Fact or friction: Jumps at ultra high frequency

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Outline I

1. Introduction
2. Theoretical setup
   - Semimartingale framework
   - Estimating quadratic variation
   - Microstructure noise and outliers
   - Estimating variance/covariance matrix
3. Simulation study
   - Simulation details
   - Results: Part I
   - Threshold estimation
   - Results: Part II

Outline II

4. Empirical application
   - Data description
   - Empirical results
   - Practical example

5. Conclusions
   - Conclusions
   - Future work

Introduction

- In recent years, high-frequency jump estimation has attracted increasing interest (e.g., Andersen, Bollerslev and Diebold 2007; Barndorff-Nielsen and Shephard 2004, 2006; Corsi and Renó 2009; Huang and Tauchen 2005)
- Jumps appear to be frequent and account for a significant proportion of total return variation (ranging about 5% – 15%).
- Most studies use sparsely sampled data, for example 5-minute data.
- We investigate the importance of the jump component with noise- and outlier-robust estimators using ultra high-frequency data.
- We find much less evidence of jumps, i.e. a substantially smaller jump proportion and fewer significant jump days.

Semimartingale framework

- We assume that a log-price X_t at time t is, potentially, of the form

$$X_t = X_0 + \int_0^t \mu_s \mathrm{d}s + \int_0^t \sigma_s \mathrm{d}W_s + \sum_{i=1}^{N_t^J} J_i, \quad t \geq 0$$

  where

- X_0 is the initial price, μ is a drift term, σ is a (stochastic) volatility process, W is a Brownian motion, while N_t^J and $(J_i)_{i \geq 0}$ represent the total number and sizes of jumps up to time t.

- X represents an underlying "efficient" price that would prevail in the absence of microstructure noise and outliers.

- We normalize time to the unit interval, $t \in [0, 1]$.
- The quadratic variation of X

$$[X]_1 = \int_0^1 \sigma_s^2 ds + \sum_{i=1}^{N_1^J} J_i^2 \equiv \text{IV} + \text{JV}.$$

- The question we investigate is how important the jump variation (JV) is relative to the integrated variance (IV).
- Let the observation times of X be equidistant time points $t_i = i/N$, for $i = 0, 1, \ldots, N$ Then, we compute log-returns by

$$\Delta_i^N X = X_{\frac{i}{n}} - X_{\frac{i-1}{n}}.$$

Realised variance and bipower variation

- The realised variance and bipower variation are defined by

$$RV_N[X] = \sum_{i=1}^{N} |\Delta_i^N X|^2 \qquad BV_N[X] = \frac{N}{N-1}\frac{\pi}{2}\sum_{i=2}^{N} |\Delta_i^N X||\Delta_{i-1}^N X|.$$

- Moreover, it holds that

$$RV_N[X] \xrightarrow{p} [X]_1 \qquad BV_N[X] \xrightarrow{p} \text{IV}.$$

- That is, bipower variation is a jump-robust estimator of the integrated variance, and $RV_N[X] - BV_N[X] \xrightarrow{p} \text{JV}$.

A high-frequency jump test

- Assume there are no jumps in X, i.e. $N_t^J \equiv 0$. Then, the following CLT holds
$$\frac{N^{1/2}(RV_N[X] - BV_N[X])}{\sqrt{0.609 \int_0^1 \sigma_s^4 ds}} \xrightarrow{d} N(0,1).$$
- Basis for performing non-parametric tests of the existence of jumps in the absence of microstructure noise and outliers.
- A feasible version is achieved by plugging in a consistent estimator of the integrated quarticity, $\int_0^1 \sigma_s^4 ds$.
- The t-statistic can be further transformed using the delta method to improve finite sample properties.

Microstructure noise and outliers

- In practice, microstructure noise (e.g., bid-ask spreads and price discreteness) cloaks the true log-price X.
- Moreover, data are contaminated with outliers (e.g., due to misplaced decimal points, errors in the data feeds etc.).
- Outliers can be hard to filter out systematically.
- We model the observed log-price Y as

$$Y_{\frac{i}{N}} = X_{\frac{i}{N}} + u_{\frac{i}{N}} + \mathbb{1}_{i \in A_N^O} O_i$$

- u is an i.i.d. microstructure noise process with $E(u) = 0$ and $\text{var}(u) = \omega^2$. Moreover, u is independent of X, i.e. $u \perp\!\!\!\perp X$.

- $(O_i)_{i \geq 0}$ are non-zero random variables, which generates the sizes of the outliers.
- A_N^O is a random set, which holds the appearance times of outliers. We assume A_N^O is a.s. finite and model it by

$$A_N^O = \left\{ \frac{[N \times T_i^O]}{N} : 0 \leq T_i^O \leq 1, i \geq 1 \right\}$$

- $(T_i^O)_{i \geq 0}$ are the jump times of another counting process N_t^O, where N_t^O is independent of N_t^J.
- The independence between N_t^J and N_t^O implies that the processes have no common jumps.
- Thus, observing both a jump and an outlier in Y over a small time interval is very unlikely.

Case I: The noiseless case

- Assume first that $u \equiv 0$, i.e. there is no noise, but there could be outliers in the data, $Y_{\frac{i}{N}} = X_{\frac{i}{N}} + 1\!\!1_{i \in A_N^O} O_i, \qquad i = 0, 1, \ldots, N$.

Theorem I

In the absence of microstructure noise but presence of outliers, the following convergence in probability holds

$$RV_N[Y] \xrightarrow{p} [X]_1 + 2 \sum_{i=1}^{N_1^O} O_i^2$$

$$BV_N[Y] \xrightarrow{p} IV + \frac{\pi}{2} \sum_{i=1}^{N_1^O} O_i^2$$

- Thus, neither estimator is consistent for the object, they are designed to estimate. Moreover, even in absence of jumps

$$RV_N[Y] - BV_N[Y] \xrightarrow{p} (2 - \pi/2) \sum_{i=1}^{N_1^O} O_i^2 > 0.$$

- Thus, a jump test based $RV_N[Y] - BV_N[Y]$ will reject the null with probability converging to 1, also under the null of no jumps!
- To estimate $[X]_1$ and the IV, we use a third estimator, the (subsampled) QRV of Christensen, Oomen and Podolskij (2010):

$$QRV_N[Y] \equiv \alpha' QRV_N(m, \overline{\lambda})[Y],$$

- Here, $\overline{\lambda} = (\lambda_1, \ldots, \lambda_k)$ with $\lambda_j \in [0,1)$ is a vector of quantiles, $\alpha = (\alpha_1, \ldots, \alpha_k)$ are quantile weights with $\alpha_j \geq 0$, $\sum \alpha_j = 1$ and $QRV_N(m, \overline{\lambda})[Y]$ is a $(k \times 1)$ vector with jth entry equal to

$$QRV_N(m, \lambda_j)[Y] = \frac{1}{N-m} \sum_{i=1}^{N-m} \frac{q_i(m, \lambda_j)}{\nu_1(m, \lambda_j)}, \text{ and}$$

$$q_i(m, \lambda) = g^2_{\lambda m}\left(\sqrt{N}|D_{i,m}Y|\right),$$

and where $D_{i,m}Y = \left(\Delta_k^N Y\right)_{(i-1)m+1 \leq k \leq im}$ for $i = 1, \ldots, n$.
- Under the assumptions of Theorem I, it holds that

$$QRV_N[Y] \xrightarrow{p} IV. \tag{1}$$

- We can further identify the jump and outlier variation by taking appropriate linear combinations of $RV_N[Y]$, $BV_N[Y]$ and $QRV_N[Y]$

$$RV_N[Y] - \left(1 - \frac{4}{\pi}\right) QRV_N[Y] - \frac{4}{\pi} BV_N[Y] \xrightarrow{p} JV$$

$$\frac{2}{\pi}\left(BV_N[Y] - QRV_N[Y]\right) \xrightarrow{p} \sum_{i=1}^{N_1^O} O_i^2$$

- An application of the delta method to the joint CLT (under no noise) of $(RV_N[Y], BV_N[Y], QRV_N[Y])$ can be used to test for jumps in the presence of outliers (see paper for details).

Case II: The noise case

- We now add the noise back and consider the simultaneous impact of noise and outliers, i.e.

$$Y_{\frac{i}{N}} = X_{\frac{i}{N}} + u_{\frac{i}{N}} + \mathbb{1}_{i \in A_N^O} O_i, \qquad i = 0, 1, \ldots, N.$$

- Well-known that standard estimators based on Y, e.g., $RV_N[Y]$, $BV_N[Y]$ or $QRV_N[Y]$ are inconsistent under noise.
- To infer the characteristics of the underlying semimartingale, we apply the pre-averaging approach, see, e.g., Jacod, Li, Mykland, Podolskij and Vetter (2009) or Podolskij and Vetter (2009a,b).

- Two ingredients are needed. First, we choose a sequence of integers

$$K = K(N) = \theta\sqrt{N} + o(N^{-1/2}), \quad \theta > 0.$$

- In the paper, we use $K = \lceil \theta\sqrt{N} \rceil$.
- The second ingredient is a pre-averaging function g, which has to satisfy some technical conditions (see paper).
- Throughout, we work with $g(x) = \min(x, 1 - x)$.
- Associated with g are some normalizing constants:

$$\psi_1^K = K \sum_{j=1}^{K} \left(h\left(\frac{j}{K}\right) - h\left(\frac{j-1}{K}\right) \right)^2, \qquad \psi_2^K = \frac{1}{K} \sum_{j=1}^{K-1} h^2\left(\frac{j}{K}\right)$$

- We then pre-average noisy returns

$$\bar{Y}_i^N = \sum_{j=1}^{K} g\left(\frac{j}{K}\right) \Delta_{i+j}^N Y .$$

- An equivalent representation

$$\bar{Y}_i^N = \frac{1}{K} \sum_{j=K/2}^{K-1} Y_{\frac{i+j}{N}} - \frac{1}{K} \sum_{j=0}^{K/2-1} Y_{\frac{i+j}{N}} ,$$

with K even and $g(x) = \min(x, 1-x)$.

- Hence, the term “pre-averaging”.

Pre-averaged RV, BV and QRV

- We define noise-robust estimators:

$$RV_N^*[Y] = \left[\frac{N}{N-K+2} \frac{1}{K\psi_2^K} \sum_{i=0}^{N-K+1} |\bar{Y}_i^N|^2 \right] - \frac{\psi_1^K}{\theta^2 \psi_2^K} \hat{\omega}^2,$$

$$BV_N^*[Y] = \left[\frac{N}{N-2K+2} \frac{1}{K\psi_2^K \mu_1^2} \sum_{i=0}^{N-2K+1} |\bar{Y}_i^N||\bar{Y}_{i+K}^N| \right] - \frac{\psi_1^K}{\theta^2 \psi_2^K} \hat{\omega}^2,$$

where $\hat{\omega}^2$ is a consistent estimator of ω^2.

- In the paper, ω^2 is estimated following Oomen (2006)

$$\hat{\omega}_{AC}^2 = -\frac{1}{N-1} \sum_{i=2}^{N} \Delta_i^N Y \Delta_{i-1}^N Y \xrightarrow{p} \omega^2.$$

- Construction of $QRV_N^*[Y]$ is slightly more involved.

$$QRV_N^*[Y] \equiv \alpha' QRV_N^*(m,\overline{\lambda})[Y],$$

where $\overline{\lambda}$ and α are as above, and the jth element of $QRV_N^*(m,\overline{\lambda})[Y]$ is given by:

$$QRV_N^*(m,\lambda_j)[Y] = \frac{1}{\theta\psi_2(N-m(K-1)+1)} \sum_{i=0}^{N-m(K-1)} \frac{q_i^*(m,\lambda_j)}{\nu_1(m,\lambda_j)}. \quad (2)$$

where

$$q_i^*(m,\lambda) = g_{\lambda m}^2\left(N^{1/4}|\overline{D}_i^N Y|\right),$$

and

$$\overline{D}_i^N Y = \{\overline{Y}_{i+(j-1)(K-1)}^N\}_{j=1}^m, \qquad \text{for} \quad i=0,1,\ldots,N-m(K-1)$$

Theorem II

Assume that the observed log-price Y obeys

$$Y_{\frac{i}{N}} = X_{\frac{i}{N}} + u_{\frac{i}{N}} + \mathbb{1}_{i \in A_N^O} O_i, \qquad i = 0, 1, \ldots, N.$$

and $E(u^4) < \infty$. Then, it holds that

$$RV_N^*[Y] \xrightarrow{p} [X]_1$$

$$BV_N^*[Y] \xrightarrow{p} IV$$

$$QRV_N^*[Y] \xrightarrow{p} IV$$

Remarks

- In contrast to the previous results, all noise-corrected estimators are also robust to outliers!
- Intuition: With a probability ”close to” one, there is at most a single outlier in the window $[i/N, (i+K)/N]$.
- The outlier influences exactly two consecutive returns with opposite sign and therefore appears with a factor $O(|g(j/K) - g((j-1)/K)|)$ in the construction of $\bar{Y}_i^N$.
- But $|g(j/K) - g((j-1)/K)| = O(1/K)$, so outliers therefore have no impact on $\bar{Y}_i^N$ asymptotically.

Theorem III

Assume that $N_t^J \equiv 0$, i.e. the observed log-price Y is a continuous semimartingale with noise and outliers. Furthermore, we assume that $E(u^8) < \infty$. As $N \to \infty$, it holds that

$$N^{1/4} \begin{pmatrix} RV_N^*[Y] - IV \\ BV_N^*[Y] - IV \\ QRV_N^*[Y] - IV \end{pmatrix} \xrightarrow{d_s} MN(0, \Sigma^*).$$

where $\xrightarrow{d_s}$ denotes stable convergence in law and Σ^* is the (unknown) conditional covariance matrix.

Noise- and outlier-robust test for jumps

- Theorem III forms the basis for a nonparametric noise- and outlier-robust test for jumps.
- Use suitably scaled measure of jumps by comparing $RV_N^*[Y]$ with either $BV_N^*[Y]$ or $QRV_N^*[Y]$, e.g.

$$\frac{N^{1/4}(RV_N^*[Y] - BV_N^*[Y])}{\sqrt{\Sigma_{11}^* + \Sigma_{22}^* - 2\Sigma_{12}^*}} \xrightarrow{d} N(0,1).$$

- In practice, a transformation (using the delta method) can improve finite sample properties of the test, e.g., a ratio- or log-based version. We found that the log-based test performs well.
- Infeasible result, as Σ^* is unknown!

Estimating Σ^*

- In order to construct a feasible jump test, we need to estimate the conditional covariance matrix Σ^*.
- We can construct estimators of the individual entries of Σ^*, for example

$$\hat{\Sigma}^*_{11} = \frac{N^{-1/2}}{\theta^2 \psi_2^2} \sum_{i=K}^{N-2K+1} |\bar{Y}_i^N|^2 \left(\sum_{l=-K+1}^{K-1} \left(|\bar{Y}_{i+l}^N|^2 - |\bar{Y}_{i+K}^N|^2 \right) \right) \xrightarrow{p} \Sigma^*_{11}$$

- Problem: The full estimated covariance matrix $\hat{\Sigma}^*$ often not positive semi-definite.
- We propose a positive semi-definite block subsample estimator of Σ^*, which has an intuitive form.

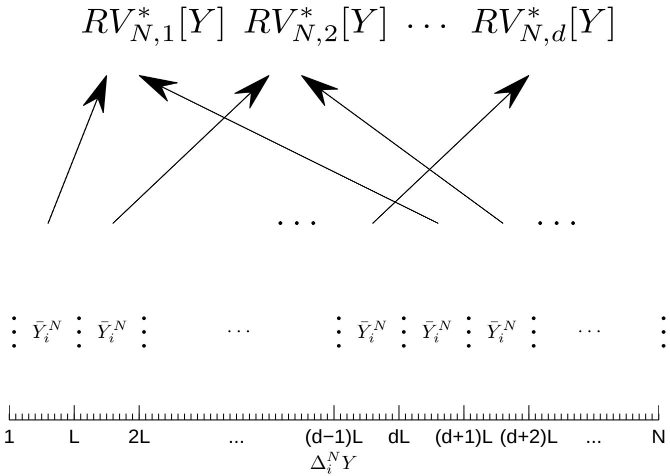
- We restrict attention to the 2×2 submatrix of Σ^* holding the covariance structure of $(RV_N^*[Y], BV_N^*[Y])$
- We choose two frequencies d and L, such that $L >> K$ and $dL = o(N)$. Here $d =$ number of subsamples, $L =$ block length.
- Let

$$RV_{N,m}^*[Y] = \frac{1}{K\psi_2^K} \sum_{i \in J_m} |\bar{Y}_i^N|^2 - \frac{\psi_1^K}{\theta^2 \psi_2^K} \hat{\omega}_{\mathrm{AC}}^2, \quad m = 1, \ldots, d$$

where

$$J_m = \{i : 0 \leq i \leq N - K + 1 \text{ and } (m - 1 + jd)L \leq i < (m + jd)L \text{ for some } j\}.$$

Illustration of subsampler



- Note that, asymptotically, $RV_{N,m}^{*}[Y]$ are mutually independent, because they are based on non-overlapping increments.
- Moreover, they satisfy the same CLT as $RV_{N}^{*}[Y]$, but with convergence rate $N^{1/4}/\sqrt{d}$.
- It is intuitive that a good proxy for Σ_{11}^{*} is given by

$$\hat{\Sigma}_{11}^{*} = \frac{1}{d}\sum_{m=1}^{d}\left(\frac{N^{1/4}}{\sqrt{d}}\left(RV_{N,m}^{*}[Y] - IV\right)\right)^{2}$$

- As the IV is unknown, we replace it with $RV_{N}^{*}[Y]$.
- We then construct $BV_{N,m}^{*}[Y]$ in a similar fashion.

- Finally, we set

$$T_{N,m} = \frac{N^{1/4}}{\sqrt{d}} \Big(RV^{*}_{N,m}[Y] - RV^{*}_{N}[Y], BV^{*}_{N,m}[Y] - BV^{*}_{N}[Y] \Big)',$$

and compute

$$(\hat{\Sigma}^{*}_{ij})_{1 \leq i,j \leq 2} = \frac{1}{d} \sum_{m=1}^{d} T_{N,m} T'_{N,m} \xrightarrow{p} (\Sigma^{*}_{ij})_{1 \leq i,j \leq 2},$$

- The estimator is positive semi-definite by construction.
- Unreported simulations show that $(\hat{\Sigma}^{*}_{ij})_{1 \leq i,j \leq 2}$ is largely unbiased if L is not too small. Also, it improves in an MSE sense by choosing larger values of d.

Simulation study

- We simulate from a number of models, including models with stochastic volatility (1- or 2- factors), leverage, jumps and outliers.
- We use $N = 10,000$ and pollute X using a noise ratio of $\gamma = 0.25$ (see, e.g., Oomen, 2006).
- Noise-robust pre-averaging estimators are computed using $\theta = \{0.10; 0.25; 0.50\}$.
- We base the QRV on absolute returns using $m = 3$ and $\lambda = 2/3$.
- This calibration is known as the *MedRV* (see, e.g., Andersen, Dobrev and Schaumburg, 2008).

Relative bias

Table: Relative bias of pre-averaging estimators.

	$RV_N^*[Y]$			$BV_N^*[Y]$			$BV_N^*[Y](\tau)$			$MedRV_N^*[Y](\tau)$		
model (down) // θ (right)	0.10	0.25	0.50	0.10	0.25	0.50	0.10	0.25	0.50	0.10	0.25	0.50
BM	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
SV	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99
SV-LEV	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99
SEV-ND	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99
SV2F-LEV	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
BMJ($n_J = 1, v_J = \frac{1}{4}$)	1.25	1.25	1.25	1.03	1.04	1.05	1.00	1.00	1.00	1.00	1.00	1.00
BMJ($n_J = 5, v_J = \frac{1}{4}$)	1.25	1.25	1.25	1.05	1.08	1.10	1.00	1.01	1.03	1.00	1.01	1.02
BMJ($n_J = 10, v_J = \frac{1}{4}$)	1.25	1.25	1.25	1.07	1.10	1.13	1.01	1.03	1.06	1.01	1.02	1.05
BMJ($n_J = 5, v_J = \frac{1}{2}$)	1.50	1.50	1.50	1.08	1.12	1.16	1.00	1.01	1.02	1.00	1.01	1.02
BM-outlier	1.00	1.00	1.00	0.99	1.00	1.00	1.01	1.00	1.00	1.00	1.00	1.00

Note. This table reports the relative bias for the pre-averaging estimators $RV_N^*[Y]$, $BV_N^*[Y]$ and $MedRV_N^*[Y]$. In the simulations, we set $N = 10,000$ and $\gamma = 0.25$. The $MedRV_N^*[Y]$ is a special case of the QRV estimator based on absolute returns, using $m = 3$ and $\lambda = 2/3$. The bias measure is equal to 1 for an unbiased IV estimator. Pre-averaging estimators based on a threshold to pre-trim $(\bar{Y}_i^N)$ are denoted with (τ).

Threshold estimation under pre-averaging

- As seen in Table 1, $BV_N^*[Y]$ is upward biased in the presence of jumps. This type of effect is also known from $BV_N[X]$.
- The bias is also present in the jump-robust $QRV_N^*[Y]$, although to a slightly lesser extent (not reported).
- In finite samples, this induces a downward bias in the estimated jump proportion and reduces the power of the jump test.
- To alleviate the bias, we experiment with a threshold in the pre-averaged return series $\bar{Y}_i^N$.
- The idea is related to the work of Aït-Sahalia and Jacod (2009), Corsi, Pirino and Renò (2010) or Mancini (2006), but there are some deviations in the workings of the threshold.

Setting the threshold

- Note that under a Brownian motion with i.i.d. noise, the asymptotic distribution (as $N \to \infty$) of $\bar{Y}_i^N$ is given by

$$N^{1/4}\bar{Y}_i^N \overset{a}{\sim} N\left(0, \psi_2\theta\sigma^2 + \psi_1\frac{1}{\theta}\omega^2\right)$$

where $\psi_1 = \lim_{K\to\infty}\psi_1^K$ and $\psi_2 = \lim_{K\to\infty}\psi_2^K$

- Thus, we can set a threshold by computing

$$\tau = q_{1-\alpha} \times \sqrt{\psi_2^K\theta\sigma^2 + \psi_1^K\frac{1}{\theta}\omega^2} \times N^{-\varpi},$$

where $q_{1-\alpha}$ is an appropriate high quantile from the $N(0,1)$ distribution and $\varpi \in (0, 0.25)$.

- Throughout, we work with $\alpha = 0.001$ and $\varpi = 0.20$, which produces good results in our simulations.
- In practice, we also use plug-in estimators of unknown quantities, i.e. we make the replacements $\sigma^2 \to \hat{IV}$, $\omega^2 \to \hat{\omega}^2_{\text{AC}}$.
- This amounts to a two-stage procedure, where IV and ω^2 are pre-estimated in order to set the threshold.
- After filtering the data, we then re-compute the estimator.
- As the original jump-robust estimator used to pre-estimate IV is slightly upward biased in the presence of jumps, the suggested procedure should also lead to conservative levels of τ.

Procedure for discarding data

- A naive threshold simply throws away all extreme observations, i.e. pre-averaged returns which satisfy

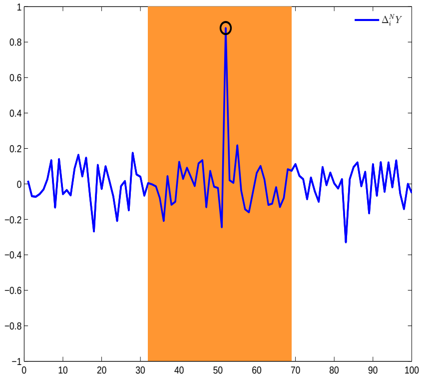
$$|\bar{Y}_i^N| > \tau.$$

- However, this tends to unnecessarily discard large amounts of data.
- Intuition: With a single "large" jump, the pre-averaging function $g(x) = \min(x, 1 - x)$ creates "humps" in $(\bar{Y}_i^N)$.
- As K increases, this induces long sequences of breaches of the threshold.

- Nonetheless, the connection between $(\Delta_i^N Y)$ and $(\bar{Y}_i^N)$ can be exploited by searching and selectively discarding noisy returns.
- Simple rule: If a breach of τ is observed, we extract the raw noisy returns that are used to construct the pre-averaged returns in that sequence.
- Then we discard the largest noisy return.
- The procedure can probably be improved, but our simulations suggest that it does a reasonable job (see Table 1).

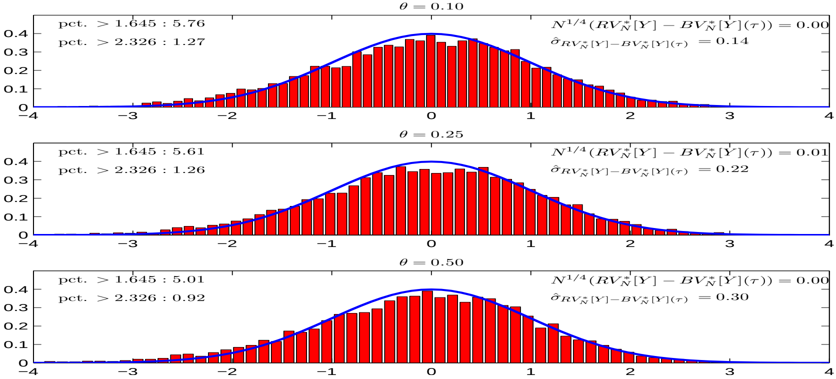
Illustration of threshold procedure

- We illustrate the mechanics below.



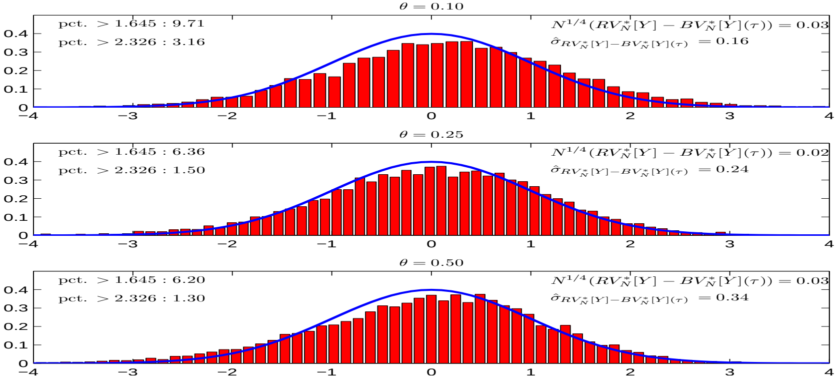
Note. To the left is the noisy return series, $(\Delta_i^N Y)$, while the pre-averaged return series, $(\bar{Y}_i^N)$, is to the right. The threshold τ is the plotted with red lines. The orange area shows the part of $(\Delta_i^N Y)$ taken out for inspection, while the black circle highlights the discarded return.

Size: model BM



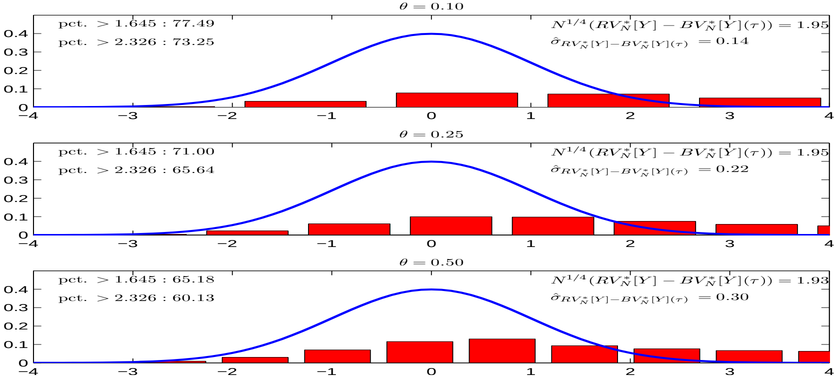
Note. We report the simulated size of the feasible log-based jump test under model BM for $\theta = \{0.10; 0.25; 0.50\}$.

Size: model SV2F-LEV



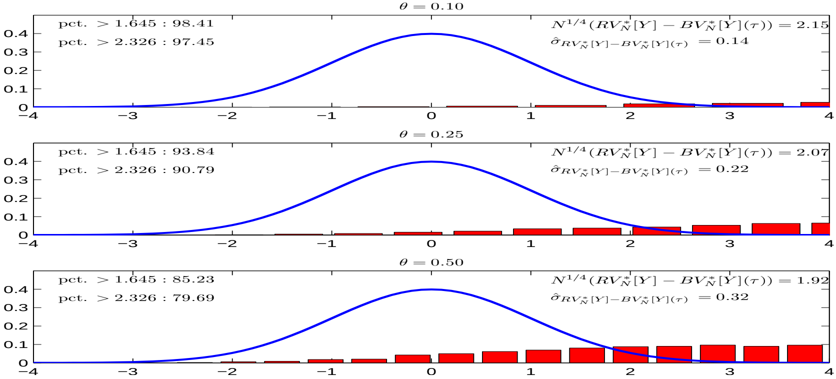
Note. We report the simulated size of the feasible log-based jump test under model SV2F-LEV for $\theta = \{0.10; 0.25; 0.50\}$.

Power: model BMJ($n_J = 1, v_J = \frac{1}{4}$)



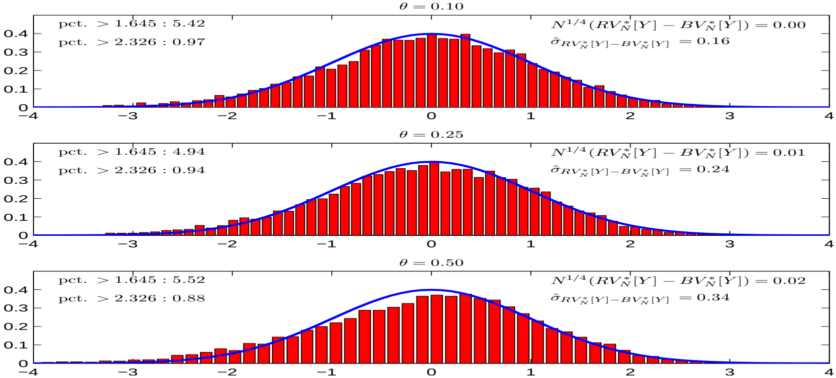
Note. We report the simulated size-adjusted power of the feasible log-based jump test under model BMJ($n_J = 1, v_J = \frac{1}{4}$) for $\theta = \{0.10; 0.25; 0.50\}$.

Power: model BMJ($n_J = 5, v_J = \frac{1}{4}$)



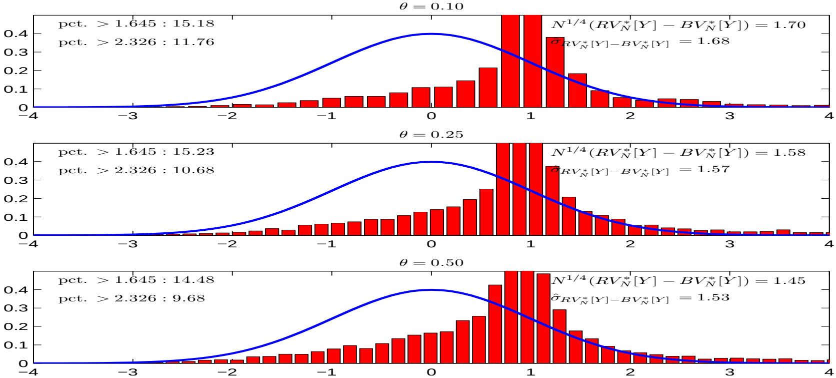
Note. We report the simulated size-adjusted power of the feasible log-based jump test under model BMJ($n_J = 5, v_J = \frac{1}{4}$) for $\theta = \{0.10; 0.25; 0.50\}$.

Size without threshold: model SV2F-LEV



Note. We report the simulated size-adjusted power of the feasible log-based jump test under model BMJ$(n_J = 5, v_J = \frac{1}{4})$ for $\theta = \{0.10; 0.25; 0.50\}$.

Power without threshold: model BMJ($n_J = 1, v_J = \frac{1}{4}$)



Note. We report the simulated size-adjusted power of the feasible log-based jump test under model BMJ($n_J = 5, v_J = \frac{1}{4}$) for $\theta = \{0.10; 0.25; 0.50\}$.

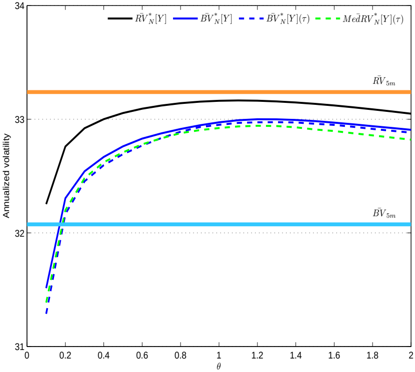
Some remarks

- The test has good size and power under model BM, but it is over-sized under stochastic volatility. The problem gets smaller, when the pre-averaging parameter θ is increased.
- Increasing θ, however, causes a slight drop in simulated power. Trade-off is in part influenced by setting a *constant* threshold.
- Nominal size is restored if we drop the threshold, but then the power of the test is eroded $\rightarrow$ Because upward bias in $BV_N^*[Y]$ and estimated standard errors deflates t-statistic.
- Practical compromise: Choose θ larger than theoretical minimum MSE choice would imply, but avoid excessive pre-averaging.

Data description

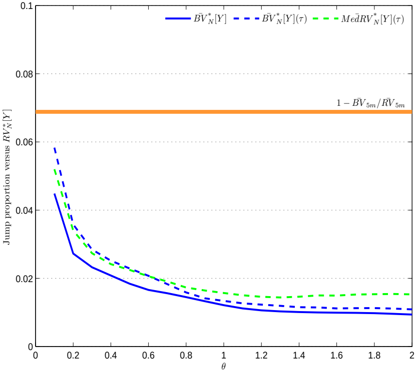
- We apply the pre-averaging technology to draw inference about jumps using a unique, extensive set of ultra high-frequency data.
- We extracted data from the NYSE TAQ database for the most recent configuration of DOW Jones (October, 2010), plus the two ETFs SPY and QQQQ.
- The data is recorded at milli-second precision and covers the sample period January, 2007 – June, 2010.
- We analyze both transaction and quotation data (only results from transaction data are reported here).
- After "light" cleaning and aggregation, we are left with a total sample size of about 4.3 billion tick-by-tick observations.

θ-signature plot - Cross-sectional average



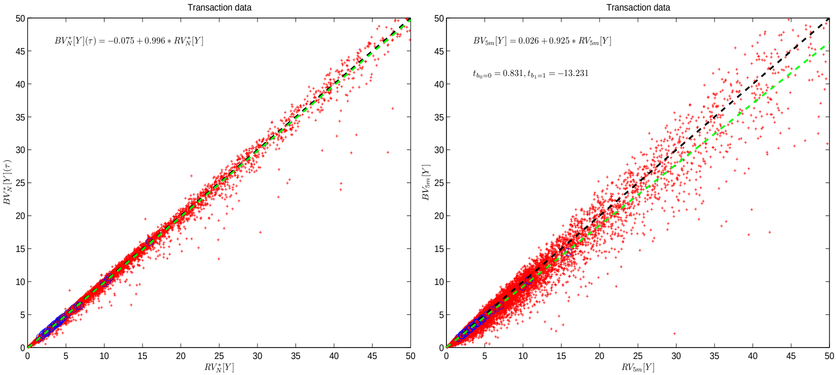
Note. We plot the average annualized volatility of the noise- and outlier-robust estimators, averaged across the cross-section of stocks included in our empirical application, as a function of θ. RV_{5m} and BV_{5m} are shown as a comparison.

Estimated jump proportion as a function of θ



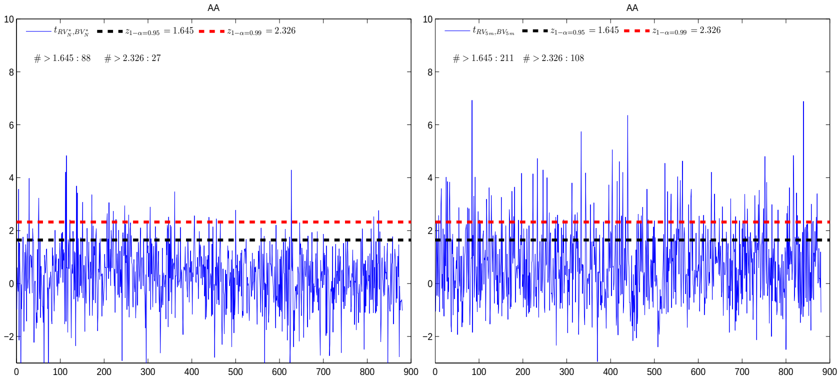
Note. We plot the average estimated jump proportion, averaged across the cross-section of stocks included in our empirical application, as a function of θ. The jump proportion estimated by using RV_{5m} and BV_{5m} is shown as a comparison.

Regression analysis



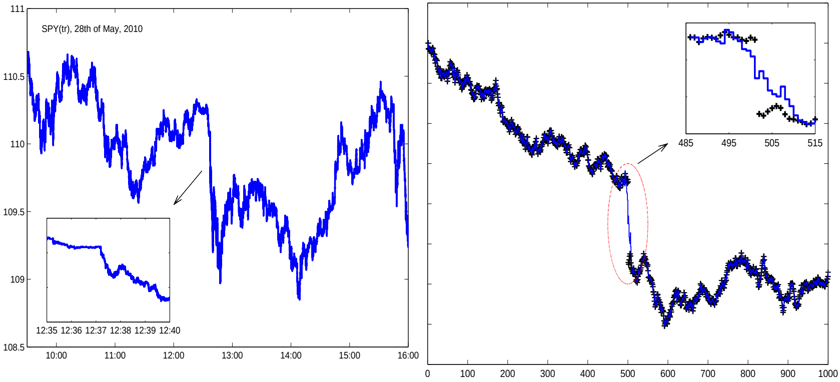
Note. The figure shows pairwise values of the average realised variance and bipower variation for each company in our selection of stocks (reported as a blue circle). We fit a regression line and test the hypothesis $b_0 = 0$ and $b_1 = 1$.

Jump test: Alcoa [AA]



Note. On the left is the noise- and outlier-robust jump test, while to the right is the low-frequency jump test based on 5-minute sampling. We also report the actual number of rejections based on the 5- and 1-% significance level.

Real and simulated sample path



Note. We plot an example of a real and simulated sample path, where a large intraday move in the price is observed over a short period of time. We then zoom into ultra high-frequency view of the sample path around the move.

Option trading with rehedging

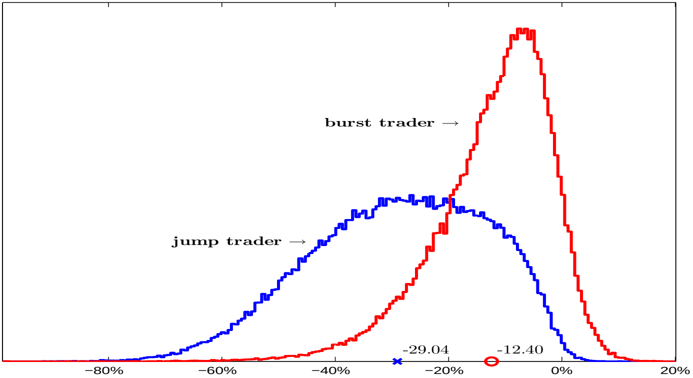
- Our results point towards a less important role for jumps than reported in previous papers. Finding could, in part, be driven by infrequent sampling, microstructure noise and data resolution.
- To illustrate the economic importance of distinguishing between a burst in volatility versus real economic jumps in financial markets, we consider an example from option trading.
- We suppose an option trader sells a short-term at-the-money call option and hedges his position in the underlying (covered call strategy) $\rightarrow$ Initially, trader is delta neutral but short gamma.
- Due to transaction costs, the trader only rehedges his delta position after every 1% move in the underlying.

Simulation details

- We simulate from a scaled Brownian motion with no drift: $X_t = \sigma W_t$. We assume the annualized volatility is 40%.
- We price the option with the Black-Scholes model. We assume the option has 1 day left to maturity and that the risk-free rate is zero.
- The initial stock price is 100 and the option is at-the-money, so strike is also 100.
- At a random position in the sample path, we place either a 2% jump in price or a 2% “burst in volatility” (cf. the simulated example path in previous figure).

- As one would expect, the trader faced with jump risk has larger losses than the trader, which faces burst risk.
- Short gamma traders face losses that are proportional to the square of the move in the underlying.
- Thus, in contrast to a jump, a burst in volatility allows the trader the valuable opportunity of rehedging his position, as the underlying moves.

Distribution of profit and loss (P&L)



Note. The plot shows the distribution of the P&L for the option trader exercising the covered call strategy with rehedging. Also reported in the figure is the average loss to the trader expressed in percent of the premium.

Conclusion

- We formulate a model, where the “efficient” price is contaminated with noise and outliers. We show that pre-averaging alleviates both sources of bias.
- We also suggest a threshold elimination procedure and propose a positive semi-definite estimator of the asymptotic covariance matrix appearing in the CLT.
- A simulation study shows these estimators are good also in finite samples.
- Using an extensive set of ultra high-frequency data, we find a much lower jump proportion and much less jumps than previously reported.

Ideas for future work / improvements

- At current, we are using a fixed value of θ, it is probably better to work with a data-driven choice. Not a simple problem! See, e.g., Hautsch and Podolskij (2010) for an MSE-based suggestion.
- Further refinement of the procedure suggested to do threshold estimation, e.g. to allow for time-varying threshold.
- Application to an OTC market, where there is no limit order book.
- Study the properties of pre-averaging, when there are potentially an infinite number of (small) jumps in the price process.