Credit Crunched Regulated Banks

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Very incomplete version

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1 Introduction

Why are apparently profitable investments not undertaken? Traditional arguments emphasize problems with asymmetric information implying credit rationing (Stiglitz and Weiss, 1981; De Meza and Webb, 1987). We provide an explanation based on the quality of information. When entrepreneurs need a bank to fund an investment, the bank can be uncertain about the correct model governing the profitability of the investment. We term this type of uncertainty as model uncertainty. If there are no frictions and the bank is risk neutral, model uncertainty does not cause a problem per se. This is due to the fact that the bank simply calculates the net present value of the investment and supports the investment if it is profitable. However, banks are subject to capital requirements set by the government. For example, the US Federal Reserve announced in December 2011 that it would implement substantially all of the Basel III rules. This implies that banks care about risk and, in particular, model uncertainty regarding the project’s profitability is important. We show that model uncertainty increases the investment distortion induced by capital requirements.

In general, agents’ preferences depend on the quality of information available and they distinguish between risk (known probabilities) and ambiguity (unknown probabilities) and may display aversion to ambiguity, just as they display aversion to risk (Knight, 1921; Ellsberg, 1961; Bossaerts et al., 2010). We use ambiguity to analyze the bank’s uncertainty regarding the profitability of the investment. We show that the impact of ambiguity on the bank’s investment decision depends on the state of the economy. In a boom, profitability tends to be less volatile implying that capital requirements are more slack. Thus, ambiguity need not be that important in this case. In a recession the project’s volatility increases implying that capital requirements become a more important determinant in the bank’s decision making.
Furthermore, we show that even if model uncertainty does not impact the risk per se, ambiguity aversion vis-a-vis capital requirements can induce the bank to neglect funding profitable projects.

To highlight the basic intuition we initially consider a one-period model. We consider a bank which has capital available to fund a risky project offered by an entrepreneur. To prevent the bank from undertaking risky investments the bank is subject to capital requirements set by the government. In particular, the bank can only undertake the project if the conditional Value at Risk, the so-called expected shortfall, of the project does not exceed a given threshold. Hence, when the bank undertakes a project, it not only considers the project’s value, it also needs to ensure that its expected shortfall satisfies the regulation requirement.

Two key features have an impact on the bank’s assessment of the project. First, the economy is either in a recession or in a boom, where a recession is characterized by a lower expected growth rate and a higher volatility. Second, the bank is uncertain about the quality of the project. To keep it simple, the bank believes that the cash flow is either generated by model 1 or by model 2, where model 1 is characterized by a relative high mean compared to model 2. Hence, the bank prefers that model 1 describes the cash flow, but fears that model 2 is the true model. That is, the bank is ambiguous about the quality of the project so we assume that the bank’s preferences are represented by smooth ambiguity preferences (Klibanoff et al., 2005). We determine an ambiguity adjusted expected shortfall (AES) and show that if the bank is ambiguity neutral, AES simply equals the model-expected value of the expected shortfall. On the other hand, if the bank is extremely ambiguity averse, AES converges to the expected shortfall in the worst case model, which in our setting is model 2. Consequently, in some circumstances an ambiguity neutral bank is willing to fund the project, whereas an ambiguity averse bank will not. Hence,
ambiguity aversion can lead to underinvestment.

The purpose of capital requirements is to induce the bank to undertake projects which are perceived as more certain. Hence, it is rational for the bank to require a relatively low return premium. Therefore, our theory leads to a different prediction regarding return premium than underinvestment based on credit rationing. If underinvestment is based on credit rationing, we expect to see increases in the return premium up to a level at which the entrepreneur rejects to turn to the bank for funding. On the other hand, if underinvestment is based on model uncertainty, we show that a higher return premium does not help the bank to comply with the regulatory threshold per se.

Our aim is to extend the model to a dynamic setting. Particularly, the economy can shift between being in a boom and in a recession. We expect dynamic effects of model uncertainty to provide important insights into a bank’s investments decisions and how these are affected by regulatory requirements. For example, a recession often has a low risk free interest rate, but, on the other hand, capital requirements are more binding in a recession. Apparently, the low interest rate should alleviate funding problems, but the tougher capital requirement may induce underinvestment. Another interesting issue is the fact that while ambiguity may not be seen as important when the economy is in a boom, the possibility of a change to a recession has spill over effects. In particular, this impacts which types of projects the bank is willing to fund. That is, banks may fund relatively risky projects in a boom, if the probability of entering a recession is low, whereas banks turn to more secure projects if a recession is getting closer.

The remainder of the paper is organized as follows. Section 2 sets up the one-period model. Section 3 presents numerical examples. Section 4 sets up the dynamic model. Section 5 discusses implications and empirical predictions. Finally, we con-
clude in Section 6.

2 The Single Period Model

Consider a one-period setting with risk neutral agents in which a bank has capital, $C_0$, available to fund a project offered by an entrepreneur. To initiate the project at time $t = 0$, the entrepreneur needs $I$ in funding from the bank. The bank’s remaining capital $C_0 - I$ is invested in the risk free asset. The entrepreneur’s project returns a cash flow $x$ after one period at time $t = 1$. At $t = 0$ the cash flow is uncertain such that the cash flow follows a log-normal distribution. Let $z$ follow a normal distribution with $\mu - \frac{1}{2} \sigma^2$ and $\sigma^2$ as the mean and the variance, respectively. Then $x = Ie^z$ is lognormally distributed with

\begin{align*}
\mathbb{E}[x] &= I e^\mu, \\
\mathbb{V}[x] &= I^2 e^{2\mu} \left( e^{\sigma^2} - 1 \right). 
\end{align*}

For simplicity we assume a constant risk-free rate equal to 0.

The bank maximizes the expected discounted profit. For simplicity, assume that $x$ is the cash flow left for paying off the bank. Since $x$ is uncertain, the bank can suffer a loss, and hence requires a return premium, $\iota$, over the risk free return. Thus, the bank’s payoff is $\min\{(1 + \iota)I, x\}$. The bank’s value of the project is given by $I$

\footnote{We assume moral hazard or adverse selection prevents the entrepreneur from getting funding by the financial market. That is, the bank can perfectly monitor the entrepreneur’s project, but the bank cannot get rid of the project by selling it to the financial market. Furthermore, we consider a risk neutral bank for simplicity; our implications would still hold assuming a risk averse bank.}

\footnote{This assumption could be relaxed. However, as managerial contracting is not the focus in our paper, we omit an elaborate description of the distribution of the cash flow between the bank and the entrepreneur.}
subtracted from the expected payoff, i.e.

\[
V(I, \iota, \mu, \sigma) = \mathbb{E}[\min\{(1 + \iota)I, x\}] - I \\
= I(\mathbb{E}[\min\{(1 + \iota), e^z\}] - 1) \\
= I(\mathbb{E}[(1 + \iota) - \max\{0, (1 + \iota) - e^z\}] - 1).
\] (3)

We recognize the max-term as the payoff of a put option with exercise price \((1 + \iota)\) and \(e^z\) as the underlying. Since the latter follows a log-normal distribution, the Black-Scholes formula gives us\(^3\)

\[
V(I, \iota, \mu, \sigma) = I(\iota - \mathcal{P}(\iota, \mu, \sigma)), \tag{4}
\]

where the value of the put option is

\[
\mathcal{P}(\iota, \mu, \sigma) = (1 + \iota)N(-d_2) - e^\mu N(-d_1),
\]

with

\[
d_1 = \frac{-\log(1 + \iota) + \mu + \frac{1}{2}\sigma^2}{\sigma}, \quad d_2 = d_1 - \sigma.
\]

For a given interest rate \(\iota\), the value of the put-option decreases in the expected rate of return of the project, \(\mu\), whereas the value of the put-option increases in the volatility of the project, \(\sigma\). Hence, from (4) it follows that the bank’s value of the project increases in the expected rate of the return of the project and decreases in the volatility.

Our model focuses on two key features which impact the project’s cash flow. First, the economy is either in a recession or in a boom. Intuitively, and in line with the literature, a recession is characterized by a lower expected growth rate and

\(^3\)The Black-Scholes formula with dividends can be found in e.g. Hull (2012).
a higher volatility.\footnote{See for example Schwert (1989), Campbell et al. (2001), Bhamra et al. (2010), and Chen (2010).} Second, the bank is uncertain about the model describing the cash flows. Specifically, the bank believes that the cash flow is generated either by model 1 or by model 2. Model 1 is characterized by a relatively high mean compared to model 2. Thus, the bank prefers that model 1 describes the cash flow, but it fears that model 2 is the true model. To set up the notation, let $i \in \{R, B\}$ denote the state of the economy (Recession, Boom), let $j \in \{1, 2\}$ denote the model for cash flows, and let $\nu$ denote the bank’s belief of the probability of model 1 being the correct model. We assume that $\mu_1 > \mu_2$ in state $i$, and by (1) it follows that model 1 generates the highest expected value of the cash flow. Table 1 summarizes the distributional characteristics.

<table>
<thead>
<tr>
<th>State</th>
<th>Model 1 Distribution</th>
<th>Model 2 Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boom</td>
<td>$N(\mu_{B1} - \frac{1}{2}\sigma_{B1}^2, \sigma_{B1}^2)$</td>
<td>$N(\mu_{B2} - \frac{1}{2}\sigma_{B2}^2, \sigma_{B2}^2)$</td>
</tr>
<tr>
<td>Recession</td>
<td>$N(\mu_{R1} - \frac{1}{2}\sigma_{R1}^2, \sigma_{R1}^2)$</td>
<td>$N(\mu_{R2} - \frac{1}{2}\sigma_{R2}^2, \sigma_{R2}^2)$</td>
</tr>
</tbody>
</table>

Table 1: Distribution of $z$ depending on state and true model. After one period the project returns a cash flow $x = Ie^z$. We normalize $\sigma_{i1}^2$ so that the project’s risk is independent of which model generate the cash flow, i.e. $\sigma_{i1}^2$ is given by (5) for $i \in \{B, R\}$. Furthermore, we assume that $\mu_{i1} > \mu_{i2}$ for $i \in \{B, R\}$ and the bank’s value is 0 in the worst case (Recession, model 2).

Generally, regulators impose conditions aiming at restricting the bank’s risk taking. This implies that model 2 is even worse, if it increases the variance of the project’s cash flow. We do not want this to drive our results. Therefore, given the risk under model 2, we normalize $\sigma_{i1}^2$ so that the project’s risk is independent of
which model generates the cash flow. That is, by (2) we require:

\[ I^2 e^{2\mu_1} \left( e^{\sigma^2_{i1}} - 1 \right) = I^2 e^{2\mu_2} \left( e^{\sigma^2_{i2}} - 1 \right) \]

implying that

\[ \sigma^2_{i1} = \log \left( 1 + e^{2(\mu_1 - \mu_2)} \left( e^{\sigma^2_{i2}} - 1 \right) \right). \] (5)

Since \( \mu_1 > \mu_2 \), our restriction implies that \( \sigma_{i1} < \sigma_{i2} \).

### 2.1 Regulation

The essence of bank regulation is to prevent banks from undertaking risky investments with potential large losses. To obtain a tractable model, we model regulation as a restriction on the riskiness of investments. A common risk measure is Value at Risk, VaR. The 1-period \( \alpha \)% VaR is defined implicitly by\(^5\)

\[
\mathbb{P}((x_1 + (C_0 - I)) - C_0 \leq -\text{VaR}) = 1 - \alpha, \quad (6)
\]

\[
\mathbb{P}(I - x_1 \geq \text{VaR}) = 1 - \alpha, \quad (7)
\]

where \( \mathbb{P} \) is the profit-and-loss distribution of the investment. In our model this implies

\[
\text{VaR} = I \left( 1 - \exp \left( \mu - \frac{1}{2} \sigma^2 + \sigma N^{-1}(1 - \alpha) \right) \right). \quad (8)
\]

Thus, VaR tells us something about the probability of getting a loss at least of size VaR. However, it does not tell us anything about the expected loss, if the cash

\(^5\)Strictly speaking, we should consider the probability of a large loss using the bank’s payoff. However, since the restriction is only relevant when the bank does not get its payment back, we have \( \mathbb{P}(\text{loss} \geq \text{VaR}) = \mathbb{P}(I - \min \{(1+\epsilon)I, x\} \geq \text{VaR}) = \mathbb{P}(\min \{(1+\epsilon)I, x\} \leq I - \text{VaR}) = \mathbb{P}(x \leq I - \text{VaR}) = \mathbb{P}(x - I \leq -\text{VaR}). \)
flow falls below the VaR threshold. Therefore, we consider the conditional VaR, the so-called expected shortfall, which is defined as\textsuperscript{6}

\[ ES = \mathbb{E}^P [\text{loss}|\text{loss} \geq \text{VaR}] \]

\[ = \mathbb{E}^P [I - x|I - x \geq \text{VaR}] . \tag{9} \]

Using our setup and (8) we get

\[ ES = \mathbb{E}^P [I(1 - e^z)|Ie^z - I \leq -\text{VaR}] \]

\[ = \mathbb{E}^P [I(1 - e^z)|e^z \leq \exp(\mu - \frac{1}{2}\sigma^2 + \sigma N^{-1}(1 - \alpha))] \]

\[ = I (1 - \mathbb{E}^P [e^z|e^z \leq K]) \]

\[ = I \left(1 - \mathbb{E}^P[e^z]\frac{N\left(\log K - \left(\mu - \frac{1}{2}\sigma^2\right)\frac{1}{\sigma} - \sigma\right)}{N\left(\log K - \left(\mu - \frac{1}{2}\sigma^2\right)\frac{1}{\sigma}\right)}\right) \]

\[ = I \left(1 - e^\mu \frac{N(N^{-1}(1 - \alpha) - \sigma)}{1 - \alpha}\right) . \tag{10} \]

where \(N\) is the cumulative distribution of the standard normal distribution, and \(N^{-1}\) is the inverse of that distribution. Intuitively, the expected shortfall is a decreasing (increasing) function of the project’s expected rate of return (volatility). Hence, given the state of the economy, we see that the expected shortfall in model 1 is lower than the expected shortfall in model 2 albeit we assume the project’s variance to be equal in the two models.

Henceforth we assume that the regulators restrict the bank’s risk taking by demanding that the expected shortfall does not exceed a threshold \(\overline{ES}\). Since regulators care about banks losing a substantial amount of their capital, this threshold

\textsuperscript{6}The conditional VaR was suggested by Artzner et al. (1999). They define certain properties that a good risk measure should have and show that the standard VaR measure does not have all of them.
should depend on the size of the bank. In theory, regulators are sufficiently sophis-
ticated so that they set up the capital requirements depending on several character-
stics of a particular bank. However, we assume for tractability that the cut-off for
the expected shortfall only depends on the size of the bank. Therefore, we define
the cut-off as

\[ ES = \rho C_0, \quad \rho \in (0, 1). \]  
(11)

That is, \( \rho \) is the regulators’ instrument. In practice, banks are heavily levered and
for many banks a large part of the liabilities consist of deposits. Thus, banks get
in trouble if they lose a relatively small amount. Regulators can deal with this by
setting \( \rho \) low. The definition of the cut-off implies that large banks, i.e., banks with
a lot of capital \( C_0 \), are allowed to invest in projects with potentially larger losses
than banks with less capital. In a dynamic model this feature is crucial as it provide
a bank with an incentive to build up capital in bad times. We return to this point
later.

The bank prefers to fund a project which maximizes the bank’s value. As noted,
the bank is uncertain about the model generating the cash flow distribution and
it takes this into account by taking the average over the models. However, it also
needs to ensure that its expected shortfall satisfies the regulation requirement. This
implies that the riskiness of the project is important for the bank. To take care of the
fact that the bank is uncertain about the model generating the cash flow distribution
we implement model uncertainty by assuming smooth ambiguity preferences as in
e.g. Klibanoff et al. (2005) and Ju and Miao (2011). A key feature of the smooth
preference framework is that it achieves separation between ambiguity, identified as
a characteristic of the decision maker’s subjective beliefs, and ambiguity attitude,
identified as a characteristic of the decision maker’s tastes. Specifically, we assume
that the bank has constant absolute ambiguity aversion with parameter $a$ when it considers satisfying the expected shortfall constraint.

Generally, constant absolute ambiguity aversion in a one-period setting implies that a risk-neutral decision maker’s ambiguity preferences over consumption, $cons$, are represented by the utility function

$$U(cons) = -\frac{1}{a} \log \left(-\mathbb{E}_\nu \left[-e^{-a\mathbb{E}_\pi[cons]}\right]\right), \tag{12}$$

where $\nu$ represent the distribution of the possible models, and $\pi$ is the distribution conditional on a model. In our setting, the bank worries about crossing the expected shortfall threshold. That is, the bank prefers a model which generates a low expected shortfall, but worries about another model which generates a high expected shortfall. In (12) a decision maker prefers a high $cons$, whereas the bank prefers a low $ES$. We circumvent this problem by using (12) on $-ES$. Hence, the bank calculates the ambiguity adjusted expected shortfall ($AES$) using its belief regarding the model generating cash flow by

$$AES = -\frac{1}{a} \log \left(\mathbb{E}_\nu \left[e^{aES}\right]\right). \tag{13}$$

Let $ES_{i,j}$ denote the expected shortfall of the project when the economy is in state $i$ conditioning on the true model being $j$. If the bank is ambiguity neutral, i.e. $a = 0$, l’Hospital’s rule implies that in state $i \in \{R, B\}$

$$AES_i \to \mathbb{E}_\nu [ES_i] = \nu ES_{i,1} + (1 - \nu) ES_{i,2}, \quad a \to 0. \tag{14}$$

That is, an ambiguity neutral bank adjusts for model uncertainty by simply taking the model-expected value of the expected shortfall. On the other hand, following Klibanoff et al. (2005) it can be shown that when the bank is extremely ambiguity
averse \((a \to \infty)\), then\(^7\)

\[
AES_i \to ES_{i,2}, \quad a \to \infty
\]

(15)

Hence, the ambiguity adjusted expected shortfall converge to the expected shortfall in the worst case model, which in our setting is model 2.

Previously we noted that the expected shortfall is highest in model 2, given the state of the economy. For most projects it is reasonable to expect that Boom is a better state than Recession. We require this to be the case in our illustrations below. Hence the project has the highest expected shortfall if the economy is in a recession and model 2 is the true model of the cash flow. That is, \(ES_{i,2} > ES_{i,1}\). Suppose that the expected shortfall overshoots the threshold set by the regulators in model 2, but not in model 1, i.e., \(ES_{i,1} < \overline{ES} < ES_{i,2}\). Hence, an ambiguity neutral bank will reject the project if it believes that model 2 is sufficiently likely. On the other hand, if model 1 is believed to be sufficiently likely, the bank accepts the project. However, if the bank has a positive belief in model 2, a sufficiently ambiguity averse bank rejects the project, since the adjusted expected shortfall increases in ambiguity aversion. We summarize these observations in the following proposition.

**Proposition 1.** Let \(ES_{i,j}\) denote the expected shortfall of the project when the economy is in state \(i\) conditioning on the true model being \(j\). If \(ES_{i,1} < \overline{ES} < ES_{i,2}\), then sufficiently high ambiguity aversion implies underinvestment.

As earlier noted, the return premium, \(\iota\), does not influence the expected shortfall, i.e., the bank cannot use the return premium to compensate for a high ambiguity aversion. This is intuitive as a higher premium benefits the bank when the investments turns out to be successful, whereas regulators care about the bank’s conditions when the investments turns out to be unsuccessful.

\(^7\)See Proposition 3 in Klibanoff et al. (2005).


3 Numerical Results

In our benchmark case we consider a bank which have been asked to fund a project of $10 mio. The expected rate of return in the four scenarios are assumed to be $\mu_{B1} = 13\%$, $\mu_{B2} = 8\%$, $\mu_{R1} = 12\%$, and $\mu_{R2} = 7\%$. In the worst case scenario (Recession, model 2) we assume a volatility of $\sigma_{R2} = 20\%$, whereas the volatility in model 2 in a boom is assumed to be 5\% lower compared to the volatility in the recession, i.e. $\sigma_{B2} = 0.95 \times 20\% = 19\%$. Finally, the volatilities in model 1 are determined by the use of (5) and implies that $\sigma_{R1} = 19.04\%$ and $\sigma_{B1} = 18.09\%$, respectively. Table 2 summarizes the parametrization.

<table>
<thead>
<tr>
<th></th>
<th>Prob(model 1) = $\nu$</th>
<th>Prob(model 2) = $1 - \nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Boom</strong></td>
<td>(0.13, 0.1809)</td>
<td>(0.08, 0.1900)</td>
</tr>
<tr>
<td><strong>Recession</strong></td>
<td>(0.12, 0.1904)</td>
<td>(0.07, 0.2000)</td>
</tr>
</tbody>
</table>

Table 2: Specification of cash flow characteristics, $(\mu_{ij}, \sigma_{ij})$, in our benchmark case. We have that $i \in \{B, R\}$ denote the state of the economy (Boom, Recession), while $j \in 1, 2$ denote the model. We normalize $\sigma_{i1}^2$ in the two states so that the project’s risk is independent of which model generate the cash flow. $\iota = 10.24\%$.

Figure 1 displays the ambiguity adjusted expected shortfall as a function of the bank’s ambiguity aversion in our benchmark case, when the economy is in a recession. If the bank is certain that one of the two models gives the true description of the project’s cash flow, the ambiguity adjusted expected shortfall equals the expected shortfall in that model. The top solid line equals the expected shortfall under the assumption that model 2 (the worst model) is the true model, while the bottom solid line equals the expected shortfall under the assumption that model 1
(the best model) is the true model. Between these lines we depict the ambiguity adjusted expected shortfall in three cases depending on the bank’s probability of model 1 being the true model, $\nu = 25\%, \ 50\%, \ and \ 75\%$, respectively. The gray vertical line depicts the threshold, $ES$ , which we assume to be 2.8.

If the bank is ambiguity neutral, $a = 0$, it follows from (14) that the ambiguity adjusted expected shortfall is the $\nu$-expected shortfall. To exemplify, in a recession we have that $ES_{R1} = 2.51$ in model 1, whereas $ES_{R2} = 3.02$ in model 2. Thus, if the bank believes there is a 25% probability of model 1 being the true model, we get $AES = 0.25 \times 2.51 + 0.75 \times 3.02 = 2.89$. Thus, the bank rejects the project. In contrast, if the bank believes model 1 is as likely as model 2, then $AES = 2.76$ implying that the bank funds the project. This illustrates that the bank rejects to fund the project, if it considers the bad model to be likely enough. However, for an ambiguity averse bank it is not sufficient that the $\nu$-expected shortfall is below the threshold. For example, when the models are equally likely, $\nu = 0.5$, the bank rejects the project, if its ambiguity aversion is slightly above 1. Intuitively, a higher belief of the good model gives more slack for ambiguity aversion. For instance, if model 1 is believed with 75% probably, the project is accepted if the ambiguity aversion is below 5. Figure 1 reveals that the ambiguity adjusted expected shortfall is particularly increasing, when the ambiguity aversion is low. As noted for $\nu = 0.5$, even a small degree of ambiguity aversion can impact the bank’s decision. That is, albeit an ambiguity neutral bank would accept the project, a moderate ambiguity averse bank may reject the project. Thus, it is particularly important to take model uncertainty into account, when a simple model-averaged expected shortfall is not far below the threshold.

The above effects discussed for a recession also hold if the economy is in a boom. The main difference is that the levels of the expected shortfalls are lower. With
Figure 1: $AES$ as a function of the bank’s ambiguity aversion, $a$, in Recession. The top (bottom) solid line equals the expected shortfall if model 2 (model 1) is the true model. The dashed line displays $AES$ for $\nu = 0.50$; the dotted line displays $\overline{ES}$. 
our parametrization, the expected shortfall in model 2 is below the threshold, so we
do not report detailed results for the boom. However, the fact that the expected
shortfall is higher in a recession compared to a boom has important implications.
Figure 2 illustrates this for the case of an equal belief of the two models. The upper
dashed line is $AES$ in a recession corresponding to the dashed line in Figure 1. The
lower solid line is the ambiguity adjusted expected shortfall in a boom, while the
threshold is illustrated with the dotted line and equals $ES = 2.8$, as before. If the
bank is ambiguity neutral, its investment policy is independent of whether a boom
or a recession prevails. However, the conclusion changes if the bank is ambiguity
averse. Suppose the bank’s ambiguity aversion equals $a = 4$. Then the ambiguity
adjusted expected shortfall in a boom is 2.65, which is clearly below the threshold,
so the bank funds the project. On the other hand, in a recession, the ambiguity
adjusted expected shortfall increases to 2.88, i.e. the bank has to reject the project.
Note that the NPV of the project in state $i \in \{R, B\}$ equals

$$NPV_i = I (\nu e^{\mu_1} + (1 - \nu) e^{\mu_2} - 1).$$

That is, in a recession we have $NPV_R = 1.00$, while in a boom $NPV_B = 1.11$.
Hence, if the bank rejects the project, as is the case in recession, underinvestment
occurs.

4 Dynamic effects of regulation and ambiguity

(incomplete)

In this section we set up a dynamic version of the model analyzed so far. We
also extend the setting so that the bank has a choice between different (mutually
exclusive) projects. For now think of these projects as a $p \in \{l, m, h\}$, where $p$
Figure 2: *AES* as a function of the bank’s ambiguity aversion, $a$. The solid (dashed) line displays $AES$ in Boom (Recession). The dotted line displays $ES$. 
indicates the NPV of a particular project. In words, we set up the bank’s dynamic decision problem, i.e., find the investment strategy which maximizes the expected cumulated value subject to regulation restriction, taking into account that the bank is uncertain about the cash flow distribution.

Recall that the bank’s uncertainty is described by the $\nu$-distribution: The bank believes that model 1 occurs with probability $\nu$, whereas model 2 is believed to occur with probability $1 - \nu$. Furthermore, in our dynamic setting we take into account that the state of the economy changes over time. To model the latter effect, we let $\lambda_i$ denote the probability that the state of the economy in the next period is not $i$, conditional on the current state being $i$, where $i \in \{B, R\}$. For example, if the current state is B (boom), then the probability of staying in the boom within the next period is $1 - \lambda_B$.

The bank must take the regulatory requirement into account. That is, the ambiguity expected shortfall is not allowed to exceed the cut-off level $\overline{ES}$. Compared to the previous section, we must adjust $AES$ so that the possibility of a change in the economy is handled. To do so we modify $AES$ depending on the current state. Suppose the current state of the economy is $i$, then the ambiguity adjusted expected shortfall is

$$AES(i) = \frac{1}{a} \log \left( \nu e^{a((1 - \lambda_i)ES(i) + \lambda_i ES(\neg i))} \right) ,$$

where $ES(i)$ ($ES(\neg i)$) is the expected shortfall given state $i$ (not state $i$) and depending on either model or model 2. Written out in detail we have

$$AES(i) = \frac{1}{a} \log \left( \nu e^{a[(1 - \lambda_i)ES(i,1) + \lambda_i ES(\neg i,1)]} + (1 - \nu) e^{a[(1 - \lambda_i)ES(i,2) + \lambda_i ES(\neg i,2)]} \right) ,$$

where $ES(i,j)$ is the expected shortfall if the state in the next period is $i$ and model $j$ prevails, $i \in \{R, B\}$ and $j \in \{1, 2\}$. 

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Given our setup, the bank wants to maximize the expected terminal value. With (16) we obtain the bank’s problem (whether to invest in project l, m, or h) at time \( t \in \{0, 1\} \) as

\[
\max_{(p_t) \in \{l, m, h\}} \mathbb{E}_0 [C_T]
\]

(17)

s.t.

\[
AES(i)_t \leq ES_t = \rho C_t, \quad \forall t \in \{0, 1, \ldots, T - 1\}
\]

(18)

and

\[
C_t = R_t - I + A_t \geq 0, \quad t \in \{1, \ldots, T\},
\]

(19)

where \( R_t \) is the (gross) return from last period’s investment, i.e., \( R_t = (C_{t-1} - I) + \min\{(1 + \phi)I, x_t(p)\} \). The last condition says that the bank must fund a new investment from its liquid funds stemming from an earlier (short term) investment as well long term investments (assets in place). Also, we have the closing condition that the bank undertakes no investment at time \( T \), all existing assets are liquidated and the proceeds are distributed to the bank owners. Finally, the definition of expected shortfall must take into account that the bank receives a payoff in each period. This payoff depends on the prevailing state of the economy. It is conceivable that the return is higher in a boom than in a recession. For simplicity, we assume that the bank receives \( A = I \) in an recession, whereas it receives \( A = mI, m > 1 \) in a boom. The fact that \( A \geq I \) and that previous investments never requires additional funds \((x \geq 0)\) makes us able to focus on underinvestment problems due to ambiguity and regulation.

\[8\]This is due to the fact that we do not consider value of dividends per se.
4.1 Projects

The bank considers investing in a one period project \( p \) at time \( t = 0, 1, \ldots, T - 1 \), respectively.\(^9\) The cash flow from each project is characterized by the underlying normal distribution as well as the state of the economy one period hence. In short,

\[
z_{p_{ij}} \sim N(\mu_{p_{ij}} - \frac{1}{2}\sigma_{p_{ij}}^2, \sigma_{p_{ij}}^2),
\]

for each project \( p \in \{l, m, h\} \) in state \( i \in \{R, B\} \) and underlying model \( j \in \{1, 2\} \).

We now set up a number of restrictions regarding the different projects. First, we assume that the projects have a net present value ordering so that

\[
0 < NPV_l < NPV_m < NPV_h,
\]

implying that without ambiguity aversion and the regulation restriction, project \( h \) is always preferred. What remain is to determine the riskiness and the uncertainty for each project.

Beginning with project \( l \) we think of this as the low NPV as well as the project with low risk and uncertainty. Indeed, we assume that there is no uncertainty for this particular project (and no risk regarding the state of the economy), i.e.,

\[
z_{lR1} = z_{lR2} = z_{lB1} = z_{lB2}.
\]

Note that if we let \( \sigma_l = \sigma_{l_{ij}} = 0 \) the project collapses to a risk free project. In this special case the bank should only earn the risk free rate of return.

We now look at the uncertain projects. As in the static setting we assume that the riskiness of a project is independent of the underlying model, see (5). To

\(^9\)The restriction to short term projects is mainly done for tractability. In principle, the model can be extended to multi-period projects, but since our focus in not on maturity issues we abstract from that here.
calculate the NPV, we need to take the subjective beliefs about the underlying cash flow model as well as the possible future state of the economy into account. Therefore, the NPV of a project, if the current state is \( i \), is

\[
NPV_p(i) = \left(1 - \lambda_i\right) \left[\nu e^{\mu_{p1}} + (1 - \nu) e^{\mu_{p2}}\right] + \lambda_i \left[\nu e^{\mu_{p-1}} + (1 - \nu) e^{\mu_{p-2}}\right].
\]  

(23)

Recall that we require the projects can be ranked according to their NPV, see (21).

To rank the medium uncertainty project and the high uncertainty project, we assume that

\[
\mu_{mi} < \mu_{hi}, i \in \{R, B\}, j \in \{1, 2\}.
\]

(24)

Based on the above we get that

\[
AES_l(i) < AES_m(i) < AES_h(i).
\]

(25)

and we can formulate a result.

**Proposition 2.** Assume the projects are as described above.

1. An ambiguity neutral bank

   (a) prefers project \( h \), if regulation is sufficiently slack, i.e., \( ES \) is high enough.

   (b) prefers project \( l \), if regulation is sufficiently restrictive.

2. For a somewhat severe, but not too restrictive, regulation, the bank

   (a) prefers project \( h \) if it is ambiguity neutral,

   (b) prefers project \( l \) if it is sufficiently ambiguity averse,

   (c) prefers project \( m \) if it is somewhat ambiguity averse.
An observation from above is that the seemingly uninteresting project $m$ becomes interesting when ambiguity aversion plays a role together with regulation. Since the regulation restriction is more binding in recession than in boom, it is conceivable that the seemingly good (high NPV) projects are not undertaking in recession. Put differently, projects which seem to have too low profit potential in a boom can become worthwhile as investment opportunities in a recession because they deliver a return potential without jeopardizing too much with downside risk. Moreover, in a dynamic setting with capital requirements it can be optimal to give up a short term high NPV project to secure greater feasibility in the future to make high NPV investments.

4.2 Numerical Results (incomplete)

The problem can now be solved by backwards induction. To be continued... Illustrate that there is a trade-off in project choice when there is also a matter of building up capital for future investments.

5 Discussion and empirical predictions (incomplete)

[based on the static model]

Our theory provides an explanation for why underinvestment occurs based on model uncertainty. Although our model is static, it is intuitive that the effects would carry over to a dynamic setting. Looking at the investment intensity over the business cycle, our model yields a rationale for why bank regulation leads to underinvestment and why this is more pronounced in recessions than in booms. Moreover, the underinvestment should be larger, the larger the model uncertainty. These observations
lead to the following predictions which can be used to test our theory.

**Prediction 1.** For industries in which model uncertainty is more likely, we expect to see that more restrictive bank regulation leads to more underinvestment.

**Prediction 2.** For industries in which model uncertainty is more likely, we expect to see that underinvestment is more severe in recessions than in booms.

**Prediction 3.** Higher model uncertainty as well as stricter regulation will postpone investments in risky projects with high NPV over a longer time.

Importantly, we do not argue that credit rationing is the key determinant for underinvestment, although it can play a role vis-a-vis model uncertainty. Thus, one way to test our model is to look at what happens with the return premium required by banks for those projects which are actually funded. If underinvestment is based on credit rationing, we should expect to see increases in the return premium up to a level at which the entrepreneur rejects to turn to the bank for funding.\(^\text{10}\) On the other hand, if underinvestment is based on model uncertainty, a higher return premium will not help the bank to comply with the regulatory threshold per se. A consequence of the requirement is that banks undertake projects which are perceived as more certain and, hence, a relatively low return premium is rational. Therefore, the two theories lead to different predictions regarding return premium.

Our aim is to extend the model to a dynamic setting, i.e. the economy can shift between being in a boom and in a recession. We expect dynamic effects of model uncertainty to provide important insights into a bank’s investments decisions and how these are affected by regulatory requirements. For example, we want to demonstrate that low interest rates need not induce more investments. The idea

\(^{10}\)Or, in a moral hazard context, the required return premium is so high that the entrepreneur cannot be induced to provide efforts to ensure a profitable project, see Tirole (2006, chapter 3).
is as follows. In a recession with low interest rates one would expect relatively more projects to be undertaken as funding appears to be cheaper. However, our model emphasizes that low interest rates go hand-in-hand with more binding capital requirements implying underinvestment. Another interesting issue is the fact that while ambiguity may not be seen as important when the economy is in a boom, the possibility of a change to a recession has spill over effects. In particular, this impacts which types of projects the bank is willing to fund. That is, banks may fund relatively risky projects in a boom, if the probability of entering a recession is low, whereas banks turn to more secure projects if a recession is getting closer.

6 Conclusion

Our theory provides an explanation for underinvestment based on model uncertainty. Intuitively, the effects would carry over to a dynamic setting. Our model yields a rationale for why bank regulation leads to underinvestment and why this is more pronounced in recessions than in booms. Moreover, the underinvestment should be larger, the larger the model uncertainty.

Furthermore, if underinvestment is based on adverse selection we expect to see increases in the return premium up to a level at which the entrepreneur rejects to turn to the bank for funding. If underinvestment is based on model uncertainty, a higher return premium will not help the bank to comply with the regulatory threshold. Consequently, banks fund less risky projects and, hence, a relatively low return premium is rational. Therefore, the two theories lead to different predictions regarding return premium.
References


