Agency Problem and Ownership Structure: Outside Blockholder As a Signal*

Sergey Stepanov† and Anton Suvorov‡

November 13, 2009

Abstract

Conventional wisdom suggests that large outside shareholders help to restrict insider opportunism and, hence, are more beneficial when the agency problem is more severe. Thus, if a firm chooses its ownership structure so as to minimize the agency cost, firms with more potential for expropriation of shareholders by insiders, are more likely to have an outside blockholder. Our model predicts that under asymmetric information about the entrepreneur’s ability/propensity to extract private benefits the outcome may be the opposite: ownership structures with an outside blockholder are chosen by the entrepreneurs that are less capable/willing to extract private benefits. Selling a large stake to an outside blockholder who would monitor the entrepreneur allows a “good” entrepreneur to separate himself from a “bad” one. The result holds regardless of whether an outside blockholder acts for the benefit of all shareholders or colludes with the entrepreneur for sharing private benefits. Our model suggests that the often documented positive relationship between the presence of an outside blockholder and firm value may arise not due to a direct effect of blockholder monitoring, but because entrepreneurs with low propensity to self-deal choose to attract an outside blockholder. Hereby, our work contributes to the endogeneity debate in the ownership-performance literature.

1 Introduction

Monitoring by an outside blockholder helps to restrict managerial (insider) opportunism and thereby mitigates the agency problem in a firm. This widespread view is often used to explain certain cross-country and intra-country regularities. In particular, it can be used to explain the prevalence of firms with large blockholders in countries where legal protection of shareholders is weak, meaning severe agency problems (see e.g. La Porta, Lopez-de-Silanes, and Shleifer (1999)). Burkart et al (2003) present the argument in a formal model, in which the initial owner of the firm chooses whether to hire a professional manager and how much of his own equity to sell.

---

*We thank Sergei Guriev for useful comments.
†New Economic School and CEFIR, Moscow. Email: sstepanov@cefir.ru
‡New Economic School and CEFIR, Moscow. Email: asuvorov@nes.ru
in the market. Subsequent monitoring by the initial owner, who essentially becomes an outside blockholder, helps reduce the rent that the manager obtains by extracting private benefits. The monitoring incentive grows with the blockholder’s stake. Since weaker shareholder protection implies a greater potential for the private benefit extraction, the equilibrium blockholder’s share is greater in weaker legal regimes.\footnote{When legal shareholder protection becomes too low, the initial owner prefers not to hire a professional manager at all and runs the firm himself.}

At the firm level, many empirical studies find that the presence or a greater equity share of a large outside shareholder is positively related to firm value or operating performance.\footnote{Using a sample of above 1400 firms from 18 emerging markets, Lins (2003) finds that large non-management blockholders increase firm value. Lehmann and Weigand (2000) run panel regressions for 361 German corporations over the time period 1991 to 1996 and conclude that the presence of the second large shareholder improves profitability as measured by the return on total assets. Maury and Pajuste (2004) study a sample of 136 Finnish listed non-financial firms that have at least one large shareholder with more than 10% of the votes over the period from 1993 to 2000. They find that a more equal distribution of votes among large blockholders has a positive effect on firm value. They also find that the identity of large shareholders matters: family firms where other large blockholders are not families have greater valuations compared to firms where other large blockholders are families, suggesting that non-family blockholders restrict private benefit extraction by families. Similarly to Maury and Pajuste (2004), Laeven and Levine (2005) pay special attention to the relative voting rights and cash flow rights as well as identities of large shareholders. They extend Maury and Pajuste’s study by looking at 900 listed firms in 13 Western European countries. Controlling for the size of the largest shareholder they find that the second large shareholder boosts corporate valuation, but only when his voting rights are large enough with respect to those of the largest shareholder. There are a few papers that stand in contrast to the above studies. Faccio et al (2001) study almost six thousands European and East Asian financial and nonfinancial corporations over the period from 1992 to 1996. For the subsample of group-affiliated corporations, controlling for the ratio of ownership to control rights of the largest shareholder, the authors find that the presence of another large shareholder who controls at least 10% of the stock increases dividend payouts in Europe, but lower them in East Asia. Anderson and Reeb (2003) studying S&P 500 firms find that the combined share of all large outside (non-family) blockholders (> 5%) has a negative effect on market and operating performance. Earle et al (2003) use panel data on Hungarian firms, listed on the Budapest Stock Exchange. They find that, while the size of the largest blockholder positively affects operating performance, adding other large shareholders does not add value (controlling for the largest shareholder’s share) and even decreases it, though this result is not statistically significant. Himmelberg et al (1999) find that a large part of the cross-sectional variation in ownership structures of US companies is explained by unobserved firm heterogeneity.

Demsetz and Villalonga (2001), extending in their work numerous previous studies on the ownership-performance relationship conclude the following: “the results from our study and from some of the studies preceding it yield unequivocal evidence for the endogeneity of ownership structure”.

1Himmelberg et al (1999) find that a large part of the cross-sectional variation in ownership structures of US companies is explained by unobserved firm heterogeneity. Demsetz and Villalonga (2001), extending in their work numerous previous studies on the ownership-performance relationship conclude the following: “the results from our study and from some of the studies preceding it yield unequivocal evidence for the endogeneity of ownership structure”.

2If in Burkart et al (2003) the parameter of shareholder protection is reinterpreted as a firm-level managerial...
We propose an agency theory model in which, contrary to the Jensen and Meckling paradigm, firms with greater agency problems are less likely to have an outside monitor. Our crucial ingredient is the asymmetry of information between the entrepreneur and the market about the ability (or propensity) of the former to extract private benefits at the expense of outside shareholders. Indeed, such characteristic of the entrepreneur is to a large extent unobservable to the market, and, hence, is difficult to control for in empirical work.

We examine the problem of the initial sole owner of the firm (entrepreneur) who wants to raise outside funds by selling equity in order to finance an investment opportunity. Under symmetric information, “good” entrepreneurs (i.e. those with a low expropriation propensity) choose not to attract an outside blockholder because they are able to raise finance anyway, and blockholder monitoring is costly (this cost is ultimately born by the entrepreneur). ”Bad” entrepreneurs can raise finance by selling just dispersed equity only when the investment opportunity is good enough. Otherwise, they need to resort to attracting an outside blockholder, because blockholder monitoring becomes necessary for convincing the investors that they will get their money back.

Thus, under symmetric information the model yields the results in line with those obtained in Holmström and Tirole (1997). In their model, entrepreneurs with high pledgeable income (one of the determinants of which is the ability of the entrepreneur to extract private benefits) can raise necessary funds without attracting an outside monitor, while firms with a lower pledgeable income need a monitor (who could be a large shareholder) in order to be able to raise finance.

Asymmetry of information changes the solution radically. When the investment opportunity is good enough, attracting an outside blockholder helps a good entrepreneur to credibly signal his type to the market, because, in this case, a bad entrepreneur prefers being priced fairly and not monitored to pretending to be good but being monitored. When the investment opportunity is not good enough, the separation becomes unfeasible in equilibrium but attracting an outside blockholder may still be necessary for a good entrepreneur in order not to be perceived a bad type (pooling equilibrium). As a result, under asymmetric information, the outside ownership concentration chosen by the good type is never below the level chosen by the bad type and, what is especially remarkable, is even higher for a range of parameters, which stands in stark contrast to the symmetric information outcome.

What is especially interesting, our result holds even when monitoring does not have any direct impact on firm value. In section 5 we allow for the possibility of collusion between the entrepreneur and the blockholder, so that monitoring does not increase the value for minority shareholders, but simply helps the blockholder to transfer a part of the entrepreneur’s private benefits into her pocket. In such setup, the described type of separating equilibrium still exists in a range of parameters, while separating equilibria of other types do not appear.

Thus, our result helps to explain a positive relationship between outside ownership concentration and firm value documented in a number of papers. In contrast to a “naïve” view propensity to self-deal, the model predicts exactly such relationship.
that explains this observation by a direct “non-equilibrium” effect of blockholder monitoring on insiders’ self-dealing, we obtain this relationship as an equilibrium outcome: good entrepreneurs choose to attract an outside blockholder in order to signal their low expropriation propensity. We should also note that our results are consistent with the observed cross-country ownership structure patterns: firms in countries with weaker shareholder protection have more concentrated ownership structures.

Our model has several interesting implications. First, the magnitude of an empirically found positive relationship between an outside blockholder’s share and firm value should never be attributed to a causal effect only: part of the effect, or even the whole effect (as our model with collusion demonstrates), may be a result of signaling. Second, as our model with collusion shows, the argument that an outside blockholder colludes with the insider or pursues her own goals at the expense of small shareholders does not provide a fully satisfactory explanation the absence of a positive relationship found in some studies (e.g. Faccio et al (2001)). Finally, our findings imply that blockholders that expropriate small shareholders should not be viewed as an “evil” only, as they may be a useful signaling device that help good entrepreneurs to convey fair values of their projects to the market.

We would like to emphasize that, though the theoretical literature on signaling in financial markets is extensive (Myers and Majluf (1984), Ross (1977), Leland and Pyle (1977), to mention a few well known papers; see also the survey by Harris and Raviv (1991)) our idea of an outside blockholder as a signal is novel – there are no such papers to our knowledge. Among this literature, the model by Leland and Pyle (1977) is probably closest to ours. In that paper a “good” entrepreneur, who values diversification due to his risk-aversion, still prefers to retain a large block of his firm’s equity since selling too much sends a negative signal to the market (“bad” entrepreneurs sell out their stakes in equilibrium). Thus, like our paper, Leland and Pyle (1977) is about signaling via the ownership structure, but their setup and implications are orthogonal to ours.

Our paper is organized as follows. Section 2 sets up the basic model. Section 3 analyzes the symmetric information benchmark. In section 4 we present the solution under asymmetric information. In section 5 we introduce the possibility of collusion between the entrepreneur and the outside blockholder and discuss our results. Section 6 discusses the relations of our results to the empirical observations. Section 7 concludes the paper.

2 The Basic Model

Consider an entrepreneur who has an investment opportunity but does not have own funds. He may be a start-up entrepreneur or the sole owner of an already large company that lacks funds to finance its further growth. The investment requires an outlay \( I < 1 \) and generates value 1. The entrepreneur can divert up to a fraction \( d \) of this value into his own pocket at no cost. The entrepreneur can be either of the two types: “good” or “bad”. The two types differ by the value
of \( d \): a good entrepreneur can divert up to \( \overline{d} \), while a bad one can divert up to \( \overline{d} > \underline{d} \), where \( \overline{d} \) and \( \underline{d} \) are exogenous. The type is initially the entrepreneur’s private information. The market only has a prior that the entrepreneur is good with probability \( \nu \).

The entrepreneur raises funds by selling equity shares: he sells \( 1 - \alpha \) and retains \( \alpha \). Diversion can be reduced via monitoring by outside shareholders. At cost \( cy \) the maximum amount that can be diverted is reduced by \( y \in [0, d] \), where \( y \) is the level of monitoring and is a choice variable.\(^5\) There is, however, a collective action problem among outside shareholders so that only a non-atomistic shareholder (blockholder) may want to choose a positive level of monitoring. Thus, we assume that if the entrepreneur wants to be monitored ex-post, he must sell a non-atomistic share \( \beta \) as a block, while the rest, \( 1 - \alpha - \beta \) is sold as dispersed equity.\(^6\) The capital market is competitive.

The timing is as follows:

At \( t = 0 \) the entrepreneur offers a pair \((\alpha, \beta)\); the investors update their beliefs and price the shares accordingly; the shares are sold and the necessary funds are raised, given that the aggregate price of all offered shares is at least \( I \); finally, \( I \) is invested in the project. The entrepreneur must satisfy the investors’ participation constraint, but he is free to raise more than \( I \) by selling extra shares. In such a case he simply pockets the extra funds.

At \( t = 1 \) the blockholder learns the type of the entrepreneur and chooses \( y \). While the assumption in italics is extreme, it looks plausible that in reality a blockholder would learn at least something about the entrepreneur’s ability to self-deal before the latter diverts everything he can. The entrepreneur observes \( y \) (hence the maximum amount he can divert) and chooses the level of diversion.

At \( t = 2 \) the returns to the shareholders and the entrepreneur’s private benefit are realized.

We set two restrictions on the parameters:

Assumption 1. \( 1 - d > I \)

Assumption 2. \( 1 - cd > I \)

As we will see below, Assumption 1 implies that, given that the market knows the type of the entrepreneur, the good type is able to raise finance without attracting an outside blockholder (non-monitored finance). Assumption 2 implies that, given that the market knows the type of

---

\(^5\)Thus, the marginal cost of reducing self-dealing is constant. One could argue that this cost should rather be convex as it should be easier to prevent or revert some obvious self-dealing opportunities and more difficult to dump expropriation further, when all easily identifiable self-dealing has already been prevented.

We have tried two other specifications. In the first alternative specification, the maximum possible \( y \) was the same for both types: \( y_{\text{max}} < \underline{d} \). Thus, essentially it was an extreme case of a convex monitoring cost such that the marginal cost of monitoring rose to infinity for any \( y > y_{\text{max}} \).

The second specification was like the first one except that the maximum possible \( y \) was larger in the bad firm.

For both alternative specifications the qualitative results of our model remained intact. However, the mechanics of the solution turned out to be significantly more complicated for these specifications. Thus, for the sake of simplicity of exposition we have decided to use the present version of the monitoring technology in the text.

\(^6\)We do not consider the possibility of selling several separate blocks. Several monitors would either coordinate their monitoring efforts or partially free-ride on each other’s efforts, but that would not change the essence of our model while introducing unnecessary complications.
the entrepreneur, the bad type is able to raise finance by attracting an outside blockholder, provided that the latter will monitor him (monitored finance).

As we will see, the blockholder’s program for determining the level of monitoring is linear. We set the following assumption on the blockholder’s behavior in case of indifference.

Assumption 3. When indifferent, the blockholder chooses the maximum level of monitoring, i.e. $y = d$.

This assumption is made for simplicity. Alternatively we could impose any level of monitoring in case of indifference – that would not alter our results. We could also allow the blockholder to select any level of monitoring when she is indifferent. That would create a difficulty with multiplicity of equilibria, since for any monitoring level chosen in case of indifference there would be a corresponding equilibrium. Such problem could be eliminated by making $c$ random and continuously distributed so that indifference would arise with probability zero. Again, such modification would not change our qualitative results.

We will first study the symmetric information benchmark. Then we will look for all pure strategy perfect Bayesian equilibria of this game satisfying the Cho-Kreps intuitive criterion.

3 Symmetric information benchmark

At $t = 1$, having observed $y$, the entrepreneur makes his diversion decision. Given that diversion is costless, he will always divert everything he can, i.e. $d - y$.

The blockholder solves the following problem:

$$\max_y \{\beta(1 - (d - y)) - cy\},$$

which yields the following solution:

$$\begin{cases} 
\text{if } \beta \geq c \text{ then } y = d \\
\text{if } \beta < c \text{ then } y = 0
\end{cases}$$

Now we turn to the entrepreneur’s problem at $t = 0$. His payoff is:

$$U = \alpha(1 - d + y(\beta, d)) + (d - y(\beta, d)) + P_b(\beta, d) + P_d(\beta, d) - I,$$

(1)

where $y(\beta, d)$ is the solution of the above blockholder’s problem, and $P_b$ and $P_d$ are the prices that the blockholder and dispersed shareholders pay for their shares. Since the capital market is competitive and the investors perfectly predict the blockholder’s action at $t = 1$, these prices are determined at $t = 0$ as follows:

$$P_b = \beta(1 - d + y(\beta, d)) - cy(\beta, d)$$

$$P_d = (1 - \alpha - \beta)(1 - d + y(\beta, d))$$
It is necessary that \( P_b(\beta, d) + P_d(\beta, d) \geq I \) for the investors to be willing to provide finance.

Plugging the expressions for the prices in the entrepreneur’s utility function, we obtain that the entrepreneur simply minimizes \( y \) subject to \( P_b(\beta, d) + P_d(\beta, d) \geq I \).

This is a rather standard result in the agency problem literature – ex-ante the entrepreneur bears all the costs and, hence, wants to create an arrangement that would minimize them. In our case, under symmetric information, the cost that the entrepreneur bears ex-ante (through price \( P_b \)) is the monitoring cost.

Hence, the entrepreneur would prefer to raise funds and offer \( \beta < c \) at the same time if feasible – in such a case he would avoid the monitoring cost. Due to Assumption 1 the good type can always do it. For example, he could sell all 100% of the shares to dispersed investors and raise \( 1 - d > I \). Hence, under symmetric information, the good type never chooses to attract an outside blockholder.

The bad type can do the same only if \( 1 - d > I \). If \( 1 - d < I \) the bad type needs to resort to using a blockholder with \( \beta \geq c \), as otherwise there will be no monitoring and even offering 100% of the shares to investors will not satisfy their participation constraint. Since, by Assumption 2, \( 1 - c d > I \), the solution with \( \beta \geq c \) is feasible.

The results of this section allow us to formulate the following proposition:

**Proposition 1** Under symmetric information, the good type never chooses to have an outside blockholder, while the bad type has to attract an outside blockholder when the investment opportunity is not good enough \((1 - d < I)\). Hence, under symmetric information, firms with a greater potential for insider expropriation are more likely to have an outside blockholder.

This result is in line with the argument presented in Holmström and Tirole (1997) or, more generally, in the book by Tirole (2006): firms with a lower pledgeable income need to resort to monitored finance in order to be able to raise funds. In our model, the difference in the pledgeable income stems from the difference in the expropriation propensity – bad entrepreneurs have lower pledgeable income.

### 4 Solution under asymmetric information

Since the blockholder observes the type of the entrepreneur before taking her monitoring decision, her problem at \( t = 1 \) and its solution remain exactly the same as under symmetric information.

The entrepreneur’s payoff as of \( t = 0 \) looks now as follows:

\[
U = \alpha(1 - d + y(\beta, d)) + (d - y(\beta, d)) + \tilde{P}_b + \tilde{P}_d - I, \tag{2}
\]

where \( \tilde{P}_b \) and \( \tilde{P}_d \) are the prices that the blockholder and dispersed shareholders pay for their shares. It must be, of course, that \( \tilde{P}_b + \tilde{P}_d \geq I \) for the entrepreneur to be able to raise funds.
The prices are now determined by the investors’ beliefs that they form upon observing \((\alpha, \beta)\) and are equal to:

\[
\begin{align*}
\tilde{P}_b & = \beta (1 - \tilde{d} + \tilde{y}(\beta, \tilde{d})) - c\tilde{y}(\beta, \tilde{d}), \\
\tilde{P}_d & = (1 - \alpha - \beta)(1 - \tilde{d} + \tilde{y}(\beta, \tilde{d})),
\end{align*}
\]

where \(\tilde{d}\) is the expected diversion and \(\tilde{y}\) is the expected monitoring upon observing \((\alpha, \beta)\). Since the decision whether to monitor or not depends only on \(\beta\), and the level of monitoring is always \(1 - d\), the expected monitoring is a function of only \(\beta\) and \(\tilde{d}\). Thus, for \(\beta \geq c\):

\[
\tilde{P}_b = \beta - c\tilde{d}, \quad \tilde{P}_d = 1 - \alpha - \beta,
\]

and for \(\beta < c\):

\[
\tilde{P}_b = \beta(1 - \tilde{d}), \quad \tilde{P}_d = (1 - \alpha - \beta)(1 - \tilde{d}).
\]

The payoff, thus, can be rewritten as:

- If \(\beta \geq c\), \(U = 1 - c\tilde{d} - I\)
- If \(\beta < c\), \(U = 1 + (1 - \alpha)(d - \tilde{d}) - I\)

These payoffs have a simple interpretation. When \(\beta \geq c\), diversion is precluded by monitoring altogether, and the only route through which the market beliefs affect the entrepreneur’s payoff is the effect on the expected blockholder monitoring, which enters price \(\tilde{P}_b\). As a result, the entrepreneur obtains the NPV of the project net of the expected monitoring cost. When \(\beta < c\), there is no monitoring, but the entrepreneur gets a premium (discount) whenever the market overvalues (undervalues) the company. Hence, the entrepreneur’s payoff is the NPV of the project plus the premium (discount) he obtains on the shares being sold.

We will consider separately two ranges of parameters: the one in which both entrepreneurs could raise non-monitored finance under symmetric information \((I < 1 - \tilde{d})\) and the one in which only the good type could raise non-monitored finance under symmetric information \((1 - \tilde{d} < I < 1 - \tilde{d})\). The sets of equilibria will be very different in the these two zones.

### 4.1 Case 1: \(I < 1 - \tilde{d}\)

First, the observation made a few lines above implies that an equilibrium in which \(\bar{\beta} \geq c\) is impossible. In such equilibrium, the bad type would obtain \(1 - c\bar{d} - I\). However, by deviating to some \(\bar{\beta} < c\), the bad type is able to get at least his symmetric information payoff, which is \(1 - I > 1 - c\bar{d} - I\).

Let us proceed now by considering possible separating and pooling equilibria.

#### 4.1.1 Separating equilibria

Given that \(\bar{\beta}\) must be smaller than \(c\) in any equilibrium, there are only two cases to consider: both \(\underline{\beta}\) and \(\bar{\beta}\) are below \(c\), and \(\bar{\beta} < c\) but \(\underline{\beta} \geq c\).
The first situation is impossible to have in equilibrium. In such separating equilibrium the bad type would get \(1 - I\), which is his symmetric information payoff. But by pretending to be the good type, he could obtain \((1 - \alpha)(\bar{d} - d) - I\), which is bigger.

Thus, the only remaining candidate for a separating equilibrium is \(\beta < c, \beta \geq c\). In such equilibrium the bad type’s payoff would be \(U = 1 - I\) and the good type would get \(U = 1 - cd - I\).

The following lemma establishes the conditions under which such equilibrium exists.

**Lemma 1** When \(I < 1 - \bar{d}\), a separating equilibrium satisfying the Cho-Kreps intuitive criterion exists iff

\[
\frac{cd}{1 - \bar{d}} \leq \frac{I(\bar{d} - d)}{1 - d}
\]

The set of all such separating equilibria is characterized by all pairs of pairs \(((a, \beta), (\pi, \bar{\beta}))\), \((a, \beta) \neq (\pi, \bar{\beta})\), such that \(\beta \geq c, \bar{\beta} < c, a \leq \min\{1 - cd - I, 1 - \beta\}, \pi \leq \min\{1 - I, 1 - \bar{\beta}\}\).

In any separating equilibrium both types raise finance.

**Proof.** See the appendix.

Figure 1 below depicts the set of all separating equilibria. For the bad type deviation is not an issue. Pretending to be good by offering \(\beta' = \beta \geq c\) leads to a loss since it yields a payoff of \(1 - cd(I - \bar{d}) < 1 - I\). Deviating to another \(\beta' < c\) does not gain him anything, given the worst out-of-equilibrium beliefs upon observing such a deviation (these beliefs clearly satisfy the Cho-Kreps criterion as the bad type would definitely deviate to another \(\beta' < c\) if he were believed to be good).

In contrast, it is unclear a priori whether the good type would want to deviate to some \(\beta' < c\) or not (deviation to \(\beta' \geq c\) does not gain anything to the good type even if he is still treated as good). On the one hand, he would suffer from the bad market beliefs but, on the other hand, he would avoid costly monitoring. Condition \(cd(I - \bar{d})/(1 - d)\) of Lemma 1 tells when the good entrepreneur values fair pricing more than the absence of monitoring and, thus, does not want to deviate. This condition has a very intuitive meaning. The left-hand side is the loss from monitoring in equilibrium. The right-hand side is simply a discount that the good entrepreneur would incur on the sold shares under the best possible deviation. The following reasoning helps to understand the latter statement. If he decides to deviate, then, in order to reduce the effect of underpricing as much as possible, the good type needs to sell as small a share as possible, provided that the investors still agree to finance. Given the bad beliefs, this share is \(I/(1 - \bar{d})\), and the discount the good type incurs is \(\bar{d} - d\) per share. Hence, we obtain the right-hand side of the condition.
4.1.2 Pooling equilibria

Given that $\beta$ must be smaller than $c$ in any equilibrium the only possibility is $\beta = \beta_p < c$. Let us denote $\hat{d} \equiv \nu d + (1 - \nu)\bar{d}$.

In such equilibrium the payoff of the bad type is $U = \alpha_p(1 - \hat{d}) + \overline{d} + (1 - \alpha_p)(1 - \hat{d}) - I = 1 + (1 - \alpha_p)(\overline{d} - \hat{d}) - I$, and the payoff of the good type is $\overline{U} = \alpha_p(1 - \overline{d}) + \overline{d} + (1 - \alpha_p)(1 - \overline{d}) - I = 1 + (1 - \alpha_p)(\overline{d} - \hat{d}) - I$.

The following lemma establishes the conditions under which such equilibrium exists.

**Lemma 2** When $I < 1 - \overline{d}$ a pooling equilibrium satisfying the Cho-Kreps criterion exists iff

$$cd \geq \frac{I(\overline{d} - \hat{d})}{1 - \hat{d}}$$

(4)

The set of all pooling equilibria satisfying the Cho-Kreps criterion is characterized by all pairs $(\alpha_p, \beta_p)$ such that $\alpha_p \in \left[ \max \left\{ 1 - \frac{I}{(1 - \overline{d})(1 - \nu)}, 1 - \frac{cd}{\overline{d} - \hat{d}} \right\}, 1 - \frac{I}{1 - \overline{d}} \right]$, $\beta_p < \min\{c, 1 - \alpha_p\}$. In any pooling equilibrium both types raise finance.

**Proof.** See the appendix. □

Similarly to the separating equilibrium case, the bad type would clearly not want to deviate. Given the bad out-of-equilibrium belief for any deviation such that $\beta' < c$, the bad type would
get $1 - I$ from such deviation, which is smaller than $\overline{U}$. If the bad type would deviate to some $\beta' \geq c$, he would get $1 - cd - I$ even if the market believes he is good, which is again smaller than $\overline{U}$.

However, it is again unclear a priori whether the good type would want to deviate. First, he could deviate to some $(\alpha', \beta')$ such that $\beta' < c$. He would suffer a more severe discount per share of the equity sold, but he could probably sell less equity, i.e. $\alpha' > \alpha_p$, which would be possible if $\alpha_p$ is not large enough. Second, he could deviate to some $(\alpha', \beta')$ such that $\beta' \geq c$. Notice that we have shown a few lines above that the bad type would never want to deviate to $\beta' \geq c$ regardless of the beliefs. Hence, in correspondence with the Cho-Kreps criterion, the market must believe that the entrepreneur is definitely good when $\beta' \geq c$ is chosen. Thus, by such deviation, the good type would gain from fair pricing but lose from monitoring.

It turns out that there always exists $\alpha_p$ that satisfies the investors’ participation constraint and makes the first type of deviation unprofitable. The same cannot be said about the second type of deviation, but when condition $cd \geq I(\overline{d} - \overline{d})/(1 - \overline{d})$ holds (and only then) there exists $\alpha_p$ that makes such deviation unprofitable. The interpretation of this condition is again intuitive. The left-hand side is the loss from monitoring is $cd$ that the entrepreneur would incur from a deviation. The right-hand side is an aggregate discount that the good entrepreneur incurs in the best for him pooling equilibrium. The best pooling equilibrium is the one in which the aggregate discount is the smallest, i.e. in which the good type sells as few shares as possible. Thus, in such equilibrium $1 - \alpha_p = I(1 - \overline{d})$, i.e. $\alpha_p$ is such that the investors agree to provide exactly $I$. The discount the good type incurs is $\overline{d} - \overline{d}$ per share. Hence, we obtain the right-hand side of the condition.

4.2 Case 2: $1 - \overline{d} < I < 1 - \overline{d}$

When the parameters is such that only the good type can raise non-monitored finance under symmetric information, separating equilibria disappear. The reason is that the entrepreneur simply cannot raise finance without attracting a blockholder when he is believed to be bad. A separating equilibrium with $\overline{\beta} \geq c$ cannot exist either because the bad type would always want to mimic the good type (remember that for any $\beta \geq c$ the entrepreneur’s payoff is $1 - cd - I$).

As far as pooling equilibria are concerned, we will have two types of them now. First, under condition (4), it will be possible to sustain the familiar pooling equilibria without a blockholder, provided that the expected diversion $\overline{\alpha}$ is not too big so that the investors still agree to finance the firm, i.e. $1 - \overline{\alpha} \geq I$. Second, another type of pooling equilibria appears, with $\overline{\beta} = \overline{\beta} \geq c$. The reason why this latter type becomes possible is that now a deviation of the bad type to $\beta' < c$, which was profitable in Case 1, is simply unfeasible due to his inability to raise non-monitored finance. Other deviations are unprofitable, given the bad out-of-equilibrium beliefs (such beliefs clearly satisfy the Cho-Kreps criterion), since for any $\beta \geq c$ the entrepreneur’s payoff is $1 - cd - I$.

The above reasoning leads us to the following lemma.
Lemma 3 When $1 - \overline{d} < I < 1 - \underline{d}$, the following is true:
- no separating equilibria exist
- there always exists a pooling equilibrium with $\overline{\beta} = \overline{\beta} = \beta_p \geq c$, any such equilibrium satisfies the Cho-Kreps criterion
- there exists a pooling equilibrium with $\overline{\beta} = \overline{\beta} = \beta_p < c$ satisfying the Cho-Kreps criterion

iff $1 - \hat{d} \geq I$ and $cd \geq \frac{I(\hat{d} - d)}{1 - d}$

Proof. The proof follows directly from Lemma 1 and Lemma 2 and the reasoning we have presented just before the lemma. ■

4.3 Equilibrium analysis. Complete picture

After considering Cases 1 and 2, the whole equilibrium analysis can be summarized in the following key proposition, which is also illustrated graphically in Figure 2.

Proposition 2

- When the monitoring cost is high ($c > \frac{\overline{d} - d}{d - d}$), both types of firms choose the same ownership structure:
  - When the investment opportunity is good ($I < 1 - \overline{d}$) both types choose not to have an outside blockholder
  - Otherwise both types may choose to have an outside blockholder (and this will be the only type of equilibrium if $I > 1 - \hat{d}$).

- When the monitoring cost is low ($c < \frac{\overline{d} - d}{d - d}$):
  - When the investment opportunity is very good ($I < \frac{cd(1 - \overline{d})}{d - d}$), both types choose not to have an outside blockholder
  - When the investment opportunity is intermediate ($\frac{cd(1 - \overline{d})}{d - d} < I < 1 - \overline{d}$), there is an equilibrium in which only the good type chooses to have an outside blockholder. Moreover, when $\frac{cd(1 - d)}{d - d} < I < 1 - \overline{d}$, this is the only type of equilibrium
  - When the investment opportunity is bad ($\overline{d} < I < 1 - \overline{d}$) both types of firms choose the same ownership structure. It may or may not contain an outside blockholder, but if $I > 1 - \hat{d}$ there are only equilibria with an outside blockholder
The dark grey area is the area where separating equilibria exist. The light grey area is the area where pooling equilibria with both types choosing to have a blockholder exist. The quadrangle bounded by the bold lines is the area where pooling equilibria with both types choosing not to have a blockholder exist.

The stark contrast with the symmetric information benchmark is that now the outside ownership concentration chosen by the good type is never below the level chosen by the bad type and, what is especially remarkable, is even higher under certain parameters. For the good type, an outside blockholder either serves a signaling device that allows him to credibly convey his type to the market (separating equilibrium) or is a necessity that he has to bear in order not to be perceived bad (pooling equilibrium with both types choosing to have an outside blockholder).

Thus, the asymmetry of information reverses the relationship between the severity of the agency problem and the presence of an outside blockholder. In other words, it is not true anymore that firms with higher pledgeable income are more likely to raise non-monitored finance. Obviously, the information asymmetry leads to inefficiency: in contrast to the symmetric information case, now the good type sometimes has to attract a blockholder, whose monitoring is costly.
In our basic model, apart from the signaling effect, a monitor has a direct positive impact on the minority shareholders’ value. Indeed, if introduced exogenously into a firm, a blockholder would eliminate diversion of profits regardless of the firm’s type. Thus, in fact, the effect of the presence of a blockholder on the stock price (dispersed shareholders’ value) consists of the direct effect, $d$, and the signaling effect, $d - d$. The sum of the two, $d$, gives the difference between the stock prices (values for dispersed shareholders) of a firm with a blockholder and a firm without a blockholder in a separating equilibrium. In the next section we introduce a model in which monitoring has no direct effect on the value for dispersed shareholders, but a firm with a blockholder is still valued higher in equilibrium due to a pure signaling effect.

5 Model with collusion (transferable $d$)

Assume the blockholder instead of reducing expropriation can collude with the entrepreneur and share the diverted amount $d$ at zero cost. In such a case blockholder monitoring does not benefit the dispersed shareholders as they obtain $1 - d$ per unit share regardless of whether monitoring took place or not.

We assume that collusion occurs after monitoring, at $t = 1.5$. The entrepreneur and the blockholder bargain over the private benefits. We assume the Nash bargaining rule for the bargaining process, with $\gamma$ being the entrepreneur’s bargaining power. Similarly to Burkart et al. (2003), we assume that if an agreement is not achieved, the blockholder simply shields $y$ from expropriation. In other words, the outside options in bargaining are:

- **Blockholder’s:** $\beta(1 - (d - y))$
- **Entrepreneur’s:** $\alpha(1 - (d - y)) + d - y$

If the agreement is reached their joint (post-monitoring) payoff is $(\alpha + \beta)(1 - d) + d$. Thus, the surplus is $y(1 - (\alpha + \beta))$. Given that the blockholder’s bargaining power is $1 - \gamma$, she obtains $\beta(1 - d) + y(\gamma\beta + (1 - \gamma)(1 - \alpha))$. Hence, she chooses $y$ by maximizing

$$\beta(1 - d) + y(\gamma\beta + (1 - \gamma)(1 - \alpha)) - cy,$$

which yields:

\[
\begin{cases}
\text{if } \beta \geq \frac{\gamma - (1 - \gamma)(1 - \alpha)}{\gamma} \equiv \hat{\beta}(\alpha) \text{ then } y = d \\
\text{if } \beta < \hat{\beta}(\alpha) \text{ then } y = 0
\end{cases}
\]

5.1 Symmetric information benchmark

Analogously to the basic model, the entrepreneur’s payoff as of $t = 0$ is:

$$U = \alpha(1 - d + y(\beta, d)) + (d - y(\beta, d)) + \gamma y(\beta, d)(1 - (\alpha + \beta)) + P_b(\beta, d) + P_d(\beta, d) - I \quad (5)$$

The prices are now:
\[ P_b = \beta(1 - d) + y(\beta, d)(\gamma\beta + (1 - \gamma)(1 - \alpha)) - cy(\beta, d) \]
\[ P_d = (1 - \alpha - \beta)(1 - d) \]

It is necessary that \( P_b(\beta, d) + P_d(\beta, d) \geq I \) for the investors to be willing to provide finance.

As in the basic model, maximizing \( U \) is equivalent to minimizing \( y \) subject to \( P_b(\beta, d) + P_d(\beta, d) \geq I \). Hence, again, the entrepreneur would prefer to raise unmonitored financing by selecting \( \beta < \hat{\beta}(\alpha) \). It turns out that collusion does not change the basic results of section 3. As in the no-collusion case, due to Assumption 1, the good type can always do it, e.g. by selling all shares to dispersed investors. The bad type can attract unmonitored finance only if \( 1 - \overline{d} > I \). If \( 1 - \overline{d} < I \) the bad type needs to resort to using a blockholder with \( \beta \geq \hat{\beta}(\alpha) \), as otherwise there will be no monitoring and even offering 100% of the shares to investors will not satisfy their participation constraint. The only difference with the basic model is that now the blockholder will get a part of the private benefit, and, hence her stake will have a higher per-share value (and the dispersed shares will have a lower value) than in the no-collusion case.

**Proposition 3** Under symmetric information, in the case when a monitor can collude with the manager for sharing private benefits, the good type never chooses to have an outside blockholder, while the bad type has to attract an outside blockholder when the investment opportunity is not good enough \( (1 - \overline{d} < I) \). Hence, under symmetric information and the possibility of the blockholder-manager collusion, firms with a greater potential for insider expropriation are more likely to have an outside blockholder.

### 5.2 Solution under asymmetric information

Under asymmetric information the entrepreneur’s payoff as of \( t = 0 \) is:

\[ U = \alpha(1 - d + y(\beta, d)) + (d - y(\beta, d)) + \gamma y(\beta, d)(1 - (\alpha + \beta)) + \tilde{P}_b + \tilde{P}_d - I \quad (6) \]

The share prices are as follows. When \( \beta \geq \hat{\beta}(\alpha) \), there is full monitoring, and the prices are:

\[ \tilde{P}_b = \beta + (1 - \gamma)(1 - (\alpha + \beta))\tilde{d} - c\tilde{d}, \quad \tilde{P}_d = (1 - \alpha - \beta)(1 - \tilde{d}) \]

When \( \beta < \hat{\beta}(\alpha) \), there is no monitoring, and the prices are:

\[ \tilde{P}_b = \beta(1 - \tilde{d}), \quad \tilde{P}_d = (1 - \alpha - \beta)(1 - \tilde{d}) \]

Using the expression for prices we obtain:

- If \( \beta \geq \hat{\beta}(\alpha) \), \( U = 1 + \gamma(d - \tilde{d})(1 - (\alpha + \beta)) - c\tilde{d} - I \)
- If \( \beta < \hat{\beta}(\alpha) \), \( U = 1 + (1 - \alpha)(d - \tilde{d}) - I \)
5.2.1 Case 1: $I < 1 - \overline{d}$. Separating equilibria

First, no separating equilibrium with $\overline{\beta} \geq \hat{\beta}(\alpha)$ can exist. In such equilibrium the bad type would obtain $1 - c\overline{d} - I$, while by deviating to $\beta < \hat{\beta}(\alpha)$ he would obtain at least $1 - I$. Second, similarly to the basic model, a separating equilibrium with both types choosing $\beta < \hat{\beta}(\alpha)$ is impossible either, since the bad type would clearly want to mimic the good type and get a premium for the shares he would sell.

Thus the only candidate for a separating equilibrium is the pair of vectors $(\alpha, \overline{\beta})$, $(\alpha, \beta)$ such that $\beta < \hat{\beta}(\alpha)$, $\beta \geq \hat{\beta}(\alpha)$. In such equilibrium, exactly as in the no-collusion case, the bad type obtains $U = 1 - I$, and the good type’s payoff is $U = 1 - cd - I$. The following lemma establishes the conditions under which such equilibrium exists and characterizes the set of all separating equilibria.

**Lemma 4** When $I < 1 - \overline{d}$, a separating equilibrium satisfying the Cho-Kreps intuitive criterion exists iff

$$
\begin{align*}
\text{(7)} & \quad \frac{cd}{I - \overline{d}} \\
& \quad \gamma > 1 - \frac{c(1 - \overline{d})}{I}
\end{align*}
$$

The set of all such separating equilibria is characterized by all pairs of pairs $((\alpha, \beta), (\overline{\alpha}, \overline{\beta}))$, $(\overline{\alpha}, \overline{\beta}) \neq (\overline{\alpha}, \overline{\beta})$, such that $\beta \geq \hat{\beta}(\alpha)$, $\overline{\beta} < \hat{\beta}(\overline{\alpha})$, $\alpha \leq \min\{1 - \frac{cd + I - \gamma d}{1 - \overline{d}(1 - \gamma)}, 1 - \overline{\beta}\}$, $\overline{\alpha} \leq \min\{1 - \frac{I - \overline{d}}{1 - \overline{\alpha}}, 1 - \overline{\beta}\}$, and $\alpha + \beta \geq 1 - \frac{c\overline{d}}{\gamma(\overline{d} - \overline{d})}$. In any separating equilibrium both types raise finance.

**Proof.** See the appendix. ■

The set of all equilibria is depicted in Figure 3. The first condition for the existence of a separating equilibrium is exactly as in the basic model – it stems from the same incentive compatibility constraint for the good type (see the proof for details). In addition, there appears another condition: $\gamma > 1 - \frac{c(1 - \overline{d})}{I}$. This condition results from the fact that, on the one hand, it must be that $\overline{\alpha} \leq \frac{I - \overline{d}}{1 - \overline{d}}$ (investors’ participation constraint), and, on the other hand, it must be that $\overline{\beta} < \hat{\beta}(\overline{\alpha}) \equiv \frac{c(I - \gamma)(1 - \overline{\alpha})}{\gamma}$. In the basic model the latter condition was simply $\overline{\beta} < c$, and, hence, did not pose a problem for the equilibrium existence. Now, however, you can find $\overline{\alpha}$ and $\overline{\beta}$ such that the two conditions hold jointly if and only if $\gamma > 1 - \frac{c(1 - \overline{d})}{I}$. The intuition is as follows. In order for $\overline{\beta} < \hat{\beta}(\overline{\alpha})$ to exist for given $\overline{\alpha}$, it is necessary and sufficient that $\hat{\beta}(\overline{\alpha}) > 0$. Otherwise, even zero share will make monitoring worthwhile for the blockholder (due to the positive fraction of the collusion surplus). The value of $\hat{\beta}(\overline{\alpha})$ increases with $\overline{\alpha}$ because larger $\overline{\alpha}$ (holding $\overline{\beta}$ fixed) lowers the overall surplus from collusion and, hence, makes monitoring less attractive. Hence, in order to ensure $\hat{\beta}(\overline{\alpha}) > 0$ we have to pick as high $\overline{\alpha}$ as possible. The largest $\overline{\alpha}$ that satisfies the investors’ participation constraint is $1 - \frac{I - \overline{d}}{1 - \overline{d}}$. For this $\overline{\alpha}$, conditions $\hat{\beta}(\overline{\alpha}) > 0$ is equivalent to $\gamma > 1 - \frac{c(1 - \overline{d})}{I}$. In other words, the blockholder’s
bargaining power must not be too large, otherwise she will find it profitable to monitor the bad entrepreneur regardless of her stake.

There are a few other differences compared to the basic model. First, $\hat{\beta}(\alpha)$ is an upward-sloping line rather than a constant. Second, the participation constraint $(P)$ depends not only on $\alpha$ but on $\hat{\beta}$ as well (see the proof of the proposition for details). Finally, the set of $(\alpha, \hat{\beta})$ is limited by the no-deviation condition for the bad type, $(IC)$. In the basic model, the bad type would never profit from a deviation (see subsection 4.1.1). When collusion is possible, pretending to be good becomes more attractive, because the monitor leaves a part of the private benefit to the manager. As follows from the payoff functions presented in the beginning of subsection 5.2, the equilibrium bad type’s payoff is $U = 1 - I$, while his payoff from mimicking the good type would be $U' = 1 + \gamma(\bar{d} - d)(1 - (\alpha + \beta)) - cd - I$. Constraint $(IC)$ is simply $U \geq U'$, i.e. it ensures that $\alpha + \beta$ is large enough. While it does constrain the set of equilibria, it does not affect the existence of a separating equilibrium, as $(IC)$ turns out to be always weaker than $\alpha + \beta \leq 1$.

![Figure 3: The set of separating equilibria in the model with collusion.](image)

5.2.2 Case 1: $I < 1 - \bar{d}$. Pooling equilibria

There can exist pooling equilibria but this is not essential for our results (since a pooling equilibrium does not generate any relationship between the firm’s type and its ownership structure). This subsection is to be written...
5.2.3 Case 2. $1 - \overline{d} < I < 1 - \overline{d}$.

Similarly to the no-collusion case there exist no separating equilibria in this range of parameters. The arguments are exactly the same as in subsection 4.2. If the entrepreneur is believed to be bad, he cannot raise unmonitored financing when $1 - \overline{d} < I$. In a separating equilibrium with $\overline{\beta} \geq \hat{\beta}(\overline{\alpha})$ the bad entrepreneur would obtain $1 - c\overline{d} - I$. But by mimicking the good type he would get either $1 + \gamma(\overline{d} - \overline{d})(1 - (\alpha + \overline{\beta})) - cd - I$, if the good type attracts a monitor, or $1 + (1 - \overline{\alpha})(\overline{d} - \overline{d}) - I$, if the good type raises unmonitored financing. In either case the bad type would clearly gain from the deviation.

There can be pooling equilibria, but again, this is not essential for our message.

To be finished...

5.3 Model with collusion: conclusions

Proposition 4 Analogous to Proposition 2. To be written.

Thus, the model in which the entrepreneur and the outside blockholder can collude yields qualitatively the same result as the basic model: entrepreneurs with lower expropriation propensity are more likely to attract outside blockholders. Notice, that in any separating equilibrium a firm with a blockholder is valued higher than a firm without a blockholder ($1 - \overline{d} > 1 - \overline{d}$) despite the fact that monitoring does not help raising dispersed shareholders’ value. This time the difference stems from a pure signaling effect.

In the next section we discuss the implication of our results for empirical work.

6 Relation to the empirical observations

A number of empirical studies have found a positive relationship between the ownership of a large outside shareholder and firm market or operating performance (see footnote 2 for a brief description of this literature). An often proposed explanation for the documented positive relations between outside blockholders and firm value is a direct impact of blockholder monitoring on insiders’ misbehavior and, hence, on the firm’s market value. This argument essentially treats the presence of an outside blockholder as an exogenous factor, while in reality ownership structures are endogenous. Our model yields a positive correlation between the presence of an outside blockholder and a firm’s market valuation as an equilibrium outcome, in which the ownership structure is endogenously determined. In any pooling equilibrium the market value of the firm’s equity from the point of view of dispersed shareholders\(^7\) is obviously the same for both types of firms. In contrast, in a separating equilibrium the dispersed equity market value (per unit share) is always higher in the firm a blockholder. This result holds even in

\(^7\)In empirical literature, a firm’s market performance is normally measured by some type of market-to-book value ratio, where the market value of equity is based on the price of shares traded in the stock market. By its nature, this stock price measures the value of equity for minority shareholders and does not incorporate any monitoring costs or benefits of control that a large shareholder personally incurs or obtains.
the collusion case, when monitoring has no direct effect on the dispersed shareholders’ value. Under no possibility of collusion, the dispersed shareholders’ value equals $1$ for a firm that has a blockholder (a good firm) and $1 - \overline{d}$ for a firm without a blockholder (a bad firm). When collusion is allowed, the corresponding values are $1 - \underline{d}$ and $1 - \overline{d} < 1 - \underline{d}$.

Our results imply that the magnitude of an empirically found positive relationship between an outside blockholder’s share and firm value should never be attributed to a causal effect only: part of the effect, or even the whole effect (as our model with collusion demonstrates), may be a result of signaling. In addition, our model with collusion shows that collusion between an outside blockholder and the insider cannot provide a fully satisfactory explanation for the absence of a positive relationship between the presence of an outside blockholder and firm value found in some studies (e.g. Faccio et al (2001)). Finally, our findings imply that blockholders that expropriate small shareholders should not be viewed as an “evil” only, as they may be a useful signaling device that help good entrepreneurs to convey fair values of their projects to the market.

We should note that our model is consistent with the cross-country differences in ownership structures. The Law and Finance empirical literature (e.g. La Porta, Lopez-de-Silanes, and Shleifer (1999)) has documented a prevalence of ownership structures with large blockholders in countries with weak legal protection of minority shareholders. In our framework, weaker shareholder protection can be modelled either through an increase in both $\underline{d}$ and $\overline{d}$ or through a decrease in $\nu$ (or both). It is, of course, unclear how exactly these parameters should be changed to properly model a change in the shareholder protection. However, we can consider two arguably plausible ways to model a decrease in the quality of the legal protection: a proportional increase of both $\underline{d}$ and $\overline{d}$ holding $\nu$ constant and a decrease in $\nu$ holding $\underline{d}$ and $\overline{d}$ constant. It can be easily shown that the first type of change increases the area where a pooling equilibrium with a blockholder exists and reduces both the zone where a separating equilibrium exists and the zone where a pooling equilibrium without a blockholder exists. The second type of change reduces the zone of a pooling equilibrium without a blockholder and has no impact on the other two zones. Thus, for both suggested ways of modelling a decrease in the legal shareholder protection, an ownership structure with a large outside shareholder is arguably more likely to appear in a weaker legal regime.

7 Conclusion

We have presented a model that predicts the choice of an entrepreneur (initial owner) who searches for external finance whether to sell a large fraction of equity to an outside blockholder or sell equity only to dispersed investors. The main messages of the paper are as follows:

- Asymmetry of information reverses the equilibrium link between the entrepreneur’s propensity to extract private benefits and outside ownership concentration. This result holds
regardless of whether an outside blockholder is supposed to act in the interest of all share-
holders or she colludes with the entrepreneur.

- Our result explains empirical observations that firms with a large outside blockholder are valued higher by the market. Such relationship is likely to arise even if an outside blockholder colludes with the manager instead of preventing expropriation, in which case the blockholder has zero direct effect on the value for minority shareholders. Thus, we show that, regardless of whether collusion is possible or not, the often documented positive correlation can be rationalized as an equilibrium relationship in which entrepreneurs with low propensity for private benefit extraction choose to attract an outside blockholder.

Another type of framework in which a direct effect of monitoring on shareholder value is not necessarily positive (and may even be negative) is a framework in which monitoring can potentially reduce managerial initiative, like in Burkart et al (1997). Arguably, a signaling effect of the kind we have obtained may appear in such a model too: as the potential for expropriation is lower in a good firm, value-reducing overmonitoring would be less likely to occur there and a good entrepreneur may be less afraid of attracting a monitor than a bad one. Such extension can be a goal for subsequent research.

Appendix

Proof of Lemma 1.

First, let us impose the restrictions on the beliefs according to the Cho-Kreps intuitive criterion.

For any possible deviation $\beta'$ of the bad type such that $\beta' < c$, the bad type would obviously gain if he were believed to be good. Thus, the Cho-Kreps criterion does not restrict beliefs for $\beta' < c$, and we can assume that any $\beta' < c$ yields the worst beliefs.

For any possible deviation $\beta'$ of the bad type such that $\beta' \geq c$, if the bad type is believed to be good he obtains $1 - cd - I < 1 - I$. Thus, he would not deviate there even if he is taken for the good type. This means that to satisfy the Cho-Kreps criterion, the belief (probability) that the entrepreneur choosing $\beta \geq c$ is bad is zero.

Now let us consider possible deviations. The equilibrium payoff of the bad type is $\overline{U} = 1 - I$. As we have just shown, he would not deviate to $\beta' \geq c$ even if taken for the good type. Assuming the worst possible beliefs for any out-of-equilibrium $\beta' < c$, a deviation to another $\beta' < c$ yields the same $1 - I$. Thus, there is no profitable deviation for the bad type.

The equilibrium payoff of the good type is

$$U = 1 - cd - I$$
The good type does not profit from deviating to another $\beta' \geq c$: even though he must be considered good by the market (due to the Cho-Kreps refinement), he would get the same $1 - cd - I$. However, he might deviate to $\beta' < c$. He would suffer from the bad market beliefs but would avoid costly monitoring. His payoff from deviating is

$$U' = \alpha'(1 - d) + d + (1 - \alpha')(1 - \overline{d}) - I = \alpha'(\overline{d} - \overline{\hat{d}}) + 1 - \overline{d} + \overline{d} - I$$

The best possible deviation is to choose the maximum possible $\alpha'$ (intuitively, since the market underprices the firm’s equity, the entrepreneur wants to sell as few shares as possible). This $\alpha'$ is obtained by making the investors’ participation constraint binding, which, given the bad beliefs, is $(1 - \alpha')(1 - \overline{d}) = I$. Substituting this into $U'$, we obtain

$$U' = 1 - \frac{I(\overline{d} - \overline{\hat{d}})}{1 - \overline{d}} - I$$

So, the necessary and sufficient condition for no deviation to be profitable is $1 - cd - I \geq 1 - \frac{I(\overline{d} - \overline{\hat{d}})}{1 - \overline{d}} - I$, or

$$cd \leq \frac{I(\overline{d} - \overline{\hat{d}})}{1 - \overline{d}}$$

Finally, the investors’ participation constraints must hold, i.e. it must be that $a \leq 1 - cd - I$ and $\overline{\sigma} \leq 1 - \frac{I}{1 - \overline{\sigma}}$.

\[\square\]

**Proof of Lemma 2.**

First, let us impose the restrictions on the beliefs according to the Cho-Kreps intuitive criterion.

For any possible deviation $\beta'$ of the bad type such that $\beta' < c$, the bad type would obviously gain if he were believed to be good. Thus, the Cho-Kreps criterion does not restrict beliefs for $\beta' < c$, and we can assume that any $\beta' < c$ yields the worst beliefs.

For any possible deviation $\beta'$ of the bad type such that $\beta' \geq c$, if the bad type is believed to be good he obtains $1 - cd - I < \overline{U} = 1 + (1 - \alpha_p)(\overline{d} - \overline{\hat{d}}) - I$. Thus, he would not deviate there even if he is taken for the good type. This means that to satisfy the Cho-Kreps criterion, the belief (probability) that the entrepreneur choosing $\beta' \geq c$ is bad is zero.

We have already established that the bad type would not want to deviate to any $\beta' \geq c$ even if the market believes he is good. Given the bad out-of-equilibrium belief for any deviation such that $\beta' < c$, the bad type would get $1 - I$ from such deviation, which is smaller than $\overline{U}$. Hence, the bad type does not want to deviate regardless of $\alpha_p$. 

21
The equilibrium payoff of the good type is

\[ U = 1 - (1 - \alpha_p)(\hat{d} - d) - I \]

Two types of deviations must be considered. First, the good type could deviate to some \( \beta' < c \).

As we know from the separating equilibrium analysis, the best the good type could get by such deviation is \( 1 - I \frac{d}{1 - d} \). Hence, the first no-deviation condition is

\[ 1 - (1 - \alpha_p)(\hat{d} - d) - I \geq 1 - I \frac{d}{1 - d} - I \]

or\n
\[ \alpha_p \geq 1 - \frac{I}{(1 - d)(1 - \nu)} \]

The investors’ participation constraint requires that

\[ \alpha_p \leq 1 - \frac{I}{1 - d} \]

It is easy to show that there always exists \( \alpha_p \) that satisfies both conditions.

Second, the good type could deviate to some \( \beta' \geq c \). Then, given the good out-of-equilibrium beliefs (due to the Cho-Kreps refinement), his payoff would be \( 1 - cd - I \). Hence, the second no-deviation condition is

\[ 1 - (1 - \alpha_p)(\hat{d} - d) - I \geq 1 - cd - I \]

or

\[ \alpha_p \geq 1 - \frac{cd}{\hat{d} - d} \]

Combining this condition with the investors’ participation constraint we obtain the necessary and sufficient condition for a pooling equilibrium satisfying the Cho-Kreps criterion to exist:

\[ cd \geq \frac{I(\hat{d} - d)}{1 - d} \]

Proof of Lemma 3.

First, let us impose the restrictions on the beliefs according to the Cho-Kreps intuitive criterion. The equilibrium payoff of the bad type is \( U = 1 - I \). Obviously, the bad type would deviate to \((\alpha', \beta')\) such that \( \beta' < \hat{\beta}(\alpha') \) if he were believed to be good upon choosing such deviation. Thus, the Cho-Kreps criterion does not set restrictions on the beliefs for \( \beta' < \hat{\beta}(\alpha') \).

Consider a deviation of the bad type to some \((\alpha', \beta')\) such that \( \beta' \geq \hat{\beta}(\alpha') \) and assume that he is believed to be good. Then his payoff would be \( U = 1 + \gamma(\hat{d} - d)(1 - (\alpha' + \beta')) - cd - I \). This is smaller than \( 1 - I \) whenever \( \alpha' + \beta' > 1 - \frac{cd}{\gamma(\hat{d} - d)} \). Thus for all \((\alpha', \beta')\) such that \( \beta' \geq \hat{\beta}(\alpha') \) and \( \alpha' + \beta' > 1 - \frac{cd}{\gamma(\hat{d} - d)} \), the market must believe that the type is bad with probability zero.

For any \((\alpha', \beta')\) such that \( \beta' \geq \hat{\beta}(\alpha') \) and \( \alpha' + \beta' \leq 1 - \frac{cd}{\gamma(\hat{d} - d)} \), beliefs can be any.

Not let us consider deviation incentives. Assuming the worst beliefs for out-of-equilibrium move such that \( \beta' < \hat{\beta}(\alpha') \), the bad type does not gain anything from such deviation. Now
assume $\beta' \geq \hat{\beta}(\alpha')$. As we have just shown, if $\alpha' + \beta' \geq 1 - \frac{cd}{\gamma(d-d')}$, the bad type does not gain from such a move regardless of the beliefs. If $\alpha' + \beta' < 1 - \frac{cd}{\gamma(d-d')}$, then, assuming the worst out-of-equilibrium beliefs, the bad type is strictly worse off from such deviation as then he would obtain $1 - cd - I < 1 - I$.

Thus, the necessary and sufficient condition for no deviation to be profitable for the bad type is

$$\alpha + \beta \geq 1 - \frac{cd}{\gamma(d-d')}$$

Given that $\beta \geq \hat{\beta}(\alpha)$ in our separating equilibrium, this condition only sets a restriction on $\alpha$ and $\beta$ but does not pose a threat to the equilibrium existence: $\beta$ can always be selected large enough for this condition to hold.

Consider now deviation incentives for the good type. His equilibrium payoff is $U = 1 - cd - I$. If he deviates to $(\alpha', \beta')$ such that $\beta' \geq \hat{\beta}(\alpha')$ than he obtains $1 + \gamma(d - \tilde{d})(1 - (\alpha' + \beta')) - cd - I$. Since $\tilde{d} \geq d$, this is no greater than $1 - cd - I$ regardless of $\tilde{d}$. Hence, such deviation is not profitable. If the good type deviates to $(\alpha', \beta')$ such that $\beta' < \hat{\beta}(\alpha')$, then, assuming the worst beliefs, he gets $1 + (1 - \alpha')(d - \tilde{d}) - I$. The best deviation is the maximum possible $\alpha'$, i.e. the one that makes the investors' participation constraint binding: $(1 - \alpha')(1 - \tilde{d}) = I$ or $\alpha' = 1 - \frac{I}{1-d}$.

Thus, the necessary and sufficient condition for no deviation to be profitable for the good type is $1 - cd - I \geq 1 - \frac{I(d-d)}{1-d} - I$, or

$$cd \leq \frac{I(d-d)}{1-d}.$$  

We must also make sure that the investors’ participation constraints are satisfied. For the good, type this constraint is $\bar{\alpha}(1 - d) + d [\gamma \beta + (1 - \gamma)(1 - \bar{\alpha})] - cd + (1 - \bar{\alpha} - \beta)(1 - d) \geq I$, which can be rewritten as $\bar{\alpha} \leq 1 - \frac{cd + I - \gamma d \beta}{1 - d}$. This constraint only sets a restriction on $\bar{\alpha}$ and $\beta$ but does not affect equilibrium existence: one can always select $\bar{\alpha}$ and $\beta \geq \hat{\beta}(\bar{\alpha})$ such that the constraint is satisfied (e.g. $\bar{\alpha} = 0$, $\beta = 1$).

For the bad type, the constraint is $\bar{\alpha} \leq 1 - \frac{I}{1-d}$. At the same time, it must be that $\bar{\beta} < \hat{\beta}(\bar{\alpha})$. To relax the latter one as much as possible, let us set $\bar{\beta} = 0$. Also, notice that $\hat{\beta}(\bar{\alpha})$ is increasing in $\bar{\alpha}$. Thus, we need that $0 < \hat{\beta}(\bar{\alpha})$ holds for $\bar{\alpha} = 1 - \frac{I}{1-d}$, which amounts to

$$\gamma > 1 - \frac{c(1 - d)}{I}.$$  

This condition together with the previously obtained $cd \leq \frac{I(d-d)}{1-d}$ constitute the necessary and sufficient condition for the existence of a separating equilibrium.

$\blacksquare$
References


