The effects of competition on investment –
Towards a taxonomy

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Abstract: Using a general two-stage framework, this paper gives sufficient conditions for increasing competition to have negative or positive effects on R&D-investment, respectively. Both possibilities arise in plausible situations, even if one uses relatively narrow definitions of increasing competition. The paper also shows that competition is more likely to increase the investments of leaders than those of laggards. When R&D-spillovers are strong, competition is less likely to increase investments. The paper also identifies conditions under which low initial levels of competition make a positive effects of competition on investment more likely.

Keywords: competition, investment, cost reduction

JEL: L13, L20, L22

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1 Introduction

Even though economists have been trying to understand the effects of the intensity of competition on R&D-investment for decades, the issue remains unsettled. While some authors argue that competitive pressure is essential to induce R&D-investments, others emphasize the Schumpeterian idea that some monopoly power is necessary for innovation. As both arguments have some merit, it is unsurprising that the theoretical analysis of the subject has been inconclusive. Depending on the definition of “competitive intensity” and the underlying oligopoly framework, investments can be increasing or decreasing functions of competitive intensity.

Understanding the sources of these different predictions is extremely difficult, because most models rely on specific functional forms. In the following, I will therefore provide a general framework that allows searching for robust predictions, because it captures many different notions of increasing intensity of competition and different types of oligopolistic interaction. To reveal the economic intuition in the most transparent fashion, I opted for simplicity in other respects: The game has two stages, with cost-reducing investment followed by product market competition. In most of the paper, I will consider a duopoly. One firm (the leader) may be exogenously more efficient than the other one (the laggard), that is, it may have lower marginal costs. The initial efficiency levels and the cost-reducing investments determine the efficiency $Y_i$ in the product market stage. Together with a competition parameter $\theta$, the efficiency levels determine the demand $D^i(Y_i,Y_j;\theta)$ and the markup $M^i(Y_i,Y_j;\theta)$ of each firm in product market equilibrium, and hence the gross profit $\Pi^i = D^i \cdot M^i$. By assumption, and in line with many examples, higher own efficiency increases both components of a firm’s profit: Lower marginal costs lead to higher demand and markup.

The framework covers many familiar cases. In particular, the competition parameter can be interpreted quite broadly. It does not necessarily refer

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1For elementary models on this topic, see Motta (2004, ch.2); Vives (forthcoming) provides a more sophisticated analysis. Similar issues are discussed in a macroeconomic context (Aghion et. al. 1997, 2001)

2Generalizations of most results to more than two firms are possible at the cost of additional notation.
to a competition policy parameter, but more generally to some parameter of the market environment capturing the intensity of competition.\textsuperscript{3} The framework applies, for instance, to a homogeneous linear Cournot model where $\theta$ is the negative of market size; a Hotelling model where $\theta$ is the inverse of transportation costs; differentiated linear Cournot or Bertrand models where $\theta$ corresponds inversely to the extent of horizontal product differentiation, as captured for instance by the demand functions of Shubik and Levitan (1980) or Singh and Vives (1984). $\theta$ may also capture a shift from Cournot to Bertrand competition or an increase in the number of firms for an otherwise given environment. The framework also covers cases with and without spillovers.

Our defining assumptions on the competition parameter $\theta$ are inspired by two common properties of these examples (and many others). First, the mark-up $M^i$ of each firm in the product market equilibrium decreases with $\theta$; competition thus has a negative mark-up effect.\textsuperscript{4} Second, the demand sensitivity effect $D^i_{\theta \theta} \equiv \frac{\partial^2 D^i}{\partial Y^i \partial \theta}$ is non-negative: The positive effect of greater efficiency on equilibrium demand ($D^i \equiv \frac{\partial D^i}{\partial Y^i}$) weakly increases with competition $\theta$.\textsuperscript{5}

In this framework, I give sufficient conditions for the effects of competition on investment to be positive and negative, respectively. I also provide conditions under which competition increases the investments of some firms (e.g., leaders) and decreases those of others (e.g., laggards). The analysis shows that there are very natural situations in which each possibility arises. Thus, searching for a general relation between competition and investment is in vain.

However, the conditions derived help to uncover the circumstances under which competition is more likely to have a positive or negative effect on a firm. Based on the general model and the set of examples, the following testable predictions emerge. First, quite generally, competition is more likely to have a positive effect on the investments of leaders than on those of laggards, and the

\textsuperscript{3}See Boone (2000) and Vives (forthcoming) for comparable approaches.

\textsuperscript{4}Boone (2000) provides a reasonable example where this property of a competition parameter is not satisfied. The ideas of the following analysis could still be applied, but at the cost of having to distinguish more cases.

\textsuperscript{5}Throughout the paper, we use subscripts to denote partial derivatives, with indices $i$ referring to $Y^i$, $y^i$, etc.
effect on laggards is quite robustly negative.\(^6\) Second, when investments have higher spillovers, increasing competition is more likely to reduce investments. Third, an inverse U-shaped relation between competition and investment is not necessarily more likely than a U-shaped relation.

A possible objection to the conclusion that competition has ambiguous effects on investment is that the approach presented here is simply too general, and that natural restrictions on the class of parameterizations might lead to more conclusive results. I show that this is not the case for two plausible candidates. First, if one identifies “increasing competition” quite narrowly with decreasing product differentiation, the possibility of negative and positive effects still arises, even for symmetric firms. Second, one might want to add a further requirement to the definition of a competition parameter, namely that competition has an unambiguously positive effect on equilibrium demand \(\left(D_i \equiv \frac{\partial D_i}{\partial \theta} \geq 0\right)\). This is often the case, because competition reduces prices. \(\frac{\partial D_i}{\partial \theta} \geq 0\) clearly works towards a positive effect on investment,\(^7\) but it is not sufficient to guarantee that competition increases investments. However, a more definite result can be obtained if one moves beyond the duopoly framework and identifies increasing competition with an increase in the number of firms. Then, there are strong forces suggesting a negative effect on per-firm investment.

The most closely related paper is Vives (forthcoming) who also considers the effects of competition on cost-reducing investments in general two-stage games.\(^8\) Vives arrives at more definite conclusions, suggesting that competition quite generally has positive effects on investment. Several reasons explain these different findings. First, Vives does not consider initial asymmetries, so that the robust negative effect of competition on laggards does

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\(^6\)This is related to, but not identical, to the concept of weak increasing dominance, which requires that leaders invest more than laggards (Cabral and Riordan 1994, Athey and Schmutzler 2001, Cabral 2002, 2008): I am arguing that increasing competition works in favor of increasing difference.

\(^7\)Intuitively, if competition leads to higher demand per firm, it becomes more attractive to increase markups by becoming more efficient.

\(^8\)In a broader sense, the paper is related to Fudenberg and Tirole (1984) and Bulow et al. (1985). These papers also consider classes of two-stage investment games, and they identify general properties of the strategic interaction guaranteeing that strategic considerations have a positive or negative effect on investment.
not show up. Second, Vives confines himself to product differentiation parameters. Third, even when increasing competition refers to lower product differentiation, there is at least one example where increasing competition has a negative effect on investment in non-degenerate parameter regions even for symmetric firms.\footnote{Importantly, however, Vives (forthcoming) contains an extension of the analysis to the case of free entry. He also allows for more than two firms and for simultaneous investment and product-market decisions.}

The approach of the paper is potentially applicable to more complex settings. For instance, I will sketch how the approach can be applied to understand the effects of competition on innovation incentives in vertical structures.

The paper is organized as follows. Section 2 introduces the analytical framework. Section 3 provides comparative statics results. Section 4 applies these results to familiar examples. Section 5 uses the general results and the examples to clarify under which circumstances a positive effect of competition is likely. Section 6 discusses welfare considerations. Section 7 sketches the extension to vertical structures, and Section 8 concludes.

\section{Set-up}

I shall consider the following class of two-stage games. In period 1, firms $i = 1, 2$ carry out a cost-reducing investment. In period 2, they engage in product-market competition. Initially, firm $i$ has marginal cost $c_i = \sigma - Y_i^0$ for some exogenous level $\sigma$ of marginal costs.\footnote{The choice of $\sigma$ is arbitrary; to simplify calculations, I usually choose $\sigma = 0$ or $\sigma = a$, where $a$ is the maximal willingness to pay for any unit of the good.} In the first stage, given $(Y_1^0, Y_2^0)$, each firm chooses its investment $y_i$. In the second stage, firm $i$ has marginal costs $c_i = \sigma - Y_i$, where $Y_i = Y_i^0 + y_i + \lambda y_j$ is the efficiency level after the investment stage and $\lambda \in [0, 1]$ is a spillover parameter. Demand of firm $i$ is $d^i(p^i, p^j; \theta)$, where $p^i$ and $p^j$ are the prices of firm $i$ and firm $j$, respectively, and $\theta$ is a competition parameter from some partially ordered set. Further, the product-market game is assumed to have a unique Nash equilibrium for
arbitrary $\theta$ and $Y = (Y_1, Y_2)$, corresponding to prices $p^i (Y_i, Y_j; \theta)$.\footnote{For price competition, $p_i (Y_i, Y_j; \theta)$ is the equilibrium price; for quantity competition, it denotes the market clearing price for equilibrium outputs.} The following quantities are thus well defined:

1. Equilibrium mark-ups $M^i (Y_i, Y_j; \theta) \equiv p^i (Y_i, Y_j; \theta) - \pi + Y_i$

2. Equilibrium demands $D^i (Y_i, Y_j; \theta) \equiv d^i (p^i (Y_i, Y_j; \theta), p^j (Y_i, Y_j; \theta); \theta)$

3. Gross equilibrium profits $\Pi^i (Y_i, Y_j; \theta) = M^i (Y_i, Y_j; \theta) \cdot D^i (Y_i, Y_j; \theta)$

I will maintain the following assumptions throughout, all of which hold in the examples to be discussed in Section 3 below.

(A1) $d^i (p^i, p^j; \theta)$ is weakly decreasing in $p^i$ and weakly increasing in $p^j$, $j \neq i$.

Thus, the firms produce (potentially imperfect) substitutes.

(A2) $p^i (Y_i, Y_j; \theta)$ is weakly decreasing in $Y_i$ and $Y_j$, $j \neq i$.

(A2) holds in most oligopoly models. Because the product market game has a unique equilibrium, the investment game reduces to a one stage game with payoff functions

$$\pi^i (y_i, y_j; \theta) = \Pi^i (Y_i^0 + y_i + \lambda y_j, Y_j^0 + y_j + \lambda y_i; \theta) - K(y_i). \quad (1)$$

(A3) $D^i (Y_i, Y_j; \theta)$ is weakly increasing in $Y_i$ and weakly decreasing in $Y_j$, $j \neq i$.

This assumption is related to (A1) and (A2). To see this, define

$$\eta^o \equiv \frac{\partial \pi^i}{\partial p^i} (p^i (Y_i, Y_j; \theta), p^j (Y_i, Y_j; \theta)) \cdot \frac{\partial p^j}{\partial Y_i} (Y_i, Y_j; \theta);$$

$$\eta^c \equiv \frac{\partial \pi^i}{\partial p^j} (p^i (Y_i, Y_j; \theta), p^j (Y_i, Y_j; \theta)) \cdot \frac{\partial p^j}{\partial Y_i} (Y_i, Y_j; \theta).$$

$\eta^o$ reflects the \textit{own-price effect} of efficiency on demand: By (A2), lower costs of firm $i$ reduce its equilibrium price $p^i$ and hence, by (A1) its demand $D^i$. 

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$\eta^c$ reflects the competitor-price effect: As $c_i$ falls, the competitor’s price falls by (A2), which reduces firm $i$’s demand $D^i_i$. As $D^i_i \equiv \frac{\partial D^i_i}{\partial Y^i_i} = \eta^o + \eta^c$, (A3) says that the own price effect dominates over the competitor price effect. Indeed, this is true in all our examples. The next assumption is slightly more problematic.

(A4) $M^i_i(Y^i_i, Y^j_j; \theta)$ is weakly increasing in $Y^i_i$ and weakly decreasing in $Y^j_j$, $j \neq i$.

As $M^i_i(Y^i_i, Y^j_j; \theta) = p^i_i(Y^i_i, Y^j_j; \theta) - \bar{c} + Y^i_i$ and $\frac{\partial M^i_i}{\partial Y^i_i} = \frac{\partial p^i_i}{\partial Y^i_i} - 1$, the first part of the assumption states that the cost reductions are larger than the induced price reductions. This holds in many, but not all, oligopoly models. Finally, I introduce two defining properties of the competition parameter.

(A5) $M^i_i(Y^i_i, Y^j_j; \theta)$ is weakly decreasing in $\theta$.

The notion that competition reduces mark-ups is standard. However, the relation between $\theta$ and demand is less clear. To see why, note that

$$\frac{dD^i_i}{d\theta} = \frac{\partial d^i_i}{\partial p^i_i} \frac{\partial p^i_i}{d\theta} + \frac{\partial d^i_i}{\partial p^j} \frac{\partial p^j}{d\theta} + \frac{\partial d^i_i}{d\theta}.$$ 

If the own price effect dominates over the competitor price effect, the sum of the first two terms are positive. However, the direct effect $\frac{dD^i_i}{d\theta} = \frac{\partial d^i_i}{\partial \theta}$ can be negative, potentially compensating the price-induced effects. Thus, equilibrium demand may rise or fall as competition increases. Moreover, as we will see below, competition may have differential impacts on the demand of leaders and laggards.

Next, consider $D^i_{\theta \theta} = \frac{\partial^2}{\partial \theta^2} (\eta^o + \eta^c)$. Clearly, $|\eta^c|$, the demand effect of higher efficiency resulting from lower competitor prices, is small for soft competition, suggesting a negative effect of $\theta$ on $\eta^c$. Indeed, the examples below

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12 For instance, it does not hold globally in a Cournot duopoly with demand generated from CES utility functions.
13 Note, however, that competition may increase the mark-up for a firm that is considerably more efficient than its competitor. With this qualification, (A5) is fulfilled in all the examples.
14 See the example in 4.1.
confirm this. However, $\eta^o$ is more likely to increase in $\theta$: Part of the effect of higher efficiency on own demand that is induced by lower own prices comes from a business-stealing effect that is absent with weak competition. In all examples, the own price effect dominates over the competitor price effect. This motivates the following assumption.

(A6) $D_{i\theta}^i \geq 0$.

We are now ready to define a competition parameter.

**Definition 1** In a duopoly model given by $D^i(Y_i, Y_j; \theta)$ and $M^i(Y_i, Y_j; \theta)$, $\theta$ is a competition parameter if (A5) and (A6) hold.

We shall illustrate the definition with specific examples in Section 4.

### 3 General comparative statics results

I will now provide general results about the effects of competition on investment. Assumptions (A1)-(A6) are not necessary to derive the results, but they are essential for the interpretation. I will suppose for simplicity that investments are chosen from some compact subset of the reals, and $\Pi^i(Y_i, Y_j; \theta)$ and $\pi^i(y_i, y_j; \theta)$ are twice continuously differentiable, even though much of the following easily generalizes to discrete choice sets and more general objective functions. Also, I assume existence and uniqueness of the equilibrium in the investment game. The following results shows that the properties of $\pi_{i\theta}^i = \frac{\partial^2 \pi^i}{\partial y_i \partial \theta}$ are essential for comparative statics. When $\pi_{i\theta}^i > 0$, $\theta$ shifts out player $i$’s reaction curve. This does not guarantee that competition increases player $i$’s investment, but there are several sets of additional conditions that lead to this outcome.

**Proposition 1** $y_i(\theta)$ is weakly increasing in $\theta$ for $i = 1, 2$ if, for $i = 1, 2$ and $j \neq i$, one of the following conditions holds:

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15 This follows from a well-known comparative statics result of Topkis (1978) for the maximizer of a supermodular function, as positivity of the relevant mixed partials for differentiable functions guarantees supermodularity.
(i) \( \pi^i_{i\theta} \geq 0 \) and \( \pi^i_{ij} = \frac{\partial^2 \pi^i}{\partial y_i \partial y_j} \geq 0 \).

(ii) \( \pi^i(y_i, y_j; \theta) \) is concave in \( y_i \). Near the equilibrium, \( \pi^i_{i\theta} \geq \frac{\pi^i_{ij}}{\pi^j_{ji}} \pi^j_{j\theta} \), and the Hahn-stability condition \( \pi^i_{ii} \pi^j_{jj} \geq \pi^i_{ij} \pi^j_{ji} \) holds.

(iii) \( \pi^i_{i\theta} \geq 0 \), \( \pi^i(y_i, y_j; \theta) \) is symmetric and concave in \( y_i; y_i(\theta) = y_j(\theta) \) in the relevant parameter range, and the Hahn stability condition holds.

**Proof.** See Appendix 1.

Importantly, by switching the signs in the inequalities \( \pi^i_{i\theta} \geq 0 \) and \( \pi^i_{ij} \geq 0 \) in (i) - (iii), one arrives at sufficient conditions for competition to have negative effects on investment. Also, for the benchmark case without spillovers (\( \lambda = 0 \)), \( \pi^i_{i\theta} = \Pi^i_{i\theta} = \frac{\partial^2 \Pi^i}{\partial Y_i \partial \theta} \), whereas, with positive spillovers \( \pi^i_{i\theta} = \Pi^i_{i\theta} + \lambda \Pi^j_{j\theta} \). Either way, the conditions of the theorem reflect properties of the gross profit function \( \Pi^i \) that are independent of the precise form of the investment cost functions.

To understand (i), recall that \( \pi^i_{i\theta} \geq 0 \) implies that reaction functions shift out as \( \theta \) increases. The supermodularity condition in (i), \( \pi^i_{ij} = \Pi^i_{ij} \geq 0 \), implies increasing reaction functions, so that the indirect effects of competition reinforce the direct effects. Thus, competition increases both players’ investments. However, unless spillovers are sufficiently large, investments are typically strategic substitutes, so that the direct and indirect effects have opposite signs. Even then, part (ii) shows that competition may still increase both players’ investments, because supermodularity is replaced by the weaker requirement that \( \pi^i_{i\theta} \geq \frac{\pi^i_{ij}}{\pi^j_{ji}} \pi^j_{j\theta} \). Also, part (iii) is applicable to investment games with strategic substitutes as long as the functions \( \pi^i \) are symmetric.

The following proposition is useful to identify such situations where competition increases the investments of one firm and decreases those of the other one, which will be shown to arise naturally when one firm is the leader and the other firm is the laggard.

**Proposition 2** Suppose for some \( i \in \{1, 2\} \) and \( j \neq i \), the following conditions hold: (a) \( \pi^i_{i\theta} \geq 0 \); (b) \( \pi^j_{j\theta} \leq 0 \); (c) \( \pi^i_{ij} \leq 0 \) and (d) \( \pi^j_{ji} \leq 0 \). Then \( y_i \) is weakly increasing in \( \theta \) and \( y_j \) is weakly decreasing.

\(^{16}\)However, the result requires additional concavity and stability requirements.
Proof. Conditions (a)-(d) imply \( \pi_{i\theta}^i \geq 0 \); \( \pi_{j\theta}^j \leq 0 \); \( \pi_{ij}^i \leq 0 \) and \( \pi_{ji}^j \leq 0 \). The result therefore follows from Theorem 5 in Milgrom and Roberts (1990) by reversing the order on the strategy space of one firm.

Intuitively, by (a) and (b), \( \theta \) has the direct effect of increasing firm \( i \)'s investment and reducing the investment of firm \( j \). By (c) and (d), these direct effects are mutually reinforcing: An increase of firm \( i \)'s investment reduces firm \( j \)'s marginal investment incentives and vice versa.

As \( \Pi^i = D^i \cdot M^i \), Proposition 1 implies the following result:

**Corollary 1** Suppose for \( i = 1, \ldots, I \),

\[
\Pi_{i\theta}^i = D_i^i \cdot M_{i\theta}^i + M_i^i \cdot D_{i\theta}^i + D_i^i \cdot M_{i\theta}^i + M_i^i \cdot D_{i\theta}^i
\]

(2)

is sufficiently large (small). Then \( y_i(\theta) \) is weakly increasing (weakly decreasing) in \( \theta \) for \( i = 1, \ldots, I \).

Here, “sufficiently large” reduces to “positive” for symmetric firms and for games with strategic complementarities (Proposition 1 (i) and (iii)). For other games, “sufficiently large” means that expression (2) must be greater than \( \frac{\pi_{ij}^i}{\pi_{ji}^j} \pi_{j\theta}^j \), which is positive (Proposition 1 (ii)). In spite of its simplicity, decomposition (2) is crucial to understand under which circumstances competition has positive effects on investments, I investigate each term in (2) separately.

The first term in (2), \( D_i^i \cdot M_{i\theta}^i \), reflects the markup effect of competition: By (A3), investment has a positive effect on demand \( (D_i^i < 0) \). Also, by (A5), \( M_{i\theta}^i \) is negative. Thus, as competition increases, mark-ups decrease, so that the positive effect of expanding demand on profits becomes smaller.

The second term, \( M_i^i \cdot D_{i\theta}^i \), reflects the demand effect of competition: By (A5), investment has a positive markup effect, \( M_i^i \). If \( D_{i\theta}^i > 0 \) the demand effect of competition on marginal investment incentives is positive, if \( D_{i\theta}^i < 0 \), it is negative.

The third term, \( D_i^i \cdot M_{i\theta}^i \), reflects the cost-pass-through effect of competition. Because \( M_{i\theta}^i = p_{i\theta}^i \), the sign of the cost-pass-through effect is positive if and only if \( p_{i\theta}^i \equiv \frac{\partial}{\partial \theta} \left( \frac{\partial \Pi_{i\theta}^i}{\partial Y_i^i} \right) \geq 0 \), that is, competition reduces the sensitivity of equilibrium prices to costs. The examples below will show that the cost-pass-through effect is ambiguous, depending on whether firms
compete à la Bertrand or à la Cournot. The fourth term, \( M^i \cdot D_i^{\theta} = M^i \cdot \frac{\partial}{\partial \theta} (\eta^c + \eta^o) \), contains \( \eta^c + \eta^o \), which aggregates the own-price effect and the competitor-price effect of higher efficiency on demand. It thus reflects the demand-sensitivity effect of competition. Under (A6), the demand-sensitivity effect is positive: As \( \theta \) increases, demand reacts more strongly to efficiency, which enhances the incentive to invest.

Summing up, the analysis in this section suggests why more intense competition does not have clear-cut effects on investment. The effect of competition on marginal investment incentives, \( \Pi^i_{\theta} \), consists of the four transmission channels just discussed. The mark-up effect is negative, whereas the demand-sensitivity effect is positive. The demand effect and the cost-pass-through effect can be positive or negative.

### 4 Examples

The following examples show how (2) helps to understand under which circumstances competition has positive or negative effects. Several of these examples are well-known, but they nevertheless are useful to identify the four transmission channels. Whenever I calculate equilibrium investment levels explicitly, the investment cost function is \( K(y_i) = y_i^2 \); importantly, however, the comparative statics also hold for more general cost functions.

#### 4.1 Inverse market size

Suppose firms are Cournot competitors, with homogeneous goods and market demand \( D(p) = a - p \) for some \( a > 0 \), and constant marginal costs \( c_i \). Define \( \theta = -a \). Hence, more intense competition corresponds to smaller demand.\(^{17}\)

Defining \( Y_i = -c_i \),

\[
D^i(Y_i; Y_j; \theta) = M^i(Y_i; Y_j; \theta) = \frac{(2Y_i - Y_j - \theta)}{3}.
\]

\(^{17}\)Boone (2007) also treats inverse market size as a competition parameter; however, as laid out in footnote, there are reasons why one might not want to do this.
Equilibrium investments can easily be calculated as
\[ y_i = \frac{1}{i} \left( -2\theta + 8Y_i^0 - 6Y_j^0 \right). \]

The effect of increasing competition on investments is thus negative. To see the economic logic behind this, note that \( D_i^i = M_i^i = \frac{2}{3} \); \( D_i^\theta = M_i^\theta = -\frac{1}{3} \). Thus, in line with (A5), the markup effect is negative. Reflecting the specific functional forms, the demand effect is identical to the markup effect and thus negative. Finally, as \( D_{i\theta} = M_{i\theta} = 0 \), the marginal effect of competition on investment is fully determined by the negative absolute demand and markup effects. Thus, \( \Pi_{i\theta} < 0 \), so that the effect of competition on marginal investment incentives is negative.

4.2 Substitutability (Shubik-Levitan)

In a market with differentiated goods, let inverse demands be
\[ p^i(q_i, q_j) = 1 - q_i - bq_j, \tag{3} \]
where \( 0 \leq b \leq 1 \) (Shubik and Levitan 1980). The corresponding demand functions \( d^i(p^i, p^j) \) satisfy \( \frac{\partial d^i}{\partial p_j} > 0 \) for \( b > 0 \); thus the goods are substitutes. For \( b = 0 \), firms are monopolists; \( b = 1 \) corresponds to homogeneous goods. Higher \( b \) corresponds to better substitutability. Thus, define \( \theta = b \).

4.2.1 Quantity competition

The middle line in Figure 1 plots investments as a function of the competition parameter for \( c_1^0 = c_2^0 = 0.5 \).\footnote{The results for the Cournot case are taken from Sacco and Schmutzler (2009), which also contains experimental evidence for the U-shape.} The line is U-shaped: Starting from a monopoly, an increase in competition first reduces investment; beyond \( \theta = 2/3 \) further increases lead to higher investments.

With small heterogeneities between firms, the qualitative pattern is similar: Competition has a U-shaped effects on leaders and laggards.\footnote{However, the level of competition from which on competition has a positive effect on investment is lower for leaders than for laggards.} For firms
that lag far behind, however, the effects of competition on investment are negative. For instance, the respective lines in Figure 1 plot the relation between competition and investments for $c_0^1 = 0.3; c_0^2 = 0.7$ for leaders (laggards). To understand this pattern, note that $D_i = M_i > 0$ (see Appendix 10). $D_i$ is negative unless firm $i$ has a very strong lead; $\frac{Y_i}{Y_j} > \frac{4\theta^2}{4\theta} (> 1.25)$. Thus, quite generally, absolute demand and markup effects are negative. As $D_i M_i > 0$, the remaining effects are positive. Hence, the U-shaped relation between competition and investment for all firms except strong laggards reflects the interplay between the negative demand and markup effects and the positive cost-pass-through and demand-sensitivity effects: Starting from low competition, greater competition, by reducing demands and markups, reduces incentives to increase efficiency. Beyond a certain threshold, the effect of competition on investment is positive, reflecting the positive demand-sensitivity and markup effects. The unambiguously negative effect for firms that are lagging far behind results because their markup and hence the positive demand-sensitivity effect $M_i D_i M_i$ is small. Therefore, the negative demand and markup effects dominate.

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20 When firms are very asymmetric, (A5) no longer applies.

21 By the same token, they have low demand, so that the positive cost-pass-through effect $D_i M_i$ is small.
4.2.2 Price competition

Figure 2 plots investments for price competition, with the same initial costs as in Figure 1. Investments decrease with competition when firms are neck-to-neck or laggards, but for the leader they increase as competition becomes very intense.

Hence, even though the fundamentals (demand and technology) are the same as for quantity competition, competition has a strictly negative effect except for strong leaders, for whom the relation is U-shaped. The economic logic nor the negative effect differs from Section 4.1. There, decreasing market size had negative absolute demand and markup effects, and the remaining effects were zero. Here, substitutability has a negative effect on investments in spite of countervailing underlying effects. To see this, note that that $D_i > 0$; $M_i > 0$; $M_{iθ} < 0$; $D_{iθ} > 0$; $M_{iθ} < 0$. Further, under symmetry $D_j > 0$ if $θ > 0.5$ (see Appendix 10). Thus, while the markup effect and the cost-pass-through effect are both negative, the demand-sensitivity effect is always positive and the demand effect is positive for intense competition.

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22 Note that, even in the symmetric case, the equilibria become asymmetric as $θ$ approaches 1.

23 More generally, $D_θ > 0$ if and only if $Y_i > Y_j$ is above a critical level that is a suitable function of $θ$. 

14
The U-shaped rather than decreasing investment function for leaders reflects the fact that the demand effect is more likely to be positive for leaders.

To understand why reducing product differentiation has a more positive effect in the Cournot case than in the Bertrand case, note that $M_i'\theta > 0$ for Cournot competition, whereas $M_i'\theta < 0$ for Bertrand competition. To see why, compare situations where products are essentially monopolists, with situations with relatively close substitutes. In the latter case, for Cournot competition, higher efficiency of a firm induces an output reduction of the competitor. Compared to the case of strong differentiation with little competitive interaction, this output reduction dampens the price-reducing effect $|p'_i|$, so that the cost-pass-through effect should be positive. Under price competition, however, greater efficiency induces lower prices of both firms, enhancing the price-reducing effect of greater efficiency. Thus, compared to the case with little product differentiation where such considerations play no role, cost reductions induce more substantial price reductions, so that $|p'_i|$ should increase. Summing up, the cost-pass-through effect works towards a positive relation between competition and investment under Cournot competition, and conversely under Bertrand competition.

### 4.3 Substitutability (Singh-Vives)

In the examples of Section 4.2, an increase in $\theta = b$ not only increases substitutability; in addition, $\theta$ shifts both demand functions inwards, so that it mixes two sources of increasing competition. An inverse demand function without this property was analyzed by Singh and Vives (1984), namely

$$p^i(q_i, q_j; \theta) = 1 - \frac{1}{1 + \theta} q_i - \frac{\theta}{1 + \theta} q_j. \quad (4)$$

It can be shown that, in both the Bertrand and the Cournot case, investment depends positively on the substitution parameter $\theta$ for this demand function, except for firms that are lagging far behind; in which case the relation may become negative.\(^\text{24}\) The main reason behind this more positive

\(^{24}\)Again, in the Bertrand case, a restriction on $b$ ($b < 0.85$) is necessary for symmetric
effect of competition on investment than in the Shubik-Levitan case is that the demand effect is now unambiguously positive (See Appendix 10).

4.4 Transportation costs

Next, consider a Hotelling duopoly. Consumers buy at most one unit of a homogeneous good, and are uniformly distributed on [0,1]. Firms are located at $q_1 = 0$ and $q_2 = 1$. Consumers incur transportation costs $t$ per unit distance in addition to the price $p^i$. Competition affects the leader’s investments positively and the laggard’s negatively, as depicted in Figure 3. This figure is drawn for $c^0_1 = c^0_2 = 0.5$ (symmetric case), $c^0_1 = 0.3$ (leader) and $c^0_2 = 0.7$ (laggard).

Simple calculations show that $M^i_\theta < 0$; $D^i_\theta > 0$; $M^i_\theta > 0$; $D^i_{i\theta} > 0$; $M^i_{i\theta} = 0$ (See Appendix 10). Crucially, $D^i_\theta > 0$ if and only if $i$ is a leader; hence the same is true of the demand effect. As a result, the sign of $\Pi^i_{i\theta}$ is determined by whether a firm is leader or laggard. Also, it is straightforward to show

---

*investment equilibria to exist.*

*We assume that transportation costs are in an intermediate range where second-order conditions hold, both firms are active and all consumers buy one unit.*

*The remaining two non-zero effects, the positive demand-sensitivity effect and the negative markup effect, happen to sum up to a positive effect for leaders, a negative effect for laggards, and they cancel out in the symmetric case.*
that $\Pi^i_{ij} < 0$. Intuitively, investments of a competitor reduce own demand and markup, thereby reducing the incentive to increase own markup and demand by investing in cost reduction.\footnote{More generally, this is true for the linear examples treated in this paper. To understand this, note that $\Pi^i_{ij} = D^i_j \cdot M^i_j + M^i_j \cdot D^i_j + M^i \cdot D^i_{ij} + M^i \cdot M^i_{ij}$. With linearity, the last two terms disappear. The first two terms are typically negative, because of (A3) and the natural assumption that $D^i_j < 0$ and $M^i_j < 0$: If competitors invest a lot, own markups and demands fall. This reduces the benefits from increasing own demands and markups by becoming more efficient.} Because $\Pi^i_{ij} < 0$, Proposition 2 can explain the differential impact of competition on the investments of the two firms: Intuitively, because competition has a positive demand effect for leaders and a negative demand effect for laggards, increasing $\theta$ has the direct effect that it raises the leader’s investment incentives and reduces those of the laggard. As investments are strategic substitutes, both effects are mutually reinforcing.

4.5 Cournot vs. Bertrand

Our framework can be adapted to understand how switching from Cournot competition to Bertrand competition affects investments. To this end, reconsider the differentiated goods examples of Section 4.2. Let $\theta \in \{0, 1\}$, where $\theta = 0$ for Cournot and $\theta = 1$ for Bertrand. Even though $\theta$ does not affect demand functions $d^i (p^i, p^j)$, it affects equilibrium demands, markups and profits. Therefore, the terms $D^i (Y_i, Y_j; \theta)$, $M^i (Y_i, Y_j; \theta)$, $\Pi^i (Y_i, Y_j; \theta)$ still make sense. Figure 4 plots the investments displayed in Figures 1 and 2 in one diagram for $c_0^1 = c_2^1 = 0.5$. Investments are thus always higher for soft (Cournot) competition, though the difference approaches zero as $b$ does.\footnote{For the Bertrand case, the figure is drawn for the parameter region where the second-order condition holds ($b < 0.933$).}

What lies behind this clear negative effect of competitive intensity (in the sense of moving from Cournot to Bertrand competition) on investments? To understand this, we compare $\Pi^i_{ij} = D^i_j \cdot M^i_j + M^i_j \cdot D^i_j + M^i \cdot D^i_{ij} + M^i \cdot M^i_{ij}$. In Figure 5, the middle line describes equilibrium demand and markup as a function of $b$ in the Cournot case. The upper line describes equilibrium demand in the Bertrand case.\footnote{Recall that a symmetric equilibrium only exists for $b < 0.923$.} The lower line describes equilibrium markup
in the Bertrand case. The figure thus shows that the markup effect is negative, that is, $M_i$ is greater for $\theta = 0$ than for $\theta = 1$, and the demand effect is positive, that is, $D_i$ is smaller for $\theta = 0$ than for $\theta = 1$. Similarly, the cost-pass-through (demand-sensitivity) effects can be obtained by comparing $M_i (D_i)$ in the Bertrand and the Cournot case.

Figure 6 shows that the demand-sensitivity effect is positive, whereas the cost-pass-through effect is negative.

Summing up, increasing competition by moving from Cournot to Bertrand competition has a negative effect on investments for two reasons. First, it
Figure 6: Cournot vs. Bertrand: Cost-pass-through and demand-sensitivity reduces the markup, which reduces the incentive to increase demand. Second, it reduces the positive reaction of markups to reducing own marginal costs. However, under Bertrand competition, equilibrium demand is higher, making mark-up increases through investments more attractive. Also, the sensitivity of equilibrium demand to efficiency is higher. Nevertheless, the negative effects dominate.

4.6 Towards a taxonomy

Table 7 summarizes the examples.\footnote{In the differentiated Bertrand and Cournot examples the number in brackets refers to the number of the underlying demand function.}

For simplicity, it only contains the symmetric cases. In line with (A5) and (A6), the markup effect is always non-positive, and the demand sensitivity effect is always non-negative, suggesting countervailing effects. The demand effect and the cost-pass through effect are ambiguous, which complicates matters further. Table 8 shows which combinations of absolute demand effects and cost-pass through effects arise in the different cases. In each case, the sign after the colon shows whether the marginal investment incentive is negative, positive, zero or U-shaped.\footnote{Again, the numbers in brackets refer to the number of the underlying demand function.} Note that there is no example
Figure 7: Summary of examples (Symmetric Case)

Figure 8: Towards a taxonomy of examples

where both the demand effect and the cost-pass through effect are negative.\textsuperscript{32}

Otherwise, however, arbitrary combinations of the two effects arise.

5 When does competition raise investments?

The examples show that, depending on the oligopoly model and the notion of competition, the effect on investment may be positive or negative. I now use the general approach of Section 3 and the examples to identify which

\textsuperscript{32}When asymmetries are allowed, some modifications are necessary. For instance, in the differentiated Bertrand example from Section 4.2, both the cost-pass-through and the demand effect are negative for laggards.
factors work towards a positive or negative effect of competition. Such factors refer to firm-specific characteristics as well as market characteristics and the underlying notion of competition.

5.1 Leaders vs. laggards

In the Hotelling case, competition increases the investments of leaders and decreases those of laggards. In the Cournot example with differentiated goods (Shubik-Levitan), competition has a negative effect on strong laggards, but a U-shaped effect for leaders, symmetric firms and firms that are not lagging behind too far. With price competition, the effect is only U-shaped for strong leaders; it is negative for all other firms. With Singh-Vives demand, the effects are positive except for strong laggards. Based on the examples, we therefore obtain the following results:

**Observation 1**: Investment tends to have a more positive effect for leaders than for laggards; and they are robustly negative for laggards.

There are two reasons why increasing competition is more likely to have a positive investment effect for leaders than for laggards, and why the effect is robustly negative for laggards. Both relate to (A6). First, the positive demand sensitivity effect $M_i^i D_i^i$ implied by (A6) is substantial only when markups are large – but when firms are lagging far behind, their markups are low. Second, because of (A6), $D_i^j$ and hence the demand effect $M_i^i D_i^i$ is more likely to be positive when a firm is efficient. Reflecting this intuition, $\Pi_i^i$ is increasing in $Y_i$ and decreasing in $Y_j$ in all the examples. Thus, starting from the perspective of a laggard, increasing his efficiency and decreasing the efficiency of the competitor until the roles of both parties are changed, must increase his investment incentives.

5.2 Spillovers

Though Section 2 applies to cases with spillovers ($\lambda > 0$), we have not treated this case in the examples. The following result suggests a tendency for spillovers to make a negative effect of competition on investments more likely.
Proposition 3 Suppose (i) investment costs are sufficiently large and (ii) \( \frac{\partial^2 \Pi}{\partial Y \partial \theta} < 0 \). As spillovers (\( \lambda \)) increase, \( \pi_{i\theta} \) falls.

**Proof.** See 9. ■

The condition \( \frac{\partial^2 \Pi}{\partial Y \partial \theta} < 0 \) seems plausible: As competition increases, the adverse effect of a more efficient competitor on own profits becomes larger in absolute value. However, closer scrutiny suggests some caution. Proceeding as in 2,

\[
\Pi_{j\theta} = D_j \cdot M_{i\theta} + M^i \cdot D_{j\theta} + D^i \cdot M_{j\theta} + M^j \cdot D_{i\theta}.
\]

For instance, the first term, \( D_j \cdot M_{i\theta} \), is positive: As competition reduces markups, it reduces the negative effect of the demand reduction following a competitor’s increase in efficiency. Nevertheless, in all our examples, at least for sufficiently symmetric firms, the remaining effects dominate, so that \( \Pi_{j\theta} < 0 \). We are left with the following, slightly tentative, conclusion.

**Observation 2:** If investments have higher spillovers, marginal investment incentives are more likely to be negatively affected by competition.

5.3 The effects of pre-existing competition

There is a quite common rough intuition that, while some competition is good for investments, “excessive competition” may have negative effects, suggesting an inverted-U relation between competition and investment. In other words, low initial levels of competition would appear to make it more likely that further increases of competition increase investments. The above examples already show that such a general statement cannot be supported in our partial equilibrium framework.\(^{33}\) In fact, the only non-monotone examples feature a U-shape. Even so, (A6) suggests two reasons why increasing competition is more likely to have positive effects when the initial level of competition is low. First, with low competition, markups and hence the demand sensitivity effect \( (M_i D_{i\theta}) \) should be high. Second, by (A6), \( D_i \) is higher when competition is intense, suggesting that the negative markup effect \( D_i M_{i\theta} \) is more pronounced when competition is intense. A potential

\(^{33}\)Aghion et al. (1997, 2001) derive an inverse U-shape from general equilibrium considerations.
countereffect arises if, $D_\theta^i > 0$, and, as for quantity competition, $M_\theta^i > 0$; because this suggests that the cost-pass-through effect $D^i M_\theta^i > 0$ is higher for higher demand. We summarize the discussion as follows:

**Observation 3:** Competition is not necessarily more likely to have a positive effect on investments when the initial level of competition is lower.

### 5.4 Positive Demand Effects

One might argue that it is “natural” for competition to have a positive effect on demand ($D_\theta > 0$): If the demand-enhancing effect of lower own price dominates over the demand-reducing effect of lower competitor prices, demand can only fall if there is direct negative demand effect.\(^{34}\) Even this does not necessarily make for less ambiguity: There are several examples where the demand effect is positive, but competition nevertheless reduces investments, even in the symmetric case. For instance, this is true for the substitution parameter in the differentiated Bertrand model of Shubik and Levitan,\(^{35}\) and it also holds when one moves from Cournot to Bertrand competition in the Shubik-Levitan case. Intuitively, while competition increases demand (and also by (A6), the sensitivity of demand to investment), it also reduces markups, which reduces investment incentives. Hence:

**Observation 4:** Even when competition increases demand, a positive effect of competition on investment does not follow.

### 5.5 The effects of the number of firms

Rather than changes in the intensity of competition for a given number of firms, consider now increases in the number of firms for an otherwise unchanged environment. Suppose there are $n \geq 2$ firms. Replace the parameter $\theta$ by $n$ and write

$$\Pi^i (Y_i, Y_{-i}; n) = M^i (Y_i, Y_{-i}; n) \cdot D^i (Y_i, Y_{-i}; n).$$

\(^{34}\)This holds, for instance, in the homogeneous Cournot example with decreasing market size.

\(^{35}\)In the Singh-Vives case, the effect is negative for sufficiently large initial levels of competition.
Apart from that, proceed as in Section 2. Write net profits as \( \pi^i (y^i, y_{-i}; n) \). For any investment level \( y \), let \( y_n \) be the \( n - 1 \)-dimensional vector consisting of entries \( y \). Finally, introduce the following weak strategic substitutes condition.

**Definition 2** The investment game satisfies **strategic substitutes at the diagonal (SSD)** if \( \frac{\partial \pi^i}{\partial y_i} (y_i, y_n; n) \) is weakly decreasing in \( y_n \) for all \( y_i \) and \( y \).

Thus, (SSD) requires player \( i \)'s investment incentives to fall as the other players' investments increase symmetrically along the diagonal. The condition is motivated by the observation that strategic substitutes typically hold in duopoly investment games with no spillovers.\(^{36}\) The following result holds.

**Proposition 4** Consider a symmetric investment game with objective functions \( \pi^i (y^i, y_{-i}; n) \) that are concave in \( y_i \) and satisfy (SSD). Suppose for all \( i \in \{1, 2, \ldots, n\} \) and \( n^L < n^H \),

\[
\frac{\partial \pi^i}{\partial y_i} (y, y_L; n^L) > \frac{\partial \pi^i}{\partial y_i} (y, y_H; n^H).
\]

For symmetric equilibria \( y_L = y(n^L) = (y_L, \ldots, y_L) \) and \( y_H = y(n^H) = (y_H, \ldots, y_H) \), \( y_L > y_H \).

**Proof.** See Appendix 9. \( \blacksquare \)

Under the conditions of Proposition 4, if an increase in the number of firms reduces marginal investment incentives of each firm, as required by (5), it also reduces investments in the symmetric equilibrium. Similar to (2), we obtain

\[
\Pi^i_{in} = D^i_n \cdot M^i_n + M^i_n \cdot D^i_n + D^i \cdot M^i_n + M^i \cdot D^i_n.
\]

Thus, as in Section 3, we can identify four transmission channels by which the number of firms affects marginal incentives. However, a higher number of firms quite robustly reduces both markups and demands, so that both the markup effect \( D^i_n \cdot M^i_n \) and the demand effect \( M^i_n \cdot D^i_n \) are negative. This

\(^{36}\)See the discussion in Section 4.4.
suggests a clearer negative effect of increasing competition on investments, unless $M_i^{in}$ and $D_i^{in}$ are positive and very large, though positive signs of $D_i^{in}$ and $M_i^{in}$ could work in the opposite direction in principle.

**Observation 5:** For symmetric firms, an increase in the number of firms tend to reduce investments per firm.

To illustrate the asymmetric case, return to the example of Section 4.2.1, and compare the investments of both firms in duopoly with the investments that each firm would have in monopoly with the same demand functions. It turns out that, if firm $i$ is the monopolist, it will invest less in monopoly than in the duopoly if and only if $\frac{17}{21} Y_i^0 - \frac{6}{7} Y_j^0 - \frac{2}{7} > 0$. Thus, interestingly, while introducing competition by a second firm always reduces investments of the former monopolist when the entrant is at least as efficient as the incumbent, entry of a less efficient firm can increase the incumbent’s investments. This is a new version of the principle that competition is more likely to have a positive effect on the investments of relatively efficient firms than on those of relatively inefficient firms (Section 5.1).

### 6 Welfare considerations

While a full welfare analysis cannot be given at this level of generality, some simple insights can be obtained. Denote the consumer surplus corresponding to prices $p_1$ and $p_2$ as $\kappa(p_1, p_2; \theta)$, and define $\kappa(Y_1, Y_2; \theta) = \kappa(p_1(Y_1, Y_2; \theta), p_2(Y_1, Y_2; \theta); \theta)$. Write welfare as

$$W(\theta) = \sum_{i=1}^{2} \pi_i(y_i(\theta), y_j(\theta); \theta) + \kappa(Y_1^0 + y_1(\theta), Y_2^0 + y_2(\theta); \theta)$$

Using the logic of the envelope theorem ($\frac{\partial n_i}{\partial y_i} = 0$ in equilibrium), $\frac{dn_i}{d\theta} = \frac{\partial n_i}{\partial y_i} \frac{\partial y_i}{d\theta} + \frac{\partial n_i}{d\theta}$. Defining

$$\Pi_i'(p^i, p^j; \theta) = (p^i - c^i) d_i'(p^i, p^j; \theta), \quad \Pi_i'(p^i, p^j; \theta)$$

\[37\] This condition is consistent with both firms producing positive outputs.
\[ \pi^i (y_i(\theta), y_j(\theta); \theta) = \tilde{\Pi}^i \left( p^i \left( Y_i^0 + y_i(\theta), Y_j^0 + y_j(\theta); \theta \right), p^j \left( Y_i^0 + y_i(\theta), Y_j^0 + y_j(\theta); \theta \right); \theta \right) - k(y_i). \]

From \( \partial \tilde{\Pi}^i / \partial p^i = 0 \), the direct effect of competition is
\[ \frac{\partial \pi^i}{\partial \theta} = \frac{\partial \tilde{\Pi}}{\partial p^i} \frac{\partial p^i}{\partial \theta} + \frac{\partial \tilde{\Pi}^i}{\partial \theta}, \]

where \( \frac{\partial \tilde{\Pi}^i}{\partial \theta} = d_i \delta. \) Even ignoring the investment effect, competition thus affects profits in two ways. First, there is a direct (positive or negative) effect via \( d_i \delta. \) Second, lower competitor prices work towards lower profits \( (\frac{\partial \tilde{\Pi}^i}{\partial p^j} \frac{\partial p^j}{\partial \theta} < 0). \) In addition, there is the investment-induced effect \( \frac{\partial \pi^i}{\partial y^j} \frac{\partial y^j}{\partial \theta}. \) If \( \frac{\partial \pi^i}{\partial y^j} < 0 \), which holds quite generally when there are no spillovers, the effect is positive or negative according as competition decreases or increases the competitor’s investments. The effect of competition on consumer surplus is

\[ \frac{d\kappa}{d\theta} = \frac{\partial \kappa}{\partial \theta} + \sum_{i=1}^{2} \frac{\partial \kappa}{\partial Y_i} \frac{\partial y^j}{d\theta}. \]

\( \frac{\partial \kappa}{d\theta} \) reflects the price-reducing effect of competition and therefore is usually positive.\(^{38} \) Because (A3) and (A4) imply that \( \frac{\partial \pi^i}{\partial y^j} < 0 \) for \( i \neq j \), the investment-induced effect on consumer surplus, \( \sum_{i=1}^{2} \frac{\partial \kappa}{\partial Y_i} \frac{\partial y^j}{d\theta} \), typically has the opposite sign as the corresponding part of the profit effect, \( \sum_{i=1}^{2} \frac{\partial \pi^i}{\partial y^j} \frac{\partial y^j}{d\theta} = \sum_{i=1,j \neq i}^{2} \frac{\partial \pi^i}{\partial y^j} \frac{\partial y^j}{d\theta}. \) Thus, if competition has a positive effect on investment, it increases consumer surplus, whereas it reduces total profits.

Several related questions remain open. First, when competition reduces investments, can the beneficial effect on firms from reducing the negative externality \( \frac{\partial \pi^i}{\partial y^j} \) be so strong that competition increases profits even when the

\(^{38} \) An obvious counterexample is the inverse market size example: Though increasing competition reduces prices, it also reduces consumer surplus. One might therefore opt for a narrower definition of increasing competition that would exclude such an example.
direct effect $\frac{\partial \pi^1}{\partial \theta}$ is negative, as one would typically expect? Second, can such a reduction in investments be so strong that consumer surplus falls?

7 An extension: Vertical Structures

The approach can potentially be extended to understand the effects of competition in more complex settings. For instance, recent literature has dealt with investment incentives in vertical structures. I will sketch very briefly how the above approach can be used to analyze how competition affects upstream innovation incentives.

For instance, consider an industry where an upstream monopolist supplies a downstream duopoly. In such a setting, one can consider the incentives for upstream cost-reducing innovations $u$. Suppose the upstream monopolist is integrated with the downstream firm $i = 1$, whereas it supplies the downstream firm 2 at an access price $a(u)$, with $a'(u) < 0$. The functional form of $a(u)$ could either result from optimization of the upstream firm, a negotiation process or from regulation. Then downstream firms will obtain profits $\Pi^i(Y_i, Y_j; \theta)$, where $Y_1 = Y_1(u), Y_2 = Y_2(a(u)), Y_1'(u) > 0, Y_2'(a) < 0$. The monopolist obtains total profits $\pi^T(u; \theta)$ from downstream activities of firm 1 and from access revenues. If the cost of supplying the necessary inputs for a downstream output of $Y$ are $C(Y; u)$ where $C$ is increasing in $Y$ and decreasing in $u$, and innovation costs are $K(u)$, total profits thus become

$$
\pi^T(u; \theta) = \Pi^1(Y_1(u), Y_2(a(u)); \theta) + a(u) \cdot D^2(Y_2(a(u)), Y_1(u); \theta) - C(Y_1(u) + Y_2(a(u)); u) - K(u).
$$

Assuming that $\frac{dY_1}{da} = 1$ and $\frac{dY_2}{da} = -1$, incentives to invest are thus

$$
\frac{\partial \pi^T}{\partial u} = \frac{\partial \Pi^1}{\partial Y_1} - \frac{\partial \Pi^1}{\partial Y_2} \cdot \frac{da}{du} - a(u) \frac{\partial D^2}{\partial Y_2} \cdot \frac{da}{du} + a(u) \frac{\partial D^2}{\partial Y_1} + a'(u) D^2(Y_2(a(u)), Y_1(u); \theta).
$$

To understand the effects of competition on upstream innovation, one needs

---

40 In principle, $a(u)$ could also depend on $\theta$. 

to understand the sign of $\frac{\partial^2 x^T}{\partial u \partial \theta}$. Taking derivatives of each of the five terms of $\frac{\partial x^T}{\partial u}$ with respect to $\theta$, one arrives at the following effects.

First, competition influences marginal cost reduction incentives of the integrated firm by affecting $\frac{\partial \Pi}{\partial Y}$. Applying our earlier discussion of simple horizontal oligopolies, this effect can be positive or negative. Second, consider $-\frac{\partial \Pi}{\partial Y} \frac{da}{da}$: Competition is likely to increase the negative effect of higher downstream competitor efficiency ($\frac{\partial \Pi}{\partial Y}$) on own profits. Thus, if upstream investments strengthen downstream competitors to some extent, stronger downstream competition has a negative effect on upstream investments. Third, consider $-a(u)\frac{\partial D^2}{\partial Y} \frac{da}{da}$: Using (A6), competition increases the effects of higher upstream efficiency on the demand of firm 2 and thereby on access revenue. Fourth, competition should reduce the absolute value of the negative effect $\frac{\partial \Pi}{\partial Y}$ of the improved own efficiency on competitor demand and thereby on the access revenue from the competitor. Finally, depending on whether competition has a positive or negative effect of on competitor demand, the positive effect of increased competitor efficiency, $a'(u)(D^2 (Y_2(a(u)), Y_1(u); \theta))$, will depend positively or negatively on competition.

Future research will explore under which circumstances the positive effects of competition on upstream innovation incentives dominate over the negative ones.

8 Conclusion

The paper has identified the channels by which competition affects investment. In the main part of the paper, increasing competition refers not to an increase in the number of firms, but to a more aggressive strategic interaction for a given number of firms, resulting for example from closer substitutability of their products. By assumption and consistent with many examples, competition reduces markups, and increases the sensitivity of equilibrium demand with respect to efficiency. Adding to these ambiguities, competition can have positive or negative effects on equilibrium demands and on the sensitivity of prices with respect to marginal costs. Unless one opts for very narrow notions of increasing competitions, the ambiguities do not disappear. Further, a positive effect of competition is more likely for leaders than for
laggards, and it is less likely when spillovers are strong. Next, no general case can be made that an inverse relation between competition and investment is more likely than a U-shaped relation. Finally, with the alternative interpretation of increasing competition as an increase in the number of firms, however, competition has a clear negative effect.

Contrary to Vives (forthcoming), I have emphasized the differential impact of competition on leaders and laggards. This suggests that it may be valuable to allow for endogenous exit decisions of firms. Firms that anticipate falling behind in an investment game may decide to exit even when they would not do so without the possibility of investment, because the investment game reinforces the asymmetry between firms. This in turn influences the investment decisions of the relatively efficient firms who face less competitors than in the case were exit is precluded.  

9 Appendix 1: Proofs

9.1 Proof of Proposition 1

(i) follows from Theorem 5 in Milgrom and Roberts (1990). (ii) follows from total differentiation of the system of first order conditions. (iii) By (i), it suffices to consider $\pi_{ij}^i < 0$. Total differentiation of the system of first order conditions shows that a negative effect of $\theta$ on investment requires $\pi_{ij}^j \pi_{ij}^i < \pi_{ij}^i \pi_{ij}^j$, and therefore, using symmetry $\pi_{ij}^i < \pi_{ij}^j$. For $\pi_{ij}^i < 0$ and symmetry, this condition is incompatible with stability.

9.2 Proof of Proposition 3

First note that

$$\frac{\partial^2 \pi^i (y_i, y_j; \theta)}{\partial y_i \partial \theta} = \frac{\partial^2 \Pi^i (Y_i^0 + y_i + \lambda y_j, Y_j^0 + y_j + \lambda y_i; \theta)}{\partial Y_i \partial \theta} + \lambda \frac{\partial^2 \Pi^i (Y_i^0 + y_i + \lambda y_j, Y_j^0 + y_j + \lambda y_i; \theta)}{\partial Y_j \partial \theta}.$$  

41 Obviously, such an extension would also involve moving beyond the duopoly case.

42 This theorem is a comparative-statics result for supermodular games.
Therefore,
\[
\frac{\partial^3 \pi^i (y_i, y_j; \theta)}{\partial y_i \partial \theta \partial \lambda} = \frac{\partial^2 \Pi^i}{\partial Y_j \partial \theta} + y_j \left( \frac{\partial^3 \Pi^i}{(\partial Y_i)^2 \partial \theta} + \frac{\partial^3 \Pi^i}{\partial Y_i \partial Y_j \partial \theta} \right) + y_i \left( \frac{\partial^3 \Pi^i}{\partial Y_i \partial Y_j \partial \theta} + \frac{\partial^3 \Pi^i}{(\partial Y_j)^2 \partial \theta} \right).
\]
Thus, if \( y_i \) and \( y_j \) are sufficiently small, \( \frac{\partial^3 \pi^i (y_i, y_j; \theta)}{\partial y_i \partial \theta \partial \lambda} < 0 \). For sufficiently large investment costs, the statement thus holds.

### 9.3 Proof of Proposition 4

As \( \frac{\partial^3 \pi^i (y^H, y^H; n^H)}{\partial y^i \partial n^H} = 0 \), (5) implies \( \frac{\partial^3 \pi^i}{\partial y^i} (y^H, y^H; n^L) > 0 \). By concavity, \( \frac{\partial^3 \pi^i}{\partial y^i} (y_i, y^H; n^L) > 0 \) for any \( y_i < y^H \). Finally, \( (SSD) \) implies \( \frac{\partial^3 \pi^i}{\partial y^i} (y_i, y^L; n^L) > 0 \). Therefore, \( y_L < y_H \) is impossible.

### 10 Appendix 2: The Examples

#### 10.1 Substitutability (Shubik-Levitan)

##### 10.1.1 Quantity competition

Define \( Y_i = 1 - c_i \), that is, \( \pi = 1 \). For \( 2Y_i \geq \theta Y_j \); \( 2Y_j \geq \theta Y_i \);\(^{43}\)
\[
D^i (Y_i, Y_j; \theta) = M^i (Y_i, Y_j; \theta) = \frac{2Y_i - \theta Y_j}{4 - \theta^2}.
\]

##### 10.1.2 Price competition

With price competition,
\[
D^i (Y_i, Y_j; \theta) = \frac{(2 - \theta^2) Y_i - \theta Y_j}{(4 - \theta^2) (1 - \theta^2)}; \quad M^i (Y_i, Y_j; \theta) = \frac{(2 - \theta^2) Y_i - \theta Y_j}{4 - \theta^2}.
\]

\(^{43}\)The following results are taken from Sacco and Schmutzler (2009).
10.2 Substitutability (Singh-Vives)

With quantity competition,
\[
D^i (Y_i, Y_j; \theta) = \frac{(1 + \theta)(2Y_i - \theta Y_j)}{(4 - \theta^2)}; \quad M^i (Y_i, Y_j; \theta) = \frac{2Y_i - \theta Y_j}{4 - \theta^2}.
\]

With price competition,
\[
D^i (Y_i, Y_j; \theta) = \frac{(2 - \theta^2) Y_i - \theta Y_j}{(4 - \theta^2)(1 - \theta)}; \quad M^i (Y_i, Y_j; \theta) = \frac{(2 - \theta^2) Y_i - \theta Y_j}{4 - \theta^2}.
\]

10.3 Hotelling

In the Hotelling model, demand functions are given by
\[
d^1(p^1, p^2; \theta) = \frac{(p^1 - p^2 + \theta)}{2\theta} \quad \text{and} \quad d^2(p^2, p^1; \theta) = \frac{(p^2 - p^1 + \theta)}{2\theta}.
\]

Defining \( Y_i = -c_i \), it is straightforward to show that
\[
D^i (Y_i, Y_j; \theta) = \frac{(Y_j - Y_i + 3\theta)}{6\theta}; \quad M^i (Y_i, Y_j; \theta) = \frac{(Y_i - Y_j - 3\theta)}{3}.
\]

Thus,
\[
D^i_\theta = \frac{(Y_i - Y_j)}{6\theta^2}; \quad M^i_\theta = -1; \quad D^i_i = -1/6\theta; \quad M^i_i = 1/3; \quad D^i_{i\theta} = 1/6\theta^2; \quad M^i_{i\theta} = 0.
\]

Simple but tedious calculations show that equilibrium investments are
\[
y_i = \frac{1}{6} + \frac{Y^0_j - Y^0_i}{2(9\theta + 1)}.
\]

References


