# Bounds On Treatment Effects On Transitions *

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**Abstract**

This paper considers identification of average treatment effects on conditional transition probabilities is considered. We show that even under random assignment only certain average treatment effects are point identified, because treated and control units drop out at different rates so that the initial comparability of treatment and controls due to randomization no longer holds. We derive sharp bounds on different average treatment effects that cannot be point identified. The bounds do not impose parametric restrictions, as e.g. proportional hazards, that would narrow the bounds or even allow for point identification. We also explore various weaker assumptions such as monotone treatment response and monotone exit rate. These weak assumptions tightens the bounds considerably.

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# 1 Introduction

We consider the effect of an intervention where the outcome is a transition from an initial to a destination state. The population of interest is a cohort of units that are in the initial state at the time origin. Treatment is assigned to a subset of the population either at the time origin or at some later time. Initially we assume that the treatment assignment is random. One main point of this paper is that even if the treatment assignment is random, only certain average effects of the treatment are point identified. This is because the random assignment of treatment only ensures comparability of the treatment and control groups at the time of randomization. At later times treated units with characteristics that interact with the treatment to increase/decrease the transition probability leave the initial state first/last, so that these characteristics are under/over represented among the remaining treated relative to the remaining controls and this confounds the effect of the treatment.

The confounding of the treatment effect by selective dropout is usually referred to as dynamic selection. Existing strategies that deal with dynamic selection rely heavily on parametric and semi-parametric models. An example is the approach of Abbring & van den Berg (2003) who use the Mixed Proportional Hazard (MPH) model (their analysis is generalized to a multistate model in Abbring, 2008) . In this model the instantaneous transition or hazard rate is written as the product of a time effect, the baseline hazard, the effect of the intervention and an unobservable individual effect. As shown by Elbers & Ridder (1982) is the MPH model nonparametrically identified, so that if the multiplicative structure is maintained, identification does not rely on arbitrary functional form or distributional assumptions. A second example is the approach of Heckman & Navarro (2007) who start from a threshold crossing model for transition probabilities. Again they establish semi-parametric identification, although their model requires the presence of additional covariates besides the treatment indicator that are independent of unobservable errors and have large support. The identified model is used to undo the confounding due to dynamic selection.

In this paper we ask what can be identified if the identifying assumptions of the semi-parametric models do not hold. We show that even under random assignment we cannot point identify many average treatment effects of interest, because of dynamic selection. However, we derive sharp bounds on various treatment effects, and show when these bounds are informative. These bounds apply e.g. if random assignment occurs at the time origin, but we want to learn the effect of the treatment on the transition probability after a number of periods, i.e. we are interested in the treatment effect dynamics. Our bounds are general, since beyond random assignment, we make no assumptions on functional form and additional covariates, and we allow for arbitrary heterogenous treatment effects as well as arbitrary unobserved heterogeneity. These bounds could be extended to unconfounded treatment assignment by creating bounds conditional on the covariates (or the propensity score) and then average over the distribution of these covariates. Besides these general bounds we show that

additional weak assumptions like monotone treatment response and monotone exit rate may tighten the bounds considerably.

There are many applications in which we are interested in the effect of an intervention on transition probabilities/rates. The Cox (1972) partial likelihood estimator is routinely used to estimate the effect of an intervention on the survival rate of subjects. Transition models are used in several fields. Van den Berg (2001) surveys the models used and their applications. These models also have been used to study the effect of interventions on transitions. Examples are Ridder (1986), Card & Sullivan (1988), Bonnal, Fougere & Serandon (1997) ,Gritz (1993), Ham & LaLonde (1996), Abbring & van den Berg (2003), and Heckman & Navarro (2007). A survey of models for dynamic treatment effects can be found in Abbring & Heckman (2007).

An alternative to the effect of a treatment on the transition rate is to consider its effect on the cdf of the time to transition or its inverse, the quantile function. This avoids the problem of dynamic selection. Fredriksson & Johansson (2008) have shown how the effect on the cdf, that is the unconditional survival probability, can be recovered even if the time-varying interventions can start at any time. From the effect on the cdf we can recover the effect on the average duration. From the effect on the cdf we cannot obtain the effect on the conditional transitions probabilities, so that this effect is not informative on the evolution of the treatment effect over time. There are good reasons why we would be interested in the effect of an intervention on the conditional transition probability or hazard rate. First, there is the close link between the hazard rate and economic theory (Van den Berg, 2001). Economic theory often predicts how the hazard rate changes over time. For example, in the application to a job bonus experiment considered in this paper labor supply and search models predict that being eligible for a bonus if a job is found, increases the hazard rate from unemployment to employment. According to these models the positive effect only exists during the eligibility period, and the effect increases shortly before the end of the eligibility period. The timing of this increase depends on the arrival rate of job offers and is an indication of the control that the unemployed has over his/her reemployment time. Any such control has important policy implications. These hypotheses can only be tested by considering how the effect on the hazard rate changes over time.

Other examples of when the evolution of the treatment effect over time is of key interest arise in different fields. For instance, two medical treatments can have the same effect on the average survival time. However, for one treatment the effect does not change over time while for the other the survival rate is initially low, e.g. due to side effects of the treatment, while after that initial period the survival rate is much higher. Research on the effects off active labor market policies (ALMP), often documents a large negative lock-in effect and a later positive effect once the program has been completed, see e.g. the survey by Kluve, Card, Fertig, Gra, Jacobi, Jensen, Leetma, Nima, Patacchini, Schmidt, van der Klauww & Weber (2007). In other cases a treatment consist of a sequence of sub-treatments assigned at pre-specified points in time to the survivors in the state. If one is interested in disentangling the sub-treatment

effects, the treatment effect over the spell has to be investigated.

In section 2 we define the treatment effects that are relevant if the outcome is a transition. Section 3 discusses their point or set identification in the case that the treatment is randomly assigned. This requires us to be precise on what we mean by random assignment in this setting. In section 4 we explore additional assumptions that tighten the bounds. Section 5 illustrates the bounds for a job bonus experiment data set. Section 6 concludes.

# 2 Treatment effects if the outcome is a transition

## 2.1 Parametric outcome models

To set the stage for the definition of a treatment effect for an outcome that is a transition, we consider the effect of an intervention in the Mixed Proportional Hazards (MPH) model. The MPH model specifies the individual hazard or transition rate $\theta(t, d(t), V)$

$$\theta(t, d(t), V) = \lambda(t)\gamma(t-\tau, \tau)^{d(t)}V$$

with $t$ as the time spent in the destination state, $\lambda(t)$, the baseline hazard, $d(t)$, the treatment indicator function in period $t$, and $V$, a scalar nonnegative unobservable that captures population heterogeneity in the hazard/transition rate and has a population distribution with mean 1. If treatment starts at time $\tau$ then $d(t) = I(t > \tau)$, i.e. we assume that treatment is an absorbing state. The nonnegative function $\gamma(t-\tau, \tau)$ captures the effect of the intervention, an effect that depends on the time until the treatment starts $\tau$ and the time treated $t-\tau$. Finally, although $\gamma$ is common to all units, the effect of the intervention differs between the units, because it is proportional to the individual $V$. The ratio of the treated and non-treated transition rates for a unit with unobservable $V$ is $\gamma(t-\tau, \tau)$ for $t > \tau$, so that in the MPH model $\gamma(t-\tau, \tau)$ is the effect of the intervention on the individual transition rate.

Let $\overline{d}(t) = \{d(s), 0 \leq s \leq t\}$ be the treatment status up to time $t$. The MPH model implies that the population distribution of the time to transition $T^{\overline{d}(T)}$ has density

$$f(t|\overline{d}(t)) = \mathbb{E}_V\left[V\lambda(t)\gamma(t-\tau, \tau)^{d(t)}e^{-\int_0^t \lambda(s)\gamma(s-\tau,\tau)^{d(s)}V\mathrm{d}s}\right]$$

and distribution function

$$F(t|\overline{d}(t)) = 1 - \mathbb{E}_V\left[e^{-\int_0^t \lambda(s)\gamma(s-\tau,\tau)^{d(s)}V\mathrm{d}s}\right]$$

The hazard/transition rate given the treatment history is

$$\theta(t|\overline{d}(t)) = \lambda(t)\gamma(t-\tau, \tau)^{d(t)}\mathbb{E}_V\left[V|T^{\overline{d}(T)} \geq t\right].$$

To define treatment effects in the MPH model we can compare units with different treatment histories $\overline{d}(t)$. Let $\overline{d}_0(t)$ and $\overline{d}_1(t)$ be two such histories. Then we can compare either the time-to-transition distribution functions in $t$, i.e. $F(t|\overline{d}_0(t))$ and $F(t|\overline{d}_1(t))$, or the transition rates in $t$, i.e. $\theta(t|\overline{d}_0(t))$ and $\theta(t|\overline{d}_1(t))$. The comparison of the transition rates is conditional on survival in the initial state up to time $t$ and the comparison of the distribution functions is not conditional on survival. As a consequence if we compare distribution functions we average over the population distribution of $V$, but if we compare transition rates we average over the distribution of $V$ for the subpopulation of survivors up to time $t$.

Let us take $\overline{d}_0(t) = 0$, i.e. the unit is in the control group during $[0, t]$, and $\overline{d}_1(t)$ arbitrary, then $F(t|\overline{d}_1(t)) > F(t|\overline{d}_0(t))$ if and only if

$$\frac{1}{\int_\tau^t \lambda(s)\mathrm{d}s} \int_\tau^t \lambda(s)\gamma(s-\tau,\tau)\mathrm{d}s > 1 \tag{1}$$

holds, i.e. if a $\lambda$ weighted average of the effect on the individual transition rate is greater than 1. Note that the comparison of the distribution functions is not confounded by the unobservable $V$. However, if we compare the transition rates in $t > \tau$

$$\theta(t|\overline{d}_0(t)) = \lambda(t)\mathbb{E}_V\left[V|T^{\overline{d}_0(T)} \geq t\right]$$

and

$$\theta(t|\overline{d}_1(t)) = \lambda(t)\gamma(t-\tau,\tau)\mathbb{E}_V\left[V|T^{\overline{d}_1(T)} \geq t\right]$$

then because

$$\mathbb{E}_V\left[V|T^{\overline{d}_0(T)} \geq t\right] > \mathbb{E}_V\left[V|T^{\overline{d}_1(T)} \geq t\right]$$

if and only if (1) holds, we have that under that condition

$$\frac{\theta(t|\overline{d}_1(t))}{\theta(t|\overline{d}_0(t))} < \gamma(t-\tau,\tau).$$

Therefore if the intervention increases the transition rate on average (as in (1), then the ratio of the population treated and control transition rates is strictly smaller than that of the individual treated and control transition rates. If the intervention decreases the transition rate on average, then the population transition rate is strictly larger than the individual rate. Hence, the effect of the intervention on the transition rate is confounded by its differential effect on the distribution of the unobservable among the treated and controls. The intuition behind this result is that the difference of the treated and control transition rates is monotonic in $V$, so that if (1) holds, treated units with a large value of $V$ are under-represented among the survivors in the initial state, while control units with a small value of $V$ are over-represented among these survivors. This dynamic selection or survivor bias is not just a feature of the MPH model. It is present in any population where the treatment and the individual characteristics interact to increase or decrease the transition probability.

Parametric and semi-parametric models for the transition rate indicate how to correct for the survivor bias in the average treatment effect. If we choose a distribution for $V$ or estimate the distribution as in Heckman & Singer (1984), we can estimate $\mathbb{E}_V\left[V|T^{\overline{d}_0(T)} \geq t\right]$ and $\mathbb{E}_V\left[V|T^{\overline{d}_1(T)} \geq t\right]$ to obtain the correction factor. Because the MPH model is nonparametrically identified this does not depend on untestable distributional assumptions. Of course it requires that the assumption that the hazard is multiplicative in the baseline hazard, the homogenous treatment effect and the spell constant unobserved effect $V$ is maintained. Without these assumptions the correction factor cannot be estimated without additional distributional assumptions.

## 2.2 Average treatment effects on transitions

In any definition of the causal effect of the treatment on the transition rate we must account for the dynamic selection or survivor bias. If we do not specify a model for the transition rate we need to find another way to make this adjustment. The approach that we take in this paper is to consider average transition rates where the average is taken in the same population for different treatment arms. The MPH model is most often normalized so that the mean of $V$ equals 1. When considering average transition rates one usually average over this population where the mean of $V$ is 1 even in later periods where due to dynamic selection the mean of $V$ is no longer 1 and depends on the treatment arm. The treatment effect identified by the MPH model therefore takes an average over a hypothetical population that at times later than the time origin partly consists of individuals who already left the state of interest and that hypothetical population is the same for every treatment arm. The latter is key in interpreting the effect as causal: by averaging over the same (hypothetical) population we have removed the survivor bias.

In this paper we do not average over the population at the time 0. Instead to define the average effect of the treatment on the transition rate at time $t$ we average over the (hypothetical) population of individuals who would have survived until time $t$ under both treatment arms. The individuals in this population have the same survival experience and any difference between the transition rates must be due to the effect of the treatment. The average is taken over a population that remains in the state of interest. Although we could discuss the definition and identification of treatment effects on transition rates in continuous time the case that time is discrete is conceptually simpler and from now on we assume that transitions occur at times $t = 1, 2, \ldots$.

As before we denote the treatment indicator in period $t$ by $d_t$ and the treatment history up to and including period $t$ by $\overline{d}_t$. Let the potential outcome $Y_t^{\overline{d}_t}$ be an indicator of a transition in period $t$ if the treatment history up to and including $t$ is $\overline{d}_t$. If treatment is an absorbing state, $\overline{d}_t$ is a sequence of 0-s until treatment starts in period $\tau$ and the remaining values are 1. It is possible that $\tau = \infty$, the unit is never treated, or $\tau = 1$, the unit is always in the treated state.

As emphasized we are interested in conditional treatment effects, i.e. treatment effects defined for the survivors in $t$. Let $\overline{d}_{0t}$ and $\overline{d}_{1t}$ be two specific treatment histories. If we average over the hypothetical subpopulation of individuals who would have survived until $t$ under both $\overline{d}_{0t}$ and $\overline{d}_{1t}$, then we define the causal effect of the intervention on the conditional transition rate as

$$\text{ATES}_t^{\overline{d}_{1t},\overline{d}_{0t}} =$$

$$\mathbb{E}\left[Y_t^{\overline{d}_{1t}} | Y_{t-1}^{\overline{d}_{1t-1}} = 0, \ldots, Y_1^{\overline{d}_{11}} = 0, Y_{t-1}^{\overline{d}_{0t-1}} = 0, \ldots, Y_1^{\overline{d}_{01}} = 0\right] -$$

$$\mathbb{E}\left[Y_t^{\overline{d}_{0t}} | Y_{t-1}^{\overline{d}_{1t-1}} = 0, \ldots, Y_1^{\overline{d}_{11}} = 0, Y_{t-1}^{\overline{d}_{0t-1}} = 0, \ldots, Y_1^{\overline{d}_{01}} = 0\right]$$

We call this treatment effect the *Average Treatment Effect on the Survivors* in $t$ ($\text{ATES}_t$). Obvious choices for $\overline{d}_{1t}$ and $\overline{d}_{0t}$ are $\overline{d}_{1t} = (0, \ldots, 0, 1, \ldots, 1)$ with the first 1 at position $\tau$, and $\overline{d}_{0t} = (0, \ldots, 0)$. If we make the usual assumption that there is no effect of the treatment before it starts[1], then $\text{ATES}_t = 0, t = 1, \ldots, \tau - 1$. The differential selection only starts after the treatment begins, so that this property of the $\text{ATES}_t$ is consistent with that fact. After the treatment starts there will be dynamic selection and the $\text{ATES}_t$ controls for that by comparing the transition rates for individuals with a common (hypothetical) survival experience. Because individuals cannot be observed in both treatment arms, we cannot hope that this treatment effect can be identified using available data.

# 3 Identification of treatment effects on transitions under random assignment

We now consider identification of the $\text{ATES}_t$ under random treatment assignment. Random assignment of treatment is the most favorable assignment mechanism. However, we need to define what we mean by random assignment in this case. Let $D_t$ be the indicator that treatment is assigned in period $t$, i.e. the unit is not treated in periods $1, \ldots, t-1$, selected for treatment in period $t$ and, because treatments is assumed to be an absorbing state, remains in the treated state in the subsequent periods. We assume that the treatment is assigned at the beginning of the period, so that the treated responses are observed in periods $t, t+1, \ldots$. We distinguish between three types of randomized assignment

**Assumption 1 (Random assignment of the time of treatment)** *For all $t$ and $\overline{d}_s, s = 1, 2, \ldots$*

$$D_t \perp Y_s^{\overline{d}_s} \qquad s = 1, 2, \ldots$$

**Assumption 2 (Sequential randomization)** *For all $t$ and $\overline{d}_s, s = t, t+1, \ldots$ with the first $t-1$ components equal to 0*

$$D_t \perp Y_s^{\overline{d}_s} \qquad s = t, t+1, \ldots | D_{t-1} = 0$$

[1] Abbring & van den Berg (2003) call this the no-anticipation assumption.

**Assumption 3 (Sequential randomization among survivors)** *For all* $t$ *and* $\overline{d}_s, s = t, t+1, \ldots$ *with the first* $t-1$ *components equal to 0*

$$D_t \perp Y_s^{\overline{d}_s} \qquad s = t, t+1, \ldots | D_{t-1} = 0, Y_{t-1}^0 = \cdots = Y_1^0 = 0.$$

Under assumption 1, the period in which the unit enters the treated state is randomly assigned. Under assumption 2, treatment is assigned randomly in period $t$ to units that have not been treated before, and under assumption, 3 the randomization is among the non-treated survivors. Random assignment of the time of treatment implies sequential randomization, which implies sequential randomization among survivors. In this paper, we focus on identification of average treatment effects under assumption 3.

In the remainder of this paper, we consider the two period case where the transition occurs in period 1, period 2 or after period 2. The reason for this is that all the main points of this paper can be illustrated in that simplified setting. For every member of the population we have a vector of potential outcomes $Y_1^1, Y_1^0, Y_2^{11}, Y_2^{01}, Y_2^{00}$, and vector of treatment indicators $D_1, D_2$. Let $Y_t$ be the observed indicator of a transition in period $t$. These observed outcomes $Y_1, Y_2$ are related to the potential outcomes by the observation rules

$$Y_1 = D_1 Y_1^1 + (1 - D_1) Y_1^0 \tag{2}$$

and

$$Y_2 = D_1 Y_2^{11} + (1 - D_1) D_2 Y_2^{01} + (1 - D_1)(1 - D_2) Y_2^{00}. \tag{3}$$

Because treatment is an absorbing state

$$D_1 = 1 \Rightarrow D_2 = 1.$$

Assumption 3 is in this case

$$D_1 \perp Y_1^1, Y_1^0, Y_2^{11}, Y_2^{01}, Y_2^{00}$$

and

$$D_2 \perp Y_2^{11}, Y_2^{01}, Y_2^{00} | D_1 = 0, Y_1^0 = 0.$$

Hence, under assumption 3 and using the observation rules we can identify from the observed transitions rates the following potential transition probabilities

$$\mathbb{E}(Y_1 | D_1 = 1) = \mathbb{E}(Y_1^1 | D_1 = 1) = \mathbb{E}(Y_1^1) \tag{4}$$

$$\mathbb{E}(Y_1 | D_1 = 0) = \mathbb{E}(Y_1^0 | D_1 = 0) = \mathbb{E}(Y_1^0) \tag{5}$$

$$\mathbb{E}(Y_2 | Y_1 = 0, D_1 = 1) = \mathbb{E}(Y_2^{11} | Y_1^1 = 0, D_1 = 1) = \mathbb{E}(Y_2^{11} | Y_1^1 = 0) \tag{6}$$

$$\mathbb{E}(Y_2 | Y_1 = 0, D_1 = 0, D_2 = 0) = \mathbb{E}(Y_2^{00} | Y_1^0 = 0, D_1 = 0, D_2 = 0) = \mathbb{E}(Y_2^{00} | Y_1^0 = 0) \tag{7}$$

$$\mathbb{E}(Y_2 | Y_1 = 0, D_1 = 0, D_2 = 1) = \mathbb{E}(Y_2^{01} | Y_1^0 = 0, D_1 = 0, D_2 = 1) = \mathbb{E}(Y_2^{01} | Y_1^0 = 0). \tag{8}$$

### 3.1 Identification of instantaneous treatment effects

The $\text{ATES}_t$ defines a number of interesting treatment effects that could be divided into two groups: instantaneous treatment effects and dynamic treatment effects. In the two period setting the two instantaneous treatment effects are

$$\text{ATES}_1^{1,0} = \mathbb{E}(Y_1^1) - \mathbb{E}(Y_1^0)$$

and

$$\text{ATES}_2^{01,00} = \mathbb{E}(Y_2^{01}|Y_1^0 = 0) - \mathbb{E}(Y_2^{00}|Y_1^0 = 0).$$

That is the average instantaneous treatment effect from treatment in the first period, and the average instantaneous treatment effect from treatment in the second period for those who survives the first period. Note that for $\text{ATES}_2^{01,00}$ the treatment in the first period is no treatment in both treatment arms, so that we only need to condition on surviving the first period under no treatment.

From equations (4) and (5) it follow that under assumption 3 we can point identify the instantaneous treatment effect

$$\text{ATES}_1^{1,0} = \text{ATE}_1^{1,0} = \mathbb{E}(Y_1^1) - \mathbb{E}(Y_1^0) = \mathbb{E}(Y_1|D_1 = 1) - \mathbb{E}(Y_1|D_1 = 0), \quad (9)$$

and from equations (7) and (8) we have

$$\text{ATES}_2^{01,00} = \mathbb{E}(Y_2^{01}|Y_1^0 = 0) - \mathbb{E}(Y_2^{00}|Y_1^0 = 0) = \quad (10)$$

$$\mathbb{E}(Y_2|Y_1 = 0, D_1 = 0, D_2 = 1) - \mathbb{E}(Y_1|Y_1 = 0, D_1 = 0, D_2 = 0).$$

### 3.2 Bounds on dynamic treatment effects on transitions

In the two period setting the dynamic treatment effect of interest is

$$\text{ATES}_2^{11,00} = \mathbb{E}(Y_2^{11}|Y_1^1 = 0, Y_1^0 = 0) - \mathbb{E}(Y_2^{00}|Y_1^1 = 0, Y_1^0 = 0),$$

that is the average treatment effect in the second period from treatment in the first period for those who survive under both treatment and no treatment in the first period. It follows directly from equations (4)-(8), which hold under assumption 3, that $\text{ATES}_2^{11,00}$ in general is not point identified. This is because the random assignment of treatment only ensures comparability of the treatment and control groups at the time of randomization. At later times treated units with characteristics that interact with the treatment to increase/decrease the transition probability leave the initial state first/last, so that these characteristics are under/over represented among the remaining treated relative to the remaining controls and this confounds the effect of the treatment. Without out any further assumption we cannot uncover this dynamic selection and point identify the average dynamic treatment effect.

It is, however, clear that the observed transitions rates place restrictions on the potential transition probabilities. We therefore turn to the second main point of this paper and derive sharp bounds on $\text{ATES}_2^{11,00}$. Sharp bounds in the sense that there exists a feasible joint distribution of the potential outcomes

which is consistent with both the upper bound and the lower bound. The sharp bounds are derived by considering the joint distribution of the potential outcomes. The upper (lower) bound is found by constructing a joint distribution of the potential outcomes which, given the restrictions from the observed quantities, maximize (minimize) $\text{ATES}_2^{11,00}$.

In order to simplify the derivations define

$$\begin{array}{rcl} p(y_1^1, y_1^0) & = & \Pr(Y_1^1 = y_1^1, Y_1^0 = y_1^0) \\ p(y_2^{01}, y_2^{00}|1,0) & = & \Pr(Y_2^{01} = y_2^{01}, Y_2^{00} = y_2^{00}|Y_1^1 = 1, Y_1^0 = 0) \\ p(y_2^{11}|0,1) & = & \Pr(Y_2^{11} = y_2^{11}|Y_1^1 = 0, Y_1^0 = 1) \\ p(y_2^{11}, y_2^{01}, y_2^{00}|0,0) & = & \Pr(Y_2^{11} = y_2^{11}, Y_2^{01} = y_2^{01}, Y_2^{00} = y_2^{00}|Y_1^1 = 0, Y_1^0 = 0) \end{array}$$

We consider an absorbing state, so that $Y_2^{10}$ is not defined. In addition as discussed above if $Y_1^1 = 1$ $Y_2^{11}$ is not defined, and if $Y_1^0 = 1$, neither $Y_2^{01}$ nor $Y_2^{00}$ is defined. The parameters of the joint distribution of the potential outcomes are then

$$\begin{array}{rl} p(y_1^1, y_1^0) & y_1^1, y_1^0 = 0, 1 \\ p(y_2^{01}, y_2^{00}|1,0) & y_2^{01}, y_2^{00} = 0, 1 \\ p(y_2^{11}|0,1) & y_2^{11} = 0, 1 \\ p(y_2^{11}, y_2^{01}, y_2^{00}|0,0) & y_2^{11}, y_2^{01}, y_2^{00} = 0, 1. \end{array}$$

We consider bounds on

$$ATES_2^{11,00} = \sum_{y_2^{00}=0,1} \sum_{y_2^{01}=0,1} p(1, y_2^{01}, y_2^{00}|0,0) - \sum_{y_2^{11}=0,1} \sum_{y_2^{01}=0,1} p(y_2^{11}, y_2^{01}, 1|0,0) \tag{11}$$

and

$$\mathbb{E}[Y_2^{11}|Y_1^1 = 0, Y_1^0 = 0] = \sum_{y_2^{00}=0,1} \sum_{y_2^{01}=0,1} p(1, y_2^{01}, y_2^{00}|0,0) \tag{12}$$

and

$$\mathbb{E}[Y_2^{00}|Y_1^1 = 0, Y_1^0 = 0] = \sum_{y_2^{11}=0,1} \sum_{y_2^{01}=0,1} p(y_2^{11}, y_2^{01}, 1|0,0). \tag{13}$$

The observed fractions, in the first period, with $D_1, Y_1$ give

$$\Pr(Y_1 = y_1|D_1 = 1) = \sum_{y_1^0=0,1} p(y_1, y_1^0) \tag{14}$$

and

$$\Pr(Y_1 = y_1|D_1 = 0) = \sum_{y_1^1=0,1} p(y_1^1, y_1) \tag{15}$$

, and the observed fractions, in the second period, with $D_2, Y_2$ give

$$\Pr(Y_2 = y_2|D_1 = 1, Y_1 = 0) = \tag{16}$$

$$\frac{\sum_{y_2^{01}=0,1} \sum_{y_2^{00}=0,1} p(y_2, y_2^{01}, y_2^{00}|0,0)p(0,0) + p(y_2|0,1)p(0,1)}{\sum_{y_1^0=0,1} p(0, y_1^0)}$$

and

$$\Pr(Y_2 = y_2|D_1 = 0, D_2 = 0, Y_1 = 0) = \tag{17}$$
$$\frac{\sum_{y_2^{11}=0,1}\sum_{y_2^{01}=0,1} p(y_2^{11}, y_2^{01}, y_2|0,0)p(0,0) + \sum_{y_2^{01}=0,1} p(y_2^{01}, y_2|0,1)p(1,0)}{\sum_{y_1^1=0,1} p(y_1^1, 0)}$$

and

$$\Pr(Y_2 = y_2|D_1 = 0, D_2 = 1, Y_1 = 0) = \tag{18}$$
$$\frac{\sum_{y_2^{11}=0,1}\sum_{y_2^{00}=0,1} p(y_2^{11}, y_2, y_2^{00}|0,0)p(0,0) + \sum_{y_2^{00}=0,1} p(y_2, y_2^{00}|0,1)p(1,0)}{\sum_{y_1^1=0,1} p(y_1^1, 0)}.$$

The bounds are obtained by minimizing and maximizing (11)-(13) under the restrictions (14)-(18), and obviously with the additional restriction that all probabilities by definition lie between zero and one. Both the outcomes in equations (11)-(13) and the restrictions are linear, so that the bounds are the solution to a LP problem.

Our main results are

**Proposition 1 (Bounds on conditional transition probabilities)** *Suppose that assumption 3 holds. Then*

$$max(0, \frac{\Pr(Y_2 = 1|D_1 = 1, Y_1 = 0)\Pr(Y_1 = 0|D_1 = 1)}{max(\Pr(Y_1 = 0|D_1 = 1) + \Pr(Y_1 = 0|D_1 = 0) - 1, 0)} -$$
$$\frac{\Pr(Y_1 = 0|D_1 = 1) - max(\Pr(Y_1 = 0|D_1 = 1) + \Pr(Y_1 = 0|D_1 = 0) - 1, 0)}{max(\Pr(Y_1 = 0|D_1 = 1) + \Pr(Y_1 = 0|D_1 = 0) - 1, 0)})$$
$$\leq \mathbb{E}[Y_2^{11}|Y_1^1 = 0, Y_1^0 = 0] \leq$$
$$min(1, \frac{\Pr(Y_2 = 1|D_1 = 1, Y_1 = 0)\Pr(Y_1 = 0|D_1 = 1)}{max(\Pr(Y_1 = 0|D_1 = 1) + \Pr(Y_1 = 0|D_1 = 0) - 1, 0)})$$

*and*

$$max(0, \frac{\Pr(Y_2 = 1|D_1 = 0, D_2 = 0, Y_1 = 0)\Pr(Y_1 = 0|D_1 = 0)}{max(\Pr(Y_1 = 0|D_1 = 1) + \Pr(Y_1 = 0|D_1 = 0) - 1, 0)} -$$
$$\frac{\Pr(Y_1 = 0|D_1 = 0) - max(\Pr(Y_1 = 0|D_1 = 1) + \Pr(Y_1 = 0|D_1 = 0) - 1, 0)}{max(\Pr(Y_1 = 0|D_1 = 1) + \Pr(Y_1 = 0|D_1 = 0) - 1, 0)})$$
$$\leq \mathbb{E}[Y_2^{00}|Y_1^1 = 0, Y_1^0 = 0] \leq$$
$$min(1, \frac{\Pr(Y_2 = 1|D_1 = 0, D_2 = 0, Y_1 = 0)\Pr(Y_1 = 0|D_1 = 0)}{max(\Pr(Y_1 = 0|D_1 = 1) + \Pr(Y_1 = 0|D_1 = 0) - 1, 0)}),$$

*and*

$$max(0, \frac{\Pr(Y_2 = 1|D_1 = 1, Y_1 = 0)\Pr(Y_1 = 0|D_1 = 1)}{max(\Pr(Y_1 = 0|D_1 = 1) + \Pr(Y_1 = 0|D_1 = 0) - 1, 0)} -$$
$$\frac{\Pr(Y_1 = 0|D_1 = 1) - max(\Pr(Y_1 = 0|D_1 = 1) + \Pr(Y_1 = 0|D_1 = 0) - 1, 0)}{max(\Pr(Y_1 = 0|D_1 = 1) + \Pr(Y_1 = 0|D_1 = 0) - 1, 0)}) -$$
$$min(1, \frac{\Pr(Y_2 = 1|D_1 = 0, D_2 = 0, Y_1 = 0)\Pr(Y_1 = 0|D_1 = 0)}{max(\Pr(Y_1 = 0|D_1 = 1) + \Pr(Y_1 = 0|D_1 = 0) - 1, 0)})$$
$$\leq ATES_2^{11,00} \leq$$
$$min(1, \frac{\Pr(Y_2 = 1|D_1 = 1, Y_1 = 0)\Pr(Y_1 = 0|D_1 = 1)}{max(\Pr(Y_1 = 0|D_1 = 1) + \Pr(Y_1 = 0|D_1 = 0) - 1, 0)}) -$$

$$max(0, \frac{\Pr(Y_2 = 1|D_1 = 0, D_2 = 0, Y_1 = 0)\Pr(Y_1 = 0|D_1 = 0)}{max(\Pr(Y_1 = 0|D_1 = 1) + \Pr(Y_1 = 0|D_1 = 0) - 1, 0)} -$$
$$\frac{\Pr(Y_1 = 0|D_1 = 0) - max(\Pr(Y_1 = 0|D_1 = 1) + \Pr(Y_1 = 0|D_1 = 0) - 1, 0)}{max(\Pr(Y_1 = 0|D_1 = 1) + \Pr(Y_1 = 0|D_1 = 0) - 1, 0)}).$$

**Proof** see Appendix A. □

Proposition 1 provides a closed form solution for the sharp bounds on $\text{ATES}_2^{11,00}$. These bounds impose no assumptions beyond sequential random assignment among survivors. In fact, we make no assumptions on functional form and additional covariates, and we allow for arbitrary heterogeneous treatment effects as well as arbitrary unobserved heterogeneity. From these general results follow two important results on point identification and on the informativeness of the bounds

**Corollary 1 (Point identification)** *1. Suppose that assumption 3 and* $\Pr(Y_1 = 0|D_1 = 0) = 1$ *hold. Then* $\mathbb{E}[Y_2^{11}|Y_1^1 = 0, Y_1^0 = 0]$ *is point identified and equal to* $\Pr(Y_2 = 1|Y_1 = 0, D_1 = 1)$.

*2. Suppose that assumption 3 and* $\Pr(Y_1 = 0|D_1 = 1) = 1$ *hold. Then* $\mathbb{E}[Y_2^{00}|Y_1^1 = 0, Y_1^0 = 0]$ *is point identified and equal to* $\Pr(Y_2 = 1|Y_1 = 0, D_2 = 0, D_1 = 0)$.

*3. Suppose that assumption 3,* $\Pr(Y_1 = 0|D_1 = 1) = 1$ *and* $\Pr(Y_1 = 0|D_1 = 0) = 1$ *hold. Then* $ATES_2^{11,00}$ *is point identified and equal to* $\Pr(Y_2 = 1|Y_1 = 0, D_1 = 1) - \Pr(Y_2 = 1|Y_1 = 0, D_1 = 0, D_2 = 0)$.

**Proof** see Appendix A. □

**Corollary 2 (Informative bounds)** *Define* $A \equiv max(\Pr(Y_1 = 0|D_1 = 1) + \Pr(Y_1 = 0|D_1 = 0) - 1, 0)$. *Suppose that assumption 3 hold. In addition if either*

$$\frac{\Pr(Y_2 = 1|D_1 = 1, Y_1 = 0)\Pr(Y_1 = 0|D_1 = 1)}{A} < 1$$

*or*

$$1 - \frac{[1 - \Pr(Y_2 = 1|D_1 = 0, D_2 = 0, Y_1 = 0)]\Pr(Y_1 = 0|D_1 = 0)}{A} > 0$$

*or*

$$1 - \frac{[1 - \Pr(Y_2 = 1|D_1 = 1, Y_1 = 0)]\Pr(Y_1 = 0|D_1 = 1)}{A} > 0$$

*or*

$$\frac{\Pr(Y_2 = 1|D_1 = 0, D_2 = 0, Y_1 = 0)\Pr(Y_1 = 0|D_1 = 0)}{A} < 1$$

*hold. Then the bounds in proposition 1 are informative on* $ATET_2^{11,00}$.

**Proof** see Appendix A. □

Corollary 1 shows that if there is no dynamic selection, i.e. if $\Pr(Y_1 = 0|D_1 = 1) = 1$ and $\Pr(Y_1 = 0|D_1 = 1) = 1$, the dynamic treatment effect $ATES_2^{11,00}$ is point identified. If everyone survive the first period we have under random treatment two directly comparable groups even in the second period. The corollary also includes two results which may seem counterintuitive: $\mathbb{E}[Y_2^{11}|Y_1^1 = 0, Y_1^0 = 0]$ is point identified if $\Pr(Y_1 = 0|D_1 = 0) = 1$, and $\mathbb{E}[Y_2^{00}|Y_1^1 = 0, Y_1^0 = 0]$ is point identified if $\Pr(Y_1 = 0|D_1 = 1) = 1$. That is the counterfactual outcome under treatment (no treatment) is point identified if no

one exits in the control (treatment) group. The intuition behind these results are that we consider the average treatment effect for those who survive the first period under both treatment and control. If $Y_1^1 = 0$ for everyone and under random assignment we have

$$\mathbb{E}[Y_2^{00}|Y_1^1 = 0, Y_1^0 = 0] = \mathbb{E}[Y_2^{00}|Y_1^0 = 0] = \mathbb{E}[Y_2|Y_1 = 0, D_1 = 0, D_2 = 0].$$

Together with similar reasoning for $\mathbb{E}[Y_2^{11}|Y_1^1 = 0, Y_1^0 = 0]$ give the results in the corollary.

Corollary 2 tells us that the bounds are informative as long as $\Pr(Y_1 = 0|D_1 = 1) = 1$ and $\Pr(Y_1 = 0|D_1 = 1) = 1$ are not too small. Even though the bounds often are informative they can be quite wide in many situations. If $\Pr(Y_1 = 0|D_1 = 1) + \Pr(Y_1 = 0|D_1 = 0) \geq 1$ it follows from proposition 1 that the width of the bounds on $ATES_2^{11,00}$ are

$$\frac{2 - \Pr(Y_1 = 0|D_1 = 1) - \Pr(Y_1 = 0|D_1 = 0)}{\Pr(Y_1 = 0|D_1 = 1) + \Pr(Y_1 = 0|D_1 = 0) - 1}.$$

In other words, the width of the general bounds is directly related to the size of $\Pr(Y_1 = 0|D_1 = 1)$ and $\Pr(Y_1 = 0|D_1 = 0)$, i.e. how large fraction that leaves the state of interest in the first period.

# 4 Identification of treatment effects on conditional transitions under additional weak assumptions

The sharp bounds in the previous section did not impose any assumptions beyond random assignment. In this section, we explore the identifying power of additional weak assumptions. To try to make the intuition behind the assumptions clear, we discuss our assumptions in the context of a medical example and relate them to the assumptions made in the popular MPH model. Again, the MPH model specifies the individual hazard rate for individual $i$ as

$$\theta(t, d(t), V_i) = \lambda(t)\gamma(t - \tau, \tau)^{d(t)}v_i \tag{19}$$

or, in a regression-type expression, the integrated hazard function for individual $i$ as

$$\log \int_0^t \lambda(t) = -\log \gamma(t - \tau, \tau)^{d(t)} - \log v_i + \varepsilon_i, \tag{20}$$

where $\varepsilon$ is $EV1$ extreme value type 1 distributed. The time to a transition $T_i^{d(T)}$ for individual $i$ is fully determined by: the baseline hazard $\lambda(t)$, the treatment path $d(T)$ , the homogenous[2] treatment effect $\gamma$, the nonnegative spell constant

[2]By homogenous we mean that given the time of treatment and the time elapsed since treatment there is one homogenous treatment effect for all individuals.

unobservable heterogeneity for individual $i$ $v_i$, and the specific draw of $\varepsilon_i$. The MPH model builds on several assumptions, most notably: proportionality of the hazard function, homogenous treatment effect, spell constant unobserved individual heterogeneity, and a single one dimensional shock which given the hazard function determines the realized time to a transition. In some applications, these assumptions are harmless and in other applications they are very restrictive. Obviously, our general bounds are sometimes wide as we impose neither one of these assumptions.

In many applications one weak assumption is monotone treatment response (MTR). The assumption has been explored by e.g. Manski (1997) and Manski & Pepper (2000). In a transition framework the assumption has to be modified. Most often, there will not be a single MTR assumption in a transition framework. For instance, one may assume positive treatment effect for all individuals in some time period and negative treatment effect for all individuals in another time period. Let $Y_{it}^{\overline{d}_t}$ be the indicator of a transition in period $t$ for individual $i$ if the treatment history up to and including $t$ is $\overline{d}_t$. In our two period example, we define three MTR assumptions appropriate for that setting: MTR with observed sign in the first period, and either negative or positive MTR in period 2 from treatment given in time period 1, as[3]

[3]Another more subtle difference compared to Manski & Pepper (2000) is that we phrase the assumptions in terms of something that is most accurately described as the average individual treatment effect. Manski & Pepper (2000) states their assumption in the form $Y_{it}^1 > Y_{it}^0$ with one single individual treatment effect. In a transition framework $Y_{it}^1 - Y_{it}^0$ could either be -1,0, or 1. It is thus reasonable to focus on the average individual treatment effect, for instance $\mathbb{E}(Y_{i1}^1) - \mathbb{E}(Y_{i1}^0)$.

**Assumption 4 (Monotone treatment response in period 1)** *For t=1 and all i*

$$\Pr(Y_1^1 = 1) \geq \Pr(Y_1^0 = 1) \Rightarrow \Pr(Y_{i1}^1 = 1) \geq \Pr(Y_{i1}^0 = 1)$$

*and*

$$\Pr(Y_1^1 = 1) \leq \Pr(Y_1^0 = 1) \Rightarrow \Pr(Y_{i1}^1 = 1) \leq \Pr(Y_{i1}^0 = 1)$$

**Assumption 5 (Positive MTR in period 2 from treatment in period 1)** *For t=2 and all i*

$$\Pr(Y_{i2}^{11} = 1 | Y_{i1}^1 = 0, Y_{i1}^0 = 0) \geq \Pr(Y_{i2}^{00} = 1 | Y_{i1}^0 = 0, Y_{i1}^1 = 0)$$

**Assumption 6 (Negative MTR in period 2 from treatment in period 1)** *For t=2 and all i*

$$\Pr(Y_{i2}^{00} = 1 | Y_{i1}^1 = 0, Y_{i1}^0 = 0) \geq \Pr(Y_{i2}^{11} = 1 | Y_{i1}^0 = 0, Y_{i1}^1 = 0)$$

For intuition behind these assumptions let us consider a medical example. The set up is as follows: time of origin is the date when the patient is diagnosed with cancer. The treatment is chemotherapy, which can start directly after the patient has been diagnosed with cancer, i.e. in time period 1, or at some later time period $t$. The transition state is death. In this context, assumption 4 means that if we observe a positive (negative) effect on average from being instantly treated with chemotherapy we conclude that all patients benefit (suffer) from being instantly treated with chemotherapy. Assumption 5 (assumption 6) implies that we assume that all patients who survive the first period benefit (suffer) in the second period from chemotherapy started in the first period.

Another source of heterogeneity in our general setting is that we have not placed any restrictions on the unobserved heterogeneity in the model. In the MPH model unobserved heterogeneity is introduced by $v_i$, the spell constant unobserved heterogeneity in the transition rate and by $\varepsilon_i$, the one dimensional idiosyncratic shock which given the transition rate determines if a transition is realized or not. Needless to say, this places restrictions on the types of unobserved heterogeneity that is plausible. One could, for instance, imagine that the shocks are multidimensional, with one shock under treatment and one shock under no treatment. As an illustration, return to the medical example, and assume that we know that chemotherapy on average is beneficial for a certain patient and that this patient receives chemotherapy and dies in time period one. One question then is what can be inferred about what would have happened to this patient if the patient would not have received chemotherapy, i.e. what can we say about $\Pr(Y_{i1}^0 = 1 | Y_{i1}^1 = 1)$. In the MPH model the answer is straightforward: as the effect of the treatment is positive on average $(\gamma(t - \tau, \tau)^{d(t)} > 1)$ and we have the single shock $\varepsilon$, it implies that we know that the patient would have died also under no treatment.

The problem with identification without the MPH model assumptions can be seen by noticing that

$$\Pr(Y_{i1}^1 = 1) < \Pr(Y_{i1}^0 = 1)$$

implies that

$$\Pr(Y_{i1}^1 = 1|Y_{i1}^0 = 1)\Pr(Y_{i1}^0 = 1) + \Pr(Y_{i1}^1 = 1|Y_{i1}^0 = 0)\Pr(Y_{i1}^0 = 0) <$$

$$\Pr(Y_{i1}^0 = 1|Y_{i1}^1 = 1)\Pr(Y_{i1}^1 = 1) + \Pr(Y_{i1}^0 = 1|Y_{i1}^1 = 0)\Pr(Y_{i1}^1 = 0).$$

As easily seen, without any further assumptions, one cannot say much about these conditional probabilities using only information on the marginal probabilities $\Pr(Y_{i1}^1 = 1)$ and $\Pr(Y_{i1}^0 = 1)$. If one nevertheless infer information from the marginal probabilities one have placed restrictions on the types of unobserved heterogeneity that is possible in the model. In fact, it may be the case that $\Pr(Y_{it}^1 = 1|Y_{it}^0 = 1) = 0$ and $\Pr(Y_{it}^0 = 1|Y_{it}^1 = 1) = 0$ even if $\Pr(Y_{it}^1 = 1) \neq 0$ and $\Pr(Y_{it}^0 = 1) \neq 0$.

In this paper we explore the identifying power of the one shock assumption made in the MPH model and weaker versions of it. For presentation reasons define for two treatment histories $\overline{d}_{st}$ and $\overline{d}_{kt}$

$$A(0) \equiv \mathbf{1}(Y_{t-1}^{\overline{d}_{st-1}} = 0, \ldots, Y_1^{\overline{d}_{s1}} = 0, Y_{t-1}^{\overline{d}_{kt-1}} = 0, \ldots, Y_1^{\overline{d}_{k1}} = 0)$$

as an indicator function taking the value one if the expression in the parenthesis is true. We explore the two assumptions

**Assumption 7 (Positively correlated shocks)** *For all $t$ and $i$ and each pair of treatment histories, denoted by $\overline{d}_{st}$ and $\overline{d}_{kt}$. If*

$$\Pr(Y_{it}^{\overline{d}_{st}} = 1|A(0) = 1) \geq \Pr(Y_{it}^{\overline{d}_{kt}} = 1|A(0) = 1)$$

*holds then*

$$\Pr(Y_{it}^{\overline{d}_{st}} = 1|Y_{it}^{\overline{d}_{kt}} = 1, A(0) = 1) \geq \Pr(Y_{it}^{\overline{d}_{st}} = 0|Y_{it}^{\overline{d}_{kt}} = 1, A(0) = 1)$$

$$\Pr(Y_{it}^{\overline{d}_{kt}} = 0|Y_{it}^{\overline{d}_{s,t}} = 0, A(0) = 1) \geq \Pr(Y_{it}^{\overline{d}_{kt}} = 1|Y_{it}^{\overline{d}_{st}} = 0, A(0) = 1),$$

*, and if*

$$\Pr(Y_{it}^{\overline{d}_{st}} = 1|A(0) = 1) \leq \Pr(Y_{it}^{\overline{d}_{kt}} = 1|A(0) = 1)$$

*holds then*

$$\Pr(Y_{it}^{\overline{d}_{kt}} = 1|Y_{it}^{\overline{d}_{st}} = 1, A(0) = 1) \geq \Pr(Y_{it}^{\overline{d}_{kt}} = 0|Y_{it}^{\overline{d}_{st}} = 1, A(0) = 1)$$

$$\Pr(Y_{it}^{\overline{d}_{st}} = 0|Y_{it}^{\overline{d}_{kt}} = 0, A(0) = 1) \geq \Pr(Y_{it}^{\overline{d}_{st}} = 1|Y_{it}^{\overline{d}_{kt}} = 0, A(0) = 1).$$

**Assumption 8 (Single dimensional shock)** *For all $t$ and $i$ and each pair of treatment histories, denoted by $\overline{d}_{st}$ and $\overline{d}_{kt}$*

$$\Pr(Y_{it}^{\overline{d}_{st}} = 1|A(0) = 0) \geq \Pr(Y_{it}^{\overline{d}_{kt}} = 1|A(0) = 0) \Rightarrow$$

$$(Y_{it}^{\overline{d}_{st}} \geq Y_{it}^{\overline{d}_{kt}}|A(0) = 0)$$

*and*

$$\Pr(Y_{it}^{\overline{d}_{st}} = 1|A(0) = 0) \leq \Pr(Y_{it}^{\overline{d}_{kt}} = 1|A(0) = 0) \Rightarrow$$

$$(Y_{it}^{\overline{d}_{st}} \leq Y_{it}^{\overline{d}_{kt}}|A(0) = 0).$$

For intuition behind these assumptions consider the medical example. Assumption 7 allows for different shocks under treatment and no treatment, but it assumes that these shocks are positively correlated. More precisely, if a randomly induced flu causes the patient to die in the first period we expect the same patient to also be exposed to the flu under no treatment. There is, however, some randomness involved, so that it may not be an exactly equally severe flu. Assumption 8 implies that all random events like exposure to a flu are the same no matter if the patient receives the treatment or not.

Combining assumption 4 with assumption 7 give

**Proposition 2 (Bounds under MTR and positively correlated shocks)** *Define* $A \equiv max(-\frac{1}{2} + \Pr(Y_1 = 0|D_1 = 0) + \frac{1}{2}\Pr(Y_1 = 0|D_1 = 1), \frac{\Pr(Y_1=0|D_1=0)}{2})$ *and* $B \equiv max(-\frac{1}{2} + \Pr(Y_1 = 0|D_1 = 1) + \frac{1}{2}\Pr(Y_1 = 0|D_1 = 0), \frac{\Pr(Y_1=0|D_1=1)}{2})$. *Suppose assumption 3, 4, and 7 hold. Then if* $\Pr(Y_1 = 1|D_1 = 1) < \Pr(Y_1 = 1|D_1 = 0)$

$$max(0, \frac{A - [1 - \Pr(Y_2 = 1|D_1 = 1, Y_1 = 0)]\Pr(Y_1 = 0|D_1 = 1)}{A}) -$$

$$min(1, \frac{\Pr(Y_2 = 1|D_1 = 0, D_2 = 0, Y_1 = 0)\Pr(Y_1 = 0|D_1 = 0)}{A})$$

$$\leq ATES_2^{11,00} \leq$$

$$min(1, \frac{\Pr(Y_2 = 1|D_1 = 1, Y_1 = 0)\Pr(Y_1 = 0|D_1 = 1)}{A}) -$$

$$max(0, \frac{A - [1 - \Pr(Y_2 = 1|D_1 = 0, D_2 = 0, Y_1 = 0)]\Pr(Y_1 = 0|D_1 = 0)}{A})$$

*and if* $\Pr(Y_1 = 1|D_1 = 1) > \Pr(Y_1 = 1|D_1 = 0)$

$$max(0, \frac{B - [1 - \Pr(Y_2 = 1|D_1 = 1, Y_1 = 0)]\Pr(Y_1 = 0|D_1 = 1)}{B}) -$$

$$min(1, \frac{\Pr(Y_2 = 1|D_1 = 0, D_2 = 0, Y_1 = 0)\Pr(Y_1 = 0|D_1 = 0)}{B})$$

$$\leq ATES_2^{11,00} \leq$$

$$min(1, \frac{\Pr(Y_2 = 1|D_1 = 1, Y_1 = 0)\Pr(Y_1 = 0|D_1 = 1)}{B}) -$$

$$max(0, \frac{B - [1 - \Pr(Y_2 = 1|D_1 = 0, D_2 = 0, Y_1 = 0)]\Pr(Y_1 = 0|D_1 = 0)}{B})$$

**Proof** see Appendix A. □

and combining assumption 4 and assumption 8 give

**Proposition 3 (Bounds under MTR and a single shock)** *Suppose assumption 3, 4, and 7 holds. Then if* $\Pr(Y_1 = 1|D_1 = 1) < \Pr(Y_1 = 1|D_1 = 0)$

$$max(0, \frac{\Pr(Y_1 = 0|D = 0) - [1 - \Pr(Y_2 = 1|D_1 = 1, Y_1 = 0)]\Pr(Y_1 = 0|D_1 = 1)}{\Pr(Y_1 = 0|D_1 = 0)}) -$$

$$\Pr(Y_2 = 1|D_1 = 0, D_2 = 0, Y_1 = 0)$$

$$\leq ATES_2^{11,00} \leq$$

$$min(1, \frac{\Pr(Y_2=1|D_1=1,Y_1=0)\Pr(Y_1=0|D_1=1)}{\Pr(Y_1=0|D=0)}) -$$
$$\Pr(Y_2=1|D_1=0,D_2=0,Y_1=0)$$

*and if* $\Pr(Y_1=1|D=1) > \Pr(Y_1=1|D_1=0)$

$$\Pr(Y_2=1|D_1=1,Y_1=0) -$$
$$min(1, \frac{\Pr(Y_2=1|D_1=0,D_2=0,Y_1=0)\Pr(Y_1=0|D_1=0)}{\Pr(Y_1=0|D_1=1)})$$
$$\leq ATES_2^{11,00} \leq$$
$$\Pr(Y_2=1|D_1=1,Y_1=0) - max(0, \frac{\Pr(Y_1=0|D=1)}{\Pr(Y_1=0|D_1=1)} -$$
$$\frac{[1-\Pr(Y_2=1|D_1=0,D_2=0,Y_1=0)]\Pr(Y_1=0|D_1=0)}{\Pr(Y_1=0|D_1=1)})$$

**Proof** see Appendix A. □

These expressions show that these weak assumptions may have strong identifying power. This will be further illustrated in our application to re-employment bonus experiment. If $\Pr(Y_1=1|D=1) \approx \Pr(Y_1=1|D=0)$ the bounds under the MTR assumption and the single shock assumption are very narrow, as we have assumed that the treated and non treated who exit during the first period have similar characteristics. Note that, if either assumption 4 or assumption 8 do not hold the bounds on the $ATES_2^{11,00}$ may be wide even if $\Pr(Y_1=1|D=1) \approx \Pr(Y_1=1|D=0)$.

The two assumptions of positive respectively negative treatment response in period 2 will effectively bound away negative respectively positive average treatment effects. For completeness are these bounds presented in Appendix B.

A third major source of heterogeneity in our general setting is that we have not placed any restrictions on the relation between $\Pr(Y_2^{11}=1|Y_1^1=0,Y_1^0=0)$ and $\Pr(Y_2^{11}=1|Y_1^1=0,Y_1^0=1)$, and no restrictions on the relation between $\Pr(Y_2^{00}=1|Y_1^1=0,Y_1^0=0)$ and $\Pr(Y_2^{00}=1|Y_1^1=1,Y_1^0=0)$. In fact, we allow for the extreme case that those who survives under both treatment and no treatment in the first period all exit under no treatment in the second period, whereas none of those who exit under treatment and survives under no treatment in the first period exit under no treatment in the second period. It means that some individuals exit relatively faster in one time period and relatively slower in another time period. The corresponding assumption in the MPH model of fixed unobserved heterogeneity obviously rules out any such heterogeneity.

We explore a weaker assumption compared to fixed unobserved heterogeneity, and explore the assumption that some individuals are inherently "weaker" than others under both treatment and no treatment as well as in all time periods. We call this monotone exit rate and define it as

**Assumption 9 (Monotone exit rate)** *For two individuals $i \neq j$, either*

$$\mathbb{E}[Y_{it}^{\overline{d}_s}|Y_{it-1}^{\overline{d}_s}=0,\ldots,Y_{i1}^{\overline{d}_s}=0] \leq \mathbb{E}[Y_{jt}^{\overline{d}_s}|Y_{jt-1}^{\overline{d}_s}=0,\ldots,Y_{j1}^{\overline{d}_s}=0]$$

*or*

$$\mathbb{E}[Y_{it}^{\overline{d}_s}|Y_{it-1}^{\overline{d}_s}=0,\ldots,Y_{i1}^{\overline{d}_s}=0] \geq \mathbb{E}[Y_{jt}^{\overline{d}_s}|Y_{jt-1}^{\overline{d}_s}=0,\ldots,Y_{j1}^{\overline{d}_s}=0].$$

*hold for all $t$ and all treatment histories $\overline{d}_s$.*

Let us once again return to the medical example. If patient A has larger chance of dying without chemotherapy compared with patient B in time period one, the monotone exit assumption implies that patient A also has larger chance of dying with chemotherapy in period one. It further means that if both patients survive until time period $t$, patient A has larger chance of dying under both chemotherapy and without chemotherapy in time period $t$. In other words, patient A is assumed to be inherently more fragile compared to patient B. In the two period case the monotone exit assumption implies that $\Pr(Y_2^{11}=1|Y_1^1=0,Y_1^0=0) \leq \Pr(Y_2^{11}=1|Y_1^1=0,Y_1^0=1)$ and $\Pr(Y_2^{00}=1|Y_1^1=0,Y_1^0=0) \leq \Pr(Y_2^{00}=1|Y_1^1=1,Y_1^0=0)$. We then have

**Proposition 4 (Bounds under monotone exit rate)** *Define $A \equiv max(\Pr(Y_1=0|D_1=1)+\Pr(Y_1=0|D_1=0)-1,0)$. Suppose that assumption 3 and assumption 9 holds. Then*

$$max(0, \frac{A-[1-\Pr(Y_2=1|D_1=1,Y_1=0)]\Pr(Y_1=0|D_1=1)}{A}-$$
$$\Pr(Y_2=1|D_1=0,D_2=0,Y_1=0))$$
$$\leq ATES_2^{11,00} \leq$$
$$\Pr(Y_2=1|D_1=1,Y_1=0)-$$
$$max(0, \frac{A-[1-\Pr(Y_2=1|D_1=0,D_2=0,Y_1=0)]\Pr(Y_1=0|D_1=0)}{A}).$$

**Proof** see Appendix A. □

For completion we report the bounds under monotone exit rate combined with MTR and positively correlated shocks, and the bounds under monotone exit rate combined with MTR and single shocks in Appendix B.

# 5 Application to the Illinois bonus experiment

## 5.1 The re-employment bonus experiment

Between mid-1984 and mid-1985, the Illinois Department of Employment Security conducted a controlled social experiment.[4] The goal of the experiment was to explore, whether bonuses paid to Unemployment Insurance (UI) beneficiaries (treatment 1) or their employers (treatment 2) reduced the unemployment of beneficiaries relative to a randomly selected control group. In this paper we focus primarily on the effect of treatment 1.

Both treatments consisted of a $ 500 bonus payment, which was about four times the average weekly unemployment insurance benefit. In the experiment,

[4] A complete description of the experiment and a summary of its results can be found in Woodbury & Spiegelman (1987).

newly unemployed claimants were randomly divided into three groups:
1. The *Claimant Bonus Group*. The members of this group were instructed that they would qualify for a cash bonus of $500 if they found a job (of at least 30 hours) within 11 weeks and, if they held that job for at least 4 months. 4186 individuals were selected for this group, of those 3527 (84%) agreed to participate.
2. The *Employer Bonus Group*. The members of this group were told that their next employer would qualify for a cash bonus of $500 if they, the claimants, found a job (of at least 30 hours) within 11 weeks and, if they held that job for at least four months. 3963 were selected for this group and 2586 (65%) agreed to participate.
3. The *Control Group*, i.e. all claimants not assigned to one of the other groups. This group consisted of 3952 individuals. The individuals assigned to the control group were excluded from participation in the experiment. In fact, they did not know that the experiment took place.

The descriptive statistics in Table 2 in Woodbury & Spiegelman (1987) confirm that the randomization resulted in three similar groups.

## 5.2 Results of previous studies

Woodbury & Spiegelman (1987) concluded from a direct comparison of the control group and the two treatment groups that the claimant bonus group had significantly smaller average unemployment duration. The average unemployment duration was also smaller for the employer bonus group, but the difference was not significantly different from zero. In the USA, UI benefits end after 26 weeks, meaning that all unemployment durations are censored at 26 weeks. Therefore note that the response variable is insured weeks of unemployment, and not weeks out of employment.

Meyer (1996) analyzed the same data but focused on the treatment effects on conditional transition rates. Besides taking care of censoring, Meyer focuses on the conditional transitions rates because labor supply and search theories suggest interesting dynamic treatment effects. The bonus is only given to the unemployed if (s)he finds a job within 11 weeks and retains it for four months. The cash bonus is also the same for all unemployed. Based on these features theory gives some interesting predictions, all investigated by Meyer (1996). The first prediction is that the transition rate during the eligibility period (first 11 weeks) will be higher in the two treatment groups compared with the control group. A second prediction is that the transition rate in the treatment groups should rise just before the end of the eligibility period, as the unemployed are in a hurry to collect the bonus.

In order to analyze these predictions, Meyer (1996) estimates a proportional hazard (PH) model with a flexible specification of the baseline hazard. He uses the treatment indicator as an explanatory variable. Since, there was partial compliance with treatment his estimator can be interpreted as a intention to

treat (ITT) estimator.[5] In his analysis Meyer (1996) controls for age, the logarithm of base period earnings, race, sex and the logarithm of the size of the unemployment insurance benefits. He finds a significantly positive effect of the claimant bonus and positive but insignificant effect of the employer bonus. A more detailed analysis of the effects for the claimant group reveals positive effect on the transition rate during the first 11 weeks in unemployment, an increased effect during week 9 and 10, and no significant effect on the transition rate after week 11. All these results are in line with the predictions from labor supply models and search theories.

### 5.3 Set identification

Meyer (1996) heavily relies on the proportionality of the hazard rate to investigate the hypothesis suggested by labor supply models and search theories. We now ask what can be said about these hypothesis if the assumptions imbedded in the MPH (PH) model do not hold, that is what can be identified relying solely on random assignment and additional weak assumptions. We follow Meyer (1996) and estimate the ITT effect. We divide time into 12 discrete periods: week 1-2, week 3-4, ... , week 23-24. The reason for this is that there is a pronounced even-odd week effect in the data, with higher transition rate during odd weeks. In this setting the theoretical predictions we wish to test could be expressed as; (*i*) positive treatment effect during the period when the bonus could be claimed (period 1-5)

$$\text{ATES}_1^{1,0}, \ldots, \text{ATES}_5^{1\ldots1,0\ldots0} > 0,$$

(*ii*) no effect once the bonus offer have expired (period 6-12)

$$\text{ATES}_6^{1\ldots1,0\ldots0}, \ldots, \text{ATES}_{12}^{1\ldots1,0\ldots0} = 0,$$

and (*iii*) intensified effect of the bonus offer at the end of the eligibility period (period 5)

$$\text{ATES}_5^{1\ldots1,0\ldots0} > \text{ATES}_4^{1\ldots1,0\ldots0}.$$

From section 3 we have that under random assignment $\text{ATE}_1^{1,0}$ is point identified, and that $\text{ATES}_2^{11,00}$ in general is not point identified. We also wish to consider bounds on $\text{ATES}_t^{1\ldots1,0\ldots0}$ for $t > 2$. It is clear that when deriving such bounds one would end up with a sequence of restrictions: one for the treatment group and one for the control group in each time period. We consider a simpler version of these bounds. Consider the bounds for time period $t$: one way of constructing such bounds is to redefine the time periods into considering $t = 0$ to $t - 1$ as the new first period and period $t$ as the new second period. The two period bounds, derived in this paper, are then directly applicable. Note

[5] The non full compliance is addressed in detail by Bijwaard & Ridder (2005). They introduce a new method to handle the selective compliance in the treatment group. If there is full compliance in the control group, their two-stage linear rank estimator is able to handle the selective compliance in the treatment group even for censored durations. In order to achieve this they assume a MPH structure for the transition rate. Their estimates indicate that the ITT estimates by Meyer (1996) underestimate the true treatment effect.

that, this procedure gives conservative bounds as we have aggregated some restrictions.

Our bounds are expressed in terms of population moments, but they could be estimated by replacing the population moments with their sample analogs, for instance

$$\Pr(Y_1 = 0|D_1 = 1) = \frac{\sum_{i=1}^{N} \mathbf{1}(D_{1i} = 1)Y_{1i}}{\sum_{i=1}^{N} \mathbf{1}(D_{1i} = 1)}.$$

Here $N$ is the number of individuals in the sample and $\mathbf{1}(\cdot)$ is an indicator function taking the value one if the expression in the parenthesis is true and zero otherwise. Inferences for set identified models have been discussed in a series of recent papers, see e.g. Chernozhukov et al. (2007) for an insightful overview of this literature. Imbens & Manski (2004) have shown how to construct confidence intervals when the identified set is an interval whose upper and lower endpoints are means (or behave like means). Our general bounds are of that type. In order to construct confidence intervals we first bootstrap (399 replicates) the variance of the two endpoints, and then apply the Imbens & Manski (2004) confidence intervals. We also apply that method to the bounds under additional restrictions.[6]

Table 1 presents the upper and the lower bound on $\text{ATES}_t$ (and their confidence intervals) for the claimant group under random assignment and combinations of additional assumptions. Figure 1 displays the same bounds, and the confidence intervals. The general bounds, which impose no assumptions beyond random assignment are labeled no. The instantaneous treatment effect on the transition rate (week 1-2) is point identified and indicates a positive treatment effect of being offered the possibility to claim a bonus. The transition rate is about 2 percentage points higher in the claimant group compared to the control group. From week 3-4 and onwards the bounds are quite wide. In fact, without further assumptions we cannot rule out that the bonus actually has a negative impact on the conditional transition rate from week 3 and onwards. However, note that until week 20 the bounds are nevertheless informative on the average treatment effect.

Next, consider what can be identified under additional weak assumptions. First, consider the plausibility of the assumptions considered in section 4. The average treatment effect is positive during the first period. Assumption 4, monotone treatment response, then implies that being offered a job bonus has positive or zero effect on the transition rate from unemployment to employment for all unemployed. It is hard to imagine that any individual would suffer from a bonus offer, so that assumption 4, most likely, is fulfilled. Assumption 8, a sin-

[6]Note that for some of these bounds intervals the upper (lower) bound is constructed by taking the maximum (minimum) value of two or more restrictions. This means that the Imbens & Manski (2004) inference in a strict sense is not applicable, see e.g. Pakes et al. (2007) and Romano & Shaikh (2008). The complication arises since with a finite sample there is some uncertainty about which restriction that is binding. One alternative is to apply the subsampling method proposed in Romano & Shaikh (2008). However, we have noticed that in our application there is little uncertainty about which of the restrictions that are binding. We therefore feel confident in applying the Imbens & Manski (2004) confidence intervals.

Table 1: Bounds on conditional transition probabilities for the Illinois job bonus experiment (claimant bonus)

| Assumptions | No [1] | MTR+PS [2] |
|---|---|---|
| Week | | |
| 1-2 | [ 0.008 ( 0.023 : 0.023) 0.037] | [ 0.007 ( 0.023 : 0.023) 0.038] |
| 3-4 | [-0.107 (-0.097 : 0.111) 0.120] | [-0.081 (-0.068 : 0.102) 0.111] |
| 5-6 | [-0.106 (-0.095 : 0.100) 0.111] | [-0.090 (-0.081 : 0.086) 0.095] |
| 7-8 | [-0.114 (-0.102 : 0.121) 0.133] | [-0.089 (-0.080 : 0.095) 0.105] |
| 9-10 | [-0.128 (-0.113 : 0.127) 0.143] | [-0.090 (-0.080 : 0.090) 0.100] |
| 11-12 | [-0.142 (-0.123 : 0.140) 0.159] | [-0.086 (-0.076 : 0.087) 0.097] |
| 13-14 | [-0.192 (-0.166 : 0.162) 0.188] | [-0.099 (-0.086 : 0.084) 0.096] |
| 15-16 | [-0.233 (-0.193 : 0.206) 0.244] | [-0.090 (-0.077 : 0.082) 0.095] |
| 17-18 | [-0.414 (-0.316 : 0.316) 0.406] | [-0.100 (-0.086 : 0.086) 0.100] |
| 19-20 | [-1.152 (-0.865 : 0.809) 1.107] | [-0.116 (-0.100 : 0.093) 0.107] |
| 21-22 | [-1.000 (-1.000 : 1.000) 1.000] | [-0.157 (-0.138 : 0.095) 0.111] |
| 23-24 | [-1.000 (-1.000 : 1.000) 1.000] | [-0.135 (-0.116 : 0.112) 0.129] |
| Assumptions | MTR+SS [3] | ME [4] |
| 1-2 | [ 0.006 ( 0.023 : 0.023) 0.039] | [ 0.008 ( 0.023 : 0.023) 0.037] |
| 3-4 | [ 0.000 ( 0.011 : 0.038) 0.056] | [-0.088 (-0.081 : 0.094) 0.103] |
| 5-6 | [-0.007 ( 0.004 : 0.046) 0.067] | [-0.075 (-0.068 : 0.075) 0.083] |
| 7-8 | [ 0.002 ( 0.013 : 0.063) 0.085] | [-0.070 (-0.063 : 0.078) 0.086] |
| 9-10 | [-0.004 ( 0.008 : 0.070) 0.084] | [-0.065 (-0.058 : 0.070) 0.077] |
| 11-12 | [-0.003 ( 0.008 : 0.063) 0.071] | [-0.057 (-0.051 : 0.063) 0.070] |
| 13-14 | [-0.013 (-0.002 : 0.057) 0.065] | [-0.061 (-0.053 : 0.057) 0.064] |
| 15-16 | [-0.008 ( 0.003 : 0.051) 0.059] | [-0.051 (-0.044 : 0.051) 0.059] |
| 17-18 | [-0.012 ( 0.000 : 0.050) 0.058] | [-0.052 (-0.045 : 0.050) 0.057] |
| 19-20 | [-0.015 (-0.003 : 0.050) 0.057] | [-0.126 (-0.048 : 0.050) 0.128] |
| 21-22 | [-0.034 (-0.021 : 0.047) 0.056] | [-1.285 (-1.000 : 1.000) 1.289] |
| 23-24 | [-0.015 (-0.002 : 0.056) 0.066] | [-1.000 (-1.000 : 1.000) 1.000] |
| Assumptions | ME+MTR+PS [5] | ME+MTR+SS [6] |
| 1-2 | [ 0.008 ( 0.023 : 0.023) 0.037] | [ 0.008 ( 0.023 : 0.023) 0.037] |
| 3-4 | [-0.071 (-0.059 : 0.094) 0.103] | [ 0.002 ( 0.014 : 0.038) 0.055] |
| 5-6 | [-0.075 (-0.068 : 0.075) 0.082] | [-0.004 ( 0.007 : 0.046) 0.068] |
| 7-8 | [-0.070 (-0.063 : 0.078) 0.086] | [ 0.005 ( 0.016 : 0.063) 0.085] |
| 9-10 | [-0.065 (-0.058 : 0.070) 0.078] | [ 0.001 ( 0.012 : 0.070) 0.083] |
| 11-12 | [-0.057 (-0.051 : 0.063) 0.071] | [ 0.002 ( 0.012 : 0.063) 0.071] |
| 13-14 | [-0.061 (-0.053 : 0.057) 0.064] | [-0.007 ( 0.004 : 0.057) 0.065] |
| 15-16 | [-0.051 (-0.044 : 0.051) 0.059] | [-0.003 ( 0.007 : 0.051) 0.059] |
| 17-18 | [-0.052 (-0.045 : 0.050) 0.058] | [-0.005 ( 0.005 : 0.050) 0.057] |
| 19-20 | [-0.055 (-0.048 : 0.050) 0.058] | [-0.009 ( 0.002 : 0.050) 0.058] |
| 21-22 | [-0.070 (-0.062 : 0.047) 0.055] | [-0.026 (-0.014 : 0.047) 0.055] |
| 23-24 | [-0.061 (-0.053 : 0.056) 0.065] | [-0.009 ( 0.003 : 0.056) 0.066] |

Notes: Bounds in parenthesis and confidence intervals in brackets. Raw indicates the difference in the raw hazard rate, and no the bounds under random assignment. MTR stands for assumption monotone treatment response, SS a single shock, PS positively correlated shocks, and ME monotone exit rate.

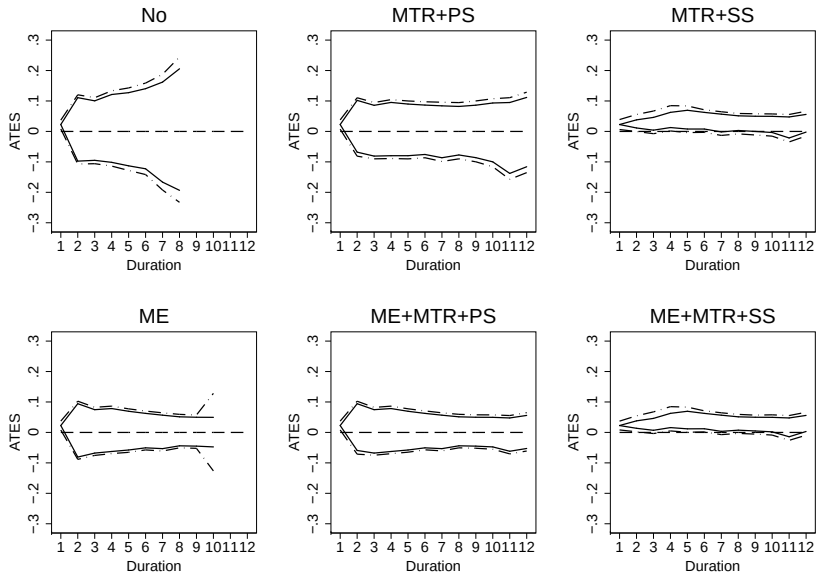


Figure 1: Bounds on conditional transition probabilities for the Illinois job bonus experiment (claimant bonus)

gle shock, means that being offered a bonus does not affect the random events influencing the arrival of employment. All random events that is not caused by the bonus offer should occur under both treatment and no treatment. We have no strong beliefs to doubt this assumption. Since assumption 7, positively correlated shocks, is weaker than assumption 8 we also explore the identifying power of this assumption. Assumption 9, monotone exit rate, implies that some unemployed individuals have a higher probability of finding employment compared to other unemployed when being offered the job bonus as well when not being offered the bonus. This assumption is fulfilled if the ranking of the individuals in terms of the characteristics that determines job offers, such as experience and job search effort, stays the same during the entire job search period. We are confident in that this is a quite good description of reality.

As expected, when imposing additional weak assumptions the bounds are tightened considerably. Assumption 7 and assumption 4 allow us to rule out very large negative and very large positive average dynamic treatment effects. Assumption ME has the same effect. Imposing assumption 8 and assumption 4 further tightens the bounds. If these assumptions hold we can rule out that the bonus offer has a negative effect on the conditional transition rates. These two assumptions together with assumption 9 give even more narrow bounds. Let us return to the three hypotheses suggested by labor models and search theories, and consider our most restrictive bounds as of model 6. We conclude that there

is a positive effect of the bonus offer on the conditional transition rate during all periods up until week 11. It confirms the first hypothesis. The upper bound increases in time period 5 (week 9-10), but the lower bound is lower than the upper bound for period 4. Hence, we cannot rule out that there is an intensified effect shortly before the bonus offer expires, but we cannot either rule out the opposite. Now consider the third hypothesis: that there is no effect on the transition rate after week 11. Obviously, as more time has passed the dynamic selection is more severe in this time period. We conclude that there actually may be a substantive positive effect on the conditional transition probabilities also after week 11. As our results diverge from the results of previous studies of the re-employment experiment we conclude that previous results based on semi-parametric models heavily rely on the imposed structure.

# 6 Conclusions

In this article, we have derived and implemented sharp bounds on conditional transitions probabilities under random assignment. We have shown that even under random assignment only instantaneous average treatment effects is point identified. Dynamic treatment effects, which requires that one study conditional transitions probabilities, are in general not point identified. Because our bounds impose no assumptions beyond the random assignment they are not sensitive to arbitrary functional form assumptions made in semi-parametric models. We have also derived bounds under additional weak assumptions such as monotone treatment response and monotone exit rate.

Our re-analysis of data from the Illinois re-employment bonus experiment shows that our bounds are informative about average treatment effects. It also demonstrates that previous semi-parametric methods to deal with dynamic selection heavily rely on structure that is imposed, as it restricts the possible types of dynamic selection. The application further shows that imposing weak assumptions may lead to quite narrowly identified bounds.

The bounds that have been derived in this paper are for a two time period setting. In future research we intend to generalize these bounds into a setting with more than two time periods. We also intend to show how our bounds, that are applicable under random treatment assignment, could be applied under unconfounded treatment assignment. In that case one way to proceed is to create bounds conditional on the covariates (or the propensity score) and then average over the distribution of these covariates.

# References


Abbring, J. (2008), The Event-History Approach to Program Evaluation, *in* D. Millimet, J. Smith & E. Vytlacil, eds, 'Advances in Econometrics , Volyme 21: Modelling and Evaluatin Treatment Effects in Econometrics', Elsevier Science.

Abbring, J. & Heckman, J. (2007), Econometric Evaluation of Social Programs, Part III: Distributional Treatment Effects, Dynamic Treatment Effects, Dynamic Discrete Choice, and General Equilibrium Policy Evaluation, *in* J. Heckman & E. Leamer, eds, 'Handbook of Econometric', Vol. 6B, North Holland, chapter 72, pp. 5145–303.

Abbring, J. & van den Berg, G. (2003), 'The Nonparametric Identification of Treatment Effects in Duration Models', *Econometrica* **71**(5), 1491–1517.

Bijwaard, G. & Ridder, G. (2005), 'Correcting for Selective Compliance in a Re-Employment Bonus Experiment', *Journal of Econometrics* **125**, 77–111.

Bonnal, L., Fougere, F. & Serandon, A. (1997), 'Evaluating the Impact of French Employment Policies on Individual Histories', *Review of Economic Studies* **64**, 683–713.

Card, D. & Sullivan, D. (1988), 'Measuring the Effect of Subsidized Training Programs on Movements In and Out of Unemployment', *Econometrica* **56**, 497–530.

Chernozhukov, V., Hong, H. & Tamer, E. (2007), 'Estimation and Confidence Regions for Parameters Sets in Econometric Models', *Econometrica* **75**, 1243–1284.

Cox, D. (1972), 'Regression Models and Life Tables (with Discussion)', *Journal of the Royal Statistical Society* **34**, 187–220.

Elbers, C. & Ridder, G. (1982), 'True and Spurious Duration Dependence: The Identifiability of the Proportional Hazard Model', *The Review of Economic Studies* **49**, 403–409.

Frchet, M. (1951), 'Sur les tableux de corrlation dont les marges sont donns', *Annales de lUniversit de Lyon A Series* **14**, 53–77.

Fredriksson, P. & Johansson, P. (2008), 'Dynamic Treatment Assignment: The Consequences for Evaluations Using Observational Data', *Journal of Business & Economic Statistics* **26**, 435–445.

Gritz, R. (1993), 'The Impact of Training on the Frequency and Duration of Employment', *Journal of Econometrics* **57**, 21–51.

Ham, J. & LaLonde, R. (1996), 'The Effect of Sample Selection and Initial Conditions in Duration Models: Evidence from Experimental Data on Training', *Econometrica* **64**, 175–205.

Heckman, J. & Navarro, S. (2007), 'Dynamic Discrete Choice and Dynamic Treatment', *Journal of Econometrics* **136**, 341–396.

Heckman, J. & Singer, B. (1984), 'A Method for Minimizing the Impact of Distributional Assumptions in Econometric Models of Duration Data', *Econometrica* **52**, 271–230.

Hoeffding, W. (1940), 'Masstabinvariante korrelationtheorie', *Schriften des Mathematischen Instituts und des Instituts fr Angewandte Mathematik und Universitt Berlin* **5**, 197–233.

Imbens, G. & Manski, C. (2004), 'Confidence Intervals for Partially Identified Parameters', *Econometrica* **72**, 1845–1857.

Kluve, J., Card, D., Fertig, M., Gra, L., Jacobi, P., Jensen, P., Leetma, R., Nima, L., Patacchini, S., Schmidt, C., van der Klauww, B. & Weber, A. (2007), *Active Labor Market Policies in Europe*, Springer.

Manski, C. (1997), 'Monotone Treatment Response', *Econometrica* **65**, 1311–1334.

Manski, C. & Pepper, J. (2000), 'Monotone Instrumental Variables: With an Application to the Returns to Schooling', *Econometrica* **68**, 115–136.

Meyer, B. (1996), 'What Have We Learned from the Illonois Reemployment Bonus Experiment?', *Journal of Labor Economics* **14**, 26–51.

Pakes, A., Porter, J., Ho, K. & Ishii, J. (2006), Moment Inequalities and Their Application. Unpublished Manuscript.

Ridder, G. (1986), 'An Event History Approach to the Evaluation of Training, Recruitment, and Employment Programmes', *Journal of Applied Econometrics* **1**, 109–126.

Romano, J. & Shaikh, A. (2008), 'Inference for Identifiable Parameters in Partially Identified Econometric Models', *Journal of Statistical Planning and Inference* **138**, 2786–2807.

Van den Berg, G. (2001), Duration Models: Specification, Identification and Multiple Durations, *in* H. J. abd Leamer E., ed., 'Handbook of Econometrics', Vol. 5, Amesterdam, North-Holland, pp. 3381–3460.

Woodbury, S. & Spiegelman, R. (1987), 'Bonuses to Workers and Employers to Reduce Unemployment: Randomized Trials in Illinois', *American Economic Review* **77**, 513–530.

# Appendix A

Introduce the following notation

$$\begin{aligned} p^1(y_1^1) &= \Pr(Y_1^1 = y_1^1) \\ p^0(y_1^0) &= \Pr(Y_1^0 = y_1^0) \\ p^{11}(y_2^{11}|y_1^1, y_1^0) &= \Pr(Y_2^{11} = y_2^{11}|Y_1^1 = y_1^1, Y_1^0 = y_1^0) \\ p^{00}(y_2^{00}|y_1^1, y_1^0) &= \Pr(Y_2^{00} = y_2^{00}|Y_1^1 = y_1^1, Y_1^0 = y_1^0). \end{aligned}$$

**Proof** (*Proposition 1.*) First, consider bounds on $\mathbb{E}[Y_2^{00}|Y_1^1 = 0, Y_1^0 = 0] = p^{00}(1|0,0)$. Start with equation (13), average out $y_2^{01}$, and rearrange gives

$$p^{00}(1|0,0)$$

$$= \frac{\Pr(Y_2 = 1|D_1 = 0, D_2 = 0, Y_1 = 0)\Pr(Y_1 = 0|D_1 = 0) - p^{00}(1|1,0)p(1,0)}{p(0,0)}.$$

Use equation (15) to substitute for $p(1,0) = \Pr(Y_1 = 0|D_1 = 0) - p(0,0)$

$$p^{00}(1|0,0) = \tag{A.1}$$

$$\frac{\Pr(Y_2 = 1|D_1 = 0, D_2 = 0, Y_1 = 0)\Pr(Y_1 = 0|D_1 = 0)}{p(0,0)} -$$

$$\frac{p^{00}(1|1,0)(\Pr(Y_1 = 0|D_1 = 0) - p(0,0))}{p(0,0)}.$$

The LP problem then consists of maximizing and minimizing equation (A.1) in $p^{00}(1|0,1)$ and $p(0,0)$. We have

$$\frac{\partial p^{00}(1|0,0)}{\partial p^{00}(1|0,1)} = \frac{-(\Pr(Y_1 = 0|D_1 = 0) - p(0,0))}{p(0,0)} = -\frac{p(1,0)}{p(0,0)} \leq 0,$$

i.e. the objective function we want to maximize/minimize is non-increasing in $p^{00}(1|0,1)$ for all values of $p(0,0)$. Then for the maximization (minimization) problem take the minimum (maximum) value of $p^{00}(1|0,1)$, and notice that $0 \leq p^{00}(1|0,1) \leq 1$ gives

$$max\ p^{00}(1|0,0) = \frac{\Pr(Y_2 = 1|D_1 = 0, D_2 = 0, Y_1 = 0)\Pr(Y_1 = 0|D_1 = 0)}{p(0,0)}, \tag{A.2}$$

with

$$\frac{\partial max\ p^{00}(1|0,0)}{\partial p(0,0)} =$$

$$-\frac{\Pr(Y_2 = 1|D_1 = 0, D_2 = 0, Y_1 = 0)\Pr(Y_1 = 0|D_1 = 0)}{p(0,0)^2} \leq 0,$$

and

$$min\ p^{00}(1|0,0) = \tag{A.3}$$

$$\frac{\Pr(Y_2=1|D_1=0,D_2=0,Y_1=0)\Pr(Y_1=0|D_1=0)}{p(0,0)}-$$
$$\frac{(\Pr(Y_1=0|D_1=0)-p(0,0))}{p(0,0)},$$

with

$$\frac{\partial min\ p^{00}(1|0,0)}{\partial p(0,0)}=$$
$$\frac{(1-\Pr(Y_2=1|D_1=0,D_2=0,Y_1=0))\Pr(Y_1=0|D_1=0)}{p(0,0)^2}\geq 0.$$

So that both for the maximization problem and for the minimization problem take $p(0,0)$ as small as possible. This unknown joint distribution is bounded by the known marginal distributions as given by equation (14) and (15). From the results in Hoeffding (1940) and Frchet (1951) we have

$$max(\Pr(Y_1=0|D_1=1)+\Pr(Y_1=0|D_1=0))-1,0)\leq p(0,0)\leq \tag{A.4}$$
$$min(\Pr(Y_1=0|D_1=1),\Pr(Y_1=0|D_1=0)).$$

Substitute the minimum value from equation (A.4) into equation (A.2)

$$max\ p^{00}(1|0,0)= \tag{A.5}$$
$$min(1,\frac{\Pr(Y_2=1|D_1=0,D_2=0,Y_1=0)\Pr(Y_1=0|D_1=0)}{max(\Pr(Y_1=0|D_1=1)+\Pr(Y_1=0|D_1=0)-1,0)}),$$

and into equation (A.3)

$$min\ p^{00}(1|0,0)= \tag{A.6}$$
$$max(0,\frac{\Pr(Y_2=1|D_1=0,D_2=0,Y_1=0)\Pr(Y_1=0|D_1=0)}{max(\Pr(Y_1=0|D_1=1)+\Pr(Y_1=0|D_1=0)-1,0)}-$$
$$\frac{\Pr(Y_1=0|D_1=0)-max(\Pr(Y_1=0|D_1=1)+\Pr(Y_1=0|D_1=0)-1,0)}{max(\Pr(Y_1=0|D_1=1)+\Pr(Y_1=0|D_1=0)-1,0)}).$$

Note that, we in equation (A.5) and (A.6) made it explicit that the probability $p^{00}(1|0,0)$ by definition lies between zero and one.

Second, consider bounds on $\mathbb{E}[Y_2^{11}|Y_1^1=0,Y_1^0=0]=p^{11}(1|0,0)$. Start with equation (12), average out $y_2^{01}$, rearrange, and use equation (14) to substitute for $p(0,1)$ gives

$$p^{11}(1|0,0)= \tag{A.7}$$
$$\frac{\Pr(Y_2=1|D_1=1,Y_1=0)\Pr(Y_1=0|D_1=1)}{p(0,0)}-$$
$$\frac{p^{11}(1|0,1)(\Pr(Y_1=0|D_1=1)-p(0,0))}{p(0,0)}.$$

Then

$$\frac{\partial p^{11}(1|0,0)}{\partial p^{11}(1|1,0)}=\frac{-(\Pr(Y_1=0|D_1=1)-p(0,0))}{p(0,0)}=-\frac{p(0,1)}{p(0,0)}\leq 0,$$

gives using similar reasoning as for the bounds on $\mathbb{E}[Y_2^{00}|Y_1^1 = 0, Y_1^0 = 0]$

$$max\ p^{11}(1|0,0) = \frac{\Pr(Y_2 = 1|D_1 = 1, Y_1 = 0)\Pr(Y_1 = 0|D_1 = 1)}{p(0,0)}, \tag{A.8}$$

with

$$\frac{\partial max\ p^{11}(1|0,0)}{\partial p(0,0)} = -\frac{\Pr(Y_2 = 1|D_1 = 1, Y_1 = 0)\Pr(Y_1 = 0|D_1 = 1)}{p(0,0)^2} \leq 0,$$

and

$$min\ p^{11}(1|0,0) = \tag{A.9}$$
$$\frac{\Pr(Y_2 = 1|D_1 = 1, Y_1 = 0)\Pr(Y_1 = 0|D_1 = 1) - \Pr(Y_1 = 0|D_1 = 1) + p(0,0)}{p(0,0)}$$

with

$$\frac{\partial min\ p^{11}(1|0,0)}{\partial p(0,0)} =$$
$$\frac{-\Pr(Y_2 = 1|D_1 = 1, Y_1 = 0)\Pr(Y_1 = 0|D_1 = 1) + \Pr(Y_1 = 1|D_1 = 0)}{p(0,0)^2} \geq 0.$$

So that again take $p(0,0)$ as small as possible. Substitute the minimum value from equation (A.4) into equation (A.8)

$$max\ p^{11}(1|0,0) = \tag{A.10}$$
$$min(1, \frac{\Pr(Y_2 = 1|D_1 = 1, Y_1 = 0)\Pr(Y_1 = 0|D_1 = 1)}{max(\Pr(Y_1 = 0|D_1 = 1) + \Pr(Y_1 = 0|D_1 = 0) - 1, 0)}),$$

and into equation (A.9)

$$min\ p^{11}(1|0,0) = \tag{A.11}$$
$$max(0, \frac{\Pr(Y_2 = 1|D_1 = 1, Y_1 = 0)\Pr(Y_1 = 0|D_1 = 1)}{max(\Pr(Y_1 = 0|D_1 = 1) + \Pr(Y_1 = 0|D_1 = 0) - 1, 0)} -$$
$$\frac{\Pr(Y_1 = 0|D_1 = 1) - max(\Pr(Y_1 = 0|D_1 = 1) + \Pr(Y_1 = 0|D_1 = 0) - 1, 0)}{max(\Pr(Y_1 = 0|D_1 = 1) + \Pr(Y_1 = 0|D_1 = 0) - 1, 0)}).$$

Third, consider bounds on $ATETS_2^{11,00}$. After substitutions the only variable appearing in equation (A.1) in the derivations of the bounds for $\mathbb{E}[Y_2^{00}|Y_1^1 = 0, Y_1^0 = 0, D_1 = 1]$ and in equation (A.7) in the derivations of the bounds for $\mathbb{E}[Y_2^{11}|Y_1^1 = 0, Y_1^0 = 0, D_1 = 1]$ is $p(0,0)$. However, in both cases the optimal value of $p(0,0)$ is the minimum value so that the bounds for $\mathbb{E}[Y_2^{00}|Y_1^1 = 0, Y_1^0 = 0, D_1 = 1]$ and $\mathbb{E}[Y_2^{11}|Y_1^1 = 0, Y_1^0 = 0, D_1 = 1]$ can be used directly when constructing bounds for $ATETS_2^{11,00}$. We then have

$$max\ ATETS_2^{11,00} = max\ p^{11}(1|0,0) - min\ p^{00}(1|0,0)$$

and

$$min\ ATETS_2^{11,00} = min\ p^{11}(1|0,0) - max\ p^{00}(1|0,0),$$

which give the results in proposition 1.

**Proof** (*Corollary 1.*) $p^{11}(1|0,0)$ is point identified if $max\ p^{11}(1|0,0) = min\ p^{11}(1|0,0)$, using proposition 1 this holds if

$$\frac{\Pr(Y_1 = 0|D_1 = 1) - max(\Pr(Y_1 = 0|D_1 = 1) + \Pr(Y_1 = 0|D_1 = 0) - 1, 0)}{max(\Pr(Y_1 = 0|D_1 = 1) + \Pr(Y_1 = 0|D_1 = 0) - 1, 0)}$$

equals zero, i.e. if $\Pr(Y_1 = 0|D_1 = 0) = 1$. In the same way $p^{00}(1|0,0)$ is point identified if $max\ p^{00}(1|0,0) = min\ p^{00}(1|0,0)$, using proposition 1 this holds if

$$\frac{\Pr(Y_1 = 0|D_1 = 0) - max(\Pr(Y_1 = 0|D_1 = 1) + \Pr(Y_1 = 0|D_1 = 0) - 1, 0)}{max(\Pr(Y_1 = 0|D_1 = 1) + \Pr(Y_1 = 0|D_1 = 0) - 1, 0)}$$

equals zero, i.e. if $\Pr(Y_1 = 0|D_1 = 1) = 1$. $ATETS_2^{11,00}$ is point identified if both $p^{00}(1|0,0)$ and $p^{11}(1|0,0)$ are point identified, i.e. if $\Pr(Y_1 = 0|D_1 = 1) = 1$ and $\Pr(Y_1 = 0|D_1 = 0) = 1$ hold.

**Proof** (*Corollary 2.*) The bounds on $ATETS_2^{11,00}$ are informative if they exclude either 1 or -1, which hold if $max\ p^{11}(1|0,0) < 1$, or $min\ p^{11}(1|0,0) > 0$, or $max\ p^{00}(1|0,0) < 1$, or $min\ p^{00}(1|0,0) > 0$ hold. Using proposition 1 it immediately follows that this hold if

$$\frac{\Pr(Y_2 = 1|D_1 = 1, Y_1 = 0)\Pr(Y_1 = 0|D_1 = 1)}{max(\Pr(Y_1 = 0|D_1 = 1) + \Pr(Y_1 = 0|D_1 = 0) - 1, 0)}$$

smaller than 1, or if

$$\frac{\Pr(Y_2 = 1|D_1 = 0, D_2 = 0, Y_1 = 0)\Pr(Y_1 = 0|D_1 = 0)}{max(\Pr(Y_1 = 0|D_1 = 1) + \Pr(Y_1 = 0|D_1 = 0) - 1, 0)} -$$

$$\frac{\Pr(Y_1 = 0|D_1 = 0) - max(\Pr(Y_1 = 0|D_1 = 1) + \Pr(Y_1 = 0|D_1 = 0) - 1, 0)}{max(\Pr(Y_1 = 0|D_1 = 1) + \Pr(Y_1 = 0|D_1 = 0) - 1, 0)}$$

largen than zero, or if

$$\frac{\Pr(Y_2 = 1|D_1 = 1, Y_1 = 0)\Pr(Y_1 = 0|D_1 = 1)}{max(\Pr(Y_1 = 0|D_1 = 1) + \Pr(Y_1 = 0|D_1 = 0) - 1, 0)} -$$

$$\frac{\Pr(Y_1 = 0|D_1 = 1) - max(\Pr(Y_1 = 0|D_1 = 1) + \Pr(Y_1 = 0|D_1 = 0) - 1, 0)}{max(\Pr(Y_1 = 0|D_1 = 1) + \Pr(Y_1 = 0|D_1 = 0) - 1, 0)}$$

larger than zero, or if

$$\frac{\Pr(Y_2 = 1|D_1 = 0, D_2 = 0, Y_1 = 0)\Pr(Y_1 = 0|D_1 = 0)}{max(\Pr(Y_1 = 0|D_1 = 1) + \Pr(Y_1 = 0|D_1 = 0) - 1, 0)}$$

smaller than one. This gives the result in the corollary.

**Proof** (*Proposition 2.*) Assumption 4 and 7 only restrict $p(0,0)$. Following the derivations of the general bounds we then we end up with the equations (A.2),(A.3),(A.8) and (A.9). Moreover, again take $p(0,0)$ as small as possible for both the maximization and the minimization problem. The restrictions as implied by assumption 4 differs whether $p^1(1) > p^0(1)$ or $p^1(1) < p^0(1)$ hold. First consider $p^1(1) > p^0(1)$. From assumption 4 we have

$$\Pr(Y_1^1 = 1) \geq \Pr(Y_1^0 = 1) \Rightarrow \Pr(Y_{i1}^1 = 1) \geq \Pr(Y_{i1}^0 = 1), \forall i$$

and from assumption 7

$$\Rightarrow \Pr(Y_{i1}^1 = 1 | Y_{i1}^0 = 1) \geq \Pr(Y_{i1}^1 = 0 | Y_{i1}^0 = 1), \forall i$$

which lead to

$$\Rightarrow \Pr(Y_{i1}^1 = 1, Y_{i1}^0 = 1) \geq \Pr(Y_{i1}^1 = 0, Y_{i1}^0 = 1), \forall i$$

and thus

$$\Rightarrow \Pr(Y_1^1 = 1, Y_1^0 = 1) \geq \Pr(Y_1^1 = 0, Y_1^0 = 1).$$

So that expressed in the short hand notation that

$$p^1(1) > p^0(1) \Rightarrow p(1,1) \geq p(0,1). \tag{A.12}$$

In the same way under assumption 7 and assumption 4

$$\begin{aligned}
&\Pr(Y_1^1 = 1) \geq \Pr(Y_1^0 = 1) \Rightarrow \Pr(Y_{i1}^1 = 1) \geq \Pr(Y_{i1}^0 = 1) \\
&\Rightarrow \Pr(Y_{i1}^0 = 0 | Y_{i1}^1 = 0) \geq \Pr(Y_{i1}^0 = 1 | Y_{i1}^1 = 0) \\
&\Rightarrow \Pr(Y_{i1}^1 = 0, Y_{i1}^0 = 0) \geq \Pr(Y_{i1}^1 = 0, Y_{i1}^0 = 1) \\
&\Rightarrow \Pr(Y_1^1 = 0, Y_1^0 = 0) \geq \Pr(Y_1^1 = 0, Y_1^0 = 1).
\end{aligned}$$

So that expressed in the short hand notation that

$$p^1(1) > p^0(1) \Rightarrow p(0,0) \geq p(0,1). \tag{A.13}$$

Using equation (14) and (15) we can rewrite equation (A.12) and (A.13) as

$$\begin{aligned}
&p(1,1) \geq p(0,1) \Leftrightarrow p^1(1) - p(1,0) = \\
&p^1(1) - (p^0(0) - p(0,0)) \geq p^1(0) - p(0,0) \Leftrightarrow \\
&2p(0,0) \geq p^1(0) - p^1(1) + p^0(0) = 2p^1(0) - 1 + p^0(0) \Leftrightarrow \\
&p(0,0) \geq -\frac{1}{2} + \Pr(Y_1 = 0 | D_1 = 1) + \frac{1}{2} \Pr(Y_1 = 0 | D_1 = 0)
\end{aligned} \tag{A.14}$$

and

$$p(0,0) \geq p(0,1) \Leftrightarrow p(0,0) \geq p^1(0) - p(0,0) \Leftrightarrow$$

$$p(0,0) \geq \frac{p^1(0)}{2} = \frac{\Pr(Y_1 = 0|D_1 = 1)}{2}. \tag{A.15}$$

Combining equation (A.14) and (A.15) and noticing that equation (A.14) is more restrictive than $p(0,0) \geq max(\Pr(Y_1 = 0|D_1 = 1) + \Pr(Y_1 = 0|D_1 = 0)) - 1, 0)$ gives

$$p(0,0) \geq \tag{A.16}$$

$$max(-\frac{1}{2} + \Pr(Y_1 = 0|D_1 = 1) + \frac{1}{2}\Pr(Y_1 = 0|D_1 = 0), \frac{\Pr(Y_1 = 0|D_1 = 1)}{2}).$$

Substituting the minimum value of $p(0,0)$ given by equation (A.16) into equations (A.2),(A.3),(A.8) and (A.9) and using

$$max\ ATETS_2^{11,00} = max\ p^{11}(1|0,0) - min\ p^{00}(1|0,0) \tag{A.17}$$

and

$$min\ ATETS_2^{11,00} = min\ p^{11}(1|0,0) - max\ p^{00}(1|0,0), \tag{A.18}$$

gives the second result in the proposition.

Second consider $p^1(1) < p^0(1)$, by similar reasoning as above under assumption 7 and assumption 4

$$p^1(1) > p^0(1) \Rightarrow p(1,1) \geq p(1,0). \tag{A.19}$$

and

$$p^1(1) > p^0(1) \Rightarrow p(0,0) \geq p(1,0). \tag{A.20}$$

Further derivations as above using equation (14), (15), (A.19) and (A.20) gives

$$p(0,0) \geq$$

$$max(-\frac{1}{2} + \Pr(Y_1 = 0|D_1 = 0) + \frac{1}{2}\Pr(Y_1 = 0|D_1 = 1), \frac{\Pr(Y_1 = 0|D_1 = 0)}{2}),$$

and the first result in the proposition follows by substituting this into equations (A.2),(A.3),(A.8) and (A.9), and using equations (A.18) and (A.17), gives the first result in the proposition.

**Proof** (*Proposition 3.*) Assumptions 4 and 7 only restrict $p(0,0)$. Again, following the derivations of the general bounds we then we end up with the equations (A.2),(A.3),(A.8) and (A.9), so that again take $p(0,0)$ as small as possible for both the maximization and the minimization problem. First consider $p^1(1) > p^0(1)$. From assumption 4 we have

$$\Pr(Y_1^1 = 1) \geq \Pr(Y_1^0 = 1) \Rightarrow \Pr(Y_{i1}^1 = 1) \geq \Pr(Y_{i1}^0 = 1), \forall i$$

and from assumption 8 we have

$$\Pr(Y_{i1}^1 = 1) \geq \Pr(Y_{i1}^0 = 1) \Rightarrow Y_{i1}^1 \geq Y_{i1}^0, \forall i$$

further

$$Y_{i1}^1 \geq Y_{i1}^0, \forall i \Rightarrow \Pr(Y_1^1 = 0, Y_1^0 = 0) = \Pr(Y_1^1 = 0) = \Pr(Y_1 = 0|D = 1)$$

or in the short hand notation that

$$p^1(1) > p^0(1) \Rightarrow p(0,0) = \Pr(Y_1 = 0|D = 1) \tag{A.21}$$

By similar argument under assumption 4 and assumption 8

$$p^1(1) < p^0(1) \Rightarrow p(0,0) = \Pr(Y_1 = 0|D = 0). \tag{A.22}$$

Substituting A.21 and A.22 for $p(0,0)$ into equations (A.2),(A.3),(A.8) and (A.9), and using equations (A.18) and (A.17), gives the result in the proposition.

**Proof** (*Proposition 4.*) First, consider $p^{00}(1|0,0)$. In deriving the general bounds we have after substitutions equation (A.1). It still holds here so that for the maximization (minimization) problem we wish to take the minimum (maximum) value of $p^{00}(1|0,1)$. In comparison with the general bounds assumption 9 places the additional restriction that $p^{00}(1|1,0) \geq p^{00}(1|0,0)$, so that for the maximization problem we have $p^{00}(1|0,1) = p^{00}(1|0,0)$, and for the minimization problem we have $p^{00}(1|0,1) = 1$. It further means that equation (A.6) still holds for the minimization problem, and for the maximization problem substituting for $p^{00}(1|0,1) = p^{00}(1|0,0)$ gives

$$p^{00}(1|0,0) =$$

$$\frac{\Pr(Y_2 = 1|D_1 = 0, D_2 = 0, Y_1 = 0)\Pr(Y_1 = 0|D_1 = 0)}{p(0,0)} -$$

$$\frac{p^{00}(1|0,0)(\Pr(Y_1 = 0|D_1 = 0) - p(0,0))}{p(0,0)},$$

and thus after rearranging

$$max\ p^{00}(1|0,0) = \Pr(Y_2 = 1|D_1 = 0, D_2 = 0, Y_1 = 0). \tag{A.23}$$

Second, consider $p^{11}(1|0,0)$. By similar reasoning equation (A.7) still holds, and for the maximization problem $p^{11}(1|0,1) = p^{11}(1|0,0)$ and for the minimization problem we have $p^{11}(1|0,1) = 1$. So that equation (A.11) still holds, and for the maximization problem substituting for $p^{11}(1|0,1) = p^{11}(1|0,0)$ gives

$$p^{11}(1|0,0) =$$

$$\frac{\Pr(Y_2 = 1|D_1 = 1, Y_1 = 0)\Pr(Y_1 = 0|D_1 = 1)}{p(0,0)} -$$
$$\frac{p^{11}(1|0,0)(\Pr(Y_1 = 0|D_1 = 1) - p(0,0))}{p(0,0)}.$$

and thus after rearranging

$$max\ p^{11}(1|0,0) = \Pr(Y_2 = 1|D_1 = 1, Y_1 = 0). \tag{A.24}$$

Combining equation (A.6),(A.11),(A.23) and (A.24),and using equations (A.18) and (A.17), gives the result in the proposition, gives the result in the proposition.

**Proof** (*Proposition 7.*) From the proof of proposition 4 we have that assumption 9 restricts $p^{00}(1|0,0)$ and $p^{11}(1|0,0)$. The proof further shows that after imposing assumption 9 $p(0,0)$ do not appear for the maximization solutions for $p^{00}(1|0,0)$ and $p^{11}(1|0,0)$. So that we have $max\ p^{00}(1|0,0)$ from equation (A.23) and $max\ p^{11}(1|0,0)$ from equation (A.24). Further assumption 9 do not restrict $p(0,0)$. However, assumption 4 and assumption 7 restrict $p(0,0)$, that is the same assumption as in the derivations of proposition 2. We can therefore use the minimization solutions for $min\ p^{00}(1|0,0)$ and $min\ p^{11}(1|0,0)$ from the proofs for proposition 2. Combining these results and noting that minimum solution for $p(0,0)$ depends on whether $\Pr(Y_1 = 1|D_1 = 1) < \Pr(Y_1 = 1|D_1 = 0)$ or $\Pr(Y_1 = 1|D_1 = 1) < \Pr(Y_1 = 1|D_1 = 0)$ give the result in the proposition.

**Proof** (*Proposition 8.*) Following similar reasoning as in the previous proof gives the result.

# Appendix B

**Proposition 5 (Bounds under positive treatment response in period2)** *Suppose assumption 3 and assumption 5 holds. Then*

$$max(0, max(0, \frac{A - [1 - \Pr(Y_2 = 1|D_1 = 1, Y_1 = 0)] \Pr(Y_1 = 0|D_1 = 1)}{A}) -$$
$$min(1, \frac{\Pr(Y_2 = 1|D_1 = 0, D_2 = 0, Y_1 = 0) \Pr(Y_1 = 0|D_1 = 0)}{A}))$$
$$\leq ATES_2^{11,00} \leq$$
$$max(0, min(1, \frac{\Pr(Y_2 = 1|D_1 = 1, Y_1 = 0) \Pr(Y_1 = 0|D_1 = 1)}{A}) -$$
$$max(0, \frac{A - [1 - \Pr(Y_2 = 1|D_1 = 0, D_2 = 0, Y_1 = 0)] \Pr(Y_1 = 0|D_1 = 0)}{A}))$$

**Proposition 6 (Bounds under negative treatment response in period2)** *Suppose assumption 3 and assumption 6 holds. Then*

$$min(0, max(0, \frac{A - [1 - \Pr(Y_2 = 1|D_1 = 1, Y_1 = 0)] \Pr(Y_1 = 0|D_1 = 1)}{A}) -$$
$$min(1, \frac{\Pr(Y_2 = 1|D_1 = 0, D_2 = 0, Y_1 = 0) \Pr(Y_1 = 0|D_1 = 0)}{A}))$$
$$\leq ATES_2^{11,00} \leq$$
$$min(0, min(1, \frac{\Pr(Y_2 = 1|D_1 = 1, Y_1 = 0) \Pr(Y_1 = 0|D_1 = 1)}{A}) -$$
$$max(0, \frac{A - [1 - \Pr(Y_2 = 1|D_1 = 0, D_2 = 0, Y_1 = 0)] \Pr(Y_1 = 0|D_1 = 0)}{A}))$$

**Proposition 7 (Bounds ME, MTR response and positively correlated shocks)** *Suppose assumption 3, 4, 7 and 9 holds. Define* $A \equiv max(-\frac{1}{2} + \Pr(Y_1 = 0|D_1 = 0) + \frac{1}{2}\Pr(Y_1 = 0|D_1 = 1), \frac{\Pr(Y_1=0|D_1=0)}{2})$ *and* $B \equiv max(-\frac{1}{2} + \Pr(Y_1 = 0|D_1 = 1) + \frac{1}{2}\Pr(Y_1 = 0|D_1 = 0), \frac{\Pr(Y_1=0|D_1=1)}{2})$ *. Then if* $\Pr(Y_1 = 1|D_1 = 1) < \Pr(Y_1 = 1|D_1 = 0)$

$$max(0, \frac{A - [1 - \Pr(Y_2 = 1|D_1 = 1, Y_1 = 0)] \Pr(Y_1 = 0|D_1 = 1)}{A}) -$$
$$\Pr(Y_2 = 1|D_1 = 0, D_2 = 0, Y_1 = 0)$$
$$\leq ATES_2^{11,00} \leq$$
$$\Pr(Y_2 = 1|D_1 = 1, Y_1 = 0) -$$
$$max(0, \frac{A - [1 - \Pr(Y_2 = 1|D_1 = 0, D_2 = 0, Y_1 = 0)] \Pr(Y_1 = 0|D_1 = 0)}{A})$$

*and if* $\Pr(Y_1 = 1|D_1 = 1) > \Pr(Y_1 = 1|D_1 = 0)$

$$max(0, \frac{B - [1 - \Pr(Y_2 = 1|D_1 = 1, Y_1 = 0)] \Pr(Y_1 = 0|D_1 = 1)}{B}) -$$
$$\Pr(Y_2 = 1|D_1 = 0, D_2 = 0, Y_1 = 0)$$
$$\leq ATES_2^{11,00} \leq$$
$$\Pr(Y_2 = 1|D_1 = 1, Y_1 = 0) -$$
$$max(0, \frac{B - [1 - \Pr(Y_2 = 1|D_1 = 0, D_2 = 0, Y_1 = 0)] \Pr(Y_1 = 0|D_1 = 0)}{B})$$

**Proposition 8 (Bounds ME, MTR response and single shocks)** *Suppose assumption 3, 4, 8 and 9 holds. Then if* $\Pr(Y_1 = 1|D_1 = 1) < \Pr(Y_1 = 1|D_1 = 0)$

$$max(0, \frac{\Pr(Y_1 = 0|D = 0) - [1 - \Pr(Y_2 = 1|D_1 = 1, Y_1 = 0)] \Pr(Y_1 = 0|D_1 = 1)}{\Pr(Y_1 = 0|D_1 = 0)}) -$$

$$\Pr(Y_2 = 1|D_1 = 0, D_2 = 0, Y_1 = 0)$$
$$\leq ATES_2^{11,00} \leq$$
$$\Pr(Y_2 = 1|D_1 = 1, Y_1 = 0) - \Pr(Y_2 = 1|D_1 = 0, D_2 = 0, Y_1 = 0)$$

*and if* $\Pr(Y_1 = 1|D = 1) > \Pr(Y_1 = 1|D_1 = 0)$

$$\Pr(Y_2 = 1|D_1 = 1, Y_1 = 0) - \Pr(Y_2 = 1|D_1 = 0, D_2 = 0, Y_1 = 0)$$

$$\leq ATES_2^{11,00} \leq$$
$$\Pr(Y_2 = 1|D_1 = 1, Y_1 = 0) -$$
$$max(0, \frac{\Pr(Y_1 = 0|D = 1)}{\Pr(Y_1 = 0|D_1 = 1)} -$$
$$\frac{-[1 - \Pr(Y_2 = 1|D_1 = 0, D_2 = 0, Y_1 = 0)] \Pr(Y_1 = 0|D_1 = 0)}{\Pr(Y_1 = 0|D_1 = 1)})$$