International competition with non-linear pricing

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Abstract

This paper models international competition between two upstream firms, one domestic and one foreign, which both serve a domestic downstream firm. The productivity of their inputs in downstream production is private information of the downstream firm. We develop the optimal non-linear pricing schemes and compare them to linear pricing, and we scrutinize the effects of trade liberalization and an increase in downstream firm heterogeneity. In particular, we show that the domestic producer is more sensitive to trade liberalization under non-linear pricing.

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1 Introduction

A well-known empirical observation is that a large part of world trade is not in final goods, but in intermediate goods. The empirical literature reports a substantial increase of vertical linkages in international production (see for example Egger and Egger, 2005, Feenstra, 1998, and Feenstra and Hanson, 1996). These intermediate goods are used as an input into the production process, either to produce another intermediate good in the value chain or to produce a final good. Due to an increasing integration of national economies, firms, even if they are not multinational, are able to source their inputs from different countries. In any case, trade in intermediate inputs leads to an international organization of firm activities, and these activities have also been investigated by the international trade literature.\(^1\)

How do domestic firms respond to an increase in foreign competition due to trade liberalization? This question has been analyzed both theoretically and empirically in a number of papers. Usually, the adjustment to an increase in import competition is considered to have potentially two effects, one on the firm size of active firms (intensive margin) and the other one on exit decisions of firms (extensive margin), and the evidence is mixed.\(^2\) In this paper, we suggest an explanation why some industries are more sensitive to trade liberalization than others. In particular, we compare the standard results under linear pricing with those under non-linear pricing in an environment of vertical relationships between independent partners. We find that the reduction in domestic output as a response to an increase in imports is stronger in an environment of vertical structures that allow firms to compete by non-linear pricing schemes.

Why is this important? Firstly, while we see a lot of outsourcing activities, all of the recent literature either considers a bilateral vertical partnership along the value chain, or

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\(^1\)A part of this literature has focussed on the general equilibrium effects of vertical fragmentation, and whether it leads to factor prices convergence or divergence as a response to fragmentation (see Grossman and Helpman, 2003, Helpman, 2004, and Helpman and Krugman, 1985). As for the international organization of the firm, the seminal papers by Antràs (2003) and Antràs and Helpman (2004, 2008) have discussed the trade-off between integrating an activity into firm boundaries or outsourcing it to an independent supplier and this trade-off depends on capital intensities in an environment of incomplete contracts and potential hold-up problems. Furthermore, Grossman and Rossi-Hansberg (2008) have taken this issue further by considering trade in tasks, suggesting that firms can outsource not only production but also service activities, and this may even lead to competition between workers within an international firm.

\(^2\)For example, Gu et al. (2003) find for the effects of NAFTA that it had no significant effect on Canadian firm size, but on exit decisions of manufacturing firms. See also Head and Ries (1999)
assumes that independent suppliers have to apply a linear pricing scheme. Both assumptions may be inappropriate: for certain tasks there may be more than just one supplier which can do the job, but also not too many. In this case, we will expect that suppliers will compete against each other, and this should give rise to strategic interactions. Secondly, we also see that firms source from different suppliers at the same time, so potential competition does not automatically lead to a natural upstream monopoly. Thirdly, the marketing literature suggests that firms make use of non-linear pricing schemes very often (see for example Bonnet et al, 2013, and Draganska et al, 2010), and it seems that the market for intermediate and customized inputs is much more suitable for such a marketing strategy than the market for final goods in which arbitrage and secondary markets may exist. Intermediate input suppliers are specialized and serve a few customers only, making arbitrage difficult. Consequently, it is the purpose of this paper to scrutinize competition between intermediate input suppliers which compete by non-linear pricing schemes. In our model, the domestic firm’s response to trade liberalization is more sensitive under non-linear pricing as it will reduce both the fixed fee and the linear component of the pricing schedule while leaving the discount scheme unchanged.

For this purpose, we use a common agency model in which two upstream firms compete against each other for input demands by a downstream firm. In the theoretical literature, common agency models have been developed both under complete and under incomplete information. The problem with models under complete information is that they may imply a multiplicity of equilibria (Bernheim and Whinston, 1986a, 1986b). We will not follow this approach for several reasons. Firstly, the common agency problem in this setup is mainly a problem of coordination, but we want to focus on the case of competition. Secondly, modern trade theory has emphasized that firm heterogeneity plays an important role in trade, but nearly all models have considered firm heterogeneity in a non-strategic setting of monopolistic competition. We want to break with this tradition and derive the non-linearity of pricing schemes as an endogenous result in a model of strategic interactions. Therefore, in our model, the downstream firm has some private cost information, and the range of the possible cost realizations defines the degree of heterogeneity. Thirdly,

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3 See McCalman (2012) for a model of monopolistic competition in which firms use a two-part tariff.  
4 For the seminal paper in the monopolistic competition setting, see Melitz (2003). An exception to the monopolistic competition approach is Long et al (2011) who consider firm heterogeneity in a trade model in which firms have private cost information and can influence the cost realization by research and development.
this setting also allows to consider the effects changes in (downstream) firm heterogeneity may play.\footnote{See Stole (1991) and Martimort (1992) for early contributions the the common agency theory under adverse selection, and Martimort (2006) for an excellent overview of the common agency literature.}

Accordingly, the paper is organized as follows: Section 2 introduces the model and shows the results for the case of competition between a domestic and a foreign input supplier with linear pricing. Section 3 extends this model to the case of non-linear pricing schemes. Section 4 considers how trade liberalization and downstream firm heterogeneity change the pricing schemes. Section 5 concludes.

## 2 The model

Our model assumes a domestic downstream firm, which produces a final good \( y \). The location of this target market is irrelevant for our analysis (except for potential welfare effects) so this could be a domestic or an international market. The inverse demand function for \( y \) is common knowledge and given by

\[
p = a - \frac{b}{2}y, \text{ where } a > 0 \text{ and } b > 0.
\]

This downstream firm needs an input \( q_i \) from at least one upstream supplier. There are two upstream firms, a domestic one labelled 1, and a foreign one, labelled 2, which sell intermediate inputs, \( q_1 \) and \( q_2 \), to the downstream firm. Without loss of generality, we normalize the marginal cost of the domestic upstream firm to zero. The foreign upstream firm’s marginal costs are also normalized to zero, but this firm has to carry a trade cost \( t \) per unit. In what follows, we will confine our analysis to competition between upstream firms and thus consider only cases in which the downstream firm will always source from both upstream firms. Furthermore, producing the input itself is too costly for the downstream firm so integration of input production is not an option. Without loss of generality, we assume that one unit of input is needed for one unit of output such that \( q_1 + q_2 = y \).

The cost function of the upstream firm(s) is common knowledge. The profitability of the downstream firm, however, is private information. The downstream firm must process the inputs and incur the processing costs. Processing the input \( q_i \) produced by
firm $i$ involves a direct cost $\omega_i > 0$ per unit, but $\omega_1$ and $\omega_2$ are known only by the downstream firm. We assume that there are cost savings in processing the two inputs together, so that the total cost of processing the bundle $(q_1, q_2)$ is given by

$$C = \omega_1 q_1 + \omega_2 q_2 - \mu q_1 q_2.$$  

The parameter $\mu > 0$ is common knowledge and measures the degree of complementarity in production. We make

**Assumption 1** $b > \mu > 0$.

Assumption 1 will guarantee that the downstream firm’s objective function is concave so that the first-order conditions are both necessary and sufficient. Furthermore, under linear prices, sourcing both inputs makes sense only if $\mu > 0$: if $\mu$ were negative, an increase in one input would increase the other input’s marginal processing cost, and the downstream firm would always be better off by sourcing only the cheaper input.

If perfect competition prevailed in the upstream market such that both upstream firms (or upstream industries) charged marginal costs, the downstream firm would employ both inputs for any given output level $y > 0$ only if the cost saving arising from complementarity achieved by mixing inputs, outweighs the difference in their direct costs:

$$\mu y \geq \max \left\{ \omega_1 - (\omega_2 + t), (\omega_2 + t) - \omega_1 \right\}.$$  

Consequently, sourcing both inputs requires a sufficient degree of complementarity compared to the relative input cost (including trade costs). We assume that the downstream firm is not able (or not permitted) to vertically integrate one or both firms, so that the upstream firms and the downstream firm play a two-stage game in pricing and input demand strategies.\(^6\) Due to our linear structure, we may redefine the privately known parameter, denoted by $\theta_i$, as the difference between the maximum willingness to pay $a$ of the inverse demand function and the marginal cost of processing the input $\omega_i$, so that $\theta_i = a - \omega_i$.\(^7\) The upstream firms know that each input’s productivity in downstream

\(^6\)This is not an unreasonable assumption because the upstream firm does not know its value for the downstream firm under incomplete information. Furthermore, it is not uncommon that providers of customized inputs have other clients in different industries, and the downstream firm may not be able to keep these clients as it has no experience in these markets.

\(^7\)Incomplete information may also have other sources which we could consider at the same time. For
production is drawn from the same c.d.f. \( F(\theta_i) \); this means that the productivities are not correlated, and no upstream firm has a natural advantage or disadvantage over the other (except for the trade cost). In particular, \( \theta_i \in [\theta, \bar{\theta}] \), where \( \bar{\theta} > \theta > 0 \). A downstream firm’s type is a pair \((\theta_1, \theta_2)\), which can be represented by a point in the rectangle \([\theta, \bar{\theta}] \times [\theta, \bar{\theta}]\). Furthermore, we assume that \( F(\theta_i) \) is differentiable in the range \([\theta, \bar{\theta}]\), so that \( f(\theta_i) = F'(\theta_i) \) exists in this range.

Since we are interested in the cases in which competition between the upstream firms prevails for all possible productivity realizations, we make

**Assumption 2**

\[
\theta > \max \left( \frac{b - \mu}{b} \bar{\theta} + r, \frac{r(3 - 2r)}{2 - r^2} \bar{\theta} + \frac{1}{2 + r} t \right)
\]

where we have defined

\[
r = \frac{4(b - \mu)}{b + \sqrt{b^2 + 8(b - \mu)}} < 1.
\]

Assumption 2 will guarantee that the demand for each input will be positive, both in the case of linear pricing and in the case of non-linear pricing by the upstream firms. In particular, it implies that the demand for the foreign input is positive even if the foreign productivity realization is worst (that is, \( \theta_2 = \bar{\theta} \)) and the domestic productivity realization is best (that is, \( \theta_1 = \bar{\theta} \)).

While our main interest is to study non-linear pricing, it is useful to start with the benchmark case of linear pricing. In the case of linear pricing, each upstream firm sets a unit price for the intermediate input it produces in the first stage. In the second stage, the downstream firm purchases the inputs and produces for its target market. The downstream firm’s profit function is given by

\[
\Pi = v(\theta_1, \theta_2, q_1, q_2) - p_1 q_1 - p_2 q_2
\]

where \( v(\theta_1, \theta_2, q_1, q_2) \) is the gross profit before paying for both inputs \( q_1 \) and \( q_2 \). The vector \((\theta_1, \theta_2)\) characterizes the downstream firm’s type. The downstream firm’s production cost example, the upstream firm may not exactly know the vertical quality of the final good when produced with its input. In this case, the downstream firm would have private information on the inverse demand intercept \( a \). Considering different \( a_i \)'s, known to the downstream firm only, instead or in addition to marginal processing costs is strategically equivalent to our approach.

\( ^8 \)The first restriction is sufficient for positive input demands under linear pricing, the second one under non-linear pricing. Neither of the restrictions implies the other.
depends on the input mix, and the gross profits of the downstream firm are given by

\[ v(\theta_1, \theta_2, q_1, q_2) = \theta_1 q_1 + \theta_2 q_2 - \frac{b}{2}q_1^2 - \frac{b}{2}q_2^2 - (b - \mu)q_1q_2. \]  

(1)

Assumption 1 is sufficient to ensure that \( v \) is concave in \((q_1, q_2)\). It implies that the demand effect is always dominant, so that the two inputs are substitutes when considering both demand and production effects at the same time.

We now solve the game in the backward induction fashion, and we show:

**Lemma 1** (Equilibrium under linear-pricing) Given Assumptions 1 and 2, the domestic firm’s input demands are strictly positive in equilibrium, and the equilibrium prices are given by

\[ p_1 = \hat{\theta}(3b - \mu)\mu + b(b - \mu)t, \quad p_2 = \hat{\theta}(3b - \mu)\mu + 2b^2t, \]

where \( \hat{\theta} = \int_{\theta} \theta f(\theta) d\theta \) is the common expected value of both \( \theta_1 \) and \( \theta_2 \).

Proof: See Appendix A.1

We are now able to discuss how trade liberalization, measured by a reduction in trade costs \( t \) for the foreign supplier, will affect the pricing behavior. We find:

\[ \frac{\partial p_2}{\partial t} = \frac{2b^2}{(3b - \mu)(b + \mu)} > \frac{\partial p_1}{\partial t} = \frac{b(b - \mu)}{(3b - \mu)(b + \mu)} > 0. \]

Both upstream firms reduce their prices following a decline in trade cost \( t \). This is not surprising because price competition implies strategic complementarity in the sense of Bulow et al. (1985). Since the foreign firm’s price cut is larger, it follows that the expected demand for its product, denoted by \( \hat{q}_2 \), rises as \( t \) falls, while the expected demand for its rival’s product, denoted by \( \hat{q}_1 \), moves in the opposite direction:

\[ \frac{d\hat{q}_1}{dt} = \frac{b^2(b - \mu)}{(2b - \mu)\mu(3b - \mu)(b + \mu)} > 0, \]

(2)

\[ \frac{d\hat{q}_2}{dt} = \frac{-b^2(b + \mu)}{(2b - \mu)\mu(3b - \mu)(b + \mu)} < 0, \]

(3)

Not surprisingly, the foreign upstream firm gains and the domestic upstream firm loses market share as a result of trade liberalization. Let \( \delta \) denote the marginal change in
domestic production per marginal change of foreign production for the case of linear pricing. Thus, $\delta$ reflects the sensitivity of domestic production to a change in foreign production due to trade liberalization, and it measures by how much domestic production declines if foreign production goes up by a marginal unit due to trade liberalization:

$$
\delta = -\frac{dq_1}{dq_2} = \frac{b - \mu}{b + \mu}.
$$

(4)

The change in domestic and foreign production has also a clear effect for the domestic supplier’s profits. Let the maximized (expected) profits of the domestic (foreign) supplier be denoted by $\pi_1^*(\pi_2^*)$. Using the envelope theorem, we find that

$$
\frac{d\pi_1^*}{dt} = p_1 \frac{\partial \hat{q}_1}{\partial p_2} \frac{\partial p_2}{\partial t} = p_1 \frac{b - \mu}{(2b - \mu)(3b - \mu)(b + \mu)} > 0
$$

(5)

so that trade liberalization will unambiguously reduce the domestic supplier’s profits. However, the impact on the foreign supplier’s profit is not clear:

$$
\frac{d\pi_2^*}{dt} = -\hat{q}_2 + (p_2 - t) \frac{\partial \hat{q}_2}{\partial p_1} \frac{\partial p_1}{\partial t} = -\hat{q}_2 + (p_2 - t) \frac{b - \mu}{(2b - \mu)(3b - \mu)(b + \mu)} \frac{b(b - \mu)t}{(3b - \mu)(b + \mu)}.
$$

(6)

The first effect is the direct effect: trade liberalization reduces the supplier’s cost per sold unit and thus increases his profits. The second effect is the strategic effect, because the domestic supplier will decrease his price in response to trade liberalization, and this harms the foreign supplier. These effects are also well-known from strategic trade policy models with price competition (see for example the seminal paper by Eaton and Grossman, 1986).

How does the potential heterogeneity of the downstream firm affect the outcome? An increase in heterogeneity can be measured by a mean-preserving spread of $\theta_i$. Since $\hat{\theta}$ stays constant, such a mean-preserving spread does not change the pricing behavior of firms unless the input demand becomes zero for some downstream types. Therefore, given Assumption 2, an increase in the downstream firm’s heterogeneity does not change equilibrium prices and expected profits of the two upstream firms.
3 Competition with Non-Linear Pricing Schemes

Let us turn to the case of competition with non-linear pricing schemes. Now, each firm \( i \) (\( i = 1, 2 \)) offers a schedule \( T_i(q_i) \) to the downstream firm which tells the downstream firm the total payment, \( T_i \), that it must make to \( i \) if it wants to buy the quantity \( q_i \). This is again a two-stage game: in the first stage, both upstream firms simultaneously specify a transfer scheme \( T_i(q_i) \) dependent on the size of the order. In the second stage, the downstream firm makes its orders and produces for the target market. The demand and cost structures are the same as before, that is, the downstream firm produces an output \( y \) using two inputs, \( q_1 \) and \( q_2 \), but its profit function is now given by

\[
\Pi = v(\theta_1, \theta_2, q_1, q_2) - T_1(q_1) - T_2(q_2) = \theta_1 q_1 + \theta_2 q_2 - \frac{b}{2} q_1^2 - \frac{b}{2} q_2^2 - (b - \mu) q_1 q_2 - T_1(q_1) - T_2(q_2).
\]

Again, we have a situation in which both \((\theta_1, \theta_2)\) are private information. Given the schedules \( T_i(q_i), i = 1, 2 \), the downstream firm chooses the input levels \( q_1(\theta_1, \theta_2) \) and \( q_2(\theta_1, \theta_2) \) to maximize its profit \( \Pi \). The upstream firms now choose non-cooperatively their schedules \( T_1(.) \) and \( T_2(.) \) to maximize their expected profits. Since orders are made simultaneously, the quantity of input purchased by the downstream firm from an upstream supplier cannot be observed by the other upstream firm, but only be anticipated. Consequently, we confine the analysis to transfer schemes of firm \( i \) which can be made dependent only on the input \( q_i \), and not on the rival input.\(^9\) In general, firm \( i \)'s optimal schedule \( T_i^*(q_i) \) depends on what it expects \( T_j^*(q_j) \) to be. Thus we must seek a Nash equilibrium pair of schedules \((T_1^*(q_1), T_2^*(q_2))\). This problem is a common agency problem under adverse selection in which the upstream firms are the principals and the downstream firm is the common agent.

In general, the Revelation Principle does not apply in a common agency context. However, given the linear-quadratic structure of the problem at hand, the Revelation Principle can be applied after a judicious transformation of variables so that upstream firm \( i \) is be-

\(^9\)Martimort (2006) calls this setup private agency as compared to public agency. Under public agency, contracts are incomplete due to the lack of centralization because firms do not coordinate their offers. Under private agency, another source of incompleteness is that each principal contracts with the agent also on different variables.
having as if it were facing a fictitious type $z_i \in [\underline{z}, \overline{z}]$ rather than $(\theta_1, \theta_2) \in [\underline{\theta}, \overline{\theta}] \times [\underline{\theta}, \overline{\theta}].$\footnote{For a first paper using this approach, see Ivaldi and Martimort (1994). Their analysis has been applied in the industrial organization literature to study theoretically and empirically competition in the mobile phone industry; see for example Miravete (2002).}

In particular, under certain assumptions about the probability distribution function, there exists a Nash equilibrium pair of schedules $(T_1^*(q_1), T_2^*(q_2))$ such that $T_i^*$ is quadratic in $q_i$ and, among all possible replies, player $i$’s best reply to a quadratic schedule $T_j(q_j)$ is a schedule $T_i(q_i)$ that is itself quadratic in $q_i.$\footnote{These authors did not require that players use only quadratic schedules; their Nash equilibrium schedules, which turn out to be quadratic, is found from working with a general space. Whether other Nash equilibria exist that are not quadratic schedules remains an open question.} For this reason, let us restrict attention to quadratic schedules. Suppose firm 2’s schedule is linear-quadratic and given by

$$T_2(q_2) = \gamma_2 + \alpha_2 q_2 + \frac{\beta_2}{2} q_2^2.$$ 

This pricing schedule has three parameters: the first one is $\gamma_2$, to which we will refer to as the fixed fee. This fixed fee has to be paid by the downstream firm upfront if it wants to source inputs from the upstream firm. The parameter $\alpha_2$ captures the linear part of the pricing schedule, and therefore we will refer to this part as the linear component price. Finally, $\beta_2$ is the parameter for the quadratic part of the schedule, and since we will show that $\beta_2 < 0$, we will refer to this part as the discount.

What is firm 1’s best reply to this schedule? Firm 1 takes $\gamma_2, \alpha_2$ and $\beta_2$ as given (as does the downstream firm). Now firm 1 can deduce that the downstream firm’s choice of $q_1$ and $q_2$ must satisfy the following two conditions if $q_1 \geq 0$ and $q_2 \geq 0$:

$$\theta_2 - \alpha_2 = (b + \beta_2) q_2 + (b - \mu) q_1, \quad (7)$$

$$\theta_1 = bq_1 + (b - \mu) q_2 + T_1'(q_1). \quad (8)$$

Eq. (7) gives the first-order condition for the downstream firm’s optimal order of the intermediate input produced by foreign firm 2, given that the pricing schedule of the foreign firm is linear-quadratic. Eq. (8) is the first-order condition for orders from the domestic firm for the general schedule $T_1(q_1)$. We will show in Proposition 1 that in equilibrium the schedule $T_1(q_1)$ must be linear-quadratic, and therefore the specifications of the pricing schedules are mutually consistent. Substituting eq. (7) into (8), we obtain, for all $(\theta_1, \theta_2)$, with $\underline{\theta} \leq \theta_i \leq \overline{\theta}$, the following necessary condition which does not directly
involve $q_2$:

$$
\theta_1 - \frac{(b-\mu)\theta_2}{b+\beta_2} + \frac{(b-\mu)\alpha_2}{b+\beta_2} = \left(b - \frac{(b-\mu)^2}{b+\beta_2}\right) q_1 + T'_1(q_1). \tag{9}
$$

We observe that the LHS of (9) involves only a linear combination of $\theta_1$ and $\theta_2$. Therefore, from the point of view of the upstream firm 1, a downstream firm’s type can be described by a scalar $z_1$ defined by

$$
z_1 \equiv \theta_1 - \frac{(b-\mu)\theta_2}{b+\beta_2}.
$$

Assumption 1 implies that $b-\mu > 0$, and we will show that in equilibrium $b + \beta_2 > b - \mu > 0$.

The definition of $z_1$ means that, from the perspective of upstream firm 1, the set of all possible downstream firm types (i.e., the sets of points in the rectangle $[\theta, \overline{\theta}] \times [\theta, \overline{\theta}]$) is partitioned into “equivalence classes” indexed by $z_1$, where $z_1$ can take on any value in the interval of real numbers denoted by $[\underline{z}_1, \overline{z}_1]$. All the firm types that belong to the same equivalence class $z_1$ will have the same input demand from upstream firm 1, even though their input demand from firm 2 might differ. Each equivalence class is represented by a straight-line segment with a positive slope equal to $d\theta_2/d\theta_1 = (b + \beta_2)/(b - \mu)$ where the slope is positive and greater than 1 due to $0 < \mu < b$ and $b + \beta_2 > b - \mu > 0$.

A similar argument applies for firm 2. From the point of view of the upstream firm 2, a downstream firm’s type can be described by a scalar $z_2$ defined by

$$
z_2 \equiv \theta_2 - \frac{(b-\mu)\theta_1}{b+\beta_1}.
$$

All the firm types that belong to the same equivalence class $z_2$ will have the same input demand from upstream firm 2, and each equivalence class is represented by a straight-line segment with slope $d\theta_2/d\theta_1 = (b-\mu)/(b+\beta_1)$ where the slope is positive and less than 1 due to $0 < \mu < b$ and $b + \beta_2 > b - \mu > 0$.

Then, for each upstream firm $i$, $i = 1, 2$, we can introduce a new random variable $z_i$ which is distributed between $\underline{z}_i$ and $\overline{z}_i$ where by definition

$$
z_i \equiv \theta - \frac{b-\mu}{b+\beta_{-i}} \overline{\theta}, \overline{z}_i \equiv \overline{\theta} - \frac{b-\mu}{b+\beta_{-i}} \theta. \tag{10}
$$

in case that all firm types have positive input demands from both upstream firms. The
random variable $z_i$ reaches its minimum, $z_i$, for the smallest realization of the downstream productivity of its own input, $\theta_i$, and the largest realization of the downstream productivity of the rival input, $\bar{\theta}$, and its maximum, $\bar{z}_i$, for the largest realization of the downstream productivity of its own input, $\theta_i$, and the smallest realization of the downstream productivity of the rival input $\bar{\theta}$. We will show that $\beta_1 = \beta_2$ in a Nash equilibrium if no downstream firm types are excluded, so that the upper bounds $\bar{z}_1$ and $\bar{z}_2$ are identical, and can be denoted by $\bar{z}$; and similarly, $\bar{z}_1 = \bar{z}_2 = \bar{z}$.

The definition of $z_1$ allows us (i) to solve for the function $T_i(q_i)$ using the Revelation Principle, where the agent is characterized by $z_i$ and the distribution of $z_i$ is known, and (ii) to prove that $b + \beta_i > 0$ in equilibrium. The result is summarized by

**Proposition 1** Assume that $z_i$ is uniformly distributed according to the c.d.f. $G(z_i) = (z_i - \bar{z})/\bar{z}$. If Assumptions 1 and 2 hold, the equilibrium pricing schemes exist which are linear-quadratic and concave, giving a discount for larger orders. Each upstream supplier offers

$$T_i = \gamma_i + \alpha_i q_i + \frac{\beta_i}{2} q_i^2,$$

where the equilibrium parameters are as follows.

$$\beta_i = \beta = \frac{b}{4} \left[ \sqrt{1 + 8 \left( \frac{b - \mu}{b^2} \right)^2} - 3 \right] < 0.$$  \hfill (11)

Furthermore, $b + \beta > b - \mu > 0$.

$$\alpha_i = \frac{1}{4 - r^2} \left[ 2(m_i + \bar{z}) + r(m_{-i} + \bar{z}) \right], \text{ where } 0 < r \equiv \frac{b - \mu}{b + \beta} < 1.$$  \hfill (12)

The fixed fees are strictly positive:

$$\gamma_i = \frac{[2\bar{z} - \bar{z} + r\alpha_{-i} - m_i]^2}{-8\beta} = \frac{-\beta}{2} [g_i(\bar{z})]^2,$$  \hfill (13)

where $m_i(m_{-i})$ denotes the marginal cost of the upstream firm $i$ (its rival firm) which is either equal to $0$ or equal to $t$, depending on whether it is the domestic or the foreign upstream firm.

The input demands are strictly positive for all productivity realizations due to Assump-
tion 2 and given by
\[ q_i(z_i) = \frac{z_i + r\alpha_i - \alpha_i}{-\beta} = \frac{\theta_i - r\theta_i + r\alpha_i - \alpha_i}{-\beta} = \frac{2z_i - \pi + r\alpha_i - m_i}{-2\beta}. \] (14)

Proof: See Appendix A.2.

Since \( \beta_i = \beta \) and the probability function of \( z_i \) is uniform, the definition of the random variable \( z_i \) as it has been assumed is confirmed for both firms. Of course, we could feel uncomfortable with doing comparative static exercises because \( \beta \) is an endogenous variable when transforming the \( \theta \)-distribution into the \( z \)-distribution. However, as we have seen in eq. (11), changes in the marginal costs or trade costs have no impact on \( \beta_i \). Furthermore, \( \beta_i = \beta < 0 \) because \((b - \mu)^2/b^2 < 1\) which means that the transfer scheme is regressive in quantity: larger demands pay a lower per unit price, and the equilibrium discount is the same for both firms irrespective of their marginal costs. Since the parameter \( \beta \) does not depend on the bounds of the distribution, any mean-preserving spread will have no bearing on the regressive part of the pricing scheme. While this result is due to the linear-quadratic specification of the model, it has the great analytical advantage that we will be able to do comparative static exercises for changes in the marginal costs and for variations of downstream firm heterogeneity.

The case of non-linear pricing features some remarkable properties. Firstly, we observe that the difference in costs between the domestic and the foreign firm will also be reflected by the fixed fees, both directly and indirectly via the \( \alpha_{-i} \). Secondly, the linear part of the price schedule, \( \alpha_i \), depends positively on both firms’ marginal costs, with a stronger dependence on own marginal costs. Thirdly, and most importantly, the strategic interaction in non-linear pricing schemes has a different quality than in linear pricing schemes. If the opponent’s linear component \( \alpha_{-i} \) increases, the fixed fee \( \gamma_i \) will increase, but neither \( \alpha_i \) nor the discount term \( \beta_i \) are modified. An increase in the opponent’s \( \alpha_{-i} \) implies that the firm’s input supply \( q_i \) has become more attractive, and instead of changing parts of its pricing schedule that depend on the quantity ordered \( q_i \), it prefers to adjust the fixed fee \( \gamma_i \) only. This means that the opponent cannot expect any adjustment at the intensive margin as in the case of linear pricing and strategic substitutes. This is, of course, not a full-fledged equilibrium analysis as we look upon the optimal adjustment of a single

\[ ^{12} \text{Note carefully that the uniform density function } g(z_i) \text{ giving rise to the c.d.f. } G(z_i) \text{ is a convolution of } f(\theta_1) \text{ and } f(\theta_2). \]
firm to a change in the rival’s linear component $\alpha_{-i}$ only. However, these observations already give us a flavor that the adjustment process under non-linear pricing has a different quality.

**4 Trade Costs and Firm Heterogeneity**

We now discuss how trade liberalization, measured by a decrease in trade cost $t$, and a mean-preserving spread will change the pricing schemes and relative input demands. Since we observe that the concave part of the pricing scheme, $\beta$, depends neither on the marginal cost of both rivals nor on the bounds of the distribution, both trade liberalization and an increase in downstream firm heterogeneity will affect only the fixed fee and the per unit component of the scheme, but not its regressive part. Furthermore, from our definition of $r$ in Assumption 2, and from equation (14) in Proposition 1, we find that $r = (b - \mu)/(b + \beta) < 1$. We find for the effect of trade liberalization:

$$\frac{\partial \alpha_1}{\partial t} = \frac{r}{4 - r^2} > 0, \frac{\partial \alpha_2}{\partial t} = \frac{2}{4 - r^2} > 0$$

Thus we conclude that trade liberalization will lead to a reduction in the per unit component of the pricing schemes. Furthermore, since $0 < r < 1$, the foreign supplier’s reduction in the linear component is larger than the domestic firm’s reduction.

For the effect of an increase in $t$ on the fixed fees $\gamma_1$ and $\gamma_2$, define

$$N_i \equiv 2z - \bar{z} + r\alpha_{-i} - m_i > 0 \text{ with } \frac{N_i}{2\beta} = q_i(z).$$

that allows us to write the fixed fees as $\gamma_i = N_i^2/(-8\beta)$. We find that

$$\frac{d\gamma_1}{dt} = \frac{N_1}{(-8\beta)} \left( r \frac{\partial \alpha_2}{\partial t} \right) = \frac{N_1}{(-8\beta)} \left[ \frac{2r}{4 - r^2} \right] > 0,$$

$$\frac{d\gamma_2}{dt} = \frac{N_2}{(-8\beta)} \left( r \frac{\partial \alpha_1}{\partial t} - 1 \right) = \frac{N_2}{(-8\beta)} \left( \frac{r^2}{4 - r^2} - 1 \right) < 0.$$

We have a clear result: trade liberalization will induce the domestic upstream firm to reduce the fixed charge. Why? This is because the downstream firm’s valuation of access

\[\text{Assumption 2 implies that } 2z - \bar{z} - t \geq 0, \text{ because } 2z - \bar{z} = (2 + r)\bar{\theta} - (1 + 2r)\bar{\theta}.\]
to the domestic input is reduced when the foreign input becomes relatively cheaper at the margin. Hence, if the domestic supplier does not reduce the access fee $\gamma_1$, some types of downstream firm may quit, relying only on the foreign inputs. In contrast, the fixed fee of the foreign upstream firm will increase.

We now turn to considering the effect of falling trade costs on the input demands by the downstream firm. As expected, trade liberalization reduces the equilibrium demand for the domestic input $q_1$. From (14), we find that

$$\frac{dq_1}{dt} = \left( \frac{r}{-2\beta} \right) \left[ \frac{d\alpha_2}{dt} \right] = \left( \frac{r}{-2\beta} \right) \left[ \frac{2}{4 - r^2} \right] > 0.$$  

(15)

The effect on production levels is similar to the case of linear pricing which we discussed in the last section (see eq. (2)). Similarly, the demand for the foreign intermediate input changes as follows:

$$\frac{dq_2}{dt} = \left( \frac{1}{-2\beta} \right) \left[ \frac{r\alpha_1}{dt} - 1 \right] = \left( \frac{1}{-2\beta} \right) \left[ \frac{r^2}{4 - r^2} - 1 \right] < 0.$$  

(16)

We summarize our results in

**Lemma 2** Trade liberalization does not change the regressive part of the pricing schemes, but (i) decreases the linear part of both pricing schemes, (ii) leads to a reduction in the domestic fixed fee, and an increase in the foreign fixed fee. Domestic input demand decreases and foreign input demand increases.

As a corollary, it follows immediately that trade liberalization reduces the expected profit of the domestic upstream firm. The effect on its expected profit is

$$\frac{d\pi_1}{dt} = \frac{d\gamma_1}{dt} + \left( \frac{d\alpha_1}{dt} \right) E(q_1) + \alpha_1 \int_{z_1}^{\bar{z}} \frac{dq_1(z_1)}{dt} dG + \beta \int_{z_1}^{\bar{z}} q_1(z_1) \frac{dq_1(z_1)}{dt} dG.$$

The change in profit is negative for $dt < 0$, and the result is similar to the case of linear pricing. What happens to the expected profit of the foreign firm? We find that

$$\frac{d\pi_2}{dt} = \frac{d\gamma_2}{dt} + \left( \frac{d\alpha_2}{dt} \right) E(q_2) + \alpha_1 \int_{z_2}^{\bar{z}} \frac{dq_2(z_2)}{dt} dG + \beta \int_{z_2}^{\bar{z}} q_2(z_2) \frac{dq_2(z_2)}{dt} dG.$$

Here, the foreign upstream firm gains from liberalization while the effect was ambiguous.
for linear pricing. The reason is that it will now be able to benefit from its increased attractiveness for the downstream firm through an increase in the fixed fee.

While we observe similar effect of a decrease in \( t \) on production levels as in the case of linear pricing, the impact on relative input demands are different. Let \( \Delta \) denote the marginal change in domestic production per marginal change of foreign production for the case of non-linear pricing. As in the case of linear pricing, \( \Delta \) reflects the sensitivity of domestic production to a change in foreign production due to trade liberalization and measures by how much domestic production declines if foreign production goes up by a marginal unit due to trade liberalization:

\[
\Delta = -\frac{dq_1}{dq_2} = \frac{r}{2 - r^2}.
\]

(17)

We now compare the relative changes in the two pricing regimes, and we can show:

**Proposition 2** \( \Delta > \delta \): The sensitivity of domestic production to changes in foreign production is larger under non-linear pricing than under linear pricing.

Proof: See Appendix A.3.

Thus, our main result from Proposition 2 is that the domestic production contraction to a unit increase in foreign production – implied by a trade cost reduction – is stronger under non-linear pricing than under linear pricing. The reason is that the foreign supplier will now also benefit through an increase in the fixed fee, and therefore the foreign supplier can appropriate liberalization profits with less distortions in input demand than in the case of linear pricing. Therefore, the sensitivity of domestic firms becomes larger as its foreign rival can benefit stronger from an increase in foreign input demand.

Considering the effect of firm heterogeneity as measured by a mean-preserving spread, we also find that the regressive part of the pricing scheme does not change with \( dz = -d\bar{\pi} > 0 \). We observe that the linear part of the pricing scheme, \( \alpha_i \), depends positively on the upper bound \( \bar{\pi} \) only, but not on the lower bound. Thus, a mean-preserving spread will unambiguously increase the linear parts of both pricing schemes.

The fixed fees \( \gamma_i \) depend negatively upon the upper bound \( \bar{\pi} \) and positively upon the lower bound \( \underline{\pi} \), but at the same time also on \( \alpha_{-i} \). Let us define the function \( \Psi_i(\underline{\pi}, \bar{\pi}) = 2\underline{\pi} - \bar{\pi} + r\alpha_{-i} \) for upstream firm \( i \). A mean-preserving spread has qualitatively the same effect on \( \gamma_i \) as on \( \Psi_i \). Differentiation yields
\[ \Psi^i = 2, \Psi = -1 + \frac{(2 + r)r}{4 - r^2} = -1 + \frac{r}{2 - r}, \]

such that the effect of a mean-preserving spread on the fixed fee charged by firm \( i \) is given by

\[ d\Psi^i = \Psi^i \frac{d\Psi}{dz} + \Psi = -3 + \frac{r}{2 - r} = -\frac{2(3 - 2r)}{2 - r} < 0 \]

(since \( r < 1 \)), which shows that a mean-preserving spread will lead to lower fixed fees. We summarize this finding in

**Proposition 3** An increase in downstream firm heterogeneity, measured by a mean-preserving spread, does not change the regressive part of the pricing schemes, but (i) increases the linear part of both pricing schemes, and (ii) leads to a reduction in the both the domestic and the foreign fixed fees.

What is the intuition behind these results? Here it is the principal-agent relationship which drives these results. Both upstream firms compete for input demand by the downstream firm, but at the same time, the combination of a discount scheme and a fixed fee allows them to reap some of the downstream firm’s profits. The least productive types determine the fixed fee as they must be held back from not sourcing from a supplier at all. A mean-preserving spread makes the least productive type less productive, so this fee must fall.\(^{14}\) At the same time, since more productive types get a rent, and the rent at the high end of the distribution increases with a mean-preserving spread, making these types more important. Although the discount scheme does not change with a mean-preserving spread, now the discount scheme covers more types of firms at both ends of the distribution. In particular, the most productive types will receive a larger discount. The increase in the linear part of the price should compensate for the necessary reduction in the fixed fee and has the intention to appropriate a part of the increased profits of the most productive types.

\(^{14}\)We restrict attention to the situation where the increase in spread is not too great, so that the assumption that all firm types are served remains valid.
5 Concluding remarks

This paper has developed a model in which a domestic and a foreign upstream firm compete for orders from a domestic downstream firm. We were able to show that trade liberalization will change the pricing schemes in a particular way: while the regressive part is not affected by the level of trade costs, the linear part is unambiguously reduced. Furthermore, the domestic fixed fee declines under an increasing competitive pressure from the foreign upstream firm. However, trade liberalization makes the foreign upstream firm’s access to the domestic market easier, and thus the foreign fixed fee increases unambiguously.

Furthermore, we find that the demand for foreign intermediate goods will increase while the demand for the domestic input will decrease. While this result is similar to the case of linear pricing, the sensitivity of domestic input demand to a change in foreign input demand is larger as the foreign supplier has now more options to appropriate the benefits from trade liberalization. In conclusion, we identify another channel that determines the vulnerability of a domestic industry to trade liberalization. If this industry is part of a vertical structure in a global value chain and produces customized inputs, non-linear pricing is possible, but implies also a larger vulnerability to trade liberalization. If arbitrage is possible, implying linear pricing, we expect a lower vulnerability to trade liberalization.

Appendix

A.1 Proof of Lemma 1

Profit maximization by the downstream firm leads to demands for intermediate inputs which are given by

\[ q_1 = \max \left( 0, \frac{b(\theta_1 - p_1) - (b - \mu)(\theta_2 - p_2)}{(2b - \mu)\mu} \right), \]
\[ q_2 = \max \left( 0, \frac{b(\theta_2 - p_2) - (b - \mu)(\theta_1 - p_1)}{(2b - \mu)\mu} \right). \]

Given Assumption 2, we will be able to confine the analysis to interior solutions. Even if both firms charge marginal costs, that is, \( p_1 = 0 \) and \( p_2 = t \), the foreign firm will receive
some demand if $\theta_1 = \bar{\theta}$ and $\theta_2 = \theta$. Assume that the equilibrium prices $p_1$ and $p_2$ are such that $q_1, q_2 > 0$ (to be verified later). Then, the upstream firms do not know $(\theta_1, \theta_2)$, but form expectations on the downstream firm’s type. Consequently, let

$$\hat{\theta} = \int_{\theta}^{\bar{\theta}} \theta f(\theta) d\theta$$

denote the common expected value of both $\theta_1$ and $\theta_2$. Given $\hat{\theta}$, each upstream firm is able to compute its expected demand, and these demands are given by

$$\hat{q}_1 = \frac{\hat{\theta}\mu - bp_1 + (b - \mu)p_2}{(2b - \mu)\mu}, \hat{q}_2 = \frac{\hat{\theta}\mu - bp_2 + (b - \mu)p_1}{(2b - \mu)\mu}. \quad (A.2)$$

Both upstream firms maximize their expected profits $p_1\hat{q}_1$ and $(p_2 - t)\hat{q}_1$ w.r.t. $p_1$ and $p_2$, respectively. This leads to Bertrand equilibrium prices

$$p_1 = \frac{\hat{\theta}(3b - \mu)\mu + b(b - \mu)t}{(3b - \mu)(b + \mu)}, p_2 = \frac{\hat{\theta}(3b - \mu)\mu + 2b^2t}{(3b - \mu)(b + \mu)}.$$

Furthermore, note that $p_1 - p_2 < t$ for all $t > 0$. Furthermore, Assumption 2 implies that

$$\frac{\theta - b - \mu}{b} \hat{\theta} > t > p_2 - \frac{b - \mu}{b}p_1 > 0$$

that guarantees that $q_2 > 0$ for all productivity combinations, and since $p_2 - \frac{b - \mu}{b}p_1 > 0$, $q_1 > 0$ as well. □

### A.2 Proof of Proposition 1

We start from the assumption that firm 1 believes that firm 2 will apply the non-linear pricing scheme

$$T_2(q_2) = \gamma_2 + \alpha_2 q_2 + \frac{\beta_2}{2} q_2^2, \text{ if } q_2 > 0.$$

Under this assumption, firm 1 can now rewrite the downstream firm’s profit as

$$\Pi = \theta_1 q_1 + \theta_2 q_2 - q_2 q_1 (b - \mu) - \frac{1}{2} b q_1^2 - \frac{b q_2^2}{2} - \gamma_2 - \alpha_2 q_2 - \frac{1}{2} \beta_2 q_2^2 - T_1(q_1),$$

if $q_2 > 0$. Assume it is optimal for the downstream firm to choose $q_2 > 0$. Then, differentiation w.r.t. $q_2$ yields the first and second order conditions
\[
\begin{align*}
\frac{\partial \Pi}{\partial q_2} &= -\alpha_2 - q_2 (b + \beta_2) + q_1 (\mu - b) + \theta_2 = 0 \quad (A.3) \\
\frac{\partial^2 \Pi}{\partial q_2^2} &= -(b + \beta_2) < 0. \quad (A.4)
\end{align*}
\]

Consequently, if \( b + \beta_2 > 0 \) and the optimal input demands are strictly positive (both to be verified later), firm 1 will anticipate that the downstream demand for input 2 is equal to

\[
q_2 = \frac{\theta_2 - \alpha_2 - q_1 (b - \mu)}{b + \beta_2}, \quad (A.5)
\]

and thus the downstream profit (net of the transfer \( T_1(q_1) \) to the upstream firm 1) can be rewritten as a function of \( q_1 \) only:

\[
\Pi = \left( \frac{(\theta_2 - \alpha_2)^2}{2 (b + \beta_2)} - \gamma_2 + q_1 \left( \frac{\alpha_2 (b - \mu)}{b + \beta_2} - \frac{\theta_2 (b - \mu)}{b + \beta_2} + \theta_1 \right) - \frac{q_1^2 (b \beta_2 + \mu (2b - \mu))}{2 (b + \beta_2)} \right) - T_1(q_1).
\]

The downstream firm’s gross profit (i.e., before subtracting the transfer \( T_1(q_1) \)) is denoted by the subscript \( G \) and given by

\[
\Pi^G = \left( \frac{(\theta_2 - \alpha_2)^2}{2 (b + \beta_2)} - \gamma_2 + q_1 \left( \frac{\alpha_2 (b - \mu)}{b + \beta_2} - \frac{\theta_2 (b - \mu)}{b + \beta_2} + \theta_1 \right) - \frac{q_1^2 (b \beta_2 + \mu (2b - \mu))}{2 (b + \beta_2)} \right).
\]

We make the following observations and assumptions:

- The term

\[
\frac{(\theta_2 - \alpha_2)^2}{2 (b + \beta_2)} - \gamma_2
\]

is the outside option of the downstream firm: this is the profit level when the downstream firm decides not to source from firm 1 at all. Consequently, this term is the reservation profit of the downstream firm when it considers whether to order from input supplier 1.

- The coefficient for the term \( q_1^2 \) does not depend on \((\theta_1, \theta_2)\). It depends only on other parameters. This fact will be used to explain later why the discount scheme does
not differ between the two upstream firms.

- We can replace the statistics

\[ \theta_1 - \frac{\theta_2(b - \mu)}{b + \beta_2} \]

by the statistics \( z_1 \). To compute the distribution of \( z_1 \) we make use of the joint density function \( \varphi(\theta_1, \theta_2) \) and the definition of \( z_1 \). We define the joint density function for \((z_1, \theta_2)\) by

\[ g(z_1, \theta_2) \equiv \varphi \left( z_1 + \frac{(b - \mu)\theta_2}{b + \beta_2}, \theta_2 \right). \]

Then the density function of \( z_1 \) is

\[ g(z_1) = \int_{\theta} \varphi \left( z_1 + \frac{(b - \mu)\theta_2}{b + \beta_2}, \theta_2 \right) d\theta_2. \]

Under the assumption that \( b + \beta_2 > 0 \), and since \( b - \mu > 0 \) (i.e. the two inputs are substitutes on aggregate) the highest value of \( z_1 \) is

\[ z \equiv \bar{\theta} - \frac{(b - \mu)\theta}{b + \beta_2} \]

and the lowest possible value of \( z_1 \) is

\[ z \equiv \bar{\theta} - \frac{(b - \mu)\bar{\theta}}{b + \beta_2} \]

- We assume \( z_1 \) is uniformly distributed so that the cumulative distribution function is

\[ G(z_1) = 1 - \frac{(z - z_1)}{(\bar{z} - z)} \]

Using the statistics \( z_1 \) allow us to rewrite downstream gross profit as

\[ \Pi^G = \frac{(\theta_2 - \alpha_2)^2}{2(b + \beta_2)} - \gamma_2 + q_1 \left( \frac{\alpha_2(b - \mu)}{b + \beta_2} + z_1 \right) - q_1^2 \frac{(b\beta_2 + \mu(2b - \mu))}{2(b + \beta_2)}. \]

Using \( z_1 \), we can now employ the Revelation Principle and the concept of the virtual surplus. First, note that
\[ \frac{\partial \Pi G}{\partial z_1} \frac{\partial z_1}{\partial \theta_1} = \frac{\partial \Pi G}{\partial z_1} = q_1(z_1) \]
due to the Envelope Theorem. Firm 1 now maximizes its expected upstream profits s.t. feasibility, incentive compatibility and individual rationality of the downstream firm. Due to Myerson (1981), this maximization problem is equivalent to maximizing the expected virtual surplus s.t. feasibility and monotonicity (see also Fudenberg and Tirole, 1991). The expected virtual surplus is given by

\[ \hat{V} = \int_{\tilde{z}}^{\bar{z}} \left\{ q_1(z_1) \left( \frac{\alpha_2(b - \mu)}{b + \beta_2} + z_1 - m_1 \right) - \frac{[q_1(z_1)]^2 (b\beta_2 + \mu(2b - \mu))}{2(b + \beta_2)} \right\} dG(z_1) \]

The following remarks are in order:

- The first part of the virtual surplus is the aggregate surplus which can be achieved by the downstream firm dealing with input supplier 1; hence, it does neither contain \( T(q_1) \) (which cancels out) nor the reservation profit of the downstream firm (which has to be subtracted). It does, however, take into account the production costs \( m_1 q_1 \). The second part, \( \left[ (1 - G(z_1)/g(z_1)) \frac{\partial \Pi G}{\partial z_1} \right] dG(z_1) \), is the efficiency loss that reflects the fact the principal has to concede greater informational rents to more efficient firm types.
- The last line has made use of our assumption that \( G(z_1) \) is uniform.

Assuming interior solutions, let us denote the optimal input supply by \( q_1(z_1) \). The first-order condition for maximizing the virtual surplus is given by

\[ \frac{\partial \hat{V}}{\partial q_1(z_1)} = \frac{\alpha_2(b - \mu)}{b + \beta_2} + 2z_1 - m_1 - \bar{z} - \frac{q_1(z_1) (b\beta_2 + \mu(2b - \mu))}{b + \beta_2} = 0, \forall z_1 \in [\tilde{z}, \bar{z}], \hspace{1em} (A.6) \]

Eq. (A.6) is necessary and sufficient only if

\[ \frac{b\beta_2 + \mu(2b - \mu)}{b + \beta_2} > 0 \hspace{1em} (A.7) \]
which requires that $b\beta + \mu (2b - \mu) > 0$ if $b + \beta > 0$ and has to be verified later. In this case, eq. (A.6) leads to an optimal input supply of

$$q_1(z_1) = \frac{\alpha_2 (b - \mu) + (2z_1 - m_1 - \bar{z}) (b + \beta_2)}{b\beta + \mu (2b - \mu)}.$$  

(A.8)

This equation can also be written as

$$z_1 = \frac{b\beta_2 + \mu (2b - \mu)}{2 (b + \beta_2)} q_1(z_1) - \frac{\alpha_2 (b - \mu)}{2 (b + \beta_2)} + \frac{(m_1 + \bar{z})}{2}.$$  

(A.9)

We observe that monotonicity is fulfilled as $q_1(z_1)$ increases with $z_1$ if condition (A.7) is fulfilled. As for implementation, we use eq. (9) from our main text:

$$z_1 + \frac{(b - \mu) \alpha_2}{b + \beta_2} - \left( b - \frac{(b - \mu)^2}{b + \beta_2} \right) q_1(z_1) - T_1'(q_1(z_1)) = 0$$

This leads to

$$T'(q_1(z_1)) = z_1 + \frac{(b - \mu) \alpha_2}{b + \beta_2} - \left( \frac{b^2 + b\beta_2 - b^2 - \mu^2 + 2b\mu}{b + \beta_2} \right) q_1(z_1)$$

$$= z_1 + \frac{(b - \mu) \alpha_2}{b + \beta_2} - \left( \frac{b\beta_2 + \mu (2b - \mu)}{b + \beta_2} \right) q_1(z_1).$$  

(A.10)

Using $q_1 = q_1(z_1)$, eqs. (A.9) and (A.10) allow us to eliminate $z_1$ and determine the marginal transfers which implement the optimal inputs as

$$T'(q_1(z_1)) = \frac{b\beta_2 + \mu (2b - \mu)}{2 (b + \beta_2)} q_1(z_1) - \frac{\alpha_2 (b - \mu)}{2 (b + \beta_2)} + \frac{(m_1 + \bar{z}) (b + \beta_2)}{2 (b + \beta_2)}$$

$$+ \frac{2(b - \mu) \alpha_2}{2(b + \beta_2)} - \left( \frac{2 [b\beta_2 + \mu (2b - \mu)]}{2(b + \beta_2)} \right) q_1(z_1),$$

such that

$$T_1'(q_1) = \frac{\alpha_2 (b - \mu) + (m_1 + \bar{z}) (b + \beta_2)}{2 (b + \beta_2)} - \frac{b\beta_2 + \mu (2b - \mu)}{2 (b + \beta_2)} q_1.$$  

(A.11)

Since $T_1'(q_1)$ is linear in $q_1$, $T_1(q_1)$ is linear-quadratic in $q_1$ such that $T_1(q_1) = \gamma_1 + \alpha_1 q_1 + \frac{\beta_1}{2} q_1^2$. Mutual consistency requires that firm 2 also expects firm 1 to employ a linear-quadratic transfer scheme, and thus eq. (A.11) implies:
\[ \beta_1 = -\frac{b\beta_2 + \mu(2b - \mu)}{2(b + \beta_2)}, \quad \beta_2 = -\frac{b\beta_1 + \mu(2b - \mu)}{2(b + \beta_1)}, \quad \text{(A.12)} \]

\[ \alpha_1 = \frac{r\alpha_2 + m_1 + \bar{z}}{2}, \quad \alpha_2 = \frac{r\alpha_1 + m_2 + \bar{z}}{2}, \quad \text{where } r = \frac{b - \mu}{b + \beta} < 1 \]

Clearly, \( \beta_1 = \beta_2 = \beta \) as can be easily verified.\(^{15}\) Thus we obtain the quadratic equation
\[ 2\beta^2 + 3b\beta + \mu(2b - \mu) = 0 \]
and the relevant solution that ensures \( b + \beta > 0 \) is given by
\[ \beta = \frac{b}{4}\sqrt{1 + 8\left(\frac{(b - \mu)^2}{b^2} - 3\right)} < 0. \]

Since \( b + \beta > 0 \), it follows that \( b\beta + \mu(2b - \mu) > 0 \).\(^{16}\) It is simple to verify that \( b + \beta > b - \mu \).

Having computed \( \beta \), we now solve for \( \alpha_1 \) and \( \alpha_2 \). We have the system
\[ 2\alpha_1 - r\alpha_2 = m_1 + \bar{z} \quad \text{(A.13)} \]
\[ -r\alpha_1 + 2\alpha_2 = m_2 + \bar{z} \quad \text{(A.14)} \]

Solving this system of equations, we get
\[ \alpha_i = \frac{1}{4 - r^2} \left[2(m_i + \bar{z}) + r(m_{-i} + \bar{z})\right]. \quad \text{(A.15)} \]

With \( m_1 = 0 \) and \( m_2 = t \),
\[ \alpha_1 = \frac{1}{4 - r^2} \left[(2 + r)\bar{z} + rt\right], \quad \text{(A.16)} \]
\[ \alpha_2 = \frac{1}{4 - r^2} \left[(2 + r)\bar{z} + 2t\right]. \quad \text{(A.17)} \]

Let us now turn to the equilibrium orders of the downstream firm. There are two alternative ways of obtaining these. The first one consists of re-writing the upstream firm 1’s optimal input supply eq. (A.8) as follows:
\[ q_1(z_1) = \frac{r\alpha_2 + (2z_1 - m_1 - \bar{z})}{[b\beta_2 + \mu(2b - \mu)]/(b + \beta)} = \frac{r\alpha_2 + (2z_1 - m_1 - \bar{z})}{-2\beta} \quad \text{(A.18)} \]

\(^{15}\)The two equations \( 2(b + \beta_2)\beta_1 = -b\beta_2 - \mu(2b - \mu) \) and \( 2(b + \beta_1)\beta_2 = -b\beta_1 - \mu(2b - \mu) \) lead to \( 2b(\beta_1 - \beta_2) = b(\beta_1 - \beta_2) \) which implies \( \beta_1 = \beta_2 \).

\(^{16}\)This is because \( 2\beta^2 + 3b\beta + \mu(2b - \mu) = 0 \) can be written as \( b\beta + \mu(2b - \mu) = -2\beta(b + \beta) > 0 \).
and, thus, by symmetry,
\[ q_i(z_i) = \frac{r\alpha_i + (2z_i - m_i - \bar{z})}{-2\beta}, \quad (A.19) \]

The second way makes use of the downstream firm’s optimal input demand equations, eq. (A.5), and symmetry,
\[
\begin{align*}
\theta_1 - bq_1 - (b - \mu)q_2 - \alpha_1 - \beta q_1 &= 0, \\
\theta_2 - bq_2 - (b - \mu)q_1 - \alpha_2 - \beta q_2 &= 0,
\end{align*}
\]

These two equations can be rewritten in matrix form as
\[
\begin{bmatrix}
-(b + \beta) & -(b - \mu) \\
-(b - \mu) & -(b + \beta)
\end{bmatrix}
\begin{bmatrix}
q_1 \\
q_2
\end{bmatrix} =
\begin{bmatrix}
\alpha_1 - \theta_1 \\
\alpha_2 - \theta_2
\end{bmatrix},
\]

and its determinant is equal to \((b + \beta)^2 - (b - \mu)^2 > 0\) because \((b + \beta) > (b - \mu)\). Then, given the equilibrium transfer schemes, the downstream input demands are given by

\[ q_i = \frac{(b + \beta) (z_i + r\alpha_{-i} - \alpha_i)}{(b + \beta)^2 - (b - \mu)^2} = \frac{z_i + r\alpha_{-i} - \alpha_i}{b + \beta - \frac{(b - \mu)^2}{b + \beta}} - \frac{\beta}{-\beta}, \quad (A.20) \]

because \(b - \frac{(b - \mu)^2}{b + \beta} = -2\beta\). Note that if we impose \(\beta = 0\), then the first line of eq. (A.20) coincides with the linear pricing case of Appendix A.1.

The two solutions (A.19) and (A.20) are identical because
\[ 2 (z_i + r\alpha_{-i} - \alpha_i) = r\alpha_{-i} + (2z_i - m_i - \bar{z}) \]
as can be verified using (A.13).

The lowest demand for input 2 is \(q_2\) for \(\theta_2 = \bar{\theta}\) and \(\theta_1 = \bar{\theta}\), i.e., when \(z_2 = \bar{z}_2 = \bar{\theta}_2 - r(\mu)\bar{\theta}_1\)

\[ q_2(z_2) = \left(\frac{1}{-\beta}\right) (z + r\alpha_1 - \alpha_2). \]

Note that \(q_2(z)\) is strictly positive iff \(z + r\alpha_1 - \alpha_2 > 0\).
As
\[ r\alpha_1 - \alpha_2 = -\frac{1}{4 - r^2} \left( (2 - r^2) t + r(2 + r)(1 - r)z \right), \]
and
\[ \bar{z} + r\alpha_1 - \alpha_2 = \frac{1}{4 - r^2} \left[ (4 - r^2) \bar{z} - r(2 + r)(1 - r)z - (2 - r^2)t \right], \]
and \( q_2(\bar{z}) > 0 \) iff
\[ (4 - r^2) \bar{z} - r(2 + r)(1 - r)z - (2 - r^2)t > 0 \]
\[ \iff \theta > \frac{[3r - 2r^2]}{(2 - r^2) \theta} + \frac{t}{(2 + r)}. \]
Finally, we prove that the equilibrium fixed charge is equal to
\[ \gamma_i = \left[ \frac{2\bar{z} - \bar{z} + r\alpha_i - m_i}{-8\beta} \right]^2. \]
Recall that
\[ II = \frac{(\theta_2 - \alpha_2)^2}{2(b + \beta_2)} - \gamma_2 - q_1 \left( \frac{\alpha_2(b - \mu)}{b + \beta_2} - \frac{\theta_2(b - \mu)}{b + \beta_2} + \theta_1 \right) \]
\[ - \frac{q_1^2 (b\beta_2 + (2b - \mu))}{2(b + \beta_2)} - T_1(q_1). \]
which is equal to
\[ \omega_2(\theta_2) + q_1(z_1) [r\alpha_2 + z_1] - \gamma_1 - \alpha_1 q_1(z_1) - \frac{\beta_2}{2} q_1^2 - \frac{q_1^2 (b\beta_2 + (2b - \mu))}{2(b + \beta_2)} \]
where
\[ \omega_2(\theta_2) = \frac{(\theta_2 - \alpha_2)^2}{2(b + \beta_2)} - \gamma_2 \]
is the surplus of the downstream firm if it deals only with firm 2. Collecting terms, and recalling that \( \beta_1 = \beta_2 = \beta \), we have
\[ \omega_2(\theta_2) - \gamma_1 + (r\alpha_2 - \alpha_1 + z_1) q_1(z_1) - \frac{1}{2(b + \beta)} q_1^2 [\beta(b + \beta) + b\beta_2 + \mu(2b - \mu)]. \]
Now,
\[ \beta(b + \beta) + b\beta_2 + \mu(2b - \mu) = \{2\beta^2 + 3b\beta + \mu(2b - \mu)\} - \beta^2 - b\beta \]
and $2\beta^2 + 3b\beta + \mu(2b - \mu) = 0$ (see eq. (A.12)). Thus,

$$\Pi(z_1, \theta_2) = \omega_2(\theta_2) - \gamma_1 + (r\alpha_2 - \alpha_1 + z_1)q_1(z_1) + \frac{\beta}{2}q_1(z_1)^2$$

But from (A.20),

$$r\alpha_2 - \alpha_1 + z_1 = -\beta q_1(z_1)$$

and thus

$$\Pi(z_1, \theta_2) = \omega_2(\theta_2) - \gamma_1 - \beta q_1(z_1)^2 + \frac{\beta}{2}q_1(z_1)^2 = \omega_2(\theta_2) - \gamma_1 - \frac{\beta}{2}q_1(z_1)^2. \quad \text{(A.21)}$$

Now look at the upstream firm 1’s marginal customer types, $z_1 = z$. Using (A.19), we find:

$$\Pi(z_1, \theta_2) = \omega_2(\theta_2) - \gamma_1 - \frac{\beta}{2} \left( \frac{r\alpha_2 + (2z_1 - m_1 - \bar{z})}{-2\beta} \right)^2 = \omega_2(\theta_2) - \gamma_1 - \frac{[r\alpha_2 + (2z_1 - m_1 - \bar{z})]^2}{(8\beta)}.$$ 

Then, $\gamma_1$ is chosen to make the marginal firm types $(z_1, \theta_2)$ indifferent between dealing with both upstream firms, or with upstream firm 2 only:

$$\gamma_1 = \frac{[r\alpha_2 + (2z_1 - m_1 - \bar{z})]^2}{(-8\beta)} > 0 \text{ if } q_1(z_1) > 0.$$

This concludes the proof.

**A.3 Proof of Proposition 2**

We do the proof by contradiction: $\delta > \Delta$ implies $(b - \mu)(2 - r^2) > (b + \mu)r$ or

$$(b - \mu) \left( 2 - \left( \frac{b - \mu}{b + \beta} \right)^2 \right) > (b + \mu) \left( \frac{b - \mu}{b + \beta} \right) \iff 2(b + \beta)^2 > (b + \mu)(b + \beta) + (b - \mu)^2$$
Define $B = b + \beta$ and $\phi(B) = 2B - B(b + \mu) - (b - \mu)^2$. $\delta > \Delta$ is equivalent to $\phi(B) > 0$. We find that $\phi(B_1(\mu)) = \phi(B_2(\mu)) = 0$ for

$$B_1(\mu) = \frac{1}{4} \left( b + \mu - \sqrt{9b^2 - 14b\mu + 9\mu^2} \right),$$

$$B_2(\mu) = \frac{1}{4} \left( b + \mu + \sqrt{9b^2 - 14b\mu + 9\mu^2} \right).$$

Furthermore,

$$\phi' = 4B - (b + \mu), \phi'' = 4,$$

showing that $\phi(B)$ is convex in $B$ and has a minimum at $\hat{B} = (b + \mu)/4$ where $B_1 < \hat{B} < B_2$. Let

$$B^*(\mu) = \frac{b}{4} \left( \sqrt{1 + \frac{8(b - \mu)^2}{b^2}} + 1 \right)$$

(A.22)
denote the $B$ implied by the optimal pricing scheme. Then, $\delta > \Delta$ if either $B^*(\mu) > B_2(\mu)$ or if $B^*(\mu) < B_1(\mu)$. As for $B^*(\mu) > B_2(\mu)$, note that only $\mu = 0$ equalizes $B^*(\mu)$ and $B_2(\mu)$ such that $B^*(0) = B_2(0) = b$. Furthermore, we find that $B''(\mu = 0) = -2/3$ and $B_2'(\mu = 0) = -1/3$, proving that $B^*(\mu) < B_2(\mu)$.

As for $B^*(\mu) < B_1(\mu)$, first note that $B^*(\mu) > b/2$, so $B^*(\mu) < B_1(\mu)$ is contradicted if

$$\frac{b}{2} > \frac{1}{4} \left( b + \mu - \sqrt{9b^2 - 14b\mu + 9\mu^2} \right) \Leftrightarrow$$

$$0 > -\frac{b - \mu}{4} - \frac{1}{4} \sqrt{9b^2 - 14b\mu + 9\mu^2},$$

which is true. □

References


