Abstract: High inter-country variability characterises the responsiveness of both output to (exogenous) shocks and employment to output contractions. We argue that inter-country differences in firm-size distributions contribute to explaining this variability. Within an open economy model, we show that competitive selection processes are an important channel through which a shock affects aggregate employment. Intra-industry selection is then shown to influence the effectiveness of active labour market policies in countering the employment and welfare effects of a negative shock.

JEL classification: E22, E64, F12, F41

Keywords: job creation, employment subsidies, competitive selection, international trade

1. INTRODUCTION

Two major sets of stylised facts emerge from the Great Recession. The first is the high inter-country variability in the responsiveness of both output to (exogenous) shocks and employment to output contractions. The second concerns a renewed policy focus on the adoption of job creation strategies and aggregate demand support via Active Labour Market Policies (ALMPs) to complement growth strategies.

The aim of this paper is twofold. First, we shall examine whether inter-country differences in intra-industry heterogeneity can help explain different performance in aggregate employment and welfare and how the degree and nature of international trade openness affect this channel. Second, we shall study how the use of this channel affects the effectiveness of ALMPs in influencing aggregate outcomes in response to exogenous shocks.

The variability among countries in output and employment performance reflects country-specific productivity responses to shocks which have typically been explained with differences in labour market institutions (e.g. employment protection laws that affect lags in laying-off workers in a recession; work-sharing agreements) and/or in aggregate economic structures (e.g., countries specialised in relatively labour intensive sectors experience higher employment responses). As documented by an extensive body of literature, however, the existence of a significant degree of intra-industry heterogeneity between firms in characteristics, behaviour and performance in international markets\(^1\) suggests that inter-country differences in firm size (and productivity) distribution can be an important channel through which a shock and policies affect aggregate outcomes. In this paper we conjecture that variations across countries in the productivity distribution of firms can contribute to explaining the observed differences in aggregate employment, and that the degree and nature of international trade openness affect this channel – which is of relevance in predicting the level and effectiveness of policy interventions aimed at increasing employment and/or offsetting the effects of negative shocks.

The fact that exposure to international competition enhances competitive selection within and across industries is now widely acknowledged by governments, and helping firms to protect and create jobs by supporting their cost competitiveness is increasingly seen as an important complement to growth strategies in a globalised environment (see, e.g., IMF, 2013). In particular, the use of Active Labour Market Policies (ALMPs), which is widespread across both advanced and industrialising economies, has intensified during the Great Recession which has seen the adoption of job creation strategies and aggregate demand support by many OECD countries to reduce structural unemployment as well as to offset the

\(^1\) For recent surveys of the theoretical and empirical literature see, respectively, Melitz and Redding (2012) and Bernard et al. (2012).
impact of negative shocks, even though a significant disparity on spending exists across countries (Kluve, 2010). These programmes consist of policies aimed at reducing search frictions (e.g. public employment services), increasing employability (e.g. training schemes), or at direct job creation. The latter include direct employment programmes in the public sectors, but also measures directed at either private employers or workers that seek to influence hiring and labour force participation – such as wage and labour subsidies.²

To address these issues and capture the relevant stylised fact discussed above, we develop a theoretical two sector model characterised by inter-firm productivity heterogeneity and an endogenous labour supply. Within this framework, we show intra-industry inter-firm heterogeneity and selection to be a channel through which shocks, by affecting average industry productivity, impact on aggregate employment and welfare. Specifically, a negative demand shock results in an anti-competitive effect that – by reallocating market shares towards less efficient firms – lowers average industry productivity, aggregate employment and welfare. We also examine how intra-industry reallocations influence the effectiveness of Active Labour Market Policies (ALMP), in the form of employment subsidies, in countering the effects of exogenous shocks on employment and welfare. In particular, we focus on the interaction between the degree of heterogeneity within and across countries and international trade openness in determining the nature of the optimal policy, its effects on aggregate employment and its effectiveness in countering the effects of exogenous shocks on employment and welfare. We find that competitive selection and intra-industry structure affect the usefulness of ALMP in countering the employment and welfare effects of a negative shock. Specifically, we show that the optimal policy entails a positive subsidy that softens competition and results in a reallocation of resources away from leisure, inter-sectorally towards the monopolistic sector, and – within the latter – away from the most efficient and towards less efficient firms. The intuition behind these results is that the subsidy policy contributes to correct the market distortion that arises from differences in mark-up between the monopolistic sector on the one hand and leisure, the outside good and the imported varieties on the other – a distortion that results in an under-consumption of the monopolistic good and in an under-supply of labour (or an over-consumption of leisure). Essentially, via the subsidy policy, the government controls the selectivity of competition with a view to maximising employment. The most effective way to do so is to induce entry –

² These policies are central to the “European Employment Strategy” to address structural unemployment and to increase labour participation and are a cornerstone of the Social Investment model of the welfare state. See Andersen and Svarer (2012) for a discussion of the Danish case. The 2013 EU Annual Growth survey, available at http://ec.europa.eu/europe2020/making-it-happen/annual-growth-surveys/index_en.htm encourages the member states to step up ALMP, paying specific attention to maintaining and even reinforcing their coverage and effectiveness. The implementation of such policies to create employment heavily featured in the ILO-IMF 2010 conference in Oslo on “The Challenges of Growth, Employment and Social Cohesion”, and have recently been advocated by the IMF (2013).
even when this entails reallocating resources towards firms lower productivity firms. Indeed, we show that the dominant policy in this context is to target subsidisation towards the domestic operations of firms only.

Our research is related to a strand of the literature that highlights the impact of intra-industry reallocations on aggregate performance. Di Giovanni and Levchenko (2013) find that the size composition of industries interacts with trade openness in determining aggregate output volatility. Several studies document how misallocations across heterogeneous production units can affect aggregate productivity and the transmission of shocks (e.g., Baily et al., 1992; Restuccia and Rogerson, 2010). Of particular interest is that different firms exhibit different cyclical patterns of net job creation (Moscarini and Postel-Vinay, 2012; Elsby and Michaels, 2013). It is therefore plausible to conjecture that intra-industry reallocations are also likely to have some impact on the aggregate employment effects of shocks. A further implication of these studies is that policy-induced distortions can be responsible for the observed inter-country variations in the strength of the inter-firm productivity-size link and for total factor productivity differences (e.g. Bartelsman et al., 2013). For instance, Garicano et al. (2013) and Gourio and Roys (2013) show that size dependent regulations affect both the firm-size distribution and the extent of industry misallocations. These papers, however, do not consider the interaction between competitive selection on the one hand and labour market policies aimed at increasing employment and trade openness on the other.

Another strand of the literature to which our work is related concerns the effects of policy on competitive selection. There still is to date a fairly small number of contributions that focus directly on the effects of policy within a heterogenous cost setting. Notable exceptions comprise Demidova and Rodriguez-Clare (2009), who focus on the effects of trade policy for a small open economy, and by Felbermayr et al (2013) who consider non-cooperative tariff policies within a two country setting. Contrary to our model, both of these papers assume a one sector economy (that is there is no outside good) and an exogenous labour supply and their focus in not on the employment creation policies. Within a static general equilibrium macro-model with imperfectly competitive goods markets, but in with no productivity heterogeneity, Molana et al (2012) examine the effects of wage and output subsidies and show that, for a small open economy, positive tax and subsidy rates exist which maximize welfare, rendering no intervention suboptimal. They also show that, within a two-country setting, a Nash non-cooperative symmetric equilibrium with positive tax and subsidy rates exists, and that cooperation between governments in setting these rates is more expansionary and leads to an improvement upon the non-cooperative solution. This paper
extends their basic framework to allow for inter-firm cost heterogeneity within the monopolistic sector. We first develop a closed economy version of the model.

The rest of the paper is organised as follows. Section 2 develops the autarkic model. Section 3 extends the model to a small open economy case and Section 4 concludes the paper.

2. AUTARKY

2.1. Demand and technology
Consider an economy consisting of two sectors, one imperfectly and one perfectly competitive, respectively producing a horizontally differentiated and a homogeneous commodity.

On the demand side, the representative consumer maximises a utility function defined over the two consumption goods and labour supply,

$$u = \left( \frac{a}{1 - \beta} \right)^{1-\beta} \left( \frac{y}{\beta} \right)^{\beta} \left( \frac{\theta h^{1+\delta}}{1 + \delta} \right), \quad 0 < \beta < 1, \quad \delta > 0, \quad \theta > 0,$$

subject to the budget constraint,

$$P_A a + P_y y = (1 - t) wh,$$

where $h$ is the time spent at work, $a$ and $P_A$ are the quantity and the price of the homogenous commodity, $y$ and $P_y$ are the quantity and price of the differentiated good, $w$ is the wage rate and $t$ is the proportional income tax rate.

Denoting by $N$ the number of consumers, the aggregate labour supply function and the demand functions for the two goods are, respectively,

$$L' = Nh = N \left( \frac{(1-t)w}{\theta P} \right)^{1/\sigma}, \quad A = Na = \frac{(1 - \beta)(1-t)wL'}{P_A}, \quad Y = N\bar{y} = \frac{\beta(1-t)wL'}{P_y},$$

where $P = P_A^{1-\beta}P_D^\beta$ is the consumer price index. $Y$ is assumed to be a CES bundle of differentiated varieties with ‘dual’ price index $P_y$, respectively given by

$$Y = \left( \int_{i \in M} y(i)^{1-\sigma} di \right)^{1/(1-\sigma)} \quad \text{and} \quad P_y = \left( \int_{i \in M} p(i)^{1-\sigma} di \right)^{1/(1-\sigma)},$$

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3 Bilbiie et al. (2008) study the efficiency properties of a one good dynamic stochastic general equilibrium model with homogenous firms and consider a number of policy interventions aimed at achieving Pareto optimality.

4 We choose disregard the lump-sum taxation for simplicity, and concentrate only on the proportional income tax which accounts for the bulk of tax revenue from the personal sector in advanced industrial economies.
where $M$ is the set of available varieties, $y(i)$ and $p(i)$ are the quantity consumed and the price of variety $i$ respectively, and $\sigma > 1$ is the constant elasticity of substitution between varieties. The demand for each variety is then

$$y(i) = Y \left( \frac{p(i)}{P} \right)^{-\sigma}, \quad i \in M. \tag{5}$$

The homogenous good is produced under perfectly competitive conditions using a constant returns to scale technology with a unit labour requirement of one, i.e. $L_A = A^\ell$ where $L_A$ and $A^\ell$ denote the labour demand by this sector and the supply of the good, respectively. The constant returns to scale technology, the zero-profit condition and free mobility of labour across the two sectors imply the equality between the wage rate and the price of the homogeneous good, hence $w = P_A$. We use this good as the numeraire and normalise $P_A = 1$, which in turn implies $w = 1$ and $P = P_y^\ell$.

In the differentiated good sector, each firm produces one variety of the good using a linear technology with increasing returns to scale. Labour is the only input and the labour requirement, to produce and market the quantities $y$ of a variety, is $l(\varphi) = \alpha + \frac{y(\varphi)}{\varphi}$ where we have dropped the variety indicator $i$ and distinguished the firm by its productivity parameter $\varphi \in [1, \infty)$. $1/\varphi$ is a firm’s marginal labour requirement and $\alpha$ is its fixed labour requirement. A firm’s profit therefore is $\pi(\varphi) = p(\varphi)y(\varphi) - (1-s)l(\varphi)$ where $s \in [0, 1]$ is the employment subsidy rate that firms receive from the government. For any given subsidy rate, the firm chooses its price to maximise profit subject to its technology and demand, but ignoring the effect of its action on the industry price index. This yields the familiar mark-up rule, $p(\varphi) = \frac{\sigma(1-s)}{(\sigma - 1)\varphi}$. Given this, and defining $r(\varphi) = p(\varphi)y(\varphi)$, it then follows that operating profits are $\pi(\varphi) = r(\varphi)/\sigma - (1-s)\alpha$.

Following Montagna (1995) and Melitz (2003) we assume that, before they can set up and start producing, a large pool $F$ of identical potential entrants make an initial ‘entry investment’ which amounts to paying a fixed entry sunk cost $f$ measured in terms of the numeraire good. This investment enables entrants to draw their technology, as embodied in the specific value of the productivity parameter $\varphi$. The draw is from a common population with a known p.d.f. $g(\varphi)$ defined over support $[1, \infty)$ with a continuous cumulative distribution $G(\varphi)$. A firm’s survival in the market will depend on the magnitude of its
\( \varphi \in [1, \infty) \) in relation to the threshold \( \varphi_c \), which satisfies \( \pi(\varphi_c) = 0 \); firms with \( \varphi \in [1, \varphi_c) \) will not enter since they would make a loss, while those with \( \varphi \in [\varphi_c, \infty) \) will succeed and all firms with \( \varphi \in (\varphi_c, \infty) \) make positive profits. Prior to entry, therefore, it is known that a fraction \( G(\varphi_c) \) of \( F \) will fail to enter while the fraction \( M \equiv (1 - G(\varphi_c))F \) will succeed. Thus, ex-post, \( M \) is the mass of varieties available to consumers. We can therefore redefine the p.d.f of the surviving (incumbent) firms over \( \varphi \in [\varphi_c, \infty) \) by \( \mu(\varphi) = \frac{g(\varphi)}{1 - G(\varphi_c)} \), which can then be used to obtain a measure of the aggregate productivity of the surviving firms as the weighted average of the productivity levels \( \varphi \in [\varphi_c, \infty) \),

\[
\bar{\varphi} = \left( \int_{\varphi_c}^{\infty} \varphi^{\sigma-1} \mu(\varphi) \, d\varphi \right)^{\frac{1}{\sigma-1}} .
\]

Using \( p(\bar{\varphi}) / p(\varphi_c) = \varphi_c / \bar{\varphi} \) and \( y(\bar{\varphi}) / y(\varphi_c) = (\bar{\varphi} / \varphi_c)^{\sigma} \) and the definition \( r(\varphi) = p(\varphi) y(\varphi) \), we obtain

\[
\frac{r(\bar{\varphi})}{r(\varphi_c)} = \left( \frac{\varphi_c}{\bar{\varphi}} \right)^{\sigma-1} \Rightarrow r(\bar{\varphi}) = \alpha \sigma (1 - s) \left( \frac{\bar{\varphi}}{\varphi_c} \right)^{\sigma-1} .
\]

All the relevant variables can then be written in terms of \( \varphi_c \) and \( \bar{\varphi} \). In particular, the industry price level, operating profits and labour demand are respectively given by

\[
P = M^{1/(1 - \sigma)} \frac{\sigma (1 - s)}{(\sigma - 1) \bar{\varphi}} ,
\]

\[
\pi(\bar{\varphi}) = \alpha (1 - s) \left[ \left( \frac{\bar{\varphi}}{\varphi_c} \right)^{\sigma-1} - 1 \right] ,
\]

and

\[
l(\bar{\varphi}) = \alpha \left( \sigma - 1 \right) \left( \frac{\bar{\varphi}}{\varphi_c} \right)^{\sigma-1} + 1 \right) .
\]

Finally, the indirect per capita utility can be written as

\[
u = \theta \frac{\delta h^{1+\delta}}{1 + \delta} ,
\]

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\(^5\) To see this, define \( \bar{\varphi} = \left( \int_{\varphi_c}^{\infty} (y(\varphi) / y(\bar{\varphi})) \mu(\varphi) \, d\varphi \right)^{\frac{1}{\sigma-1}} \) and note that the weight \( y(\varphi) / y(\bar{\varphi}) \) is given by \( (\varphi / \bar{\varphi})^\sigma \) which can be substituted back in the definition of \( \bar{\varphi} \) to obtain (6).
which is monotonically increasing in \( h \). Thus, maximising \( u \) is equivalent to maximising both welfare and employment.

### 2.2. General equilibrium and policy analysis

The entry process continues until the expected net profit of entry is zero, \( M \pi(\tilde{\phi}) - Ff = 0 \), which we write as

\[
M \left( r(\tilde{\phi}) - (1-s)l(\tilde{\phi}) \right) - Ff = 0. \tag{11}
\]

The labour market equilibrium condition, the aggregate market clearing condition for the differentiated good and the homogeneous good market equilibrium condition are, respectively

\[ L_s + Ml(\tilde{\phi}) = L^e, \tag{12} \]
\[ Mr(\tilde{\phi}) = \beta(1-t)L^e, \tag{13} \]
\[ A + fF = A^s, \tag{14} \]

where \( A = (1-\beta)(1-t)L^e \), and \( A^e = L_s \). Finally, the government budget constraint is

\[ sMl(\tilde{\phi}) = tL^e. \tag{15} \]

Note that one of the market clearing conditions, e.g. the homogeneous goods market equilibrium in (14), can be obtained from the rest and is therefore redundant.

The above equations complete the model, that consists of 14 equations and 14 unknowns: \( F, M, L^e, L_s, l(\tilde{\phi}), A, A^e, P_t, r(\tilde{\phi}), y(\tilde{\phi}), \pi(\tilde{\phi}), \bar{\phi}, \phi_e \) and either the tax rate \( t \) or the subsidy rate \( s \). In order to obtain explicit solutions, we adopt the Pareto distribution and let

\[
G(\phi) = 1 - \phi^{-\gamma} \quad \text{and} \quad g(\phi) = \gamma \phi^{-(1+\gamma)}, \quad \phi \in [1, \infty), \tag{16}
\]

where the shape parameter \( \gamma \) provides an inverse measure of dispersion: the higher is \( \gamma \) the more homogeneous are the firms.\(^6\) Using this, \( 1 - G(\phi_e) = \phi_e^{-\gamma} \) and (16) imply

\[
\bar{\phi}^{\gamma - 1} = \left( \frac{\gamma}{1 + \gamma - \sigma} \right) \phi_e^{\sigma - 1}. \tag{17}
\]

\(^6\) In the Pareto distribution, both mean and variance are negatively related to the shape parameter \( \gamma \). Thus, the smaller is \( \gamma \), the higher is the average firm efficiency and the higher is the productivity dispersion (i.e. the lower is the density of firms at lower productivity levels). It is in this sense that we argue that the value of \( \gamma \) captures the efficiency of the distribution: a “more efficient distribution of firms” is one with a higher average productivity and a higher dispersion – i.e. one with a smaller \( \gamma \). To obtain meaningful results we impose \( \gamma > \sigma - 1 \).
Then, making use of the above results and (17), we can write the expected free-entry profit condition as

\[ M\alpha(1-s)\left[\frac{\sigma-1}{1+\gamma-\sigma}\right] - Ff = 0. \]  

(18)

Given (16), \( M = (1-G(\varphi_c))F \) implies \( M = \varphi_c^{-\gamma} F \) which can then be substituted into (18) to obtain the value of the equilibrium productivity cut-off,

\[ \varphi_c = \left[ \frac{\alpha(\sigma-1)(1-s)}{f(1+\gamma-\sigma)} \right]^{\gamma}. \]  

(19)

Thus, \( \frac{\partial \varphi_c}{\partial \gamma} < 0 \): the minimum productivity required to survive in equilibrium is negatively related to the degree of heterogeneity of firms (which is inversely related to \( \gamma \)). Moreover, \( \frac{\partial \varphi_c}{\partial s} < 0 \): a higher subsidy reduces the productivity of the marginal firm – by helping firms, a higher subsidy softens competition making it easier to survive in equilibrium.

For a given \( s \) and treating \( t \) as endogenous, the model can be solved (see the Appendix for details) to express all endogenous variables in terms of \( s \). The corresponding equilibrium tax rate is given by

\[ t = \frac{\beta(1+\gamma\sigma-\sigma)s}{\beta(1+\gamma\sigma-\sigma)s + \gamma\sigma(1-s)} < s. \]  

(20)

The indirect utility function in (10) is a monotonic function of \( h \) which is in turn given by \( h = \left(\frac{1-t}{\theta P} \right)^{v/\delta} \) from (3). Evaluating \( h \) using (20) and the solution for \( P \) we find it to be concave in \( s \), such that

\[ s^{\text{opt}}(\gamma) = \frac{(1-\beta)\gamma}{\gamma\sigma - \beta(1+\gamma\sigma-\sigma)} > 0 \]  

(21)

maximises \( h \), and hence \( u \), inducing optimum welfare and employment. Thus, the optimal subsidy is positive and since \( \frac{\partial \varphi_c}{\partial s} < 0 \), we have: \( \text{7} \)

**Proposition 1:** The optimal policy entails *softening* competitive selection by reducing the minimum productivity required to survive in equilibrium.

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\( \text{7} \) Note that since in the CES case the mark-up does not depend on productivity, the optimal subsidy is homogenous across firms.
As is clear from (21), \( \frac{ds^{opt}}{d\gamma} > 0 \). Hence:

**Proposition 2:** The more skewed is the productivity distribution towards less efficient firms, the larger is the optimal subsidy.

We can think of the subsidy as an instrument the government uses to control the productivity cut-off: substitution of (21) into (19) reveals that \( \frac{\partial \varphi^{opt}}{\partial \gamma} < 0 \) as long as \( 0.5 < \beta < 1 \), that is the lower is the degree of firm heterogeneity (and the more skewed is the productivity distribution towards less efficient firms), the smaller is the optimal value of the productivity cut-off (and the lower will be the optimal average productivity in the industry. Thus:

**Proposition 3:** The optimal productivity cut-off level is lower the lower is degree of firm heterogeneity.

In essence, given the shape of the welfare function, the objective of the government is maximise aggregate employment. In order to do so, via the policy, the government triggers a reallocation of expenditure across goods and leisure, and a reallocation of resources across sectors and within the monopolistic industry sector – by manipulating the selectivity of competition and the size of the industry. As noted, the productivity cut-off is decreasing in \( s \): by helping firms, an increase in subsidy softens competition, making it easier to survive in equilibrium. Hence, an increase in \( s \) – whilst worsening the efficiency composition of the industry – works towards increasing entry: as discussed in the Appendix, the mass of firms is concave in \( s \), but reaches a maximum at a subsidy level that exceeds the value that maximises employment. Thus, increases in subsidy up to the optimum level lead to an expansion in the mass of firms in the industry – that also works towards an increase in aggregate employment. These effects on aggregate employment are however partially offset by the fact that, as is clear from (20), the tax rate rises with the subsidy – which has a negative impact on labour supply. Taken together, these forces underpin the concavity of the welfare function with respect to the subsidy: the optimal subsidy corresponds to the threshold level of subsidy beyond which the effect of a higher tax dominates the positive effects on employment of the subsidy. Note however, a higher value of \( \gamma \) is associated with a lower mass of firms – which results in a lower aggregate employment. Hence, at high values of \( \gamma \) the optimal subsidy is higher, i.e. \( \frac{ds^{opt}}{d\gamma} > 0 \). This reflects the fact that with less efficient firms, on average, a higher degree of intervention is required to raise employment. In fact, the skewedness of the

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8 Hence, the optimal subsidy – that maximises employment – entails an ‘intermediate’ level of product variety, suggesting that increases in subsidies eventually result in a trade-off between employment and variety. At some point, further increases in the mass of firms in the industry occur at the expense of a lower employment level.
productivity distribution matters in determining the effectiveness of the subsidy policy. To see this let us write the optimal employment level as $L^\text{opt} = L(\gamma, s^\text{opt}(\gamma))$. Thus, 
\[
\frac{dL^\text{opt}}{d\gamma} = \frac{\partial L^\text{opt}}{\partial \gamma} + \frac{\partial L^\text{opt}}{\partial s^\text{opt}} \frac{ds^\text{opt}}{d\gamma}.
\]
Determining the properties of this analytically is unwieldy but our numerical analysis show that $\frac{dL^\text{opt}}{d\gamma} < 0$, $\frac{d^2 L^\text{opt}}{d\gamma^2} > 0$. Hence, ceteris paribus, a fall in the degree of productivity heterogeneity (i.e. a higher value of $\gamma$) will result in a lower aggregate employment in equilibrium, despite a higher optimal subsidy rate.

Note that $\lim_{\gamma \to \infty} s^\text{opt} = \frac{1}{\sigma}$ and $\lim_{\gamma \to \infty} \phi^\text{opt} = 1$. Thus, given that $p(\phi) = \frac{\sigma(1-s)}{(\sigma-1)\phi}$, 
\[
p(\phi^\text{opt})\bigg|_{\gamma \to \infty} = 1 = w.
\]
Hence, when all firms draw the same productivity level with probability one, the optimal subsidy eliminates the mark-up margin $\sigma/(\sigma-1)$. That is, the optimal subsidy eliminates the monopolistic distortion as $\gamma \to \infty$. It also follows that the extent to which the subsidy can do this is directly related to $\gamma$ – that is the more homogenous are firms and the more effective is the subsidy in addressing the monopolistic distortion. It is straightforward to show that the fact that the differentiated good is priced at a mark-up while leisure and the homogenous good are not results in a wedge between the marginal rate of substitution and the marginal rate of transformation between leisure and the homogenous good on the one hand and the differentiated on the other. As a result, the market outcome is characterised by a sub-optimal level of the consumption of $Y$ and an excessive consumption of the homogenous good and leisure. The subsidy contributes to correcting this distortion by reducing the share of employment in the homogenous good sector\(^9\)

\[
\frac{L_d}{L} = \frac{(1-\beta)\gamma\sigma}{\gamma\sigma + \beta(1+\gamma\sigma-\sigma)\frac{s}{1-s}},
\]
which is negatively related to $s$.

In sum, in equilibrium with $s>0$, a country with a lower degree of productivity heterogeneity (i.e. a higher value of $\gamma$) has a lower efficiency cut-off, a lower average industry productivity and a lower aggregate employment. The optimal policy entails a positive subsidy that softens competition and results in a reallocation of resources across the two production

\(^9\)See Bilbiie et al. (2008) and Molana et al (2013) for further discussion. The effects of cost heterogeneity on the optimality of the market solution has been examined by Dhingra and Morrow (2012) for the CES and VES case and by Nocco et al (2013) for quasi-linear demands. Dhingra and Morrow (2012) show that in a one-sector-heterogeneous-CES world, the market solution corresponds to the first best – i.e. the way the market allocates resources across firms does not matter (and the optimal policy is laissez faire). When, as in our case, mark-ups differ across sectors, the monopolistic distortion leads to inefficient market allocations that can be corrected by policy. As we show, the extent to which the policy offsets the distortion depends on the degree of heterogeneity.
sectors, away from leisure and – within the monopolistic sector – away from the most efficient and towards less efficient firms. In the next section we examine the implications of trade openness for the optimal policy and how they interact with competitive selection forces to determine the impact of exogenous (international) shocks.

3. A SMALL OPEN ECONOMY

3.1 Model setup

In this section we extend the model to consider an open economy that trades both goods with the rest of the world. We assume the country to be ‘small’ in the sense that it cannot affect the total aggregate demand for its exports and the price of its imports. The homogenous good (which we retain as the numeraire) is freely traded, whilst the differentiated good is traded at a per-unit ( iceberg) trade cost, $\tau > 1$. Labour is assumed to be internationally immobile. Foreign variables will be denoted by an asterisk. As shown in the Appendix, in a manner similar to Demidova and Rodriguez-Clare (2013), we obtain the small open economy (SOE) model from a two-country model consisting of a Home and a Foreign country. The Home country then becomes the SOE while the Foreign country represents the rest of the world. This is achieved by letting the labour force, $N$, in the Home country become very small relative to that of the Foreign country, whose variables are treated as exogenous. 10 For ease of exposition, in what follows we shall only report the relevant results rather than the step-by-step procedure and outline the details in the Appendix.

Consumers’ preferences are described by the utility function in (1), with the sub-utility $Y$ and its dual price index now respectively given by

$$Y = \left( \int_{i \in M} y_d(i)^{\frac{1}{\gamma-1}} di + \sum_{i \in M, \iota \in M'} y^*_d(i)^{\frac{1}{\gamma-1}} di \right)^{\frac{1}{\frac{1}{\gamma-1}}}, \quad P_t = \left( \int_{i \in M} p_d(i)^{\frac{1}{\gamma-1}} di + \sum_{i \in M, \iota \in M'} p^*_d(i)^{\frac{1}{\gamma-1}} di \right)^{\frac{1}{\frac{1}{\gamma-1}}}, \quad (23)$$

where the subscripts $d$ and $x$ refer to domestically produced varieties and exported varieties, respectively – thus, e.g., $y^*_d$ and $p^*_d$ are the quantity and price of foreign exported (i.e. imported) varieties consumed by Home consumers. The solutions for $L$, $A$ and $Y$ resulting from the maximisation of the utility in (1) subject to the relevant constraints are as in (3), while demand for the domestic and foreign varieties of the differentiated good are respectively given by

$$y_d(i) = Y \left( \frac{p_d(i)}{P_t} \right)^{-\sigma}, \quad y^*_d(i) = Y \left( \frac{p^*_d(i)}{P_t} \right)^{-\sigma}. \quad (24)$$

10 This is done by first scaling the model by the foreign labour force $N^*$ and then letting $N/N^*$ shrink. Demidova and Rodriguez-Clare (2013) assume an inelastic labour supply and do not have an outside good. Hence, their wage rate is endogenous. Flam and Helpman (1987), within a homogenous firms setting, consider a SOE which can however affect the price index in the rest of the world.
The possibility of trade implies that firms in the monopolistic sector will have to decide after entry whether to produce for the domestic market only or to also export. In equilibrium, only a subset of active firms will export. This is because, in addition to the fixed entry cost \( f \) (that cover the cost of product innovation required to develop a variety of the good), an exporting firm incurs a fixed operating export cost \( \alpha_e \) – which exceeds that required for domestic production, \( \alpha_d \) – as well as variable trade cost that raises the marginal cost of exporting.

By virtue of the SOE assumption, we assume foreign demand for a domestic variety to be given by \( y_*(\varphi) = K^*(p_*(\varphi))^{-\sigma} \), where \( K^* \) is a constant that incorporates both total expenditure on the differentiated good and the price index in the Foreign country. Thus, allowing for the subsidy rate for domestic sales (\( s_d \)) and for exports (\( s_x \)) to differ, a firm’s profits from domestic and export sales are then given by

\[
\pi_d(\varphi) = p_d(\varphi)y_d(\varphi) - (1-s_d)wl_d(\varphi), \\
\pi_x(\varphi) = p_x(\varphi)y_x(\varphi) - (1-s_x)wl_x(\varphi),
\]

where

\[
l_d(\varphi) = \alpha_d + \frac{y_d(\varphi)}{\varphi}, \\
l_x(\varphi) = \alpha_x + \frac{\tau y_x(\varphi)}{\varphi},
\]

which reflect the fact that to deliver one unit of the good to the foreign market a firm needs to ship \( \tau > 1 \) units. Maximisation of (25) subject to the relevant demand functions then implies the following optimal price rules for a firm with productivity \( \varphi \) serving the two markets

\[
p_d(\varphi) = \frac{\sigma(1-s_d)w}{(\sigma-1)\varphi}, \quad p_x(\varphi) = \frac{\sigma(1-s_x)\tau w}{(\sigma-1)\varphi}.
\]

Thus, the existence of the iceberg trade cost implies that, for given subsidy rates, equilibrium prices in the export market are a multiple – by a constant factor of proportionality \( \tau \) – of those in the domestic market. In other words, for an exporting firm, \( \frac{p_x(\varphi)}{p_d(\varphi)} = \frac{(1-s_x)\tau}{(1-s_d)}. \)

After having paid the fixed entry cost \( f \), firms draw their productivity from the Pareto distribution in (16). A firm will be able to operate in the domestic market only if its productivity draw can generate a level of variable profit sufficient to cover the fixed production cost \( \alpha_d \). For the firm to become an exporter, its productivity needs to be sufficiently high to generate a variable profit that can cover the higher fixed export cost.

\[11\] Despite the SOE assumption, due to the monopoly power that each firm in the monopolistic sector has in its market niche, the quantity of output sold by each firm in the foreign market will be a function of its price.
\( \alpha_r (> \alpha_d) \) and the variable iceberg trade cost. The competitive selection process that follows entry will result in the emergence of two productivity cut-offs, defined by 
\[
\varphi_d = \sup \{ \varphi : \pi_d(\varphi_d) = 0 \} \quad \text{and} \quad \varphi_x = \sup \{ \varphi : \pi_x(\varphi_x) = 0 \},
\]
that correspond to the productivity of the marginal firms that survive in the domestic market and that of the marginal exporters, respectively. Thus, the possibility of international trade, and the fact that trade is costly, gives rise to an additional selection process between exporting and non-exporting firms – with only relatively more productive firms being able to export – that results in a partitioning of firms determined by the export cut-off. Hence, for a given mass of entrants \( F \), a mass \( M = (1 - G(\varphi_d))F = \varphi_d^{-}F \) of firms will sell in the domestic market, and a mass \( M_x = (1 - G(\varphi_x))F = \varphi_x^{-}F \) of firms will also export. Following the same procedure as in autarky, we obtain the corresponding average productivities
\[
\bar{\varphi}_d^{\sigma-1} = \left( \frac{\gamma}{1 + \gamma - \sigma} \right) \varphi_d^{\sigma-1}, \quad \bar{\varphi}_x^{\sigma-1} = \left( \frac{\gamma}{1 + \gamma - \sigma} \right) \varphi_x^{\sigma-1}. \quad (28)
\]
In the general equilibrium, the entry process continues until the expected net profit of entry is driven to zero, i.e. \( M \pi_d(\bar{\varphi}_d) + M_x \pi_x(\bar{\varphi}_x) - FF = 0 \). The zero profit conditions for the marginal domestic and exporting firms and for the foreign firms whose products are imported require, respectively, \( \pi_d(\varphi_d) = 0 \), \( \pi_x(\varphi_x) = 0 \), and \( \pi_x(\varphi_x^*) = 0 \). By virtue of the SOE assumption, we treat the foreign country’s equilibrium mass of firms and domestic productivity cut-off point as exogenous. However, as in Demidova and Rodriguez-Claire (2009, 2013), we allow for the foreign export cut-off \( \varphi_x^* \) to be endogenous – i.e. the mass of foreign varieties that is imported by domestic consumers is endogenous and affected by what happens in the SOE.\(^{12}\) Then, as is shown in the Appendix, the zero-profit conditions of the marginal firms imply that
\[
\beta (1-t) Nh \left( (\sigma - 1) \varphi_d P_{t/y} \right)^{\sigma-1} - \alpha_d \left( \sigma (1-s_d) \right)^\sigma = 0, \quad (29)
\]
\[
K^* \left( \frac{(\sigma - 1) \varphi_x}{\tau} \right)^{\sigma-1} - \alpha_x \left( \sigma (1-s_x) \right)^\sigma = 0, \quad (30)
\]
\[
\beta (1-t) Nh \left( (\sigma - 1) \varphi_x^* P_{t/y} \right)^{\sigma-1} - \alpha_x^* \left( \sigma (1-s_x^*) \right)^\sigma = 0. \quad (31)
\]
\(^{12}\) In Demidova and Rodriguez-Claire (2013) these features of the model emerge starting from a symmetric two-country model and letting the share of labour in the domestic economy go to zero. In their model, this is possible because of an exogenous labour supply and the absence of a government budget constraint. In our model, due to these additional ‘complications’, letting the population share go to zero would lead to equilibrium relationships that are not consistent with the two-country model. Hence, we setup and solve the model by making the behavioural assumptions discussed above instead.
As is standard in these models, the above imply a positive relationship between pairs of cut-offs, \( \varphi_d \) and \( \varphi_x \) on the one hand and \( \varphi_s \) and \( \varphi_s^* \) on the other, which is affected by the relevant trade cost and the relative wages.\(^{13}\) However, as noted above, the SOE assumption requires \( \varphi_s^* \) to be taken as exogenous and \( \varphi_s \) is only affected by the subsidy rate and trade cost. More explicitly, equation (30) directly implies

\[
\varphi_s = \left( \frac{\alpha_y}{K} \right)^{\frac{1}{\sigma-1}} \frac{\tau}{(\sigma-1)} \left( \sigma(1-s_s) \right)^{\frac{\sigma}{\sigma-1}},
\]

whilst equations (29) and (31) can be combined to obtain

\[
\varphi_d = \left( \frac{\alpha_d}{\alpha_s} \right)^{\frac{1}{\sigma-1}} \left( \frac{1-s_d}{1-s_s} \right)^{\frac{\sigma}{\sigma-1}} \left( \frac{\varphi_s^*}{\tau} \right).
\]

Equation (32) suggests that an employment subsidy to the export operations of firms reduces the export cut-off, i.e. it makes it easier for firms to cover the fixed and variable costs of exports. Equation (33) reflects the competitive pressure that home firms face from foreign firms in the domestic market.\(^{14}\) A ceteris paribus increase in the minimum productivity of foreign exporters toughens the competition faced by Home country’s firms in domestic market and results in an increase in the minimum productivity required to survive in the domestic market. However, for a given foreign export productivity cut-off, an increase in domestic subsidy reallocates market shares away from foreign country’ firms and towards home firms, by reducing the minimum efficiency required to survive in the domestic market and by increasing \( \varphi_s^* \) – which makes it more difficult for foreign firms to access it. This result reflects the fact that the subsidy contributes to addressing an additional distortion that arises from the fact that while home varieties are purchased at a mark-up, foreign varieties are imported at the international price. Akin to what was discussed in the closed economy model with respect to the allocation of expenditure between the monopolistic good and the homogenous good and/or leisure, this implies that Home consumers allocate too little expenditure on Home varieties relative to imported ones.

The labour market clearing condition is now

\[
L_A + M_{d}\left( \hat{\varphi}_d \right) + M_x\left( \hat{\varphi}_x \right) = L',
\]

where, as in autarky, \( L_A = A' \) and \( L' = N \left( \frac{1-\delta}{\theta \beta P_Y} \right)^{\frac{1}{\delta}} \). The balanced government budget constraints is

\(^{13}\) See the two-country version of the model outlined in the Appendix.

\(^{14}\) It is worth noting that the above relationships are unaffected by the shape of the productivity distribution function.
Finally, the model is closed by the trade balance equation which equates the value of imports and exports for the country,

\[ \left( A^* - A - fF \right) + M_x r_x (\hat{s}_x) = M_x^* r_x^* (\hat{s}_x^*) \]  

A concise version of the model described above can be formulated in 7 equations (outlined in the Appendix) which determine \( \varphi_d, \varphi_x, \varphi_x^*, F, P_t, h \) and the tax rate \( t \), treating the subsidy rates as exogenous.

### 3.2 Policy analysis

In this section we study the effects of optimal employment subsidy policy in response to an exogenous (international) shock on the equilibrium of the model. Although the model can be characterised analytically, given the complexity of the algebra involved, we resort to numerical solutions to illustrate the effects of external shocks and policy. Our calibration parameters are consistent with those widely used in the literature for this type of model.\(^{15}\)

As a benchmark, it is useful to consider the case in which the government is not active, for which we set \( s_d = s_x = t = 0 \). This will enable us to isolate the effects of the size of \( \gamma \), the shape parameter of the productivity distribution, on the impact of a negative shock. Table 1 below gives the equilibrium solutions for two values of \( \gamma \) before and after a demand shock.

As is clear from the table, in the no-policy equilibrium case, a reduction in \( \gamma \) (that results in an increase in firm heterogeneity, with the distribution becoming less ‘skewed’ towards low productivity levels) will increase the domestic cut-off, i.e. \( (\partial \varphi_d / \partial \gamma) < 0 \): thus, a SOE with a more efficient productivity distribution of firms will be characterised by a higher minimum productivity required to survive in the Home market. This essentially amounts to a toughening of market competition that reallocates market shares towards more efficient firms and leads to greater entry and a larger mass of surviving firms and a larger extensive margin of export (defined as the ratio of exporters to the total population of firms). Note that, as can be seen from equation (32), the export cut-off is unaffected by \( \gamma \). Hence, a change in the shape parameter of the distribution will affect the mass of exporters only via a change in the mass of firms that survive in the industry and not by altering the selection into exporting. The implication of this is that the increase in the population of firms ensuing from a fall in \( \gamma \) will result in an increase in the extensive margin of exports from the initial equilibrium. Both the higher average efficiency of the industry and the larger mass of surviving firms translate to a lower price index and a higher employment – both working towards an increase in welfare.

\(^{15}\) See e.g. Felbermayr et al. (2011).
Thus, a country with a higher degree of firm heterogeneity (in which there is a lower density of smaller, less efficient firms) will be characterised by a higher level of economic activity, with a larger extensive margin of export.

Table 1. Effects of a foreign demand shock on equilibrium at different values of $\gamma$

<table>
<thead>
<tr>
<th>Variables</th>
<th>No Policy Pre-Shock</th>
<th>No Policy Post-Shock ($%\Delta K^*= -10%$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_d$</td>
<td>0.765958</td>
<td>0.825452</td>
</tr>
<tr>
<td>$\phi_y$</td>
<td>1.243678</td>
<td>1.291368</td>
</tr>
<tr>
<td>$\phi_y^*$</td>
<td>2.943318</td>
<td>2.904461</td>
</tr>
<tr>
<td>$F$</td>
<td>0.138317</td>
<td>0.127915</td>
</tr>
<tr>
<td>$h$</td>
<td>0.415184</td>
<td>0.412617</td>
</tr>
<tr>
<td>$P_y$</td>
<td>1.209356</td>
<td>1.228252</td>
</tr>
<tr>
<td>$M$</td>
<td>0.342436</td>
<td>0.33132</td>
</tr>
<tr>
<td>$M_s$</td>
<td>0.06590</td>
<td>0.05362</td>
</tr>
<tr>
<td>$M_s / M$</td>
<td>0.19244</td>
<td>0.16185</td>
</tr>
</tbody>
</table>

*Parameters values reported here are scaled versions of the solutions.

In what follows we consider a negative demand shock (a 10% reduction in $K^*$), whose effects are given in the third and fourth columns of Table 1. We have also examined the case of a negative trade shock (in the form of 10% increase in trade cost), but we do not discuss it here as its effects are qualitatively identical to those of a demand shock. For a given value of $\gamma$, the negative shock reduces the domestic productivity cut-off and increases the export cut-off, i.e. it softens competition for firms in the domestic market and toughens it for their export. These changes lead to a market and resource reallocation towards less efficient firms, resulting in a contraction in the total mass of entrants, surviving firms and exporters. Thus, the negative shock has an anti-competitive effect that translates into a lower average industry productivity. As can be seen from inspection of the table, however, the effects of the shock are more severe at the lower value of $\gamma$. The reason for this is that, given its source, the international shock has a stronger and more direct effect on exporters; hence, given that a more efficient country will have a larger share of exporters, a larger share of firms will be directly negatively affected by the shock. Despite this, however, the post-shock level of average efficiency, mass of firms, extensive margin of exports, employment and welfare are still higher when $\gamma$ is smaller.

Thus, this analysis confirms that (intra-industry) competitive selection is a channel for the transmission of shocks, through which they affect aggregate industry productivity, aggregate employment and welfare. In particular, in a SOE, a negative shock has a selection-softening effect on less efficient firms, and overall anti-competitive effects that result in a

---

16 Hence, qualitatively, the negative shock has the opposite effects on selection of trade liberalisation in the standard model à la Melitz (2003).
lower average productivity – and which are stronger in a relatively more efficient country (i.e. one with a smaller value of $\gamma$).

Moving on to consider the effects of policy in response to the shock, the two graphs in Figure 1 below plot, for a ‘high’ and a ‘low’ value of $\gamma$, the level of per-capita employment (which is proportional to welfare) against the subsidy rate. Three different subsidy policies are considered: a uniform subsidy (i.e. $s_u = s_d = s_x > 0$, given by the blue line); a domestic-only subsidy given to labour employed in production for domestic sales (i.e. $s_d > 0$, $s_x = 0$, given by the black line); and an export-only subsidy given to labour employed in production for exports (i.e. $s_x > 0$, $s_d = 0$, given by the red line). The straight line in both graphs corresponds to the pre-shock no-subsidy equilibrium in which the government is not policy active. Starting from this equilibrium, as discussed earlier, an exogenous demand shock results in a fall in employment (corresponding to the start of the three concave curves in either graphs).

As is clear from the figure, subsidies can be used to offset the employment effects of the shock. As in autarky, in all cases there is a positive level of subsidy that maximises employment and welfare. Different policies, however, entail different optimal subsidy rates and result in different employment (and welfare) gains. Whilst the domestic-only and the uniform subsidy take the economy to a level of activity (and welfare) that is higher than the pre-shock one, the export-only subsidy fails to do so. A uniform subsidy will be more effective initially – taking the economy back to the pre-shock level ‘faster’ than a domestic-only subsidy – but will result in a lower optimal level of welfare.

**Figure 1. Effects of a negative foreign demand shock and optimal subsidy**

![Graphs showing response of employment and welfare to labour subsidy](image)

**Fig 1.1 – 10% negative foreign demand shock**
Straight line is the pre-shock level with no subsidy

**Fig 1.2 – 10% negative foreign demand shock**
Straight line is the pre-shock level with no subsidy

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As a short-hand, we shall henceforth refer to these as ‘uniform’, ‘domestic-only’ and ‘export-only’ subsidies, respectively.
To see the effects of the policy in more detail, the first three columns of Table 2 below give the equilibrium values of the variables at different policy optima for a given value of $\gamma$. The pre-shock values at the two different levels of $\gamma$ are also repeated for ease of comparison. As is clear from the table, $s_d^{opt} > s_u^{opt} > s_x^{opt}$, with the corresponding levels of employment/welfare being $h(s_u^{opt}) > h(s_x^{opt}) > h(s_x^{opt})$. Thus, the dominant policy from a welfare point of view is a wage subsidy targeted to domestic sales – with the least preferred option being to target subsidisation to exports.

Underpinning these results is the effects that each subsidy policy has on competitive selection relative to the post-shock equilibrium (that can be seen by comparing the relevant columns of Table 2, for a given $\gamma$). As implied by equations (32) and (33), all subsidies to domestic firms increase $\varphi_x^*$ – thus raising the minimum productivity required of foreign firms to penetrate the home market and reallocating market shares away from foreign firms. However, whilst both the export-only and the uniform subsidies reduce $\varphi_x$ and increase $\varphi_d$, making it easier to export from and more difficult to survive in the domestic market, the domestic-only subsidy has virtually no effect on $\varphi_x$ but reduces $\varphi_d$. Thus, whilst the former two subsidies trigger an intra-industry reallocation away from relatively less efficient and towards relatively more efficient producers (a reallocation that is clearly relatively stronger in the export-only subsidy case), a domestic-only subsidy does the opposite by softening the competitive pressure on relatively less efficient firms. Consistently, at the optimum, for a given value of $\gamma$, among the different policy scenarios, a domestic-only subsidy is associated with the lowest value of $\varphi_d$, the highest value of $\varphi_x$ and also of $\varphi_x^*$. In all cases, the policy increases the mass of firms operating in the industry, reduces the price index and increases employment, but the domestic-only subsidy leads to a contraction of the extensive margin of exports that does not occur in the other two cases.

**Table 2. Optimal subsidies after a negative foreign demand shock at different values of $\gamma$**

<table>
<thead>
<tr>
<th>Variables</th>
<th>$\gamma=3.4$</th>
<th>$\gamma=3.3$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Post-shock (no policy)</td>
<td>Post-shock $s_u = s_d = s_x$ only</td>
</tr>
<tr>
<td>$\varphi_u$</td>
<td>0.755847</td>
<td>0.768666</td>
</tr>
<tr>
<td>$\varphi_x$</td>
<td>1.291368</td>
<td>1.119310</td>
</tr>
<tr>
<td>$\varphi_x^*$</td>
<td>2.904461</td>
<td>3.407759</td>
</tr>
<tr>
<td>$F$</td>
<td>0.127915</td>
<td>0.169495</td>
</tr>
<tr>
<td>$h$</td>
<td>0.412617</td>
<td>0.418257</td>
</tr>
</tbody>
</table>

18 We have verified that in the post-shock equilibrium all optimal subsidies are higher than in the pre-shock one.
In sum, as discussed in the autarkic case, via the subsidy, the government controls the selectivity of competition with a view to maximising employment. The most effective way to do so is to trigger a reallocation of resources: across sectors (away from the homogeneous sector and leisure and towards the monopolistic good), across firms within the monopolistic sector (towards relatively less productive firms), and across countries (towards domestic producers). Ultimately, subsidisation of home firms acts as a protection against international competition, generating a home market effect that relocates production towards the country. This home market effect is strengthened by the endogeneity of labour supply – given that the subsidy works towards increasing aggregate employment and the size of the SOE’s market. Given its differential effects on the selectivity of competition, the domestic-only subsidy is the dominant policy: despite the fact that it entails the highest tax rate, at \( s = s_d^{opt} \), the equilibrium is characterised by the highest level of economic activity compared to the alternative cases considered.

As discussed earlier, the drop in economic activity following the negative shock is larger at lower values of \( \gamma \). This is illustrated in the second graph of Figure 1 and in the last section of Table 2. As in the previous case, \( s_d^{opt} > s_u^{opt} > s_x^{opt} \). However, as in autarky, all optimal subsidy increase in \( \gamma \) – i.e. the optimal subsidies are lower in a country with a smaller \( \gamma \). Furthermore, \( h(s^{opt}, \gamma_{low}) > h(s^{opt}, \gamma_{high}) \) for all \( s^{opt} = (s_u^{opt}, s_d^{opt}, s_x^{opt}) \) – i.e. with a more efficient productivity distribution all optimal policies set in response to a negative shock are associated with higher levels of employment and welfare.

4. CONCLUSIONS

We have argued that inter-country differences in firm size (and productivity) distribution can contribute to explaining differences between countries regarding the relationship between output and employment. Within a theoretical model characterised by firm heterogeneity and an endogenous labour supply we have examined the role of intra-industry heterogeneity and selection on the level of economic activity. By extending the autarkic model to a small open
economy setting, we have shown how it can act as a channel through which shocks, by affecting average industry productivity, impact on employment and welfare. In particular, a negative demand shock results in an anti-competitive effect that – by reallocating market shares towards less efficient firms – lowers average industry productivity, aggregate employment and welfare. Countries with a ‘more efficient distribution of firms’ are shown to weather out the shock better than less efficient ones (in the sense of regaining higher levels of welfare) despite being more strongly affected by a negative trade shock (due to their higher export participation rate).

The model also shows that the use of ALMPs (in the form of wage subsidies) to sustain employment entails positive optimal subsidies that in most cases take economy to higher levels of employment and welfare than the post-shock ones, despite the fact that the employment subsidy has a negative effect on consumer disposable income via the proportional income taxation.

Furthermore, a uniform policy (that does not discriminate between production for domestic markets and for exports) is dominated, from a welfare point of view, by a policy that targets the labour employed in production for the domestic market only (and hence is biased towards the less efficient firms).

A key testable hypothesis emerging from the model is that in countries with a higher degree of firm heterogeneity – i.e. with a firm size distributions that is more skewed towards smaller (and less efficient) firms – a negative shock should have a stronger negative effect on aggregate employment. Other testable hypotheses emerging from the theoretical model concern the role of ALMP and how it is influenced by the shape of the size and productivity distribution of firms.
References


## Table A1: Notation used in the model setup (Home country only)

<table>
<thead>
<tr>
<th>Description</th>
<th>Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Per capita consumer utility</td>
<td>$u$</td>
</tr>
<tr>
<td>Fixed cost of production of the differentiated good (closed economy)</td>
<td>$\alpha$</td>
</tr>
<tr>
<td>Fixed cost of production of the differentiated good (domestic &amp; export)</td>
<td>$\alpha_d$ &amp; $\alpha_s$</td>
</tr>
<tr>
<td>Budget share of $y$ &amp; $a$</td>
<td>$\beta$ &amp; $1-\beta$</td>
</tr>
<tr>
<td>Labour supply elasticity (inverse of real wage elasticity of supply)</td>
<td>$\delta$</td>
</tr>
<tr>
<td>Productivity distribution shape parameter (Pareto)</td>
<td>$\gamma$</td>
</tr>
<tr>
<td>Firm Level Productivity (differentiated good sector)</td>
<td>$\varphi$</td>
</tr>
<tr>
<td>Productivity cut-off for marginal firms (closed economy)</td>
<td>$\varphi_c$</td>
</tr>
<tr>
<td>Productivity cut-offs for marginal firms (non-exporting &amp; exporting)</td>
<td>$\varphi_d$ &amp; $\varphi_s$</td>
</tr>
<tr>
<td>Average productivity (closed economy)</td>
<td>$\bar{\varphi}$</td>
</tr>
<tr>
<td>Average productivity (non-exporting &amp; exporting)</td>
<td>$\bar{\varphi}_d$ &amp; $\bar{\varphi}_x$</td>
</tr>
<tr>
<td>Scale coefficient of labour supply</td>
<td>$\theta$</td>
</tr>
<tr>
<td>CES elasticity of substitution</td>
<td>$\sigma$</td>
</tr>
<tr>
<td>Profit of a firm producing the differentiated good (closed economy)</td>
<td>$\pi$</td>
</tr>
<tr>
<td>Profit of a firm producing the differentiated good (domestic &amp; export)</td>
<td>$\pi_d$ &amp; $\pi_x$</td>
</tr>
<tr>
<td>Iceberg trade cost for exporting firms</td>
<td>$\tau$</td>
</tr>
<tr>
<td>Per capita demand for homogenous good</td>
<td>$a$</td>
</tr>
<tr>
<td>Aggregate demand for homogenous good</td>
<td>$A$</td>
</tr>
<tr>
<td>Aggregate supply of homogenous good</td>
<td>$A'$</td>
</tr>
<tr>
<td>Mass of entrants</td>
<td>$F$</td>
</tr>
<tr>
<td>Per capita labour supply</td>
<td>$h$</td>
</tr>
<tr>
<td>Fixed entry cost</td>
<td>$f$</td>
</tr>
<tr>
<td>Labour requirement for producing the differentiated good (closed economy)</td>
<td>$l$</td>
</tr>
<tr>
<td>Labour requirement for producing the differentiated good (domestic &amp; export)</td>
<td>$l_d$ &amp; $l_x$</td>
</tr>
<tr>
<td>Aggregate labour supply</td>
<td>$L' = Nh$</td>
</tr>
<tr>
<td>Consumer population size</td>
<td>$N$</td>
</tr>
<tr>
<td>Mass of varieties of differentiated good produced (mass of surviving firms)</td>
<td>$M$</td>
</tr>
<tr>
<td>Mass of varieties of differentiated good produced and exported</td>
<td>$M_x$</td>
</tr>
<tr>
<td>Consumer price index</td>
<td>$P$</td>
</tr>
<tr>
<td>price of the homogeneous good</td>
<td>$P_A$</td>
</tr>
<tr>
<td>CES price index for $Y$</td>
<td>$P_Y$</td>
</tr>
<tr>
<td>Variety prices set by a firm producing the differentiated good (closed economy)</td>
<td>$p$</td>
</tr>
<tr>
<td>Variety prices set by a firm producing the differentiated good (non-exporting &amp; exporting)</td>
<td>$p_d$ &amp; $p_s$</td>
</tr>
<tr>
<td>Revenue of a firm producing the differentiated good (closed economy)</td>
<td>$r$</td>
</tr>
<tr>
<td>Revenue of a firm producing the differentiated good (domestic &amp; export)</td>
<td>$r_d$ &amp; $r_x$</td>
</tr>
<tr>
<td>Labour subsidy received by differentiated good producers</td>
<td>$s$</td>
</tr>
<tr>
<td>Labour subsidy received by differentiated good producers (domestic &amp; export)</td>
<td>$s_d$ &amp; $s_x$</td>
</tr>
<tr>
<td>Income tax rate</td>
<td>$t$</td>
</tr>
<tr>
<td>Wage rate</td>
<td>$w$</td>
</tr>
<tr>
<td>Demand for a variety of differentiated good (closed economy)</td>
<td>$y$</td>
</tr>
<tr>
<td>Domestic and foreign demand for a domestically produced variety of differentiated good</td>
<td>$y_d$ &amp; $y_x$</td>
</tr>
<tr>
<td>Aggregate demand for differentiated good (CES)</td>
<td>$Y$</td>
</tr>
</tbody>
</table>
A1. Solution of the closed economy model

Making use of \( \tilde{\phi}^{\sigma^{-1}} = \left(\frac{\gamma}{1 + \gamma - \sigma}\right) \phi_c^{\sigma^{-1}} \) in (17), \( r(\tilde{\phi}) = \alpha \sigma (1 - s) \left(\frac{\phi}{\phi_c}\right)^{\sigma^{-1}} \) and

\[
l(\tilde{\phi}) = \rho \alpha \sigma \left(\frac{\phi}{\phi_c}\right)^{\sigma^{-1}} + \alpha \]

are written respectively as \( r(\tilde{\phi}) = \alpha \sigma (1 - s) \left(\frac{\gamma}{1 + \gamma - \sigma}\right) \) and

\[
l(\tilde{\phi}) = \alpha \left(\frac{1 + \gamma \sigma - \sigma}{1 + \gamma - \sigma}\right), \]

which are then substituted in (13) and (15) to obtain

\[
M \alpha \sigma (1 - s) \left(\frac{\gamma}{1 + \gamma - \sigma}\right) = \beta (1 - t) L
\]

\[
sM \alpha \left(\frac{1 + \gamma \sigma - \sigma}{1 + \gamma - \sigma}\right) = tL
\]

For any given \( L \) these solve for \( t \) and \( M \)

\[
t = \frac{\beta (1 + \gamma \sigma - \sigma) s}{\beta (1 + \gamma \sigma - \sigma) s + \gamma \sigma (1 - s)}
\]

\[
M = \frac{\beta (1 + \gamma - \sigma)}{L \alpha (\beta (1 + \gamma - \sigma) s + \gamma \sigma (1 - s))}
\]

The per capita indirect utility is obtained from (1) and (10), i.e.

\[
u = \theta \delta h^{1+\delta},
\]

\[
h = \left(\frac{1 - t}{\theta P^\beta_Y}\right)^{1/\delta},
\]

hence any argument that maximises \( h \) will also maximise \( u \). Using the expression for \( P_Y \) in (7) and noting that \( h = L/N \), we obtain

\[
L = N \left[\frac{1-t}{\theta (M^{1/(1-\sigma)} \frac{\sigma(1-s)}{(\sigma-1)\tilde{\phi}})^\beta}\right]^{1/\delta}
\]

and replacing \( L \) using the equation for \( M/L \) derived above yields

\[
M = N \left[\frac{\beta (1 + \gamma - \sigma)}{\alpha (\beta (1 + \gamma - \sigma) s + \gamma \sigma (1 - s))}\right] \left[\frac{1-t}{\theta (M^{1/(1-\sigma)} \frac{\sigma (1-s)}{(\sigma-1)\tilde{\phi}})^\beta}\right]^{1/\delta}
\]
Substituting for $t$ from the solution derived above and recalling that
\[ \tilde{\phi}_{\sigma-1} = \left( \frac{\gamma}{1+\gamma-\sigma} \right)^{\phi_c^{\sigma-1}} \]
and $\phi_c = \left[ \frac{\alpha(\sigma-1)(1-s)}{f(1+\gamma-\sigma)} \right]^{1/\gamma}$, we can use the above to obtain the solution for $M$ and $L$:

\begin{align*}
M &= \Omega \left( \beta (1+\gamma-\sigma) \right)^{\frac{\delta(\sigma-1)}{\beta-\delta(\sigma-1)}} (1-s) \left[ \frac{(\sigma-1)(\gamma+f(1-\gamma))}{\gamma[\beta-\delta(\sigma-1)]} \right] \left[ \gamma \sigma (1-s) + \beta (1+\gamma \sigma - \sigma) s \right]^{\frac{(1+\delta)(\sigma-1)}{\beta-\delta(\sigma-1)}}, \\
L &= \alpha \Omega \left( \beta (1+\gamma-\sigma) \right)^{\frac{-\beta}{\beta-\delta(\sigma-1)}} (1-s) \left[ \frac{(\sigma-1)(\gamma+f(1-\gamma))}{\gamma[\beta-\delta(\sigma-1)]} \right] \left[ \gamma \sigma (1-s) + \beta (1+\gamma \sigma - \sigma) s \right]^{\frac{-\beta+\delta(\sigma-1)}{\beta-\delta(\sigma-1)}},
\end{align*}

where

\[ \Omega = \left( \frac{\alpha}{\Lambda N} \right)^{\frac{\delta(\sigma-1)}{\beta-\delta(\sigma-1)}} (\gamma \sigma)^{\frac{(\sigma-1)}{\beta-\delta(\sigma-1)}} \text{ and } \Lambda = \theta^{1/\delta} \rho^{\beta/\delta} \left( f / (1+\gamma-\sigma) \right)^{\frac{(\sigma-1)}{\beta-\delta(\sigma-1)}}. \]

Inspection of these reveals that both $M$ and $L$ are concave in $s$, with $L$ reaching a maximum at a lower level of $s$ than $M$ does. It is straightforward to show that the value of $s$ which maximises $L$, and hence welfare, is

\[ s^{opt} = \frac{(1-\beta)\gamma}{\gamma \sigma - \beta(1+\gamma \sigma - \sigma)}. \]

**Derivation of equation (20):**

\[ \frac{l(\tilde{\phi})}{y(\tilde{\phi})} = \frac{(1+\gamma \sigma - \sigma)(1+\gamma-\sigma)^{1/(\sigma-1)}}{(\sigma-1)^{1/\gamma} \phi_c^{\sigma-1}}. \]

As shown above, $l(\tilde{\phi}) = \alpha \left( \frac{1+\gamma \sigma - \sigma}{1+\gamma-\sigma} \right)$. To obtain the expression for the denominator, recall that

\[ y(\tilde{\phi}) = \left( \tilde{\phi} / \phi_c \right)^{\sigma}, \quad y(\phi_c) = \alpha (\sigma-1) \phi_c, \quad \text{and } \phi_{\sigma-1} = \left( \frac{\gamma}{1+\gamma-\sigma} \right)^{\phi_c^{\sigma-1}} \]

which can be used to write

\[ y(\tilde{\phi}) = \alpha (\sigma-1) \left( \frac{\gamma}{1+\gamma-\sigma} \right)^{\frac{1}{\sigma/(\sigma-1)}} \phi_c. \]

It is then straightforward to use this to find the expression for the above ratio. Given that

\[ \phi_c = \left[ \frac{\alpha(\sigma-1)(1-s)}{f(1+\gamma-\sigma)} \right]^{1/\gamma}, \]

$\phi_c$ is decreasing in $s$. Hence $\frac{l(\tilde{\phi})}{y(\tilde{\phi})}$ is increasing in $s$. 


### A2. Setting up the two country and the small open economy models

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<th>No</th>
<th>Description</th>
<th>Equation for Home Country</th>
<th>Equation for Foreign Country</th>
</tr>
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<tbody>
<tr>
<td>(A1)</td>
<td>Consumer’s utility function</td>
<td>[ u = \left( \frac{a}{1 - \beta} \right)^{1 - \beta} \left( \frac{v}{\beta} \right)^{\beta} - \theta h^{\alpha \delta} ]</td>
<td>[ u^* = \left( \frac{a^<em>}{1 - \beta} \right)^{1 - \beta} \left( \frac{v^</em>}{\beta} \right)^{\beta} - \theta h^{\alpha \delta} ]</td>
</tr>
<tr>
<td>(A2)</td>
<td>Aggregate labour supply</td>
<td>[ L' = Nh = N \left( 1 - t \right) \frac{w}{\theta P} ]</td>
<td>[ L'^* = N'h^* = N'^* \left( 1 - t^* \right) \frac{w^<em>}{\theta P^</em>} ]</td>
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<tr>
<td>(A3)</td>
<td>Aggregate demand for homogenous good</td>
<td>[ A = Na = \frac{N (1 - \beta)(1 - t)wh}{P_A} ]</td>
<td>[ A'^* = N'^<em>a'^</em> = \frac{N'^* (1 - \beta'^<em>)(1 - t'^</em>)w^<em>h^</em>}{P_A^*} ]</td>
</tr>
<tr>
<td>(A4)</td>
<td>Aggregate demand for differentiated good</td>
<td>[ Y = Ny = \frac{N\beta (1 - t)wh}{P_Y} ]</td>
<td>[ Y'^* = N'^<em>\beta'^</em>(1 - t'^*)w^<em>h^</em> ]</td>
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<tr>
<td>(A5)</td>
<td>Consumer price index</td>
<td>[ P = P_A^{\alpha - \beta} P_Y^{\beta} ]</td>
<td>[ P^* = P_A^{\alpha - \beta} P_Y^{\beta} ]</td>
</tr>
<tr>
<td>(A6)</td>
<td>Production function for the homogenous good</td>
<td>[ A^* = L_A ]</td>
<td>[ A'^* = L_A^* ]</td>
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<td>(A7)</td>
<td>Productivity distribution in the differentiated good sector</td>
<td>[ G(\phi) = 1 - \phi^{-\gamma} \text{ and } g(\phi) = \gamma \phi^{-1 - \gamma}, \quad \phi \in [1, \infty) ]</td>
<td>[ G'^<em>(\phi) = 1 - \phi'^{-\gamma} \text{ and } g'^</em>(\phi) = \gamma \phi'^{-1 - \gamma}, \quad \phi \in [1, \infty) ]</td>
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<tr>
<td>(A8)</td>
<td>Mass of varieties of differentiated good (mass of surviving firms)</td>
<td>[ M \equiv (1 - G(\phi_d)) F; \quad M = \phi_d^{-\gamma} F, \quad \phi \in [\phi_d, \infty) ]</td>
<td>[ M^* \equiv (1 - G'^<em>(\phi_d^</em>)) F'^<em>; \quad M^</em> = \phi_d'^{-\gamma} F'^<em>, \quad \phi \in [\phi_d^</em>, \infty) ]</td>
</tr>
<tr>
<td>(A9)</td>
<td>Mass of varieties of differentiated good exported</td>
<td>[ M_x \equiv (1 - G(\phi_x)) F; \quad M_x = \phi_x^{-\gamma} F, \quad \phi \in [\phi_x, \infty) ]</td>
<td>[ M_x^* \equiv (1 - G'^<em>(\phi_x^</em>)) F'^<em>; \quad M_x^</em> = \phi_x'^{-\gamma} F'^<em>, \quad \phi \in [\phi_x^</em>, \infty) ]</td>
</tr>
<tr>
<td>(A10)</td>
<td>Average productivity cut-offs are proportional to marginal firms’ cut-offs</td>
<td>[ \bar{\phi}_d^{-1} = \left( \frac{\gamma}{1 + \gamma - \sigma} \right) \phi_d^{-1}, \quad \bar{\phi}_x^{-1} = \left( \frac{\gamma}{1 + \gamma - \sigma} \right) \phi_x^{-1} ]</td>
<td>[ \bar{\phi}_d'^{-1} = \left( \frac{\gamma'}{1 + \gamma' - \sigma} \right) \phi_d'^{-1}, \quad \bar{\phi}_x'^{-1} = \left( \frac{\gamma'}{1 + \gamma' - \sigma} \right) \phi_x'^{-1} ]</td>
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### Table A2 continued

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<tr>
<td>(A11)</td>
<td>Productivity distribution of the surviving firms in the differentiated good sector</td>
<td>( \mu_d(\varphi) = \frac{g(\varphi)}{1 - G(\varphi_d)} ), ( \mu_*(\varphi) = \frac{g(\varphi)}{1 - G(\varphi)} )</td>
<td>( \mu^<em>_d(\varphi) = \frac{g^</em>(\varphi)}{1 - G^<em>(\varphi_d)} ), ( \mu^</em>_<em>(\varphi) = \frac{g^</em>(\varphi)}{1 - G^*(\varphi)} )</td>
</tr>
</tbody>
</table>
| (A12)| CES aggregation of differentiated goods consumed (domestically produced and imported) | \[ Y = \left( \int_{\varphi \in [\varphi_d, \infty)} M \mu_d(\varphi) y_d(\varphi)^{1-\sigma} d\varphi + \right. \\
|     |     | \left. \int_{\varphi \in [\varphi^*, \infty)} M^* \mu^*_d(\varphi) y^*_d(\varphi)^{1-\sigma} d\varphi \right)^{\frac{1}{1-\sigma}} \] | \[ Y^* = \left( \int_{\varphi \in [\varphi^*_d, \infty)} M^* \mu^*_d(\varphi) y^*_d(\varphi)^{1-\sigma} d\varphi + \right. \\
|     |     | \left. \int_{\varphi \in [\varphi^*_*, \infty)} M \mu_*(\varphi) y_*(\varphi)^{1-\sigma} d\varphi \right)^{\frac{1}{1-\sigma}} \] |
| (A13)| CES price index for \( Y \)                                                                                           | \( P_Y = \left( \int_{\varphi \in [\varphi_d, \infty)} M \mu_d(\varphi) p_d(\varphi)^{1-\sigma} d\varphi + \right. \\
|     |     | \left. \int_{\varphi \in [\varphi^*, \infty)} M^* \mu^*_d(\varphi) p^*_d(\varphi)^{1-\sigma} d\varphi \right)^{\frac{1}{1-\sigma}} \) | \( P^*_Y = \left( \int_{\varphi \in [\varphi^*_d, \infty)} M^* \mu^*_d(\varphi) p^*_d(\varphi)^{1-\sigma} d\varphi + \right. \\
|     |     | \left. \int_{\varphi \in [\varphi^*_*, \infty)} M \mu_*(\varphi) p_*(\varphi)^{1-\sigma} d\varphi \right)^{\frac{1}{1-\sigma}} \) |
| (A14)| Demand for a domestically produced differentiated good facing a firm with a given productivity | \( y_d(\varphi) = Y \left( \frac{P_d(\varphi)}{P_Y} \right)^{-\sigma}, \quad \varphi \in [\varphi_d, \infty) \) | \( y^*_d(\varphi) = Y^* \left( \frac{P^*_d(\varphi)}{P^*_Y} \right)^{-\sigma}, \quad \varphi \in [\varphi^*_d, \infty) \) |
| (A15)| Demand for an imported differentiated good facing a firm with a given productivity | \( y^*_x(\varphi) = Y \left( \frac{P_x(\varphi)}{P^*_Y} \right)^{-\sigma}, \quad \varphi \in [\varphi^*_x, \infty) \) | \( y^*_x(\varphi) = Y^* \left( \frac{P^*_x(\varphi)}{P^*_Y} \right)^{-\sigma}, \quad \varphi \in [\varphi^*_x, \infty) \) |
| (A16)| Labour requirement for producing the differentiated good by a firm with a given productivity for its domestic production | \( l_d(\varphi) = \alpha_d + \frac{y_d(\varphi)}{\varphi}, \quad \varphi \in [\varphi_d, \infty) \) | \( l^*_d(\varphi) = \alpha^*_d + \frac{y^*_d(\varphi)}{\varphi}, \quad \varphi \in [\varphi^*_d, \infty) \) |
Table A2 continued

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<td>(A17)</td>
<td>Profit of a firm with a given productivity for its domestic production</td>
<td>[\pi_d(\varphi) = p_d(\varphi) y_d(\varphi) - (1 - s_d) w l_d(\varphi), \quad \varphi \in [\varphi_d, \infty)]</td>
<td>[\pi^<em>_d(\varphi) = p^</em>_d(\varphi) y^<em>_d(\varphi) - (1 - s^</em>_d) w^* l^<em>_d(\varphi), \quad \varphi \in [\varphi^</em>_d, \infty)]</td>
</tr>
<tr>
<td>(A18)</td>
<td>Price set by a firm with a given productivity for its domestic production</td>
<td>[p_d(\varphi) = \sigma \left(1 - s_d\right) w \left(\sigma - 1\right)^{-1} \varphi, \quad \varphi \in [\varphi_d, \infty)]</td>
<td>[p^<em>_d(\varphi) = \sigma \left(1 - s^</em>_d\right) w^* \left(\sigma - 1\right)^{-1} \varphi, \quad \varphi \in [\varphi^*_d, \infty)]</td>
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<tr>
<td>(A19)</td>
<td>Labour requirement for producing the differentiated good by a firm with a given productivity for its export production</td>
<td>[l_s(\varphi) = \alpha_s + \frac{\tau y_s(\varphi)}{\varphi}, \quad \varphi \in [\varphi_s, \infty)]</td>
<td>[l^<em>_s(\varphi) = \alpha^</em>_s + \frac{\tau^* y^<em>_s(\varphi)}{\varphi}, \quad \varphi \in [\varphi^</em>_s, \infty)]</td>
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<td>(A20)</td>
<td>Profit of a firm with a given productivity for its export production</td>
<td>[\pi_s(\varphi) = p_s(\varphi) y_s(\varphi) - (1 - s_s) w l_s(\varphi), \quad \varphi \in [\varphi_s, \infty)]</td>
<td>[\pi^<em>_s(\varphi) = p^</em>_s(\varphi) y^<em>_s(\varphi) - (1 - s^</em>_s) w^* l^<em>_s(\varphi), \quad \varphi \in [\varphi^</em>_s, \infty)]</td>
</tr>
<tr>
<td>(A21)</td>
<td>Price set by a firm with a given productivity for its export production</td>
<td>[p_s(\varphi) = \sigma \left(1 - s_s\right) w \left(\sigma - 1\right)^{-1} \varphi, \quad \varphi \in [\varphi_s, \infty)]</td>
<td>[p^<em>_s(\varphi) = \sigma \left(1 - s^</em>_s\right) w^* \left(\sigma - 1\right)^{-1} \varphi, \quad \varphi \in [\varphi^*_s, \infty)]</td>
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<tr>
<td>(A22)</td>
<td>Aggregating domestic price of differentiated good</td>
<td>[\int_{\varphi \in [\varphi_s, \infty)} M \mu_d(\varphi) p_d(\varphi)^{1-\sigma} d\varphi = M p_d(\tilde{\varphi}_d)^{1-\sigma}]</td>
<td>[\int_{\varphi \in [\varphi_s^<em>, \infty)} M^</em> \mu^<em>_d(\varphi) p^</em>_d(\varphi)^{1-\sigma} d\varphi = M^* p^<em>_d(\tilde{\varphi}_d^</em>)^{1-\sigma}]</td>
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<tr>
<td>(A23)</td>
<td>Aggregating imported price of differentiated good</td>
<td>[\int_{\varphi \in [\varphi_s^<em>, \infty)} M^</em> \mu^<em>_s(\varphi) p^</em>_s(\varphi)^{1-\sigma} d\varphi = M^* p^<em>_s(\tilde{\varphi}_s^</em>)^{1-\sigma}]</td>
<td>[\int_{\varphi \in [\varphi_s^<em>, \infty)} M^</em> \mu^<em>_s(\varphi) p^</em>_s(\varphi)^{1-\sigma} d\varphi = M^* p^<em>_s(\tilde{\varphi}_s^</em>)^{1-\sigma}]</td>
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<tr>
<td>(A24)</td>
<td>CES price index for differentiated good in terms of aggregates</td>
<td>[P_i = \left(M p_d(\tilde{\varphi}_d)^{1-\sigma} + M^* p^<em>_s(\tilde{\varphi}_s^</em>)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}]</td>
<td>[P^<em>_i = \left(M^</em> p^<em>_d(\tilde{\varphi}_d^</em>)^{1-\sigma} + M^* p^<em>_s(\tilde{\varphi}_s^</em>)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}]</td>
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<tr>
<td>(A25)</td>
<td>Aggregating revenue of domestic sales of differentiated good</td>
<td>[\int_{\varphi \in [\varphi_s, \infty)} M \mu_d(\varphi) r_d(\varphi) d\varphi = M r_d(\tilde{\varphi}_d)]</td>
<td>[\int_{\varphi \in [\varphi_s^<em>, \infty)} M^</em> \mu^<em>_d(\varphi) r^</em>_d(\varphi) d\varphi = M^* r^<em>_d(\tilde{\varphi}_d^</em>)]</td>
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<tr>
<td>(A26)</td>
<td>Aggregating revenue of exports of differentiated good</td>
<td>$\int_{\phi \in [\phi_0, \infty)} M_s \mu_s(\phi) r_s(\phi) d\phi = M_s r_s(\tilde{\phi}_s)$</td>
<td>$\int_{\phi \in [\phi_0, \infty)} M^<em>_s \mu^</em>_s(\phi) r^<em>_s(\phi) d\phi = M^</em>_s r^<em>_s(\tilde{\phi}^</em>_s)$</td>
</tr>
<tr>
<td>(A27)</td>
<td>Aggregating profit of domestic sales of differentiated good</td>
<td>$\int_{\phi \in [\phi_0, \infty)} M \mu_d(\phi) \pi_d(\phi) d\phi = M \pi_d(\tilde{\phi}_d)$</td>
<td>$\int_{\phi \in [\phi_0, \infty)} M^* \mu^<em>_d(\phi) \pi^</em>_d(\phi) d\phi = M^* \pi^<em>_d(\tilde{\phi}^</em>_d)$</td>
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<tr>
<td>(A28)</td>
<td>Average profit for domestic sales</td>
<td>$\pi_d(\tilde{\phi}_d) = \frac{r_d(\tilde{\phi}_d)}{\sigma} - (1-s_d) w \alpha_d$</td>
<td>$\pi^<em>_d(\tilde{\phi}^</em>_d) = \frac{r^<em>_d(\tilde{\phi}^</em>_d)}{\sigma} - (1-s^<em>_d) w^</em> \alpha^*_d$</td>
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<tr>
<td>(A29)</td>
<td>Aggregating profit of exports of differentiated good</td>
<td>$\int_{\phi \in [\phi_0, \infty)} M_s \mu_s(\phi) \pi_s(\phi) d\phi = M_s \pi_s(\tilde{\phi}_s)$</td>
<td>$\int_{\phi \in [\phi_0, \infty)} M^<em>_s \mu^</em>_s(\phi) \pi^<em>_s(\phi) d\phi = M^</em>_s \pi^<em>_s(\tilde{\phi}^</em>_s)$</td>
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<tr>
<td>(A30)</td>
<td>Average profit for exports</td>
<td>$\pi_s(\tilde{\phi}_s) = \frac{r_s(\tilde{\phi}_s)}{\sigma} - (1-s_s) w \alpha_s$</td>
<td>$\pi^<em>_s(\tilde{\phi}^</em>_s) = \frac{r^<em>_s(\tilde{\phi}^</em>_s)}{\sigma} - (1-s^<em>_s) w^</em> \alpha^*_s$</td>
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<tr>
<td>(A31)</td>
<td>Labour used in the differentiated sector for domestically used production</td>
<td>$\int_{\phi \in [\phi_0, \infty)} M \mu_d(\phi) l_d(\phi) d\phi = M l_d(\tilde{\phi}_d)$</td>
<td>$\int_{\phi \in [\phi_0, \infty)} M^* \mu^<em>_d(\phi) l^</em>_d(\phi) d\phi = M^* l^<em>_d(\tilde{\phi}^</em>_d)$</td>
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<td>(A32)</td>
<td>Labour used in the differentiated sector for export production</td>
<td>$\int_{\phi \in [\phi_0, \infty)} M_s \mu_s(\phi) l_s(\phi) d\phi = M_s l_s(\tilde{\phi}_s)$</td>
<td>$\int_{\phi \in [\phi_0, \infty)} M^<em>_s \mu^</em>_s(\phi) l^<em>_s(\phi) d\phi = M^</em>_s l^<em>_s(\tilde{\phi}^</em>_s)$</td>
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<tr>
<td>(A33)</td>
<td>Labour used in the differentiated sector for export production</td>
<td>$\int_{\phi \in [\phi_0, \infty)} M_s \mu_s(\phi) l_s(\phi) d\phi = M_s l_s(\tilde{\phi}_s)$</td>
<td>$\int_{\phi \in [\phi_0, \infty)} M^<em>_s \mu^</em>_s(\phi) l^<em>_s(\phi) d\phi = M^</em>_s l^<em>_s(\tilde{\phi}^</em>_s)$</td>
</tr>
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</table>
Specification of the two country model used in the analysis

Using (A16) and (A18), (A17) can be written as \( \pi_d = \frac{r_d}{\sigma} - (1 - s_d) w d \). Using this expression for profit, the zero profit condition (ZPC) of marginal non-exporting Foreign firms and exporting Home and Foreign firms implies

\[
\varphi = \varphi_d \Rightarrow \pi_d (\varphi_d) = 0 \Rightarrow r_d (\varphi_d) = \sigma (1 - s_d) w d \alpha_d
\]

(E1)

Using a similar approach we obtain

\[
\varphi = \varphi^*_d \Rightarrow \pi^*_d (\varphi^*_d) = 0 \Rightarrow r^*_d (\varphi^*_d) = \sigma (1 - s^*_d) w^*_d \alpha^*_d
\]

(E1*)

\[
\varphi = \varphi_s \Rightarrow \pi_s (\varphi_s) = 0 \Rightarrow r_s (\varphi_s) = \sigma (1 - s_s) w_s \alpha_s
\]

(E2)

\[
\varphi = \varphi^*_s \Rightarrow \pi^*_s (\varphi^*_s) = 0 \Rightarrow r^*_s (\varphi^*_s) = \sigma (1 - s^*_s) w^*_s \alpha^*_s
\]

(E2*)

The zero expected profit of entry for the Home and Foreign country require

\[
M \pi_d (\hat{\varphi}_d) + M_s \pi_s (\hat{\varphi}_s) - FP_d f = 0,
\]

(E3)

\[
M^* \pi^*_d (\hat{\varphi}^*_d) + M^*_s \pi^*_s (\hat{\varphi}^*_s) - F^* P^*_d f^* = 0.
\]

(E3*)

The balanced government budget constraints (equating the subsidy bill with tax revenue) for the Home and Foreign country are

\[
s_d w M_d + s_s w M_s x_s = N w h,
\]

(E4)

\[
s^*_d w^* M^*_d + s^*_s w^* M^*_s x^*_s = N^* w^* h^*.
\]

(E4*)

The CES price indices in (A13) and the aggregations in (A22) and (A23) imply

\[
P = \left( M \left( p_d (\hat{\varphi}_d) \right)^\frac{1}{1-\sigma} + M^* \left( p^*_d (\hat{\varphi}^*_d) \right)^\frac{1}{1-\sigma} \right)^\frac{1}{1-\sigma},
\]

(E5)

\[
P^* = \left( M^* \left( p^*_d (\hat{\varphi}^*_d) \right)^\frac{1}{1-\sigma} + M_s \left( p_s (\hat{\varphi}_s) \right)^\frac{1}{1-\sigma} \right)^\frac{1}{1-\sigma}.
\]

(E5*)

Per-capita labour supplies, given by (A2), are

\[
h = \left( \frac{1 - \tau}{\theta P^\beta} \right)^\frac{1}{\delta},
\]

(E6)

\[
h^* = \left( \frac{1 - \tau^*}{\theta P^*}\right)^\frac{1}{\delta}.
\]

(E6*)
The other equilibrium conditions which should hold are the labour market equilibrium
conditions,
\[ L_d + Ml_d + M_s l_s = Nh, \]  
(E7)
\[ L'_d + M^*l'_d + M^*_s l'_s = N^* h^*, \]  
(E7*)
the global market equilibrium condition for the homogenous good
\[ P_A (A + Ff) + P'_d (A' + F' f') = P_A A' + P'_d A'', \]  
(E8)
and the global trade balance,
\[ P_A (A' - A - Ff) + M_s r_s (\bar{\phi}_s) = M'_s r'_s (\bar{\phi}'_s). \]  
(E9)

It can be shown that (E8) and (E9) are satisfied – by the implication of Walras law – if
(E1) to (E7*) hold.

Finally, since the homogenous good is competitively produced with CRS, freely traded
and used as numeraire, we have \( P_A = P'_d = 1, \ w = P_s = 1 \) and \( w^* = P^* = 1. \) After appropriate
substitutions explained below, which take account of the latter normalisations, 12 equations
(E1)-E(6*) can be shown to be expressed in the form of (E1)'-E(6*)' below which determine the
12 unknowns \((\varphi_d, \varphi_s, \varphi'_d, \varphi'_s, P_1, P'_1, F, F', h, h', t, t')\) on the assumption that the subsidy rates
are treated as exogenous policy instruments. It should then be ensured that this solution satisfies
labour resource conditions in (E7)-(E7*).

\[
\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} N\beta(1-t)h\left(P_d\right)^{\sigma-1}\left(\varphi_d\right)^{\sigma-1}\left(1-s_d\right)^{-\sigma} = \sigma d
\]  
\( (E1)' \)
\[
\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} N^*\beta(1-t')h^*\left(P'_d\right)^{\sigma-1}\left(\varphi'_d\right)^{\sigma-1}\left(1-s'_d\right)^{-\sigma} = \sigma d^*
\]  
\( (E1*)' \)
\[
\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} N^*\beta(1-t')h^*\left(P'_y\right)^{\sigma-1}\left(\varphi'_x\right)^{\sigma-1}\tau^{1-\sigma}\left(1-s'_x\right)^{-\sigma} = \sigma x^*
\]  
\( (E2)' \)
\[
\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} N\beta(1-t)h\left(P_y\right)^{\sigma-1}\left(\varphi_x\right)^{\sigma-1}\tau^{1-\sigma}\left(1-s_x\right)^{-\sigma} = \sigma x
\]  
\( (E2*)' \)
\[
(1-s_d)\varphi_d^y\left(\gamma\left(1+\gamma-\sigma\right)\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\frac{N\beta(1-t)h\left(P_d\right)^{\sigma-1}\left(\varphi_d\right)^{\sigma-1}\left(1-s_d\right)^{-\sigma} - \alpha_d}{\sigma} + \right) +
\]
\[
(1-s_s)\varphi_x^y\left(\gamma\left(1+\gamma-\sigma\right)\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\frac{N^*\beta(1-t')h^*\left(P'_y\right)^{\sigma-1}\left(\varphi'_x\right)^{\sigma-1}\tau^{1-\sigma}\left(1-s'_x\right)^{-\sigma} - \alpha_s}{\sigma} \right) = f
\]  
\( (E3)' \)
In addition to the above, in order to examine welfare, we substitute in the per capita utility

\[ u = \left( \frac{a}{1 - \beta} \right)^{1 - \beta} \left( \frac{y}{\beta} \right)^{\beta} \frac{\partial h^{1/\delta}}{1 + \delta} \]

using \( a = (1 - \beta)(1 - t)h \), \( y = \frac{\beta(1 - t)h^*}{P_Y} \) and \( h = \left( \frac{1 - t}{\partial P_Y^\beta} \right)^{1/\delta} \), to obtain

\[ u = \left( \frac{\partial \delta}{1 + \delta} \right)^{1/\delta} = \left( \frac{\partial \delta}{1 + \delta} \right)^\delta \left( \frac{1 - t}{1 + \delta} \right)^{(1-\delta)/\delta} \]

Hence, \( u \) is a monotonically increasing function.
of \( h \) which is in turn an increasing function of the after tax real wage \( \frac{1-t_i}{P_i^\theta} \). By the same token,

\[
u^* = \left( \frac{\partial \delta}{1 + \delta} \right) h^{\delta + \gamma} = \left( \frac{\partial \delta}{1 + \delta} \right) \left( \frac{1-t_i}{P_i^\theta} \right)^{\delta + \gamma}.
\]

While the model is fully determined by (E1)’-(E6)’ and (E7)-(E7’), its properties are better understood if it is further reduced. For instance, using (E1)’ and (E1*)’ to eliminate \( (h, h^*) \) from (E2)’-(E3)’ we obtain

\[
\left( \frac{\varphi_x}{\varphi_d} \right)^{\sigma-1} = \tau^{\sigma-1} \left( \frac{1-s_x}{1-s_d} \right)^{\alpha_x} \left( \frac{\alpha_x}{\alpha_d} \right),
\]

(E2’’)

\[
\left( \frac{\varphi_x}{\varphi_d} \right)^{\sigma-1} = \tau^{\sigma-1} \left( \frac{1-s_x}{1-s_d} \right)^{\alpha_x} \left( \frac{\alpha_x}{\alpha_d} \right),
\]

(E2*’’)

\[
\left( \alpha_x (1-s_x) \varphi_x^{\sigma-\gamma} + \alpha_x (1-s_x) \varphi_x^{\sigma-\gamma} \right) \left( \frac{\alpha_x}{1+\gamma-\sigma} \right) = f^*,
\]

(E3’’)

\[
\left( \alpha_x (1-s_x) \varphi_x^{\sigma-\gamma} + \alpha_x (1-s_x) \varphi_x^{\sigma-\gamma} \right) \left( \frac{\alpha_x}{1+\gamma-\sigma} \right) = f^*,
\]

(E3*’’)

which determine \( (\varphi_d, \varphi_x, \varphi_d^*, \varphi_x^*) \). This recursive nature of the model is important in understanding how the trade cost, fixed costs, the shape parameter of the productivity distribution and the subsidy rates determine the productivity cut-offs. It also shows that the latter do not depend on the relative size of the two countries embodied in \( N / N^* \), which is important when making one country small relative to the other by letting \( N / N^* \) reduce.

**Specification of the small open economy (SOE)**

We obtain the SOE from the two country model above by (i) reducing \( N / N^* \) to make the Home country small relative to the Foreign country; (ii) making the Home country relatively more open by increasing \( \tau^* \); and (ii) treating \( (\varphi_d^*, P_r^*, F^*, t^*) \) as exogenous and dropping equations (E1)’, (E3)’, (E4)’, (E5)’ and (E6)’. This leads to characterising the SOE model by 6 equations (E1)’, (E2)’, (E2*)’, (E3)’, (E4)’, (E5)’ and (E6)’ which determine \( (\varphi_x, \varphi_x, \varphi_x^*, h, P_r, F, t) \). Note that \( \varphi_x^* \) ought to be treated as endogenous so as to allow for the adjustment of the mass of imported varieties consumed at Home (in the SOE). Examining these equations, we note that there is no direct channel which allows us to obtain the limiting case as \( N / N^* \rightarrow 0 \). In such circumstances, where we resort to numerical solutions, we can overcome the problem by first making the two country model asymmetric by reducing \( N / N^* \) and ensuring that the extent of relative openness captured by \( \tau / \tau^* \) is raised to offset the impact of the change
in $N/N^*$ and that the solution remains plausible. The result is an asymmetric two country model in which the foreign country is sufficiently large and faces a relatively higher barrier to export to its small trade partner (Home country). We then calibrate the SOE using this as the initial condition, treating the variables of the Foreign country except $\phi_i^*$ as exogenous, and solving equations (E1)', (E2)', (E2')', (E3)', (E4)', (E5)' and (E6)' to determine $(\varphi_d, \varphi_i^*, h, P_f, F, t)$, ensuring that the trade balance is satisfied.

We examine two types of exogenous shocks: a demand shock and a general trade shock. The demand shock is formulated as a change in the demand for exports which is identified by writing the demand for exports facing the firms in the SOE as

$$y_x(\varphi) = \left[ N^* \beta (1-t) h^* \left( P_f t \right)^{\sigma^{-1}} \right] \left( P_s(\varphi) \right)^{-\sigma}$$

where the term in square brackets is now exogenous, and is denoted by $K^*$ in the text, i.e. $K^* = N^* \beta (1-t) h^* \left( P_f t \right)^{\sigma^{-1}}$. A typical negative shock then can be captured by a drop in $h^*$ representing a rise in unemployment in the rest of the world. The negative trade shock, on the other hand, is simply captured by a rise $\tau$. To have the impact of these two shocks of the same scale, we reduce $h^*$ and $\tau^{\sigma^{-3}}$ by 10% respectively. As discussed in the text, the effects of these shocks are qualitatively identical.