#### Predictable Dynamics in Market, Size and Value Betas

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Abstract: This paper examines to what extent dynamics in market, size and value betas are predictable out-of-sample. Constructing ex-ante market and factor neutral portfolios, i.e. zero-beta portfolios, we provide evidence that the dynamics in all three betas are predictable. We are able to construct zero-beta portfolios based on the forecasted betas that even out-of-sample have no significant exposure to neither the market nor the size and value factors. However, this result is highly dependent on the particular specification used to model the dynamics in betas and we conclude that more flexible specifications provide superior results. Finally, we find that the out-of-sample reduction in portfolio risk, both for size, value and industry sorted portfolios, is larger for models including the size and value factors in comparison with models relying only on the market factor.

# 1 Introduction

Many active portfolio strategies build on estimates of market beta as well as betas related to the famous size and book-to-market factors developed by Fama & French (1992, 1993). The main examples are market neutral strategies and similar strategies involving multiple sources of beta risk, i.e. market and factor neutral strategies. These investment strategies involve simultaneously taking long and short positions while at the same time creating exposure to assets or portfolios that are identified as over- or undervalued. Accurate estimates of betas are crucial also for other directional long-short strategies not aiming at zero beta risk exposure but instead allowing managers to tilt towards value or growth portfolios, to shift from small-medium capitalization stocks to large capitalization stocks and from a net long position to a net short position, i.e. different types of beta-timing strategies. Therefore, if the temporal variation in beta risk can be predicted, the performance of such portfolio strategies can be improved. Conversely, unintended exposure to beta risk can severely hurt the performance of long-short portfolios inherently relying on the construction of portfolios with a target for beta risk.<sup>2</sup> There is also an important link between the dynamics in beta and classical portfolio optimization because restrictions on large covariance matrices are in practise often imposed in the form of an underlying factor structure involving market and alternative betas.

There is an ongoing academic discussion on the nature of the time-variation in market beta risk. In brief, there are three different approaches to model market beta dynamics in the literature. The first is purely data-driven rolling sample estimators (Fama & Macbeth, 1973, Officer, 1973). The second is to assume an implicit link between beta dynamics and an underlying unobserved state variable (Fabozzi & Frances, 1978, Ohlsson & Rosenberg, 1982, Bos & Newbold, 1984). The third is to assume an explicit link between beta dynamics and macroeconomic or fundamental variables (Shanken, 1990, Ferson & Harvey, 1991, 1993, Harvey, 1991). Ghysels & Jaquier (2005) argues in favor for purely data-driven filters and claim that

<sup>&</sup>lt;sup>1</sup>In general, an alternative or exotic beta risk is a sensitivity to a systematic risk factor that provides a non-zero expected return after controlling for market beta risk. Potential sources of such beta return include size and value effects, volatility, commodites, exposure to peak risk and corporate default risk.

<sup>&</sup>lt;sup>2</sup>This is a problem especially for market and factor neutral strategies because these ex-ante beta-neutral positions tend to go long in portfolios with underestimated betas and short in portfolios with overestimated betas. This in turn is a consequence of the fact that the target beta of zero is far below the average beta of one. Because of these estimation errors the ex-post portfolio betas are then on average positive, not zero (Fama & Macbeth, 1973). This is in contrast to portfolio strategies for example targeting an average beta, which are much less sensitive to estimation errors in betas (Ghysels & Jaquier, 2005).

neither aggregate nor firm-specific variables has any predictive power for the time-variation in market beta out-of-sample. Jostova & Philipov (2005) successfully use a mean-reverting stochastic specification of market beta dynamics in an out-of-sample hedging application for five U.S. industry portfolios.

The novelty of this study comes from the fact that it explores to what extent the dynamics in size and value betas are predictable out-of-sample, while previous research has exclusively focused on the predictability of market beta dynamics. We employ either purely data-driven filters or stochastic time-series specifications of the dynamics in beta risk. Constructing exante market and factor neutral portfolios, i.e. zero-beta portfolios, we provide evidence that the dynamics in all three betas are predictable. We are able to construct zero-beta portfolios that even out-of-sample have no significant exposure to neither the market nor the size and value factors. However, this result is highly dependent on the particular specification used to model the dynamics in betas. We conclude that more flexible specifications provide superior results. Finally, we also evaluate the beta predictions by measuring the out-of-sample reduction in portfolio volatility of the zero-beta portfolios compared with the original portfolios. We find that the out-of-sample reduction in portfolio risk, both for size, value and industry sorted portfolios, is larger for models including the size and value factors in comparison with models relying only on the market factor.

The data used are monthly returns from 10 size, 10 value and 10 industry portfolios taken from Kenneth French's data library. The sample period stretches from 1926:7 to 2004:12 and the out-of-sample evaluation period is 1990:1 to 2004:12.

The remainder of the paper is organized as follows. Section 2 discusses the different specifications of beta dynamics employed. Section 3 details the evaluation methods. Section 4 contains the results and section 5 concludes.

# 2 Beta dynamics

The beta dynamics is applied to two different factor models. The market model with a proxy for the market portfolio as the sole factor states that the nominal excess return on any asset i is given by:

$$r_t^i = \beta_t^{iM} r_t^M + \varepsilon_t^i, \tag{1}$$

where  $r_t^i$  and  $r_t^M$  are nominal excess returns on asset i and the market portfolio and  $\beta_t^{iM}$  is the beta of asset i with the market. Fama & French (1992) propose to augment the market model

with a size and a value factor:

$$r_t^i = \beta_t^{iM} r_t^M + \beta_t^{iSMB} r_t^{SMB} + \beta_t^{iHML} r_t^{HML} + \varepsilon_t^i, \tag{2}$$

where  $r_{t+1}^{SMB}$  and  $r_{t+1}^{HML}$  are nominal returns on the Fama-French SMB (size) and HML (value) portfolios and  $\beta_t^{iSMB}$  and  $\beta_t^{iHML}$  the corresponding conditional size and value betas.

The different specifications of beta dynamics explored in this paper are summarized in Table I.

Table I: Different assumptions of beta dynamics

Model	Description
CONST	Expanding window betas
R60M	Moving window 60 months betas
KFMR	Kalman filtered mean-reverting betas
KFRW	Kalman filtered random-walk betas
HF2S	Hamilton filtered two-state betas
HF3S	Hamilton filtered three-state betas

The different specifications are discussed below.

#### 2.1 Specifications of purely data-driven filters

The main data-driven modeling approaches is either to use overlapping or non-overlapping blocks of data to calculate beta. We use the standard 60 months overlapping window filter. We refer to this filter as R60M.

#### 2.2 Specifications of stochastic beta risk

#### 2.2.1 Kalman filter approach

For the Kalman-filter based beta specifications, we assume that the conditional market, size and value betas are forecasted through latent mean-reverting AR(1)-processes:

$$\beta_{t+1}^{iM} = \beta^{iM} + \phi^{M} \left( \beta_{t}^{iM} - \beta^{iM} \right) + \eta_{t+1}^{iM}$$
 (3)

$$\beta_{t+1}^{iSMB} = \beta^{iSMB} + \phi^{SMB} \left( \beta_t^{iSMB} - \beta^{iSMB} \right) + \eta_{t+1}^{iSMB}$$
 (4)

$$\beta_{t+1}^{iHML} = \beta^{iHML} + \phi^{HML} \left( \beta_t^{iHML} - \beta^{iHML} \right) + \eta_{t+1}^{iHML}$$
 (5)

This model may be viewed as an extension to a multivariate state-equation of similar models in, among others, Adrian & Franzoni (2004), Brooks, Faff & McKenzie (2002) and Berglund & Knif (1999). We also consider a random walk version, i.e. we place the restrictions  $\phi^M = \phi^{SMB} = \phi^{HML} = 1$  on the AR-parameters. In the following these two models are referred to as KFMR and KFRW, respectively. The conditional betas are related to the asset returns through the measurement equation:

$$r_{t+1}^{i} = \beta_{t+1}^{iM} r_{t+1}^{M} + \beta_{t+1}^{iSMB} r_{t+1}^{SMB} + \beta_{t+1}^{iHML} r_{t+1}^{HML} + \varepsilon_{t+1}^{i}.$$
 (6)

The forecasts of beta are obtained directly from the equations (3) - (5). The Kalman filter based time-varying beta models are estimated by maximum likelihood following Hamilton (1994), page 385.

#### 2.2.2 Hamilton filter approach

For the Hamilton-filter based beta specifications we assume that the *within state* betas are determined by the equations:

$$r_{t+1}^{i} = \beta_{s}^{iM} r_{t+1}^{M} + \beta_{s}^{iSMB} r_{t+1}^{SMB} + \beta_{s}^{iHML} r_{t+1}^{HML} + \varepsilon_{st+1}^{i}$$
 (7)

for states s = 1, ..., S. Here,  $\beta_s^{iM}$  is the (constant) within state market beta of asset i in state s and similarly  $\beta_s^{iSMB}$  and  $\beta_s^{iHML}$  are the (constant) within state size and value betas. This assumption implies the following overall regression with time-varying betas:

$$r_{t+1}^{i} = \sum_{s=1}^{S} 1_{\{X_{t+1}=s\}} \left[ \beta_{s}^{iM} r_{t+1}^{M} + \beta_{s}^{iSMB} r_{t+1}^{SMB} + \beta_{s}^{iHML} r_{t+1}^{HML} \right] + \varepsilon_{t+1}^{i}$$
 (8)

where  $\varepsilon_{t+1}^i = \sum_{s=1}^S 1_{\{X_{t+1}=s\}} \varepsilon_{st+1}^i$ . The conditional market, size and value betas are forecasted by probability weighted averages of within state betas:

$$\beta_{t+1}^{iM} = \operatorname{E}_{t}\left[\sum_{s=1}^{S} 1_{\{X_{t+1}=s\}} \beta_{s}^{iM}\right] = \sum_{s=1}^{S} \operatorname{Pr}_{t}\left(X_{t+1}=s\right) \beta_{s}^{iM}$$
(9)

$$\beta_{t+1}^{iSMB} = \sum_{s=1}^{S} \Pr_{t} \left( X_{t+1} = s \right) \beta_{s}^{iSMB}$$

$$\tag{10}$$

$$\beta_{t+1}^{iHML} = \sum_{s=1}^{S} \Pr_{t} \left( X_{t+1} = s \right) \beta_{s}^{iHML}$$

$$\tag{11}$$

where  $X_t$  is the unobservable Markov-switching state variable,  $\Pr_t(X_{t+1} = s)$  is the conditional probability that the state of the market is s in the next period. Note that the one-step-ahead probabilities for each state reflecting the uncertainty about the next period's state,

 $\Pr_t(X_{t+1} = s)$ , are directly obtained from the Hamilton filter during estimation. This is why we denote these betas "Hamilton filtered" betas, i.e. the Hamilton filter is explicitly used in the calculations of the forecasted betas. We assume that S = 2 or S = 3, i.e. that the number of states are two or three, and that the error terms  $\varepsilon_{st+1}^i$  are Normal distributed with a constant variance that is equal across states.<sup>3</sup> In the following these two models are referred to as HF2S and HF3S, respectively. Related specifications for market beta can be found in Ramchand & Susmel (1998), Huang (2000) and Galagedera & Shami (2004) and for Fama-French betas in Coggi & Manescu (2004). The main methodological difference is that these papers are not concerned with forecasts of betas per se and therefore do not employ the methodology developed in Equations (9)-(11). The Hamilton filter based time-varying beta models are estimated by maximum likelihood following Hamilton (1994), page 692.

#### 2.2.3 Differences between the beta specifications

The different specifications of beta dynamics can broadly be categorized with respect to their flexibility. In this sense the constant beta specification is the least flexible because it does not allow for any time-variation in beta and its "memory" stretches all the way back to the beginning of the sample. At the other extreme, the random walk model is the most flexible because it only remembers one period back. The remaining beta specifications fall somewhere in between with the rolling regression as the second most flexible. The mean-reversion betas are forced to return to a constant mean and the regime-switching betas are forced to take on values between the lowest and highest within regime betas. In turn, the mean-reversion level and the within regime betas are estimated using data from the beginning of the sample. Therefore we consider these specifications less flexible than the rolling regression and random walk betas but more flexible than the constant betas.

# 3 Evaluation methods

We use two different methods for evaluation of the different models. Both methods are constructed from a hedge or zero-beta portfolio in relation to our original portfolios (size, book-to-market, industry). In the first method we regress the expost return of the zero-beta portfolio

<sup>&</sup>lt;sup>3</sup>We also estimated models with a state-dependent idiosyncratic variance, but this generalization in general actually worsened the hedging performance. This result may be interpreted as if there is no out-of-sample exploitable correlation between the value of the beta and the level of idiosyncratic variance.

on the factors. In the second method we look at the out-of-sample reduction in volatility of the hedge portfolios compared to holding the portfolio itself.

#### 3.1 Hedge portfolio

Our empirical application designed to evaluate the usefulness of the forecasted betas is an outof-sample experiment of portfolio formation. Based on forecasted betas we construct long-short portfolios with zero exposure to the market (and to size and value portfolios). This strategy is implemented by selling short the market a dollar amount equal to the forecasted market betas for each dollar invested in the corresponding test portfolios, i.e. the industry portfolios etc. Therefore, the expost returns from the zero-beta portfolios are

$$\varepsilon_{T+1}^{i} = r_{T+1}^{i} - \hat{\beta}_{T+1|T}^{iM} r_{T+1}^{M}, \tag{12}$$

where  $r_{T+1}^i$  and  $r_{T+1}^M$  are the excess returns on portfolio i and the market portfolio in period T+1 and  $\hat{\beta}_{T+1|T}^{iM}$  is the one-period-ahead forecasted market beta. Incorporating size and value factors the expost returns are

$$\varepsilon_{T+1}^{i} = r_{T+1}^{i} - \hat{\beta}_{T+1|T}^{iM} r_{T+1}^{M} - \hat{\beta}_{T+1|T}^{iSMB} r_{T+1}^{SMB} - \hat{\beta}_{T+1|T}^{iHML} r_{T+1}^{HML}$$
(13)

where  $\hat{\beta}_{T+1|T}^{iSMB}$  and  $\hat{\beta}_{T+1|T}^{iHML}$  are the one-period-ahead forecasted size and value betas. It follows that the ex ante expected excess return on the zero-beta portfolios are zero and that idiosyncratic volatility is the only remaining source of uncertainty.

For each out-of-sample period T+1 the betas are forecasted based on an estimation of the model from the beginning of the sample up to and including period T. In order to simulate a real-time out-of-sample forecast situation, all parameters are updated when new information arrives, i.e. all models are reestimated each month during the evaluation period. All betas are forecasted one-period-ahead using the six models of beta in Table I.

#### 3.1.1 Ex post portfolio betas

The ex post betas of the zero-beta portfolio returns, i.e.  $\varepsilon_{T+1}^i$  in (12) and (13), are estimated by running time-series regressions of the zero-beta returns onto the factors:

$$\varepsilon_{T+1}^{i} = b^{iM} r_{T+1}^{M} + v_{T+1}$$
 (14)

$$\varepsilon_{T+1}^{i} = b^{iM} r_{T+1}^{M} + b^{iSMB} r_{T+1}^{SMB} + b^{iHML} r_{T+1}^{HML} + \upsilon_{T+1}. \tag{15}$$

where  $b^{iM}$ ,  $b^{iSMB}$  and  $b^{iHML}$  are the beta coefficients to be estimated. The portfolio returns  $\varepsilon_{T+1}^i$  calculated in (12) are ex ante market neutral and the portfolio returns calculated in (13)

are ex ante market and factor neutral. Therefore, our null hypothesis are that the estimated coefficients in (14) and (15) are zero. This is like an absolute measure that can be compared across all portfolios and specifications, and it will test if it is possible to construct portfolios with no significant factor exposure ex post.

#### 3.1.2 Reduction in ex post portfolio volatility

The reduction in total volatility is measured by the difference in volatility between the original portfolio returns and the zero-beta portfolio returns, i.e. between  $r_{T+1}^i$  and  $\varepsilon_{T+1}^i$  in (12) and (13). Since the betas in (12) and (13) are estimated out-of-sample, there is no guarantee that the ex post volatility of the zero-beta portfolio is lower than the ex post volatility of the original portfolio. Usually, however, using the estimated betas at least some of the systematic risk is removed from the original portfolio, leaving a zero-beta portfolio with lower volatility. The reduction in ex post volatility may alternatively be interpreted as the pseudo- $R^2$  and the difference in volatility reduction between the market model and the Fama-French three factor model may consequently be interpreted as the change in pseudo- $R^2$  when incorporating the size and value factors. This is more like a relative measure that can compare the specifications across the same type of assets or portfolios.

#### 4 Results

We first use the absolute measure to differentiate between different beta specifications, which, in particular is a comparison between constant betas (the model CONST is used) and dynamic betas. Next, we look at the value of augmenting the market model with factor portfolios based on size and book-to-market characteristics and if this difference is robust to different beta specifications.

We analyze 10 size, 10 value and 10 industry portfolios from Kenneth French's data library (see http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/). The data are monthly and stretches from 1926:7 to 2004:12. The out-of-sample evaluation period is 1990:1 to 2004:12.

#### 4.1 Factor exposure ex post

The estimation of the ex post portfolio betas are presented in Table III. The results are very clear: the two dynamic models R60M and KFRW are by far the best. Notice that the two best models have the shortest memories compared to the other models. There is almost no difference

between the market model and the three-factor model, which means that it is most convenient to construct a market neutral portfolio just using the market model. The best dynamic models are also good from an absolute point view, since they show that for most portfolios it is possible to construct a hedge that has zero sensitivity out of sample. They perform extremely good for the size portfolios with no significant betas and only slightly worse for the industry portfolios. The constant beta model, which could be considered as a benchmark, is performing poorly both absolutely, with several significant betas, and relative to R60M and KFRW. However, if one is constrained to use the constant model, then the three factor model is better even for constructing market neutral strategies. We can conclude that even if the more complicated model KFRW is somewhat better that the common model R60M, since it has fewer significant betas and lower estimates, one is tempted to consider R60M as the "best" model due to its simplicity.

#### 4.2 Volatility reduction ex post

In Table II we show the reduction in average volatility of the hedge portfolio in relation to the unhedged portfolio for three different types of portfolios. For example 28.5 for the CONST beta-model and the market model is the per centage reduction in average volatility of the hedge portfolio (using (12)) compared to just holding the industry portfolio. We can see that for all specifications the hedge reduces volatility on average for both size, growth/value and industry portfolios (see the rows labeled Market and Fama-French in each panel in Table II). The average reduction in volatility ranges from 28.5% (Market with constant betas) up to 77.6% (Fama-French model using R60M for size portfolios). Notice that the market model is always a more effective hedge for the characteristic sorted portfolios compared to the industry sorted portfolios. Thus, betas contain useful information for hedging purposes out-of-sample and this is independent of the beta specification.

Looking at the value of augmenting the market model with the Fama-French factors, we find that for all specifications of beta, the hedging performance of the Fama-French model is superior to the market model (see last row in Table II). As expected the efficiency gain from incorporating the Fama-French factors is smallest for industry portfolios (the relative gain varies between 14.0 and 28.2 percentage) and largest for size portfolios (between 46.4 and 49.1 percentage). These results indicate that the Fama-French size and value factors provide additional out-of-sample information for hedging purposes over and above the information contained in market beta for both growth/value, size and industry portfolios.

Some points can also be made about the results for individual portfolios (see Tables IV-VI in Appendix). First, we note that the market model has large difficulties hedging the Energy and Utilities portfolios. For the Fama-French model, the situation improves, but these two portfolios are still clearly the most difficult to hedge. However, from Table IV we can see that R60M and KFRW produce hedge portfolios for these two industries that have no significant factor exposure ex post. For the Fama-French model smaller cap (value) portfolios are the most difficult to hedge and larger cap (growth) portfolios the easiest (see Tables V-VI). Just looking at R60M vs KFRW the former model is slightly superior when it comes to reduce ex post volatility.

# 5 Conclusion

In this study we evaluate to what extent the dynamics in market, size and value betas are predictable out-of-sample using two factor models: the market model and the Fama-French three factor model. The specification of the beta dynamics varies from constant betas to Kalman and Hamilton filtered betas. The evaluation of the different models is based on a hedge or zero-beta portfolio in relation to our original portfolios. We compare the portfolios along two dimensions. First we regress the ex post return of the zero-beta portfolio on the factors. This is an absolute comparison across all portfolios and specifications, which tests if it is possible to construct portfolios with no significant factor exposure ex post. Secondly, we analyze the out-of-sample reduction in volatility of the hedge portfolios compared to holding the portfolio itself. This like a relative measure that compares the specifications across the same type of assets or portfolios.

We find that the dynamic models, moving window 60 months betas (R60M) and Kalman filtered random-walk betas (KFRW), are better than all other specifications. For these two models it is possible to construct a hedge that has zero sensitivity out of sample for most portfolios. For all specifications the hedge reduces volatility on average, which means that betas contain useful information for hedging purposes out-of-sample. Looking at the value of augmenting the market model with the Fama-French factors, we find that for all specifications of beta, the hedging performance of the Fama-French model is superior to the market model. These results indicate that the Fama-French size and value factors provide additional out-of-sample information for hedging purposes over and above the information contained in market beta for both growth/value, size and industry portfolios.

Thus, the general advice is to use a dynamic three factor model where the dynamic sensitivi-

ties are estimated by R60M or KFRW. Even if the more complicated model KFRW is somewhat better than the common model R60M, the latter might be considered a "better" model due to its simplicity.

# Appendix

Table II: Average percentage reduction in volatility of hedged portfolios compared to unhedged portfolios and the difference between the Fama-French hedge and the market hedge.

PA]	PANEL A: 10 SIZE SORTED PORTFOLIOS.							
Beta-model:	CONST	R60M	KFMR	KFRW	HF2S	HF3S		
Market	50.5	52.3	51.2	48.1	51.3	51.0		
Fama-French	74.3	77.6	75.7	74.8	75.1	75.8		
Difference	23.8	25.3	24.5	26.7	23.8	24.8		
PANEL B: 1	PANEL B: 10 BOOK-TO-MARKET SORTED PORTFOLIOS.							
Beta-model:	CONST	R60M	KFMR	KFRW	HF2S	HF3S		
Market	40.3	47.9	48.2	50.0	46.1	47.3		
Fama-French	59.9	63.3	63.5	64.2	61.7	61.9		
Difference	19.6	15.5	15.3	14.2	15.6	14.6		
P	ANEL C: 1	0 INDUS	STRY PO	RTFOLI	OS.			
Beta-model:	CONST	R60M	KFMR	KFRW	HF2S	HF3S		
Market	28.5	33.0	31.6	33.7	31.3	31.8		
Fama-French	32.5	41.0	40.5	41.2	36.0	36.4		
Difference	4.0	8.0	8.9	7.5	4.7	4.6		

Table III. Results for individual portfolios: Ex post beta exposure.

PANEL A: Market. Bold faced number are significant on the 5% level.

SIZE MARKET	P01 P02 P03 P04 P05 P06 P07 P08 P09 P10	CONST -0.557 -0.263 -0.226 -0.166 -0.136 -0.158 -0.136 -0.029 -0.103 0.038	R60M 0.038 0.063 0.040 0.037 0.046 0.023 0.014 0.046 -0.009 -0.014	KFMR -0.352 -0.146 -0.142 -0.107 -0.058 -0.089 -0.073 0.003 -0.034 0.014	KFRW 0.001 -0.036 -0.030 -0.031 -0.009 0.011 0.003 0.010 -0.007 -0.007	HF2S -0.397 -0.204 -0.154 -0.102 -0.074 -0.047 -0.056 0.001 -0.055 0.019	-0.251 -0.154 -0.141 -0.093 -0.069 -0.042 -0.055 -0.003 -0.037 0.031
BM MARKET	P01 P02 P03 P04 P05 P06 P07 P08 P09 P10	CONST 0.110 -0.001 0.021 -0.198 -0.182 -0.229 -0.381 -0.442 -0.528 -0.598	R60M -0.016 -0.015 -0.022 -0.055 -0.048 -0.039 -0.030 -0.017 0.036 0.068	KFMR 0.037 -0.027 0.006 -0.083 -0.099 -0.135 -0.164 -0.139 -0.150 -0.339	KFRW -0.019 -0.002 0.011 0.012 0.012 -0.008 0.005 -0.009 0.008 -0.009	HF2S 0.003 -0.027 0.004 -0.107 -0.118 -0.125 -0.206 -0.230 -0.256 -0.364	HF3S 0.020 -0.019 -0.011 -0.106 -0.092 -0.138 -0.213 -0.151 -0.164 -0.292
IND MARKET	NoDur Durbl Manuf Enrgy HiTec Telcm Shops Hlth Utils Other	CONST -0.185 -0.148 -0.256 -0.308 0.399 0.510 -0.058 -0.172 -0.483 -0.186	R60M -0.117 -0.043 -0.041 -0.070 0.151 0.087 -0.038 -0.139 0.015 -0.031	KFMR -0.032 -0.103 -0.187 -0.300 0.168 0.167 -0.036 -0.136 -0.352 -0.139	KFRW -0.011 -0.013 0.008 -0.032 0.142 0.011 -0.008 -0.052 0.001 0.009	HF2S -0.119 -0.113 -0.219 -0.279 0.172 0.259 -0.027 -0.127 -0.365 -0.136	HF3S -0.100 -0.097 -0.179 -0.290 0.217 0.295 -0.043 -0.136 -0.370 -0.120

PANEL B: Fama-French. Bold faced number are significant on the 5% level.

SIZE		CONST	R60M	KFMR	KFRW	HF2S	HF3S
FF	P01	-0.199	-0.012	-0.065	-0.024	-0.138	-0.083
		-0.403	0.036	-0.213	0.007	-0.355	-0.196
		-0.564	0.039	-0.167	0.036	-0.335	-0.160
	P02	-0.014	0.006	0.033	0.006	-0.020	0.002
		-0.167	0.032	0.053	0.012	0.009	0.023
		-0.288	0.039	0.006	0.025	-0.144	-0.086
	P03	-0.026	-0.003	0.011	0.006	0.002	-0.001
		-0.083	0.001	-0.021	-0.005	-0.039	-0.022
		-0.113	0.039	-0.007	0.015	-0.030	0.008
	P04	0.030	0.005	0.035	0.015	0.055	0.038
		-0.067	-0.024	0.006	-0.006	-0.012	0.048
		-0.033	0.048	0.044	0.019	0.053	0.042
	P05	0.010	0.014	0.010	-0.008	0.025	0.032
		-0.040	-0.029	-0.028	-0.001	-0.044	-0.022
		-0.039	0.031	-0.005	-0.006	0.009	0.023
	P06	-0.011	-0.001	0.008	-0.008	-0.004	0.003
		-0.023	-0.050	-0.041	-0.014	0.021	0.016
		-0.024	-0.007	0.011	-0.015	-0.015	-0.021
	P07	-0.015	-0.002	0.004	-0.002	-0.007	-0.004
		-0.016	-0.029	-0.016	-0.020	-0.015	-0.029
		0.061	0.013	0.037	-0.002	0.067	0.062
	P08	0.053	0.018	0.054	0.005	0.072	0.075
		0.123	-0.019	0.083	-0.005	0.089	0.094
		0.066	-0.004	0.069	0.000	0.081	0.085
	P09	-0.007	-0.014	0.014	-0.026	0.009	0.012
		0.060	0.019	0.032	0.029	0.033	0.034
		0.062	0.009	0.039	-0.018	0.091	0.071
	P10	0.008	0.000	0.007	0.013	0.002	0.002
		-0.097	0.019	-0.016	-0.009	-0.047	-0.021
		-0.045	-0.001	-0.001	0.010	-0.029	-0.029

ВМ	P01	CONST	R60M	KFMR	KFRW	HF2S	HF3S
FF		-0.136	-0.003	-0.042	0.004	-0.064	-0.068
		-0.175	-0.009	-0.088	0.002	-0.109	-0.095
	P02	-0.186	0.042	-0.028	0.026	-0.061	-0.052
		0.004	0.009	-0.007	0.027	0.002	-0.009
		-0.050	-0.008	-0.040	-0.060	-0.062	-0.049
	P03	0.259	0.103	0.119	0.047	0.208	0.169
		0.122	0.003	0.073	0.011	0.119	0.103
		-0.052	0.001	0.000	-0.006	-0.036	-0.071
	P04	0.385	0.112	0.135	0.060	0.326	0.265
		-0.009	-0.041	0.042	0.005	0.043	0.016
		0.044	-0.033	0.045	0.067	0.021	0.005
	P05	0.286	0.042	0.156	0.044	0.275	0.237
		0.058	-0.043	0.091	0.004	0.079	0.071
		0.106	-0.037	0.060	0.009	0.076	0.108
	P06	0.289	0.027	0.299	0.035	0.298	0.322
		0.005	-0.034	0.047	-0.005	0.024	0.020
		0.043	0.013	0.033	0.061	0.026	0.015
	P07	0.056	0.026	0.088	0.039	0.106	0.105
		-0.012	-0.017	-0.015	-0.017	0.006	-0.016
		0.015	0.013	-0.008	0.006	0.010	-0.001
	P08	0.064	0.032	0.123	-0.023	0.084	0.091
		-0.028	-0.002	-0.017	-0.013	-0.007	0.002
		0.052	-0.010	0.047	-0.002	0.044	0.039
	P09	0.043	0.041	0.067	-0.007	0.065	0.035
		-0.059	0.050	-0.030	0.004	-0.005	0.012
		-0.148	-0.019	-0.075	-0.007	-0.142	-0.144
	P10	-0.265	0.033	-0.111	-0.006	-0.136	-0.102
		0.035	0.095	0.035	0.042	0.038	0.039
		-0.337	0.013	-0.145	0.009	-0.259	-0.276
		-0.219	0.097	-0.131	0.079	-0.193	-0.157

IND		CONST	R60M	KFMR	KFRW	HF2S	HF3S
FF	NoDur	-0.018	-0.105	-0.056	-0.046	-0.011	-0.026
		-0.152	0.064	0.007	0.090	-0.161	-0.159
		0.398	0.053	-0.044	-0.027	0.146	0.087
	Durbl	-0.018	0.018	-0.045	0.025	0.002	0.026
		-0.283	-0.089	-0.116	-0.066	-0.343	-0.265
		0.128	0.068	0.040	0.053	0.117	-0.055
	Manuf	-0.099	-0.038	-0.090	-0.030	-0.102	-0.107
		-0.001	-0.031	0.013	0.000	0.058	-0.005
		0.264	0.026	0.131	-0.037	0.147	0.115
	Enrgy	-0.104	-0.060	-0.151	-0.043	-0.115	-0.090
		0.174	-0.145	0.188	-0.069	0.193	0.103
		0.315	-0.037	0.061	-0.025	0.229	0.249
	HiTec	0.034	0.148	0.091	0.125	0.219	0.196
		0.165	0.029	-0.006	-0.034	0.015	0.065
		-0.504	0.107	-0.032	0.067	-0.183	-0.043
	Telcm	0.395	0.097	0.124	0.027	0.242	0.232
		-0.118	-0.046	-0.056	-0.033	-0.026	0.047
		-0.243	-0.069	-0.074	-0.077	-0.208	-0.218
	Shops	0.021	-0.007	0.016	-0.006	0.011	0.032
		-0.133	-0.121	-0.119	-0.128	-0.151	-0.168
		0.392	0.101	0.170	0.015	0.294	0.241
	Hlth	-0.232	-0.113	-0.090	-0.051	-0.196	-0.157
		-0.252	0.032	-0.035	-0.006	-0.238	-0.249
		0.076	0.098	0.067	-0.014	0.008	-0.001
	Utils	-0.174	0.023	-0.172	0.024	-0.112	-0.120
		0.090	0.042	0.070	0.005	0.052	0.125
		0.451	0.004	0.077	-0.008	0.294	0.297
	Other	0.123	-0.003	0.099	0.003	0.045	0.114
		-0.259	-0.048	-0.095	-0.036	-0.229	-0.148
		0.141	0.011	0.099	-0.010	-0.005	0.085

Table IV: Results for individual industry portfolios: Reduction in volatility.

PANEL A: Market

Beta-model:	CONST	R60M	KFMR	KFRW	HF2S	HF3S
NoDur	19.4	23.5	28.8	28.4	23.4	26.2
Durbl	37.7	38.1	37.8	36.2	37.9	37.7
Manuf	43.9	49.8	46.5	49.2	45.4	46.7
Enrgy	9.2	12.6	8.9	12.6	9.4	9.5
· HiTec	44.6	49.0	49.4	49.6	48.1	49.2
Telcm	29.4	39.0	37.3	38.0	36.6	35.4
Shops	39.8	40.8	41.7	40.9	41.8	41.7
Hlth	21.0	23.9	22.5	26.3	24.3	24.4
Utils	-6.0	4.8	-4.0	6.1	-1.5	-2.0
Other	46.1	47.6	47.6	49.5	47.5	48.9

PANEL B: Fama-French.

Beta-model:	CONST	R60M	KFMR	KFRW	HF2S	HF3S
NoDur	18.6	34.7	35.4	35.5	31.0	26.9
Durbl	37.8	41.6	41.3	39.7	36.0	37.2
Manuf	48.3	55.0	53.0	54.5	49.5	50.2
Enrgy	16.6	17.0	14.5	16.7	16.5	12.5
HiTec	52.4	59.6	61.5	61.1	57.8	60.0
Telcm	31.7	40.7	40.7	40.0	36.2	35.7
Shops	33.4	43.3	44.3	44.0	35.4	38.4
Hlth	18.5	28.5	29.3	32.0	21.2	23.0
Utils	10.1	23.3	20.0	21.5	15.6	16.4
Other	57.6	66.0	65.2	67.0	60.7	63.4

Table V: Results for individual size portfolios: Reduction in volatility. PANEL A: Market.

Beta-model:	CONST	R60M	KFMR	KFRW	HF2S	HF3S
P01 (small)	13.5	22.4	17.4	14.3	16.8	18.4
P02	32.1	33.8	32.3	22.7	31.7	30.1
P03	39.1	40.6	39.4	33.3	39.6	38.0
P04	43.5	44.3	43.0	37.5	43.5	43.1
P05	49.8	50.2	49.6	45.8	50.0	49.0
P06	55.7	57.1	56.9	56.6	57.7	57.7
P07	60.4	62.2	61.4	60.9	62.0	62.0
P08	66.1	65.8	66.0	65.1	66.1	66.0
P09	70.8	72.9	72.3	72.2	72.3	72.8
P10 (big)	73.6	73.5	73.3	73.1	73.6	73.2

PANEL B: Fama-French.

Beta-model:	CONST	R60M	KFMR	KFRW	HF2S	HF3S
P01 (small)	53.0	63.0	58.0	55.9	57.1	61.0
P02	74.1	78.8	76.1	74.5	76.1	76.6
P03	80.2	82.8	82.7	82.8	80.8	81.6
P04	78.0	79.8	79.3	78.5	78.4	79.2
P05	76.1	77.4	76.6	75.4	76.1	76.6
P06	72.1	73.7	72.0	72.0	72.8	72.0
P07	73.7	76.4	74.4	72.8	73.2	72.8
P08	73.7	77.0	74.4	74.0	73.5	73.4
P09	75.5	78.0	74.9	74.3	75.5	75.4
P10 (big)	86.3	89.0	88.4	88.0	87.4	88.8

Table VI: Results for individual book-to-market portfolios: Reduction in volatility.

PANEL A: Market.

Beta-model:	CONST	R60M	KFMR	KFRW	HF2S	HF3S
P01 (low)	61.2	61.9	62.3	62.2	62.7	62.4
P02	66.5	66.0	67.2	67.8	67.4	67.8
P03	59.9	60.2	60.1	60.1	60.2	60.4
P04	45.9	48.6	50.3	50.2	48.5	48.8
P05	40.9	42.8	44.3	45.1	43.2	43.6
P06	46.1	50.4	50.0	53.0	50.0	49.7
P07	27.1	39.1	39.0	42.5	35.5	36.7
P08	20.3	36.6	38.1	39.6	31.4	35.5
P09	21.3	41.8	45.5	47.0	37.0	40.1
P10 (high)	13.6	31.0	25.7	32.6	24.6	27.5

PANEL B: Fama-French.

Beta-model:	CONST	R60M	KFMR	KFRW	HF2S	HF3S
P01 (low)	72.2	77.4	75.2	76.1	75.6	75.4
P02	60.8	67.4	69.2	69.4	64.4	66.3
P03	55.8	65.1	67.1	66.0	59.1	60.2
P04	54.3	59.6	60.1	58.8	56.6	56.8
P05	55.3	59.0	55.0	59.4	54.3	53.9
P06	63.3	61.4	63.2	62.1	61.7	61.2
P07	60.9	60.5	61.4	64.1	63.1	62.2
P08	68.3	68.8	68.2	69.6	68.8	69.9
P09	59.9	62.1	63.2	63.8	62.3	63.1
P10 (high)	48.5	51.8	52.8	52.3	50.9	49.9

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