Abstracts

Zero-sum games with multidimensional payoffs via set optimization

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New (solution) concepts and results for two-person zero-sum matrix games with multi-dimensional payoff will be presented. Such games were already studied by Shapley, Blackwell and Aumann some 60 years ago, but a general theory which parallels the elegant van-Neumann approach for the one-dimensional payoff case is still missing.
In the first part of the talk, a set optimization approach will be discussed and the problem reformulated as a complete lattice-valued one. This already leads to a new solution concept which may serve as an appropriate generalization of min-max solutions to the vector-valued case. It then turns out that two different equilibrium concepts need to be considered: the (old) Shapley equilibrium which is not interchangeable and thus not appropriate for worst-case insurance, and the new set optimization concept which fills this gap. Existence results, examples and an interpretation will be given.
In the second part of the talk, relationships to (scalar) multi-matrix games will be discussed. The main insight is that zero-sum games with vector payoffs share several features with bi- and multi-matrix (non zero-sum) games with scalar payoffs–a prime example is due to Aumann and Maschler. A transformation between these classes of games is discussed along with a new interpretation for both classes of games.
The theory works even if the two players have different preference relations generated by two potentially different convex cones. This situation is rarely considered in the vector optimization literature, but standard in the economics literature and essential for the transformation between zero-sum vector games and non-zero sum scalar games.

This work includes some new material and is otherwise based on

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A weight set decomposition method for the weighted Tchebycheff scalarization

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In general, solving the weighted Tchebycheff scalarization problem is an adequate way to calculate at least weakly efficient solutions to discrete multiobjective optimization problems. Under the assumption of uniform nondominance, this technique guarantees to find nondominated solutions [1, 3]. Similar to the Dichotomic approach when dealing with the weighted sum scalarization, Ralphs et al. [2] introduced an algorithm for generating all nondominated solutions of biobjective integer programs utilizing the weighted Tchebycheff scalarization. We generalize this approach to an arbitrary number of objectives to find subsets of the efficient set of a discrete multiobjective optimization problem, and state properties regarding a decomposition of the parameter set of the weighted Tchebycheff scalarization. Combining both and limiting to three objectives, we propose an algorithm that returns this decomposition of the parameter set and generates therefore all nondominated solutions. We further provide a running time analysis and explore methods to relax the initially made assumption.

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Robustness Indicators for Multi-Objective Integer Linear Programming.

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Decision uncertainty is a robustness concept in continuous single and multi-objective optimization, which takes into account the inaccuracy of the implementation of a solution (also called implementation error). Due to technical limitations a solutions might not be feasible for implementation. Implementing a solution in the neighborhood of the computed optimal solution can lead to a smaller or larger loss in solution quality. To limit this loss one can, for example, optimize the worst solution in the neighborhood (minmax). Eichfelder et al. (2017) investigate decision uncertainty for continuous multiobjective problems using set-valued optimization problems as robust counterpart. We transfer this concept to multi-objective combinatorial optimization problems. In the discrete context decision uncertainty is due to the fact that certain items (edge, element, ...) of a Pareto optimal solution might turn out to be unavailable in the implementation phase. Instead of reoptimizing the problem one is interested in repairing a solution by substituting the flawed item. Thus, the originally selected Pareto optimal solution is substituted by a neighboring solution. The neighborhood structure is thereby defined by a combinatorial adjacency structure of the problem. We propose robustness indicators, which are based on those neighborhoods. They are mainly of two types, cardinality indicators and quality indicators. In a first case study we evaluate the robustness indicators on randomly generated instances of the cardinality constrained knapsack problems. Furthermore, by using these indicators as a quality measure for the representation problem we construct representative subsets of the non-dominated set that are robust against decision uncertainty.

Determination of the nadir point for multiobjective discrete optimization problems

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Computing the nadir point is a challenging issue for multiobjective problems with more than two objectives. We present theoretical results that characterize the type of nondominated points that can contribute to the definition of the nadir components and identify search zones where such points may lie. Then, we propose a simple algorithm based on these results. Our experiments show that our algorithm clearly outperforms state of the art algorithms for multiobjective discrete optimization problems.

Tackling complexity: Perspectives for exact methods in MOO.

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Large dimensional problems with many objectives are usually difficult to engage by exact methods. Difficulties are multifold, because exhaustive search is not affordable and the presence of many objective generates a multidimensional Pareto set, hard to be explored by decision makers. Therefore the solutions found by heuristics possibly are suboptimal and the problem structure may remain largely unknown, even in rough lines.
In such a situation, a possible approach could be attempting a decomposition of the many-variables-many-objectives problem into a product of small dimensional, few objectives subproblems by analysing the sensitivity of variables in affecting objectives. Considering moderate dimensional problems allows for the use of exact methods generating geometric surrogates of the Pareto set, that can reveal the global structure and other critical features. The remaining problem is to compose a satisfactory global solution set for the original problem. If some parameters controlling the smoothness of the functions at hand are available it is possible to estimate the accuracy of the approximation of the solution proposed.

Fundamental challenges in solving Scenario-based Multiobjective Optimization Problems

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In many real-world optimization problems, decision-makers are faced with several (typically conflicting) objectives that should be optimized simultaneously. In particular, when the numbers of objectives and variables are high, finding globally Pareto optimal solutions is demanding. Besides having multiple conflicting objectives, the presence of uncertainty in almost all real applications brings more complexity. Even worse, in reality, a correctly-defined complete sample space and/or (reliable) probability distributions, which are required in classical probability-based approaches such as stochastic programming, are not available. In this case, scenario-based multiobjective optimization methods can be applied, where scenarios represent plausible future states. However, in this structure, the performance of a decision should be evaluated in terms of all objective functions under conditions of all scenarios which is introducing an additional dimension in the solution process and more complexity for a decision-maker (DM) to handle. Our main concern is how to solve scenario-based multiobjective optimization problems (SBMOOPs) as they will easily turn into large optimization problems with tens or hundreds of objectives. In this talk, two potential ways of solving SBMOOPs and fundamental challenges faced in each of them will be discussed. A traditional way is the use of scalarization functions. However, the DMs preferences should be taken into consideration in finding a preferred Pareto optimal solution. In SBMOOPs, the elicitation of required preference information such as aspiration levels from the DM for all the objectives in all scenarios can be very laborious or beyond human capabilities. Indeed, we cannot expect all the preferences, for all the objectives under conditions of all scenarios, to be available from the DM and, thus, we have to deal with incomplete preferences. It is not straightforward to apply the existing approaches, developed for incomplete preference handling, to high-dimensional problem such as SBMOOPs. Another way of dealing with a high-dimensional problems is decomposing the original problem into a set of subproblems with fewer objectives and variables in a meaningful manner. If this happens, DMs need to consider smaller amounts of information at a time and decomposing can help DMs not only in understanding and solving the problems but also in analysing the sensitivity of variables in different objectives in each scenario. After having solved the subproblems, a set of globally Pareto optimal solutions for the original problem can be built. In the last part of this talk, we review a recently proposed decomposition-based method called ANOVA-MOP. We discuss its potential and main challenges in solving scenario-based multiobjective optimization problems.
Keywords: Scenario-based multiobjective optimization, uncertainty, incomplete preferences, dimensionality reduction, decomposition, ANOVA-MOP.

A branch-and-Benders-cut algorithm for bi-objective integer programming: application to a stochastic facility location problem.

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Motivated by applications in disaster relief and public facility location under demand uncertainty, we develop a branch-and-Benders-cut algorithm for bi-objective two-stage stochastic programming, where the integer variables only appear in the first stage. As test case we use a bi-objective stochastic facility location problem. The considered objectives are cost and uncovered demand, whereas the demands at the different population centers are uncertain but their probability distributions are known. The latter information is used to produce a set of scenarios. We apply a Benders' type decomposition approach which is known as the L-shaped method for stochastic programming and we embed it into a recently developed branch-and-bound framework for bi-objective integer optimization. We analyze and compare different optimality cut generation strategies and we show how they affect the lower bound set computation scheme. Finally, we compare the branch-and-Benders-cut approach to a straightforward branch-and-bound implementation based on a deterministic equivalent formulation of the original two-stage stochastic program.

One-Exact ε-Pareto Sets.

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It is well-known that the polynomial-time solvability of a certain singleobjective problem determines the class of multiobjective optimization problems that admit a polynomial-time computable ε-Pareto set. Similarly, in this talk, we characterize the class of problems having a polynomial-time computable ε-Pareto set that optimizes one objective function exactly by the efficient solvability of an appropriate singleobjective problem. This class includes important problems such as multiobjective shortest path and spanning tree and the approximation guarantee we provide is, in general, best possible.

Furthermore, for biobjective problems from this class, we provide an adaptive algorithm that computes a one-exact ε-Pareto set of cardinality at most twice the cardinality of a smallest such set.

Multi-criteria decision making via multivariate quantiles

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A novel approach for solving a multiple judge, multiple criteria decision making (MCDM) problem is proposed. The presence of multiple criteria leads to a non-total order relation. The ranking of the alternatives in such a framework is done by reinterpreting the MCDM problem as a multivariate statistics one and by applying the concepts in [1]. A function that ranks alternatives as well as additional functions that categorize alternatives into sets of "good" and "bad" choices are presented. The paper shows that the properties of these functions ensure a reasonable decision making process.

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Nonconvex Pareto Navigator for Computationally Expensive Multiobjective Optimization Problems

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Interactive multiobjective optimization methods enable a decision maker to actively take part in the solution process, direct it with one's preferences and at the same time gain insight about the interdependences among the objectives and learn about the feasibility of preferences. We discuss navigation based methods which are a special kind of interactive methods, where the decision maker can see how objective function values change in real time based on the preferences. We formally characterize interactive navigation methods and discuss their desirable properties. We then introduce a new interactive method called Nonconvex Pareto Navigator, which operates in the objective space. It extends the Pareto Navigator method for nonconvex problems. It needs a set of Pareto optimal solutions as input and generates an approximation of the Pareto optimal set in the objective space. The decision maker can then navigate on the approximation and direct the search for interesting regions. In this way, the decision maker can conveniently learn about the conflicting objectives and adjust one's preferences, if needed. We introduce special cones to generate the approximation with the PAINT method. The cones enable extrapolation beyond the given Pareto optimal solutions. Navigation methods are well-suited for computationally expensive problems, because navigating on the approximation is computationally inexpensive.

Short bio: Kaisa Miettinen is Professor of Industrial Optimization at the University of Jyvaskyla. Her research interests include theory, methods, applications and software of nonlinear multiobjective optimization including interactive and evolutionary approaches. She heads the Research Group on Industrial Optimization and is the director of the thematic research area called Decision Analytics utilizing Causal Models and Multiobjective Optimization (DEMO, www.jyu.fi/demo). She has authored over 170 refereed journal, proceedings and collection papers, edited 14 proceedings, collections and special issues and written a monograph Nonlinear Multiobjective Optimization. She is a member of the Finnish Academy of Science and Letters, Section of Science and has served as the President of the International Society on Multiple Criteria Decision Making (MCDM). She belongs to the editorial boards of eight international journals. She has previously worked at IIASA, International Institute for Applied Systems Analysis in Austria, KTH Royal Institute of Technology in Stockholm, Sweden and Helsinki School of Economics, Finland. In 2017, she received the Georg Cantor Award of the International Society on MCDM for independent inquiry in developing innovative ideas in the theory and methodology.