Asymptotic Theory for Integrated Processes

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DGPE & CREATES PhD Course

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Course objective and content

Recent developments in time-series econometrics have been dominated by the analysis of integrated processes, i.e., non-stationary processes with stationary differences. This has led to a vast literature on unit root testing and cointegration analysis.

This course provides an introduction to the asymptotic distribution theory associated with such models and methods. Important tools in the analysis are weak convergence to Brownian motion, the continuous mapping theorem, stochastic integrals, and projections in function spaces. These tools are applied to derive and characterise the asymptotic properties of various estimation and testing procedures in this area, such as the Dickey-Fuller tests and its various extensions, and Johansen's reduced rank regression estimator and trace test for cointegration. Also discussed is the asymptotic efficiency and optimality of such procedures, based on the theory of local asymptotic likelihood analysis.

The course will enable the participants, first, to grasp the econometric literature on unit roots and cointegration, and second, to contribute to this literature in the form of extensions to the existing methods.

Course outline and reading list

The course consists of four lectures, which explain the theory and illustrate the results using Monte Carlo simulations; some exercises will also be discussed during the lectures. The lectures are based on lecture notes, which will be made available before the course starts, and which also contain the exercises. The topics discussed in each lecture, together with a selective bibliography, are listed below.

Lecture 1: Introduction and functional limit theory

Topics:

- Monte Carlo simulation of least-squares estimation of first-order autoregression
- Review of stationary asymptotics for simple regression and first-order autoregression
- Weak convergence on function spaces
- Brownian motion and the invariance principle
- The continuous mapping theorem
- Unit root asymptotics for simple regression and first-order autoregression

Readings:

- Chapters 1 and 5 of Boswijk, H. P. (2004), *Asymptotic Theory for Integrated Processes* (manuscript in preparation; Part I of the manuscript will be made available).
- Phillips, P. C. B. and Z. Xiao (1998), "A Primer on Unit Root Testing", *Journal of Economic Surveys*, 12, 423–469.

Lecture 2: Dependence, local power and optimality of unit root tests

Topics:

- Dependent errors in unit root regressions: consequences and remedies
- Stochastic integrals, Itô's lemma, weak convergence to stochastic integrals
- Near-integration, local power of unit root tests
- Local asymptotic likelihood analysis, optimality, non-Gaussian errors

Readings:

- Chapters 4 and 6 of Boswijk, H. P. (2004), *op. cit*.
- Haldrup, N. and M. Jansson (2006), "Improving Size and Power in Unit Root Testing", in *Palgrave Handbook of Econometrics, Volume 1: Econometric Theory*, 252–277.
- Phillips, P. C. B. (1987), "Time Series Regression with a Unit Root", *Econometrica*, 55, 277–301.
- Phillips, P. C. B. and Z. Xiao (1998), *op. cit*..
- Rothenberg, T. J. and J. H. Stock (1997), "Inference in a Nearly Integrated Autoregressive Model with Nonnormal Innovations", *Journal of Econometrics*, 80, 269–286.

Lecture 3: Deterministic components, regression with integrated processes

Topics:

- Deterministic components; L^2 projections
- Similar tests
- GLS detrending
- Multiple regression with integrated, stationary and deterministic regressors

Readings:

- Elliott, G., T. J. Rothenberg and J. H. Stock (1996), "Efficient Tests for an Autoregressive Unit Root", *Econometrica*, 64, 813–836.
- Haldrup, N. and M. Jansson (2006), *op. cit*.
- Park, J. Y. and P. C. B. Phillips (1988, 1989), "Statistical Inference in Regressions with Integrated Processes", *Econometric Theory*, 4, 468–497, and 5, 95–131.
- Phillips, P. C. B. and Z. Xiao (1998), *op. cit*.

Lecture 4: Optimal inference on cointegration

Topics:

- Locally asymptotically quadratic and locally asymptotically mixed normal likelihood ratios
- Optimal inference on cointegration in triangular systems
- Cointegrated vector autoregressions; Granger representation theorem
- The Johansen reduced-rank regression procedure and trace test
- Cointegration inference in the presence of near-unit roots
- Optimal inference on cointegration for non-Gaussian and heteroskedastic time series

Readings:

- Elliott, G. (1998), "On the Robustness of Cointegration Methods when Regressors Almost Have Unit Roots", *Econometrica*, 66, 149–158.
- Jeganathan, P. (1995), "Some Aspects of Asymptotic Theory with Applications to Time Series Models", *Econometric Theory*, 11, 818–887.
- Johansen, S. (1991), "Estimation and Hypothesis Testing of Cointegration Vectors in Gaussian Vector Autoregressive Models", *Econometrica*, 1551–1580.
- Phillips, P. C. B. (1991), "Optimal Inference in Cointegrated Systems", *Econometrica*, 59, 283–306.

Further background reading

- Davidson, J. (1994), *Stochastic Limit Theory*. Oxford: Oxford University Press.
- Hamilton, J. D. (1994), *Time Series Analysis*. Princeton: Princeton University Press.
- Johansen, S. (1995), *Likelihood-Based Inference in Cointegrated Vector Autoregressive Models*. Oxford: Oxford University Press.