Price-Matching leads to the Cournot Outcome

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June 18, 2013

Abstract

We study the effects of price-matching in a duopoly setting in which each firm selects both its price and output, simultaneously. We show that the availability of a price-matching option leads to the Cournot outcome in this setting. This result is a stark contrast to the one obtained in the standard Bertrand competition that the market price in the presence of a price-matching option ranges from the monopolistic price to the Bertrand price. Our result suggests that the effect of price-matching depends on whether the output is a choice variable for the firms.

Keywords: Price matching

JEL Classifications: L00, L01, L02, D4

1 Introduction

Retail businesses often use price-matching guarantees: if a product the seller carries is sold for a cheaper price by some other seller, then the seller will offer the product for the same low price. The retail giant Walmart, for example, offers price-matching guarantees frequently, especially during the Christmas shopping seasons.

Since Salop (1986) price-matching has largely been considered an anti-competitive practice: a price-matching firm warns its competitors that it will not be undersold; it thus eliminates the rivals’ incentive to undercut the price. For this reason, the market price ranges from the monopolistic to the Bertrand price when the firms have options to price-match (Salop, 1986). Doyle (1988) further argues that the market price is most likely to be the monopolistic price because it alone survives the process of iterative elimination of weakly dominated strategies. However, both Salop (1986) and Doyle (1988) overlook the fact that

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the output decision is strategic.\(^1\) Particularly, both papers assume implicitly that the firms’ output adjusts automatically to the market demand. This assumption is perhaps realistic if the production takes place after firms receive pre-orders. In all other cases, because the production is costly, the firms must choose their output carefully. Therefore, in this paper we assume that the output decision is strategic for the firms. Specifically, we study a duopoly model in which each firm selects its output, price, and price-matching option, simultaneously.

We show that the Cournot price is the unique equilibrium outcome in the setting in which the firms make both production and pricing decisions. The key reason for our result is that at an equilibrium the total supply equals the market demand corresponding to the market price at this equilibrium. As a result, a firm can earn positive profit even when its rival underprices it – the reason why the Bertrand equilibrium is not an equilibrium in our setting. In addition, the availability of price-matching options allows the firms to protect their market share. Subsequently, each firm responds to the other’s strategy in the same way as it would in the Cournot competition.

From the influential paper of Kreps and Scheinkman (1983), it is well-known that the Cournot outcome is the only subgame perfect equilibrium (SPE) in the setting in which the firms first select their output and then set their prices. However, if the firms select their output and price at the same time, there is no (pure) equilibrium as long as the cost of production is positive. The reason for this result is that, as in the Bertrand competition, the firms have an incentive to underprice the other slightly as long as the market price does not equal the marginal cost. But if both firms price their product at the marginal cost, each firm has incentive to increase its price and to set its production so that it sells its full production at the new price. Thus, there is no pure equilibrium if the firms select their output and price at the same time. However, if firms have options to price-match, then the only equilibrium outcome is the Cournot one as we show in this paper. Thus, price-matching is vital for our main result.

The closest study to ours is Tumennasan (forthcoming): he studies the effects of price-matching in Kreps and Scheinkman (1983)’s two-stage setting in which the firms select their output (which is interpreted as capacity) in the first stage and make their pricing decisions in the second stage. Our setting, on the other hand, is static, i.e., the firms both choose their output and make pricing decisions simultaneously. Tumennasan (forthcoming) shows that the effects of price-matching varies with the production cost. Specifically, if the cost of production is “high,” price-matching weakly lowers the market price. If the cost of

\(^1\)To the best of our knowledge almost the whole literature on price-matching does not model the firms’ output decisions explicitly. One exception is Tumennasan (forthcoming), which we will discuss in greater detail later.
production is “low,” the effects of price-matching is ambiguous because the set of SPE prices is a set that includes the Cournot price in its interior. The current paper, on the other hand, demonstrates that the Cournot price is the only equilibrium price if the firms make their production and pricing decision at the same time. Tumennasan (forthcoming) and this paper together show that the timing of the output and pricing decisions plays a vital role in how price-matching affects the market price.

There are some papers that argue that price-matching is not necessarily anti-competitive: Corts (1995) extends the price-matching policy to the price beating policy and restores the Bertrand price as the unique equilibrium.\(^2\) Also, Hviid and Shaffer (1999) introduce hassle costs, i.e., consumers have to bear certain costs to convince a price-matching firm that there is a lower price in the market. In their model, a firm can steal the other’s market share by underpricing because customers save the hassle costs by buying from the price cutter, thus, restoring the Bertrand price.\(^3\) Moorthy and Winter (2006) introduce cost heterogeneity among firms and show that only a low cost firm uses price-matching to signal that it is low priced.

The paper is organized as follows: Section 2 lays out the model. Section 3 proves our main result that price-matching leads to the Cournot outcome.

2 Model

Two identical firms produce a homogeneous product; the market demand for this product is \(P(x)\) or \(D(p) = P^{-1}(p)\) where \(x\) and \(p\) are quantity and price, respectively.

Each firm \(i\) selects (i) its output \(k_i \in \mathbb{R}_+\), (ii) announced price \(p_i \in \mathbb{R}_+\), and (iii) price-matching option \(o_i \in \{0, 1\}\) where 1 means “match” and 0 means “do not match.” The buyers are aware of the firms’ choices. As a result, the price-matching options allow the firms to alter the actual price of its product. In particular, if a firm does price match, then it sells its product for the lowest price on the market. On the other hand, if the firm does not price match, then it sells its product for its announced price. The effective price of firm \(i\) is the price the firm sells its product for, i.e., \(p_e^i(p_1, o_1, p_2, o_2) := (1 - o_i)p_i + o_i \min\{p_1, p_2\}\). The effective prices are instrumental for our analysis as they, along with the outputs, determine the sales quantity of the firms. To simplify the notation, we often write \(p_e^i\) instead of \(p_e^i(p_1, o_1, p_2, o_2)\).

\(^2\)Kaplan (2000) further extends the strategy set to include effective price strategies and restores the possibility of monopoly pricing.

\(^3\)Dugar and Sorensen (2006) take the model of Hviid and Shaffer (1999) to an experimental lab and find a significantly different price than the Bertrand price.
We now formulate the sales quantity of the firms. Let $p_i^e$ and $p_j^e$ be the effective prices for firms $i$ and $j$ ($\neq i$), respectively. Then firm $i$ sells

$$x_i(p_1^e, p_2^e, k_1, k_2) = \begin{cases} 
\min \{k_i, D(p_i^e)\} & \text{if } p_i^e < p_j^e \\
\min \{k_i, \max \{D(p) - k_j, \frac{D(p)}{2}\}\} & \text{if } p_i^e = p_j^e = p \\
\min \{k_i, \max \{0, D(p_i^e) - k_j\}\} & \text{if } p_i^e > p_j^e.
\end{cases} \quad (1)$$

The above formulation implicitly assumes that the firms split the market if they announce the same price as long as each firm’s output is sufficiently large. In addition, the efficient rationing rule is used, i.e., the consumers with a higher valuation buy from the firm with the lower effective price.

Let the strategy profile be $((k_1, p_1, o_1), (k_2, p_2, o_2))$. Then the profit of firm $i = 1, 2$ is

$$\pi_i(k_1, p_1, o_1, k_2, p_2, o_2) = p_i^e x_i(p_1^e, k_1, p_2^e, k_2) - c k_i$$

where $c > 0$.

Observe that the cost function is assumed to be linear in the formulation of the profit. We emphasize here that this assumption is not important in our analysis: the main result of the paper is valid as long as the cost function is strictly increasing and convex. We also maintain the following assumption throughout the paper.

**Assumption 2.1.** $P(x)$ is strictly positive on some bounded interval $(0, \bar{x})$ on which it is twice continuously differentiable, strictly decreasing, and concave. For $x \geq \bar{x}$, $P(x) = 0$.

We now turn our attention to the standard Cournot competition. Thanks to Assumption 2.1, one can easily show that the profit function $P(x+y)x-cx$ is concave on $[0, \bar{x}-y]$. Let $r(y)$ be the Cournot best response to the rival’s production $y$, i.e., $r(y) = \arg \max_{0 \leq x \leq \bar{x}-y} P(x+y)x-cx$.

The following lemma, which is instrumental in our analysis, is from Kreps and Scheinkman (1983).

**Lemma 2.2.** (a) The Cournot best response $r_b$ is nonincreasing in $y$. In addition, $r_b$ is continuously differentiable and strictly decreasing over the range where it is strictly positive (b) $r'(y) \geq 1$, with strict inequality for $y$ with $r(y) > 0$, so that $y + r(y)$ is nondecreasing in $y$.

**Proof.** See Kreps and Scheinkman (1983). \qed

Due to Assumption 2.1, there is a unique Cournot duopoly equilibrium with each firm supplying $x^e$. Let $p^e := P(2x^e)$ and $\pi^e := (p^e - c)x^e$. Furthermore, the monopolistic price and
half of the monopolistic quantity play a role in our analysis. Let \( p^m := \arg \max_p (p - c) D(p) \), \( x^m := D(p^m)/2 = \arg \max_x (P(2x) - c)x \), and \( \pi^m := (p^m - c)x^m \).

### 3 Main Result

In this section, we prove our main result: the only equilibrium outcome in our model is the one that results in the Cournot outcome. We will first study some properties of equilibria, which in turn will lead to our main conclusion.

The first key property is that both firms set the same effective price at any equilibrium. If this is not the case, then the firm with the higher price must not be price-matching. Then the firm with the lower price must already net the monopolistic profit. Otherwise, it has an incentive to adjust its price (and output) slightly toward the monopolistic price. When the firm with the lower price nets the monopolistic profit, the firm with the higher price nets a non-positive profit because there is no demand beyond what the monopolist can provide. However, the firm with the higher price can net half of the monopolistic profit by setting its price to the monopolistic price and its output to half of the monopolistic quantity. Consequently, both firms set the same effective price at any equilibrium. In addition, the range for the equilibrium effective price is \((c, P(0))\). We collect these results below.

**Lemma 3.1.** If \((p^e_1, p^e_2)\) is an equilibrium effective price pair, then \(p^e_1 = p^e_2 \in (c, P(0))\).

**Proof.** Let \(((k_1, p_1, o_1), (k_2, p_2, o_2))\) be an equilibrium strategy pair.

First we show that \(k_i > 0\) for \(i = 1, 2\). On the contrary, suppose \(k_i = 0\) for some \(i \in \{1, 2\}\). Clearly, \(i\) nets 0 profit. In this case, \(j\) is essentially a monopolist. It thus must be that \(k_j = 2x^m\), \(p_j = p^m\) and \(\pi_j(k_1, p_1, o_1, k_2, p_2, o_2) = 2\pi^m\). If \(i\) sets its output to \(x^m\) and price to \(p^m\), then \(i\) nets \(\pi^m\), which is positive. Thus, \(k_i > 0\) for \(i = 1, 2\).

If \(p^e_i < c\) for some \(i\), then the firm with the lowest announced price must net a strictly negative profit because both firms produce a strictly positive output. Thus, \(p^e_i \geq c\) for \(i = 1, 2\). We now dispose of the \(p^e_i = p_i = c\) case. Suppose that \(p^e_i = p_i = c\). In this case, firm \(i\) nets a non-positive profit. In addition, \(i\) must not net a negative profit because \(i\) nets 0 profit by producing 0 output. Thus, \(i\) nets 0 profit. Observe now that firm \(i\) has a profitable deviation unless \(p_j = c\), where \(j \neq i\). Specifically, if \(p_j > c\), then by setting its price to \(c + \epsilon\), where \(\epsilon > 0\) satisfies that \(c + \epsilon < p_j\), and its output to \(D(c + \epsilon)/2\) firm \(i\) nets \(\epsilon D(c + \epsilon)/2\), a positive profit. Thus, \(p_j = p_i = c\) which implies that both firms make 0 profit (recall that the firms can always net 0 profit by setting their output to 0). Furthermore, it must be that \(k_i + k_j \leq D(c)\). Otherwise, one of the firms cannot sell its full capacity; thus, this firm earns a negative profit. Without loss of generality, assume that the firm with the higher output is
firm $i$. Then by setting its price to $c + \delta$, where $\delta > 0$ satisfies $D(c + \delta) - k_j > 0$, and its output to $D(c + \delta) - k_j$, $i$ nets $\delta (D(c + \delta) - k_j)$, a positive profit. This is a contradiction. Thus, each firm $i$ must set its effective price strictly higher than $c$, i.e., $p_i^e > c$.

Now let us show that $p_i^e = p_j^e$. On the contrary, suppose that $c < p_i^e < p_j^e$. This means that $p_i < P(0)$ and $o_j = 0$. As a result, firm $i$ nets $p_i \min\{k_i, D(p_i)\} - c k_i$. First observe that $p_i < P(0)$. Otherwise, firm $i$’s profit is $p_i \min\{k_i, D(p_i)\} - c k_i$. Because $p_i \in (c, P(0))$ and $D(p_i) > 0$, observe that firm $i$’s profit for a given price $p_i$ is maximized at output $D(p_i)$. In other words, it must be that $k_i = D(p_i)$. Subsequently, firm $j$’s profit is non-positive because $D(p_j) - k_i < 0$, for all $p_j > p_i$ and $k_i = D(p_i)$. However, if firm $j$ sets its price to $p_i$ and its output to $D(p_i)/2$, then it nets $(p_i - c)D(p_i)/2$, a positive profit. This is a contradiction.

Finally, let us show that $p_i^e = p_j^e < P(0)$. On the contrary, suppose that $p_i^e = p_j^e \geq P(0)$. Then neither firm’s profit is not positive. But if a firm sets its price to $p_i$ (which is strictly lower than $P(0)$ by definition) and its output to $x^m$, then the firm nets $c k_i$. This is a contradiction.

We use the terminology market price at an equilibrium to refer to the common effective price the firms offer at this equilibrium. The following lemma asserts that, at an equilibrium, each firm supplies half of the market demand corresponding to the market price at this equilibrium. The key observation for this result is that at any equilibrium the total market supply equals the market demand corresponding to the market price at this equilibrium. Indeed if there is an excess supply (or excess demand), then one of the firms have an incentive to decrease (increase) its output without altering the market price. Now if the two firms do not supply the same output, then the one with the lower output has an incentive to increase its output while leaving the market price unaltered.

**Lemma 3.2.** If the market price at an equilibrium is $p \in (c, P(0))$, then each firm’s output is $D(p)/2$ at this equilibrium.

**Proof.** We first show that $k_1 + k_2 = D(p)$. Recall that $p \in (c, P(0))$, due to Lemma 3.1. If $k_1 + k_2 < D(p)$, then each firm $i$ nets $(p - c)k_i$. But firm $i$ can net $(p - c)(D(p) - k_j)$ by changing only its output to $D(p^*) - k_j$. This is a profitable deviation since $D(p) - k_j > k_i$. If $k_1 + k_2 > D(p)$, then there is some excess output in the market. Let $j$ be the firm with the higher capacity. Then $j$ earns $p \min\{k_j, \max\{D(p)/2, D(p) - k_i\}\} - c k_j$. Since $k_j \geq k_i$ and $k_1 + k_2 > D(p)$ (by assumption), $k_j > \max\{D(p)/2, D(p) - k_i\}$. This means that firm $j$ does not sell its entire output at $p$. If $j$ decreases its output slightly but keeps its effective price at
$p$, then $j$'s income does not change but its cost of production decreases. Hence, firm $j$ has a profitable deviation which would be a contradiction. Thus, it must be that $k_1 + k_2 = D(p)$. Finally, let us show that $k_1 = k_2 = D(p)/2$. Suppose $k_i < D(p)/2$. Then it must be that $k_j > D(p)/2 > D(p) - k_j = k_i$. Consequently, firm $i$ nets $(p - c)k_i$. If firm $i$ increases its output slightly to $k_i + \epsilon$ ($\epsilon > 0$ is small enough) and sets its effective price to $p$, then it earns $(p - c)(k_i + \epsilon) > (p - c)k_i$. This is a contradiction.

Lemmas 3.1 and 3.2 imply that each firm nets the same profit at a given equilibrium. In other words, for a given market price at an equilibrium, each firm nets half of the total market profit corresponding to this market price.

In the lemma below, we first show that both firms price-match at each equilibrium. Indeed if a firm does not price-match, then the other firm, by slightly undercutting the equilibrium market price, can steal the market share of the non-price-matcher. The lemma also establishes a new and a tighter upper bound on the equilibrium market prices at the monopolistic price. If the market price strictly exceeds $p^m$, then each firm has an incentive to set its price to the monopolistic price and its output to half of the monopolistic quantity. The deviating firm nets half of the monopolistic profit, which is superior to half of the total market profit corresponding to any price. Lastly, the lemma shows that, in the case that the market price at an equilibrium is strictly below the monopolistic price, both firms’ announced prices equal the market price. If the firms set different prices in this case, then the firm with the lower price has a profitable deviation. Specifically, this firm increases its price slightly and decreases its output to half of the market demand corresponding to its new price. At the deviation, the firm nets half of the total market profit corresponding to the altered price. But this profit increases as the price approaches the monopolistic price. Thus, both firms must have the same announced prices at a given equilibrium if the market price is strictly below the monopolistic price.

**Lemma 3.3.** Consider any equilibrium strategy profile $((k_1, p_1, o_1), (k_2, p_2, o_2))$.

(i) Both firms must price match.

(ii) The equilibrium market price $p := p_1^e = p_2^e$ does not exceed $p^m$, i.e., $p \leq p^m$.

(iii) If $p < p^m$, then $p_1 = p_2 = p$.

Proof. Due to Lemma 3.1, it must be that $p_1^e = p_2^e = \min\{p_1, p_2\} := p > c$. In addition, it must be that $k_1 = k_2 = D(p)/2$ (Lemma 3.2). Thus, both firms net $(p - c)D(p)/2$.

To the contrary of (i), suppose $o_i = 0$. This and the result that $p_1^e = p_2^e = p > c$ (Lemma 3.1) imply that $p_i = p$. Now firm $j$, by setting its price to $p - \epsilon$, where $\epsilon > 0$, and its output...
to \( D(p - \epsilon) \), nets \((p - \epsilon - c)D(p - \epsilon)\), which is a profitable deviation if \( \epsilon \) is sufficiently small. Therefore, \( o_i = 1 \).

To the contrary of (ii), suppose \( p > p^m \). If firm \( i \) sets its price to \( p^m \) and its output to \( x^m \), then it nets \((p^m - c)x^m = (p^m - c)D(p^m)/2\). But by the definition of \( p^m \), \((p - c)D(p)/2 < (p^m - c)D(p^m)/2\). Hence, \( i \) has a profitable deviation which is a contradiction.

To prove (iii), suppose that \( p < p^m \) and \( p_1 = p < p_j \). From the first part of this lemma we already know that \( o_j = 1 \). Thanks to Lemma 3.2, firm \( i \) nets \((p - c)D(p)/2 \). If firm \( i \) sets its price to \( \bar{p} = \min\{p_j, p^m\} \) and its production to \( D(\bar{p})/2 \), then it nets \((\bar{p} - c)D(\bar{p})/2\). As the function \((y - c)D(y)/2\) is concave and maximized at \( p^m \), it must be that \((\bar{p} - c)D(\bar{p})/2 > (p - c)D(p)/2\). Therefore, \( i \) has a profitable deviation which is a contradiction. Thus, \( p_1 = p_2 = p \) if \( p < p^m \).

We are now ready to prove our main result that the only equilibrium in our setting is the one which results in the Cournot outcome. To see this, first recall that each firm’s output at an equilibrium is half of the market demand that corresponds to the market price at this equilibrium. Let this quantity be \( x^* \). Then each firm’s profit is \((P(2x^*) - c)x^*\). If a firm sets its price to \( P(x^* + r(x^*)) \) and its output to \( r(x^*) \) and does not price-match, it sells \( r(x^*) \) for \( P(x^* + r(x^*)) \). The reason for this result is that the other firm cannot sell more than its output, \( x^* \). Clearly, \((P(2x^*) - c)x^* < (P(x^* + r(x^*)) - c)r(x^*)\) by the definition of \( r(\cdot) \) as long as \( x^* \neq x^c \). Thus, the market price must be the Cournot price.

**Theorem 3.4.** There exists only one equilibrium. Particularly, at the equilibrium it must be that \((k_i, p_i, o_i) = (x^c, p^c, 1)\) for both \( i = 1, 2 \).

**Proof.** First let us show that the strategy profile in which both firms price match and set their output to \( x^c \) and their price to \( p^c \) is an equilibrium. At this equilibrium, each firm nets \((p^c - c)x^c \). Suppose firm \( i \) deviates unilaterally by setting its effective price to \( p \in [c, p^c] \) and its production to \( k \). Then it earns

\[
p \min \{k, D(p) - x^c\} - ck
\]

because \( D(p) - x^c > D(p)/2 \), for all \( p < p^c \). For any given \( p \), observe that the above expression is maximized at \( k = D(p) - x^c \). Thus, firm \( i \) earns at most \((p - c)(D(p) - x^c)\) by setting its price to \( p \). Now let us find the \( \max_{p \leq p^c} (p - c)(D(p) - x^c) \). Denote \( x = D(p) - x^c \). Then our problem is equivalent to \( \max_{x \geq x^c} (p - c)(D(p) - x^c) \). This expression, by the definition of \( r(\cdot) \), is maximized at \( x = r(x^c) = x^c \). Thus, \( \max_{p \leq p^c} (p - c)(D(p) - x^c) = (p^c - c)x^c \). Therefore, there is no profitable deviation for firm \( i \) in which its price does not exceed \( p^c \). Suppose firm \( i \) deviates unilaterally by setting its effective price to \( p > p^c \) and its production to \( k \). Then
it earns
\[ p \min \{k, \max\{D(p) - x^c, 0\}\} - ck. \]

Similarly to the previous case one can see that firm \( i \) does not have any incentive to increase its price above \( p^c \). This completes the proof that the proposed strategy profile is an equilibrium.

We now show that there is no equilibrium in which the resulting effective price is different from \( p^c \). On the contrary, suppose there is an equilibrium at which the effective price is \( p^* \neq p^c \). Due to Lemma 3.1, \( p^* \in (c, P(0)) \). When this result is combined with Lemma 3.2, we obtain that firm \( i \) nets \( (p^* - c)D(p^*)/2 \) or \( (P(2x^*) - c)x^* \) where \( x^* = D(p^*)/2 \). Now if firm \( i \) sets its price to \( p = P(x^* + r(x^*)) \) and its output to \( D(p) - x^* \), then it earns \( (p - c) \max\{0, (D(p) - x^*)\} = (P(x^* + r(x^*)) - c)r(x^*) \). The last equality is due to the fact that \( x^* < D(0) \) and that \( r(x) > 0 \) for all \( x \leq D(0) \). Now observe that, by the definition of \( r(\cdot) \), we obtain that \( (P(x^* + r(x^*)) - c)r(x^*) > (P(2x^*) - c)x^* \). Hence, no \( p^* \neq p^c \) is an equilibrium effective price.

We can show the uniqueness of the equilibria by combining that \( p^* = p^c \) is the unique equilibrium effective price with Lemmas 3.2 and 3.3(iii).

\[ \square \]

References


