frame f, she has to behave as when she faces the menu $\{x_f, y_f\}$ and so as predicted by representation (1) under frame f. A key property here is that the decision maker adopts one of the existing frames, as opposed to forming a new one not already used by some alternative in the menu. Thus, choices continue to be affected by the exogenously given frames.

Our theory has several applications. A model where the decision maker is sensitive to exogenously given frames opens the door to studying strategic framing. For instance, in an electoral debate a candidate may strategically postpone mentioning specific aspects of his agenda to leverage recency effects for swaying voters. Another application is to thinking formally about the idea of emphasizing or making some aspects more salient. For instance, if a buyer's weight function α is decreasing, then a seller interested in (de)emphasizing some aspects of his product may do so by presenting them earlier (later). The point here is that the structure of our model determines the structure of optimal framing.[7] More generally, our model may allow us to formally talk about rhetoric and its concerns with how to arrange the points of an argument in the most persuasive manner (i.e., what the classics called *dispositio*). This could offer a novel angle on persuasion that is fundamentally different from the more standard strategic information provision, because here *what* is said remains

[6]Krosnick and Alwin (1987) examine the possibility that the first alternative on a menu "may establish a cognitive framework or standard of comparison that guides interpretation of later" alternatives.

[7]The contracting literature has examined strategic framing in buyer-seller relationships, where framing is assumed to influence the buyer's willingness to pay (see, e.g., Ostrizek and Shishkin (2018), Salant and Siegel (2018)). Our model can provide a foundation for how this influence works.

constant while *how* things are said is what matters. Finally, our theory can offer new insights into phenomena like the endowment effect (Thaler (1980), Kahneman et al. (1991)). Suppose that every time a decision maker becomes the owner of an object, she represents its attributes in her mind using the frame that maximizes her evaluation of the product. This reframing may be conscious or not—for instance, because of *regret avoidance* or *ex-post rationalization*. Either way, it will in general raise her willingness to accept above her initial willingness to pay for the same physical object.

In ongoing work, we develop several generalizations of representation (1). First, some evidence suggests that the effects of the presentation position of an attribute may depend on the nature of the attribute itself. Thus, some attributes are more prone to cause order effects. Second, the attributes need not influence choice probabilities in an additively separable way (i.e., the Luce value of an alternative need not be additively separable).[8] Third, our framework may provide a new perspective for understanding randomness in choice. Suppose we have a decision maker that chooses deterministically, but before choosing she randomly selects an order of the attributes to use to analyze the different alternatives. If the observer has no access to the used order, the choices might look random while they are in fact the outcome of a partially deterministic process.

Related Literature. The importance of framing effects for decision making has been recognized at least since the seminal work of Kahneman and Tversky (1979) and Tversky and Kahneman (1981). These papers brought to economics the key idea that the *way* choice problems are presented, not only their factual content, can significantly affect choices. Since then, the literature has evolved in two directions. Some papers developed general frameworks to think about framing effects (Salant and Rubinstein (2008), Bernheim and Rangel (2009), Salant (2011)). The rest of the literature, instead, has focused on modeling specific ways in which a problem presentation can influence choice, in order to have more applicable theories. Our paper belongs to this second strand.

This literature considers several forms of framing. Following Kahneman and Tversky (1979), many papers have investigated the effects of presenting choices as a gain or a loss. Another important form of framing is the phenomenon called 'mental-accounting' in relation to saving and investment decisions (Thaler (1985), Thaler (1990)). More recently, several papers have modeled salience effects, where the weights given to different attributes when making a choice can depend on how one attribute *stands out* from the others in the menu (e.g., Kőszegi and Szeidl (2012), Bordalo et al. (2012), Bordalo et al. (2013b), Bordalo et al. (2013a)). Rubinstein and Salant (2006) analyzed how the position of alternatives on a list impacts behavior. Our work differs from these papers by being the first—to the best of our knowledge—to study theoretically framing as the presentation order of the attributes

[8] That said, models that are additively separable in the attributes are often used in practice (see, e.g., Kumar and Gaeth (1991), Ericson and Starc (2016), Allen and Rehbeck (2016)).

of choice alternatives. As such, it can be interpreted as a salience theory that complements the existing one: While in those papers the salience of an attribute depends on the degree to which it varies across alternatives and in relation to other attributes, in our paper salience depends on the presentation position of an attribute.[9] Our framework can allows for asymmetric treatment of gains and losses, but adding this would obscure the gist of the paper. Finally, in some settings attribute-order effects can be an underlying driver of the phenomenon studied by Rubinstein and Salant (2006) (see Section 4). Thus, the present paper can be seen as advancing our understanding of list-order effects.

From a more conceptual standpoint, our paper also relates to the cognitive-science literature that studies how people seem to form their preferences at the moment of elicitation (Lichtenstein and Slovic (2006)). Indeed, one interpretation of our model is that the decision maker has well-defined tastes for each attribute. However, when it comes to combining these tastes to form an overall evaluation of an alternative, she lets the presentation order of the attributes influence her evaluation. In this way, elicitation procedure can define her choices at the very moment she has to make them. Although this may undermine the discovery of the decision maker's true tastes, our analysis shows how observing choices across frames can overcome this issue.

Our paper also relates to the vast literature on random choice, which has received a lot of interest recently.[10] Closest to ours are several papers that include observable characteristics into models of choice (Lancaster (1966), McFadden (1973), Gorman (1980), Allen and Rehbeck (2016)). The key difference is that we add not only observable attributes, but also their presentation order as part of the dataset to study its influence on choice. Gul et al. (2014) propose a related, but different, approach where the decision maker *subjectively* frames multi-attribute alternatives. Their elegant analysis identifies how the decision maker treats alternatives as more or less substitute to each other based on the subjective similarity of attributes (as in the classic red-bus-blue-bus problem). This approach, however, is silent about the role of objective and exogenous frames of attributes. It seems possible that exogenous frames interact with subjective frames, which would establish an interesting connection between our and their work.

[9] In their study of health-insurance decisions, Ericson and Starc (2016) argue that their evidence "suggests that theories of salience that only rely on the attributes of choice (rather than how they are presented) miss important elements of salience."

[10] It is impossible to have a full list of random-choice papers here, so we will focus on the closest ones.

2 Model

2.1 Choice Domain

The objects of choice are called *items*. Each item is exhaustively described by the attributes in a set A, with typical element denoted by a. We assume that $|A| = N$ is finite and $N \geq 3$.[11] Each attribute can take multiple values: Let Q_a be the set of possible values of each $a \in A$. Assume that $|Q_a| \geq 2$ for all $a \in A$. The description of an item consists of a *list* of values, one for each attribute.

We want to allow for the possibility that the order in which attributes are presented affects the choice of an item. To this end, we introduce the notion of *attribute-order frame*. Let F be the set of all bijections from $\{1, \ldots, N\}$ to A. For every frame $f \in F$, $f(i)$ is the attribute presented in the ith position of the description of the item. Thus, under the frame f an item can be written as

$$x_f = (x_{f(i)})_{i=1}^N,$$

where $x_{f(i)} \in Q_{f(i)}$ is the value of the attribute in position i. The set of all items under frame f is

$$X_f = \times_{i \in N} Q_{f(i)},$$

and $X = \cup_{f \in F} X_f$ is the set of all possible items. Subsets of items—which can be viewed as lists themselves—will be called choice menus. The items in a menu need not be described using the same frame. However, it will be important in the first part of the analysis to consider menus that use a consistent frame, that is, menus whose items are all presented using the same frame f. We will refer to such a menu as an *f-consistent* menu and adopt the notation

$$M_f \subseteq X_f.$$

We will denote a generic menu by $M \subseteq X$. If a menu is not f-consistent for any $f \in F$, we will refer to it as an *inconsistent* menu.

Example 1. Suppose items are health plans. Oversimplifying, assume that an health plan is described by its copay, its deductibles, and its premium. That is, $A = \{c, d, p\}$. Each attribute can be either high or low: $Q_a = \{h, l\}$. A frame is the order in which the attributes are presented in the description of a plan. Therefore, although each item is an element of $\{h, l\}^3$, the order in which attributes are presented may matter. One frame f may have the order $\{c, d, p\}$, while another f' may be $\{p, c, d\}$. A plan with a high premium, a high copay, and low deductibles may be presented as $x_f = (h_c, l_d, h_p)$ or as $x_{f'} = (h_p, h_c, l_d)$. Often, health plans are presented to potential customers using a table where, for instance, each row represents an attribute and each column a plan. Thus, if we interpret this table as the menu,

[11] It is possible to conduct the analysis with $N = 2$ at the cost of stronger separability axioms.

we always have an f-consistent menu, where f is given by the order of the table rows.

The primitive data consists of choice probabilities from menus of items. For every $M \subseteq X$ and $x \in M$, we assume that we can observe the probability that a decision maker chooses x from M. This probability is denoted by

$$p(x, M),$$

which has the usual interpretation of the random-choice literature.[12] Note that the frames of each item in a menu are assumed to be part of the dataset. Standard models implicitly assume that frames are irrelevant as far as choice behavior is concerned. This paper aims to relax this assumption and impose structure on how attribute-order frames can affect choice. Thus, our dataset consists of

$$\mathcal{P} = \{p(x, M) : \text{ for all } x \in M \text{ and } M \subseteq X\}.$$

2.2 Representation

We will start by considering a simple representation of $\mathcal{P}$ restricted to menus that are frame consistent. Denote by $\mathcal{P}^c$ the subset of the entire dataset $\mathcal{P}$ that is given by

$$\mathcal{P}^c = \{p(x, M_f) : \text{for all } x \in M_f, M_f \subseteq X_f, f \in F\}.$$

Definition 1. An attribute-order Luce (AOL) representation of $\mathcal{P}^c$ consists of a weight function $\alpha : \{1, \ldots, N\} \to \mathbb{R}_{++}$ and an attribute utility function $u_a : Q_a \to \mathbb{R}$ for every $a \in A$ that satisfy

$$p(x, M_f) = \frac{\exp\{\sum_{i=1}^N \alpha(i) u_{f(i)}(x_{f(i)})\}}{\sum_{y \in M_f} \exp\{\sum_{i=1}^N \alpha(i) u_{f(i)}(y_{f(i)})\}}.$$

We will refer to an AOL representation as defined by the pair (α, u), where $u = (u_a)_{a \in A}$. Intuitively, an AOL representation can be interpreted as follows. The decision maker derives a utility from each attribute in accordance with its value. Given this, he aggregates these attribute-specific utilities in a way that depends on the position of the attribute in the description of the item using the weights α. Thus, attributes that come early in the description can receive higher or lower weight than attributes that come later. It is immediate to see that if $\alpha(i) = \alpha$ for all $i \in \{1, \ldots, N\}$, we recover a standard Luce model of random choice where order frames become irrelevant.

Inconsistent menus raise separate intricacies related to how the decision maker deals with the different frames of the items on the menu. We will return to these issues later.

[12] See, for example, Luce (1959), Block and Marschak (1960), Marschak (1974), Gul and Pesendorfer (2006), Manzini and Mariotti (2014), and Apesteguia and Ballester (2018).

3 Axiomatic Characterization

We seek to obtain a characterization of AOL representations of $\mathcal{P}^c$ in terms of properties of the dataset. To this end, we introduce the following axioms. The first is standard in random-choice models (see, e.g., McFadden (1973)).

Axiom 1 (Positivity)**.** *$p(x, M) > 0$ for all $x \in M$ and $M \subseteq X$.*

The next axiom simply rules out attributes that never affect choices.

Axiom 2 (Non-Triviality)**.** *For every $a \in A$, there exists $q, q' \in Q_a$ such that, for every $f \in F$, if $f(i) = a$, $x_{f(i)} = q$, $x'_{f(i)} = q'$, and $x_{f(j)} = x'_{f(j)}$ for all $j \neq i$, then*

$$p(x_f, \{x_f, x'_f\}) \neq p(x'_f, \{x_f, x'_f\}).$$

The next axiom starts to add structure to the model. It requires that Luce's usual independence of irrelevant alternatives hold *as long as* we fix the frame used to describe all items in a menu.

Axiom 3 (f-IIA)**.** *For every $f \in F$, $M_f \subseteq X_f$, and $x_f, y_f \in M_f$,*

$$\frac{p(x_f, \{x_f, y_f\})}{p(y_f, \{x_f, y_f\})} = \frac{p(x_f, M_f)}{p(y_f, M_f)}.$$

Axiom 3, however, allows for the possibility that the choice probabilities depend on the frame. Since this axiom refers only to data in $\mathcal{P}^c$, it does not impose restrictions on the rest of the dataset. Section 4 will explain why IIA may not hold for frame-inconsistent menus when framing matters.

By standard arguments, Axioms 1 and 3 are equivalent to the general Luce representation

$$p(x, M_f) = \frac{e^{v_f(x)}}{\sum_{y \in M_f} e^{v_f(y)}}$$

for some function $v_f : X_f \to \mathbb{R}$. The remaining axioms aim to specialize each v_f to

$$v_f(x) = \sum_{i=1}^{N} \alpha(i) u_{f(i)}(x_{f(i)}).$$

First, we introduce a condition that delivers additive separability. To this end, it is convenient to work with log-likelihoods of choice:

$$\ell(x, M) = \ln(p(x, M)).$$

Note that if $x, y \in M_f$, then

$$\ell(x, M_f) - \ell(y, M_f) = v_f(x) - v_f(y).$$

Intuitively, the next axiom requires that the log-likelihood of choosing x_f over y_f should only depend on the attributes that differ between the two, not on the level of other attributes.

Axiom 4 (f-Additive Separability)**.** *Fix $f \in F$ and any $j, k \in \{1, \ldots, N\}$. For all x_f, x'_f, y_f, y'_f that satisfy $x_{f(i)} = y_{f(i)}$ and $x'_{f(i)} = y'_{f(i)}$ for $i = j, k$ and $x_{f(i)} = x'_{f(i)}$ and $y_{f(i)} = y'_{f(i)}$ for all $i \neq j, k$, we have*

$$\ell(x_f, \{x_f, x'_f\}) - \ell(x'_f, \{x_f, x'_f\}) = \ell(y_f, \{y_f, y'_f\}) - \ell(y'_f, \{y_f, y'_f\}).$$

While the previous axioms restricted choices within each frame, the next axiom relates choices across frames in order to identify their effect on behavior. In words, suppose the alternatives x_f and y_f differ only in the first attribute. Now, re-frame them by only shifting this attribute to a later position i, obtaining $x_{f'}$ and $y_{f'}$. Then, the difference in the log-likelihood of choosing x_f over y_f should be proportional to the log-likelihood difference for $x_{f'}$ and $y_{f'}$, where the proportionality factor can depend only on the position i.

Axiom 5 (Attribute-Position Independence)**.** *For all $f, f' \in F$ that satisfy $f(i) = f'(1)$ for some $i \neq 1$ the following holds: If x_f, $\hat{x}_f$, $y_{f'}$, and $\hat{y}_{f'}$ satisfy $x_{f(i)} \neq \hat{x}_{f(i)}$, $x_{f(i)} = y_{f'(1)}$, $\hat{x}_{f(i)} = \hat{y}_{f'(1)}$, $x_{f(k)} = \hat{x}_{f(k)}$ for $k \neq i$, $y_{f'(k)} = \hat{y}_{f'(k)}$ for $k \neq 1$, and $\ell(y_{f'}, \{y_{f'}, \hat{y}_{f'}\}) \neq \ell(\hat{y}_{f'}, \{y_{f'}, \hat{y}_{f'}\})$, then*

$$\frac{\ell(x_f, \{x_f, \hat{x}_f\}) - \ell(\hat{x}_f, \{x_f, \hat{x}_f\})}{\ell(y_{f'}, \{y_{f'}, \hat{y}_{f'}\}) - \ell(\hat{y}_{f'}, \{y_{f'}, \hat{y}_{f'}\})} = r(i).$$

The last axiom requires that changing the position of an attribute does not transform it from a good into a bad.

Axiom 6 (Monotonicity)**.** *Let f, f', x_f, $\hat{x}_f$, $y_{f'}$, and $\hat{y}_{f'}$ satisfy the properties of Axiom 5. Then, $\ell(x_f, \{x_f, \hat{x}_f\}) > \ell(\hat{x}_f, \{x_f, \hat{x}_f\})$ if and only if $\ell(y_{f'}, \{y_{f'}, \hat{y}_{f'}\}) > \ell(\hat{y}_{f'}, \{y_{f'}, \hat{y}_{f'}\})$.*

We can now state our characterization result.

Theorem 1. *Axioms 1–6 hold if and only if $\mathcal{P}^c$ has an AOL representation (α, u).*

3.1 Proof of Theorem 1

We divide the proof into two parts, which correspond to the following lemmas.

Lemma 1. *Axioms 1–4 hold if and only if for every $f \in F$ there exist non-constant functions $w^f_{i,f(i)} : Q_{f(i)} \to \mathbb{R}$ for every $i \in \{1, \ldots, N\}$ such that, for all $x_f \in X_f$,*

$$v_f(x_f) = \sum_{i=1}^{N} w^f_{i,f(i)}(x_{f(i)}). \tag{2}$$

Moreover, $(w^f_{i,f(i)})_{i=1}^N$ and $(\hat{w}^f_{i,f(i)})_{i=1}^N$ represent the same choice probabilities under frame f if and only if $w^f_{i,f(i)} = \beta_f \hat{w}^f_{i,f(i)}$ for some $\beta_f > 0$ and all $i \in \{1, \ldots, N\}$.

Proof. We have that

$$\ell(x_f, \{x_f, x'_f\}) - \ell(x'_f, \{x_f, x'_f\}) = \ell(y_f, \{y_f, y'_f\}) - \ell(y'_f, \{y_f, y'_f\})$$

if and only if

$$v_f(x_f) - v_f(x'_f) = v_f(y_f) - v_f(y'_f),$$

which means that

$$v_f(x_f) > v_f(x'_f) \Leftrightarrow v_f(y_f) > v_f(y'_f).$$

Given the restrictions on x_f, x'_f, y_f, and y'_f assumed in Axiom 4, this last expression means that how v_f ranks the attributes in positions j and k is independent of the values of the other attributes. Axiom 2 ensures that each dimension of the frame f can matter for choices. By standard arguments (Debreu (1960)), it follows that it has to be possible to write v_f in an additively separable form in positions and the attribute assigned by f to that position, which corresponds to expression (2). Additively separable forms are unique only up to positive affine transformations, which in this case can depend on f: Therefore, if we have two such representations with functions v_f and $\hat{v}_f$ of all choices under the frame f, we must have

$$v_f = \beta_f \hat{v}_f + \xi_f,$$

where $\beta_f > 0$ and $\xi_f \in \mathbb{R}$. Given the exponential formula for the representation of $p(x, M_f)$, we can always normalize $\xi_f = 0$ for every $f \in F$ without loss of generality. □

Now we would like to specialize each function in the sum of (2) to take the form

$$w^f_{i,f(i)}(x_{f(i)}) = \alpha(i) u_{f(i)}(x_{f(i)}).$$

Lemma 2. *Axioms 1–6 hold if and only if there exists a function $\alpha : \{1, \ldots, N\} \to \mathbb{R}_{++}$ and non-constant functions $u_a : Q_a \to \mathbb{R}$ for every $a \in A$ such that, for every $i \in \{1, \ldots, N\}$ and $f \in F$, we have*

$$w^f_{i,f(i)}(x_{f(i)}) = \alpha(i) u_{f(i)}(x_{f(i)}), \quad x_{f(i)} \in Q_{f(i)}.$$

Proof. Take any $f, f' \in F$, $i \neq j$, and x_f, $\hat{x}_f$, $y_{f'}$, and $\hat{y}_{f'}$ that satisfy the premise of Axiom 5. Then,

$$\begin{aligned}\ell(x_f, \{x_f, \hat{x}_f\}) - \ell(\hat{x}_f, \{x_f, \hat{x}_f\}) &= \sum_{k=1}^{N} \left[w^f_{k,f(k)}(x_{f(k)}) - w^f_{k,f(k)}(\hat{x}_{f(k)}) \right] \\ &= w^f_{i,f(i)}(x_{f(i)}) - w^f_{i,f(i)}(\hat{x}_{f(i)}),\end{aligned}$$

$$\begin{aligned}\ell(y_{f'}, \{y_{f'}, \hat{y}_{f'}\}) - \ell(\hat{y}_{f'}, \{y_{f'}, \hat{y}_{f'}\}) &= \sum_{k=1}^{N} \left[w^{f'}_{k,f'(k)}(y_{f'(k)}) - w^{f'}_{k,f'(k)}(\hat{y}_{f'(k)}) \right] \\ &= w^{f'}_{1,f'(1)}(y_{f'(1)}) - w^{f'}_{1,f'(1)}(\hat{y}_{f'(1)}).\end{aligned}$$

By Axiom 2, these differences are not always equal to zero.

Using the implication of Axiom 5, we have

$$w^f_{i,f(i)}(x_{f(i)}) - w^f_{i,f(i)}(\hat{x}_{f(i)}) = r(i)\left[w^{f'}_{1,f'(1)}(y_{f'(1)}) - w^{f'}_{1,f'(1)}(\hat{y}_{f'(1)})\right],$$

where $r(i) > 0$ by Axiom 6 for all i. Note that the term in the square brackets depends on the frame only via $a = f'(1)$ for the particular frame where attribute a appears in 1st position. Therefore,

$$w^f_{i,f(i)}(x_{f(i)}) - w^f_{i,f(i)}(\hat{x}_{f(i)}) = r(i)\left[w^{f'}_{1,f(i)}(x_{f(i)}) - w^{f'}_{1,f(i)}(\hat{x}_{f'(i)})\right].$$

By holding f fixed and varying the position i for which $x_{f(i)} \neq \hat{x}_{f(i)}$, we obtain that for all $i \in \{1, \ldots, N\}$

$$w^f_{i,f(i)}(x_{f(i)}) - w^f_{i,f(i)}(\hat{x}_{f(i)}) = r(i)\left[w^{f'}_{1,f(i)}(x_{f(i)}) - w^{f'}_{1,f(i)}(\hat{x}_{f'(i)})\right],$$

where $r(1) = 1$ as a normalization. Given this, let $u_a = w^{f'}_{1,a}$ for all $a \in A$, where again f' is the particular frame that puts attribute a in the first position.

Therefore, for every $x_f, \hat{x}_f \in X_f$, we can write

$$\sum_{i=1}^{N} w^f_{i,f(i)}(x_{f(i)}) - \sum_{i=1}^{N} w^f_{i,f(i)}(\hat{x}_{f(i)}) = \sum_{i=1}^{N} r(i)u_{f(i)}(x_{f(i)}) - \sum_{i=1}^{N} r(i)u_{f(i)}(\hat{x}_{f'(i)}).$$

This implies that, for all $x_f \in X_f$,

$$\sum_{i=1}^{N} w^f_{i,f(i)}(x_{f(i)}) = \sum_{i=1}^{N} r(i)u_{f(i)}(x_{f(i)}) + \zeta_f$$

for some $\zeta_f \in \mathbb{R}$. Exploiting again the exponential representation for choice probabilities, we can normalize $\zeta_f = 0$ without loss of generality.

We conclude that for all $f \in F$ and $x_f \in X_f$, we can write

$$v_f(x_f) = \sum_{i=1}^{N} r(i)u_{f(i)}(x_{f(i)}).$$

$\square$

3.2 Uniqueness

It is worth noting that AOL representations have strong uniqueness properties. This is perhaps unsurprising given that Axiom 5 imposes a lot of structure on the model.

Proposition 1 (Uniqueness)**.** *Suppose that (α, u) and (α', u') are AOL representations of the same dataset $\mathcal{P}^c$. Then, $\alpha = b\alpha'$ for some $b > 0$ and $u_a = \gamma u'_a + \zeta_a$ where $\gamma > 0$ and $\zeta_a \in \mathbb{R}$ for all $a \in A$.*

Proof. Consider first α and α'. The proof of Lemma 2 shows that the only degree of freedom for the weight function involves normalizing it to 1 for $i = 1$. Therefore, α and α' can differ only with respect to this renormalization, which implies that $\alpha = b\alpha'$ for some $b > 0$.

Consider now u and u'. By non-triviality, for every $a \in A$, there exists $q, q' \in Q_a$ such that $u_a(q) > u_a(q')$ and $u'_a(q) > u'_a(q')$. Let $x = (q, (z_{a'})_{a' \neq a})$ and $x' = (q', (z_{a'})_{a' \neq a})$. Then,

$$\ell(x, \{x, x'\}) = b[u_a(q) - u_a(q')] = u'_a(q) - u'_a(q').$$

Therefore, since $b > 0$ is arbitrary, $u_a = \gamma u'_a + \zeta_a$ for some $\gamma > 0$ and $\zeta_a \in \mathbb{R}$. $\square$

3.3 Types of Responses to Attribute-Order Frames

We now show how observable attribute-order effects can be mapped into properties of our representation. For now, we will focus on primacy and recency effects.

To this end, we first need to define how the data reveals that the decision maker prefers one value of an attribute to another value.

Definition 2 (Revealed Attribute Preference)**.** Consider $x_a, y_a \in Q_a$ with $x_a \neq y_a$. Let $x = (x_a, (z_{a'})_{a' \neq a})$ and $y = (y_a, (z_{a'})_{a' \neq a})$ for arbitrary values $z_{a'}$ of the attributes other than a. Then, x_a is revealed preferred to y_a—denoted by $x_a \succ y_a$—if $p(x, \{x, y\}) > \frac{1}{2}$.

Given an AOL representation, it is easy to see that $x_a \succ y_a$ if and only if $u_a(x_a) > u_a(y_a)$.

We can now formally define primacy and recency effects in terms of our observable data. Intuitively, suppose x_f dominates y_f on the attribute in position i and is dominated on attribute in position $i + 1$, while they share the value of all other attributes. Then changing the frame by swapping the position of the two attributes should decrease the probability of choosing x over y in the case of primacy effects, while it should increase it in the case of recency effects.

Definition 3 (Primacy and Recency Effects)**.** Let x_f and y_f satisfy $x_{f(i)} \succ y_{f(i)}$, $y_{f(i+1)} \succ x_{f(i+1)}$, and $x_{f(j)} = y_{f(j)}$ for all $j \neq i, i+1$. Let $x_{f'}$ and $y_{f'}$ satisfy $x_{f'(i)} = x_{f(i+1)}$, $x_{f'(i+1)} = x_{f(i)}$, $y_{f'(i)} = y_{f(i+1)}$, $y_{f'(i+1)} = y_{f(i)}$, $x_{f'(k)} = x_{f(k)}$, and $y_{f'(k)} = y_{f(k)}$ for all $k \neq i, i+1$. Then, $\mathcal{P}^c$ exhibits a primacy (recency) effect if

$$p(x_{f'}, \{x_{f'}, y_{f'}\}) < (>) \; p(x_f, \{x_f, y_f\}).$$

We can now map these phenomena into properties of our AOL representation.

Proposition 2. *Let (α, u) be an AOL representation of $\mathcal{P}^c$. Then, $\mathcal{P}^c$ exhibits a primacy (recency) effect if and only if α is strictly decreasing (increasing).*

Proof. Consider the primacy effect—the argument is the same for the recency effect. Suppose first that α is strictly decreasing. Recall that $x_a \succ y_a$ if and only if $u_a(x_a) > u_a(y_a)$. Then, using the representation, we have

$$\begin{aligned}
\ln\left(\frac{p(x_{f'},\{x_{f'},y_{f'}\})}{1-p(x_{f'},\{x_{f'},y_{f'}\})}\right) &= \sum_{i=1}^{N}\alpha(i)u_{f'(i)}(x_{f'(i)}) - \sum_{i=1}^{N}\alpha(i)u_{f'(i)}(y_{f'(i)}) \\
&= \alpha(i)[u_{f'(i)}(x_{f'(i)}) - u_{f'(i)}(y_{f'(i)})] \\
&\quad +\alpha(i+1)[u_{f'(i+1)}(x_{f'(i+1)}) - u_{f'(i+1)}(y_{f'(i+1)})] \\
&< \alpha(i+1)[u_{f'(i)}(x_{f'(i)}) - u_{f'(i)}(y_{f'(i)})] \\
&\quad +\alpha(i)[u_{f'(i+1)}(x_{f'(i+1)}) - u_{f'(i+1)}(y_{f'(i+1)})] \\
&= \alpha(i)[u_{f(i)}(x_{f(i)}) - u_{f(i)}(y_{f(i)})] \\
&\quad +\alpha(i+1)[u_{f(i+1)}(x_{f(i+1)}) - u_{f(i+1)}(y_{f(i+1)})] \\
&= \ln\left(\frac{p(x_f,\{x_f,y_f\})}{1-p(x_f,\{x_f,y_f\})}\right).
\end{aligned}$$

Therefore, $p(x_{f'},\{x_{f'},y_{f'}\}) < p(x_f,\{x_f,y_f\})$.

Conversely, suppose that $p(x_{f'},\{x_{f'},y_{f'}\}) < p(x_f,\{x_f,y_f\})$. Then, using the representation, we have

$$\begin{aligned}
\sum_{i=1}^{N}\alpha(i)[u_{f'(i)}(x_{f'(i)}) - u_{f'(i)}(y_{f'(i)})] &= \ln\left(\frac{p(x_{f'},\{x_{f'},y_{f'}\})}{1-p(x_{f'},\{x_{f'},y_{f'}\})}\right) \\
&< \ln\left(\frac{p(x_f,\{x_f,y_f\})}{1-p(x_f,\{x_f,y_f\})}\right) \\
&= \sum_{i=1}^{N}\alpha(i)[u_{f(i)}(x_{f(i)}) - u_{f(i)}(y_{f(i)})].
\end{aligned}$$

Therefore,

$$\begin{aligned}
&\alpha(i)[u_{f'(i)}(x_{f'(i)}) - u_{f'(i)}(y_{f'(i)})] + \alpha(i+1)[u_{f'(i+1)}(x_{f'(i+1)}) - u_{f'(i+1)}(y_{f'(i+1)})] \\
<&\alpha(i)[u_{f(i)}(x_{f(i)}) - u_{f(i)}(y_{f(i)})] + \alpha(i+1)[u_{f(i+1)}(x_{f(i+1)}) - u_{f(i+1)}(y_{f(i+1)})],
\end{aligned}$$

which is equivalent to

$$[\alpha(i)-\alpha(i+1)]\{[u_{f(i+1)}(x_{f(i+1)}) - u_{f(i+1)}(y_{f(i+1)})] - [u_{f(i)}(x_{f(i)}) - u_{f(i)}(y_{f(i)})]\} < 0.$$

Since $u_{f(i+1)}(x_{f(i+1)}) < u_{f(i+1)}(y_{f(i+1)})$ and $u_{f(i)}(x_{f(i)}) > u_{f(i)}(y_{f(i)})$, this implies that $\alpha(i) > \alpha(i+1)$ for all $i = 1, \ldots, N-1$. □

We can now use our theory to characterize decision makers who are susceptible to attribute-order frames in different degrees. To this end, we need a behavioral definition of exhibiting comparatively more primacy (recency) effects. Clearly, through the lens of our model, behavior differences between individuals are driven either by how they weigh an attribute based on its presentation position or by their tastes for attributes. Since we are interested in the former effect, we should only compare individuals who have the same tastes for attributes.

To define this property of two individuals, we have to compare their choice data from the same choice menus. Let these datasets for individual 1 and 2 be $\mathcal{P}_1^c$ and $\mathcal{P}_2^c$.

Definition 4 (Same Attribute Tastes)**.** Decision maker 1 and 2 exhibit the same tastes for attributes if $\mathcal{P}_1^c$ and $\mathcal{P}_2^c$ have the following property. There exists $\gamma > 0$ such that, if for every $a \in A$ and $x_a, y_a \in Q_a$ we let $x = (x_a, (z_{a'})_{a' \neq a})$ and $y = (y_a, (z_{a'})_{a' \neq a})$ for arbitrary values $z_{a'}$ of the attributes other than a, then

$$\ell_1(x, \{x, y\}) = \gamma \ell_2(x, \{x, y\}).$$

This definition is motivated by the observation that, if we always normalize $\alpha^i(1) = 1$,

$$\ell_i(x, \{x, y\}) = u_a^i(x_a) - u_a^i(y_a).$$

Therefore, $\ell_1(x, \{x, y\}) = \gamma \ell_2(x, \{x, y\})$ implies that $u_a^1 = \gamma u_a^2 + \zeta_a$ where $\zeta_a \in \mathbb{R}$ for all $a \in A$. By Proposition 1, the revealed tastes for attributes of AOL decision makers are unique up to affine transformations. Therefore, individual 1 and 2 have the same tastes. We denote this property by $\mathcal{P}_1^c =_u \mathcal{P}_2^c$.

We can now define when the behavior of an individual exhibits more susceptibility to primacy or recency effects.

Definition 5 (Comparative Primacy Effect)**.** Suppose $\mathcal{P}_1^c =_u \mathcal{P}_2^c$. Decision maker 1 is more susceptible to primacy effect than decision maker 2 is if the following holds. Let x_f and y_f satisfy $y_{f(i)} \succ x_{f(i)}$, $x_{f(i+1)} \succ y_{f(i+1)}$, and $x_{f(j)} = y_{f(j)}$ for all $j \neq i, i+1$. Then,

$$p_2(x, \{x, y\}) = \frac{1}{2} \quad \Rightarrow \quad p_1(x, \{x, y\}) \leq \frac{1}{2}.$$

Definition 6 (Comparative Recency Effect)**.** Suppose $\mathcal{P}_1^c =_u \mathcal{P}_2^c$. Decision maker 1 is more susceptible to recency effect than decision maker 2 is if the following holds. Let x_f and y_f satisfy $y_{f(i)} \succ x_{f(i)}$, $x_{f(i+1)} \succ y_{f(i+1)}$, and $x_{f(j)} = y_{f(j)}$ for all $j \neq i, i+1$. Then,

$$p_2(x, \{x, y\}) = \frac{1}{2} \quad \Rightarrow \quad p_1(x, \{x, y\}) \geq \frac{1}{2}.$$

The next result maps these behavioral comparisons into properties of our AOL representation.

Proposition 3. *Suppose $\mathcal{P}_1^c =_u \mathcal{P}_2^c$. Decision maker 1 is more susceptible to recency (primacy) effect than decision maker 2 is if and only if their AOL representations (α^1, u) and (α^2, u) satisfy*

$$\frac{\alpha^1(i+1)}{\alpha^1(i)} \geq (\leq) \frac{\alpha^2(i+1)}{\alpha^2(i)}, \quad i = 1, \dots, N-1.$$

Proof. Fix $i = 1, \dots, N-1$ and let x_f and y_f satisfy $y_{f(i)} \succ x_{f(i)}$, $x_{f(i+1)} \succ y_{f(i+1)}$, and $x_{f(j)} = y_{f(j)}$ for all $j \neq i, i+1$. Then, $p_2(x, \{x, y\}) = \frac{1}{2}$ implies that

$$0 = \ell_2(x, \{x, y\}) - \ell_2(y, \{x, y\}) \;\; = \;\; \sum_{i=1}^{N} \alpha^2(i)[u^2_{f(i)}(x_{f(i)}) - u^2_{f(i)}(y_{f(i)})]$$

$$
\begin{aligned}
&= \alpha^2(i)[u^2_{f(i)}(x_{f(i)}) - u^2_{f(i)}(y_{f(i)})] \\
&\quad +\alpha^2(i+1)[u^2_{f(i+1)}(x_{f(i+1)}) - u^2_{f(i+1)}(y_{f(i+1)})],
\end{aligned}
$$

and so

$$
\frac{\alpha^2(i)}{\alpha^2(i+1)} = \frac{u^2_{f(i+1)}(x_{f(i+1)}) - u^2_{f(i+1)}(y_{f(i+1)})}{u^2_{f(i)}(y_{f(i)}) - u^2_{f(i)}(x_{f(i)})}.
$$

Therefore, $p_1(x, \{x, y\}) \geq \frac{1}{2}$ if and only if

$$
\begin{aligned}
0 \leq \ell_1(x, \{x, y\}) - \ell_1(y, \{x, y\}) &= \sum_{i=1}^{N} \alpha^1(i)[u^1_{f(i)}(x_{f(i)}) - u^1_{f(i)}(y_{f(i)})] \\
&= \alpha^1(i)[u^1_{f(i)}(x_{f(i)}) - u^1_{f(i)}(y_{f(i)})] \\
&\quad +\alpha^1(i+1)[u^1_{f(i+1)}(x_{f(i+1)}) - u^1_{f(i+1)}(y_{f(i+1)})],
\end{aligned}
$$

which is equivalent to

$$
\frac{\alpha^1(i)}{\alpha^1(i+1)} \leq \frac{u^1_{f(i+1)}(x_{f(i+1)}) - u^1_{f(i+1)}(y_{f(i+1)})}{u^1_{f(i)}(y_{f(i)}) - u^1_{f(i)}(x_{f(i)})}.
$$

Since $\mathcal{P}^c_1 =_u \mathcal{P}^c_2$, we know that $u^1_a = \gamma u^2_a + \zeta_a$ where $\gamma > 0$ for all $a \in A$. Therefore,

$$
\frac{u^1_{f(i+1)}(x_{f(i+1)}) - u^1_{f(i+1)}(y_{f(i+1)})}{u^1_{f(i)}(y_{f(i)}) - u^1_{f(i)}(x_{f(i)})} = \frac{u^2_{f(i+1)}(x_{f(i+1)}) - u^2_{f(i+1)}(y_{f(i+1)})}{u^2_{f(i)}(y_{f(i)}) - u^2_{f(i)}(x_{f(i)})}.
$$

We conclude that $p_1(x, \{x, y\}) \geq \frac{1}{2}$ if and only if $\frac{\alpha^1(i+1)}{\alpha^1(i)} \geq \frac{\alpha^2(i+1)}{\alpha^2(i)}$. A similar argument establishes the case of primacy effects. □

4 General Menus

In many situations decision makers encounter menus whose alternatives are all framed in the same way. This is the case whenever the menu is essentially described as a table where each column contains an alternative and each row an attribute (or vice versa). This is often the case for health plans or investment products, such as ETFs or mutual funds. However, it can happen that alternatives are presented to a decision maker each in a different order of attributes. This gives rise to an inconsistent menu. Thus, we aim to extend our theory to also cover these cases.

The main idea is as follows. In order to choose from the menu, the decision maker still has to somehow compare its alternatives. She may do so by considering each attribute for all alternatives sequentially according to some order—this seems reasonable especially for complex alternatives with many attributes. Therefore, effectively the decision maker may be reframing the menu at hand to render it consistent with some frame f. In this case, her choice frequencies should match those revealed in the sub-dataset $\mathcal{P}^c$ for f-consistent menus.[13] To formalize this, for every general menu $M \subseteq X$, define $\hat{M}_f$ as the reframing of M according to frame f. That is, $x_f \in \hat{M}_f$ if and only if x_f is obtained by reordering the attributes of some $x \in M$ according to frame f. We can express this property again in terms of observables.

Definition 7. Given menu $M \subseteq X$, the decision maker reframes M according to frame $f \in F$ if $p(x, M) = p(x_f, \hat{M}_f)$ for every $x \in M$.

The decision maker can reframe all alternative using any attribute-order frame she wants. However, a plausible assumption is that this frame is prompted by the presentation order of one of the alternatives already in the menu. For instance, if the first (or last) alternative on the menu mentions first its quality and then its price, the decision maker may follow this cue and first compare the quality across all alternatives and then their prices. Here, first or last refers to the order in which the alternatives are listed, which could be from left to right in the columns of a table or from top to bottom in the rows of a table. The point is that we can again express this phenomenon in terms of observables—the specific mechanisms are not essential here.

Definition 8. Given menu $M \subseteq X$, the decision maker adopts the frame f of alternative $\overline{x}_f \in M$ if $p(x, M) = p(x_f, \hat{M}_f)$ for all $x \in M$.

We can express these cases using our AOL representation, thereby providing a tool for modeling these reframing phenomena. Indeed, note that $p(x, M) = p(x_f, \hat{M}_f)$ for every $x \in M$ if and only if

$$p(x, M) = \frac{\exp\{\sum_{a \in A} \alpha(f^{-1}(a)) u_a(x_a)\}}{\sum_{y \in M} \exp\{\sum_{a \in A} \alpha(f^{-1}(a)) u_a(y_a)\}},$$

where f^{-1} is the inverse function of the frame f. Thus, suppose we elicit the parameters α and u of some decision maker using $\mathcal{P}^c$ and we have reasons to believe (possibly based on some tests) that she always adopts the frame of the first alternative on every menu. Then. we can predict her choice probabilities for every inconsistent menu.

The role of items as cues for how to reframe inconsistent menus can be one reason for failures of Luce's IIA for such menus. This is intuitive. Consider the initial menu $\{y_{f'}, z_{f'}\}$ and the expanded menu $M = \{x_f, y_{f'}, z_{f'}\}$. Suppose that, when facing inconsistent menus, the decision maker uses the first item on the menu to reframe the others. Then, we may have

$$\frac{p(y_{f'}, \{y_{f'}, z_{f'}\})}{p(z_{f'}, \{y_{f'}, z_{f'}\})} \neq \frac{p(y_{f'}, M)}{p(z_{f'}, M)} = \frac{p(y_f, \{y_f, z_f\})}{p(z_f, \{y_f, z_f\})}.$$

[13] Of course, in reality when decision makers face inconsistent menus, their behavior may exhibit no specific regularity or other properties. In this case, it is harder to develop a theory that can account for such behavior.

Thus, our model provides a way to understand violations of IIA as resulting from attribute-order framing effects. Of course, IIA may fail for many other reasons.[14]

5 Applications

This section is very preliminary. Its purpose is to showcase the tractability of the model and some ideas that seem worthwhile studying further.

5.1 Endowment Effect

The endowment effect is a phenomenon that relates the willingness to pay (WTP) for acquiring an object and the willingness to accept (WTA) for giving up possession of the same object (Thaler (1980)). Standard choice theory would predict that there should be no difference between the two (i.e., $WTA = WTP$). Nevertheless, evidence suggests that subjects often exhibit a WTA that exceeds their WTP (see, e.g., Kahneman et al. (1991)). Our framework can offer some new insights into this phenomenon.

To simplify the exposition and focus on the intuitions, we consider the deterministic counterpart of our model, where the utility of x_f is defined as follows:

$$v(x_f) = \sum_{i=1}^{N} \alpha(i) u_{f(i)}(x_{f(i)}).$$

Moreover, suppose the decision maker's overall utility is quasi-linear in money: The utility of good x under frame f and of the amount of money m is

$$U(x_f, m) = v(x_f) + m.$$

Let $v(0)$ be the utility of not having the good. Note that $v(0)$ does not depend on the frame, which is reasonable because there is no good to be evaluated in this case.

Now, suppose we present good x to our decision maker under frame f. Then, her willingness to pay under frame f is

$$WTP_f = v(x_f) - v(0).$$

Clearly, if when thinking about giving up possession of the good the decision maker evaluates it under the same frame f, she behaves perfectly rationally in the sense that $WTP_f = WTA_f$.

However, evidence from cognitive science suggests that this might not be the case. Well-known biases—like *regret avoidance* or *wishful thinking*—suggest that decision makers tend

[14] Our model can also violate the classic Regularity Axiom, which requires that the choice probability of an alternative (weakly) decreases when the menu expands (see, e.g., Block and Marschak (1960)). Intuitively, this is because the larger menu may cue a frame that is more favorable to that alternative.

to use the most favorable description of an action they took or an object they bought to avoid negative feelings. Our model opens the door to formalizing this idea. Suppose that, once she owns the object and the effect of the seller's framing has waned, our decision maker reorders its attributes in the most favorable frame to mentally describe it. That is, she adopts f^* such that

$$f^* \in \arg\max_{f' \in F} v(x_{f'}).$$

Given this, her WTA becomes

$$WTA_{f^*} = v(x_{f^*}) - v(0).$$

By definition, we must have

$$WTA_{f^*} \geq WTP_f$$

for every $f \in F$, with strict inequality for some f unless α is constant. Thus, our decision maker would exhibit the endowment effect whenever she tries to avoid regretting her decisions. It is worth mentioning that the avoidance of negative feelings seems to be a potential cause of the endowment effect (Zhang and Fishbach (2005)).

Our model can potentially add few insights. If the decision maker remembers how she reframed the good after experiencing it for the first time, the endowment effect should disappear because she cannot be manipulated by changing the frame when choosing the good again. In other words, experience should eliminate the endowment effect in our model. This is consistent with evidence suggesting that market experience seems to eliminate the endowment effect (List (2003)) as well as evidence suggesting that the effects of attribute-order framing disappear for subjects who had experience with the choice alternatives (Kumar and Gaeth (1991), Levin and Gaeth (1988)).[15]

5.2 Strategic Framing

This section briefly showcases how our model can offer novel insights about advertisement. These are complementary to the view that advertisement exists to provide information about the available products to consumers, who have well-defined fixed tastes. Here, instead, that information is held constant. What changes is *how* information is presented to consumers, which is obviously an important aspect of advertisement.

Sellers usually can choose how to present their products. Through the lens of our model, this means they can choose the order of attributes $f \in F$. Suppose consumers in a market can be described by our model. For simplicity, assume that they all have the same weight

[15] This does not reduce the importance of studying framing effects, as many and consequential choices in life happen infrequently and with little to no experience.

function α and tastes $u = (u_a)_{a \in A}$, and the sellers know (α, u). If choosing a frame is costless, then a seller of product x will choose f^x such that

$$f^x \in \arg\max_{f \in F} v(x_f).$$

Clearly, different sellers may prefer different frames, which leads to interesting strategic considerations. Intuitively, each sellers tries to emphasize specific attributes of her product by positioning them in her description so as to leverage the consumers' sensitivity to attribute-order frames. For this strategy to be successful, the consumers must end up using the frame originally chosen by the seller. However, according to our treatment of general menus (Section 4), this may require that the seller's product appears in a specific position (e.g., the first one) on the list that the consumers see. Thus, our model can justify why sellers may be willing to pay a premium for being listed first on, say, the webpage of an online store.

More generally, the sellers have an incentive to invest so as to ensure that their preferred frame becomes the leading frame in the industry. Combining this with the consumers' avoidance of negative feelings mentioned in the previous section, we see how these two forces can boost one another. Once a consumer adopts one of the frames chosen by the sellers, it will indeed be the frame that renders the corresponding alternative as desirable as possible. This removes any incentive for the consumer to mentally reframe her choice. In turn, this can make it even harder for other sellers to compete with the seller that has emerged as the leader. This logic is consistent with what is known as the *pioneering advantage*: Carpenter and Nakamoto (1989) find a gap between the market shares of pioneers and later entrants that cannot be explained by switching costs, but rather seems to arise from the process whereby consumers form their preferences.

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