Incentives through Consumer Learning

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Abstract

Consumer inexperience can—in the form of uncertainty about the match value—play an important role for solving a moral hazard problem between a firm and its customers. Inexperienced consumers learn about their match value from the outcomes of their transactions. It may be rational for them to stop trading even if they are convinced that the firm always exerts high effort. The firm then exerts high effort in order to reduce the loss of inexperienced consumers. Serving inexperienced consumers credibly signals commitment to high effort. We discuss the implications of this mechanism for advertising, product design and consumer education.

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1 Introduction

An increasing literature highlights the adverse effects of consumer inexperience on market outcomes and welfare. For example, many consumers do not consider hidden add-on prices when purchasing the base good (such as hotel accommodations). This motivates firms to shroud information, which creates an inefficiency (Gabaix and Laibson 2006). Prices may be too high in search markets if some consumers are imperfectly informed about product quality or have to incur costs in order to become informed (e.g. Armstrong and Chen 2009, Armstrong et al. 2009). Inexperienced consumers may also be exploited if they do not anticipate their future preferences (Spiegler 2011, Part 1), or if they do not exactly know their demand type when trading with firms (Courty and Li 2000, Matthews and Persico 2007, Inderst and Peitz 2012). When seeking financial advice, they may be unaware of the conflict of interest that sellers of financial products face, which in many cases results in the exploitation of consumers (Inderst and Ottaviani 2012).

In this paper, we argue that consumer inexperience can—in the form of uncertainty about their match value—also play an important role for promoting market efficiency when there is a moral hazard problem between a firm and its customers. Consider a firm that may exert either high or low effort. Its customers privately observe either a good or bad outcome, which is an imperfect signal about the firm’s effort. The consumers’ match value (or type) is defined as the probability of observing a good outcome when the firm exerts effort. Inexperienced consumers learn about their match value from the outcomes of their transactions. A good outcome indicates a high expected valuation for the firm’s good, while a bad outcome indicates a low expected match value. Hence, it may be rational for inexperienced consumers to stop trading even if they are convinced that the firm always exerts high effort. The firm then exerts high effort in order to reduce the loss of inexperienced consumers. Serving inexperienced consumers credibly signals the firm’s commitment to high effort. Consumer learning therefore creates dynamic incentives that are absent when all consumers are perfectly informed about their match value. To illustrate, consider the following two examples:

1. An inexperienced consumer dines in a French restaurant. If she is disappointed by the food, this may have two different reasons: either because she does not like the French cuisine in general, or because the chef exerted low effort.

2. The manager of a company purchases the service of a consulting firm. This firm has substantial expertise in some domains (e.g. finding new markets, optimize advertising), but less so in others (e.g. human resource management). If the realized increase in profits is low after the consultants’ recommendation has been implemented, this may be due to low effort or due to a mismatch between the company’s
problems and the consultants’ expertise.

Our approach is reminiscent to the “reputation” mechanism\(^1\) where consumers learn about a firm’s type (competent or inept, committed to effort or opportunistic) from the outcomes of past transactions. There, the firm chooses actions in order to shift consumers’ beliefs in a favorable direction and to reap the rewards of high effort. The same happens in our model, but we reverse the object of consumers’ uncertainty. This yields a more natural interpretation of the reputation mechanism. Given that the firm is long-lived and consumers are only short-lived, consumers’ uncertainty about their match value seems more plausible than uncertainty about the firm’s type (especially in the light of the literature cited above). Moreover, this source of incomplete information will not dry out over time as long as there is some fluctuation in the consumer population. At a later stage, we will compare our approach to other repeat-purchase mechanisms in more detail.

We show that the scope for a high-effort equilibrium (an equilibrium where the firm exerts high effort in each period) strictly increases in the share of inexperienced consumers. When all consumers perfectly know their type and the firm always provides high effort, it is rational to exert a punishment for bad outcomes only for those consumers who are indifferent between trading or not with the firm (all other consumers have a unique best-response, which is independent of previous outcomes). Hence, if the distribution over consumers’ types is continuous, a high-effort equilibrium does not exist. In contrast, inexperienced consumers learn about their type and may strictly prefer to trade (not to trade) with the firm after observing a good (bad) outcome. The mass of customers who rationally exert a punishment for bad outcomes therefore increases in the share of inexperienced consumers.

A crucial feature of our mechanism is the interaction of the firm’s effort decisions with its pricing strategy. The repeat-purchase mechanism generates dynamic incentives only if the firm continuously trades with all inexperienced consumers except those who observed a bad outcome. In principle, consumers could force the firm to charge a certain price by refusing to trade at any other price. However, such an implicit contract between the firm and consumers is difficult to establish and therefore unrealistic. We characterize the circumstances under which a high-effort equilibrium exists where consumers behave passively: they trade with the firm if and only if the expected payoff from trade is weakly positive. Such an equilibrium has the property that the moral hazard problem is resolved without reputational concerns (learning about the firm’s type as in the reputation literature, feedback through recommendation systems as in Dellarocas 2003), explicit contracts (warranties, performance pay) or implicit contracts between the firm and consumers (cor-

\(^1\)See Bar-Isaac and Tadelis (2008) and for an in-depth review.
ominated punishment by consumers as in Klein and Leffler 1981). Consumer learning then constitutes the simplest and probably cheapest remedy for the moral hazard problem.

It turns out that a good where such an equilibrium exists has the attributes of a niche product: some consumers really like the good and observe a good outcome with high probability, all other consumers strongly dislike it and observe a good outcome with low probability. If the firm exerts high effort, observing a bad outcome is a strong signal for consumers of having a very low type. It then does not pay off for the firm to serve disappointed, inexperienced consumers. A high-effort equilibrium where consumers behave passively then exists if additionally the share of inexperienced consumers and their expected type is sufficiently large relative to the cost of effort.

The model generates a number of implications for industrial organization. First, we introduce a new kind of incomplete information on the side of consumers that generates a reputation effect. Second, advertising in our model can play both a functional and an informational role: it informs consumers about the existence of a product and thereby increases the mass of young, inexperienced consumers who become repeat customers if and only if they observe a good outcome. For old and experienced consumers a large advertising campaign then credibly signals commitment to high quality. Third, the interaction between product design and advertising content is important for effort incentives. An ideal marketing mix for the efficacy of our mechanism is a niche product design and advertising that provides no information beyond the existence of the product. Finally, the model implies that consumer education can have a negative impact on quality since it reduces the mass of consumers who rationally exert a punishment for bad outcomes.

The rest of the paper is organized as follows: the next section outlines the framework of the model. In Section 3, we show how the presence of inexperienced consumers increases the scope for high-effort equilibria. Section 4 establishes a sufficient condition for the existence of a high-effort equilibrium where consumers behave passively. In Section 5, we illustrate traditional repeat-purchase mechanisms and compare them to the present approach. The last section discusses the implications of our results.

2 The Model

Basic Framework. Time is discrete and indexed by $t \in \{1, 2, \ldots\}$. In each period, a new cohort of consumers of mass 1 is born. Consumers live for two periods, i.e., there are “young” and “old” consumers in each period. All cohorts are identical. In each period, consumers may trade with a firm $A$. If a consumer trades with $A$ in period $t$, she pays up front the price $p^t \in [0, 1]$ and privately observes either a good, $A_1$, or a bad outcome, $A_0$, depending on $A$’s effort (the next paragraph explains how effort translates into outcomes). If a consumer does not trade with $A$, her payoff in this period is 0. Let $e^t$ be $A$’s effort
in period $t$, which can be either high ($e^t = 1$) or low ($e^t = 0$) and is not observed by consumers. The cost of high effort is $c > 0$ and that of low effort is 0. The payoffs for a consumer associated with a good and a bad outcome are 1 and 0, respectively. $A$'s payoff in period $t$ is $p^t$ times the mass of consumers who trade with it in period $t$ minus effort cost $ce^t$. $A$ maximizes the expected discounted sum of profits. Its discount rate is $\delta_A \in (0, 1)$. The discount rate of consumers is $\delta_C = 0$.\footnote{This assumption is made for simplicity. It prevents consumers from experimentation. If $\delta_C = 0$, a consumer trades with $A$ only if her expected payoff from trade weakly exceeds 0. If $\delta_C > 0$ a young, inexperienced consumer may also trade with $A$ if the expected payoff from trade is slightly negative.}

**Consumers.** Each consumer has a type $r \in (0, 1)$. If a consumer with type $r$ trades with $A$ in period $t$, the probability of a good outcome for her is $r$. In case of $e^t = 0$, the probability of a good outcome is 0. At the birth of a consumer, nature draws her type according to the continuous\footnote{In several examples, we will deviate from this assumption for convenience reasons.} distribution function $F(r)$ with density $f(r)$. A consumer may be experienced or inexperienced. An experienced consumer exactly knows her type $r$, while an inexperienced consumer only knows the distribution function $F$. Let $\lambda$ be the share of inexperienced consumers in each cohort. Denote by $\tilde{r} = E_F(r)$ the expected type of young, inexperienced consumers. Consumers are Bayesian and update beliefs about their type. If an inexperienced consumer trades with $A$ and observes a good outcome, her expected type becomes $\tilde{r}_{A1} = E_F(r | A1)$. If she observes a bad outcome and believes that $A$ has exerted high effort with probability one, then her expected type is $\tilde{r}_{A0} = E_F(r | A0)$.\footnote{Note that a consumer can be both young and experienced (she already knows her type in the first period of her life) or she can be both old and inexperienced (after one observation there is still some uncertainty about her type).} Figure 2(d) below illustrates the distribution over expected types for $\lambda > 0$ and a uniform distribution $F$.\footnote{The assumption that all inexperienced consumers have the same expectation about types makes the model tractable. We could introduce different classes of consumers with distributions $F^1, F^2, \ldots$ (I did that in previous versions of the paper), but this would make the model cumbersome and would not generate any additional insights.}

**Strategies and equilibrium.** The sequence of events in each period $t$ is as follows: (i) young consumers are born; for each young consumer nature draws the type $r$ according to $F(r)$; (ii) $A$ chooses $p^t$; (iii) each consumer decides whether to trade with $A$ or not; (iv) $A$ chooses $e^t$; (v) outcomes occur, payoffs are realized and old consumers die. Let $\sigma^{y,t} = (\sigma^{y,t}_y, \sigma^{o,t}_o)$ ("y" for "young" and "o" for "old") be the strategy of an experienced consumer of type $r$ who is born in period $t$, where $\sigma^{y,t}_y$ maps $p^t$ into the set of mixed actions, $\sigma^{y,t}_y : [0, 1] \rightarrow \Delta(\{0, 1\})$, and $\sigma^{o,t}_o$ maps $p^t, p^{t+1}$ and the outcome observed in period $t$ into the set of mixed actions, $\sigma^{o,t}_o : [0, 1]^2 \times \{A0, A1, \emptyset\} \rightarrow \Delta(\{0, 1\})$. Define the
strategy $\sigma^t_A$ of inexperienced consumers who are born in period $t$ accordingly. Denote by $h^t = \{(1,1), (p^t, e^t),\ldots, (p^{t-1}, e^{t-1})\}$ the private history of $A$ in period $t$ and by $H$ the set of all such histories. Let $\sigma^A = (\sigma^A_p, \sigma^A_e)$ be $A$’s strategy (“$p$” for “pricing” and “$e$” for “effort”), where $\sigma^A_p$ is the pricing strategy, $\sigma^A_p : H \to \Delta([0,1])$, and $\sigma^A_e$ is the $A$’s effort strategy, $\sigma^A_e : H \to \Delta([0,1])$. In Section 3 and 5, an equilibrium of this game refers to a perfect Bayesian equilibrium. In Section 4, we fix the consumers’ strategies in order to exclude coordination on an implicit contract. This strategy may not be subgame-perfect. Hence, in Section 4, we refer to Bayesian Nash equilibria. An equilibrium is called high-effort equilibrium if on the equilibrium path $A$ exerts high effort with probability 1 in each period.

### 3 The Scope for High-Effort Equilibria

In this section, we show how the distribution $F$ and the share of inexperienced consumers $\lambda$ determine the scope for a high-effort equilibrium. Let us start with the case where all consumers are experienced, i.e., $\lambda = 0$. Suppose that $A$ exerts high effort in each period. Since $F$ is continuous, almost all consumers have a unique best-response in each period: a consumer with type $r$ will trade with $A$ if $r - p^t > 0$, and not trade with $A$ if $r - p^t < 0$. Consequently, the mass of consumers for whom it is rational to trade (not to trade) with $A$ after observing a good (bad) outcome is zero. So if $c > 0$, a high-effort equilibrium does not exist.

On the contrary, inexperienced consumers update the belief about their type after trading with $A$. Given that $A$ always exerts high effort, a bad outcome indicates that her expected type is $\tilde{r}_{A0} = \tilde{r}$. So if $p^t \in [\tilde{r}_{A0}, \tilde{r}]$ as well as $e^t = 1$ in each period $t$, it is rational for inexperienced consumers to trade with $A$ in the first period of their life, and to trade (not to trade) with $A$ after observing a good (bad) outcome. When all inexperienced consumers follow this strategy, $A$ loses the amount $\lambda \tilde{r} p^t$ in period $t + 1$ if it deviates to low effort in period $t$. We then only have to make sure that $A$’s pricing strategy is such that $p^t \in [\tilde{r}_{A0}, \tilde{r}]$ in each period $t$. This can be accomplished by an implicit contract where consumers trade with $A$ if and only if $p^t$ takes on a prespecified value (for example, $\tilde{r}$, which is optimal in the sense that it maximizes effort incentives). We therefore get the following result.

**Proposition 1** If and only if $c \leq \delta_A \lambda \tilde{r}^2$, there exists a high-effort equilibrium.

**Proof.** We prove the “if” statement. Consider the following strategy profile. $A$ charges $p^t = \tilde{r}$ in each period $t$ and exerts high effort in period $t$ if and only if $p^t = \tilde{r}$; a consumer trades with $A$ if and only if (i) her type (or expected type) is weakly larger than $\tilde{r}$, and (ii) $p^t = \tilde{r}$. We show that this profile constitutes an equilibrium. Consumers can never
deviate profitably from this strategy. A cannot gain by charging a price different from \( \tilde{r} \) (since its revenue then is 0). A young, inexperienced consumer observes a good outcome with probability \( \tilde{r} \). The continuity of \( F \) ensures that \( \tilde{r}_{A0} < \tilde{r} \) so that an inexperienced consumer who observed a bad outcome will not trade with \( A \). By exerting low effort in period \( t \), \( A \) gains \( c \) in period \( t \) and her continuation payoff decreases by \( \delta_A \lambda \tilde{r}^2 \). Hence, it does not pay off for \( A \) to exert low effort. Consequently, a high-effort equilibrium exists.

We prove the “only if” statement. Assume by contradiction that a high-effort equilibrium exists and \( c > \bar{c} \lambda \tilde{r}^2 \). In this equilibrium, it is rational for a consumer with type (or expected type) \( r \) to trade with \( A \) in period \( t \) only if \( r - p^t \geq 0 \), and it is rational for her not to trade with \( A \) in period \( t \) only if \( r - p^t \leq 0 \). Consider any period \( t \) and let \( p^t \) be given. We show that it does not pay off for \( A \) to exert effort in period \( t \) and \( t + 1 \). If \( p^t > \tilde{r} \), no young, inexperienced consumer trades with \( A \) in period \( t \) so that no consumer changes the belief about her type after observing the outcome. Consequently, \( A \)’s effort in period \( t \) has no consequence for its continuation value so that it does not pay off to exert effort in period \( t \). The same is true for period \( t + 1 \) if \( p^{t+1} > \tilde{r} \). If \( p^t \leq \tilde{r} \) and \( p^{t+1} \in [\tilde{r}_{A0}, \tilde{r}] \), then by not exerting effort in period \( t \), \( A \) gains \( c \) in period \( t \) and its continuation value decreases by at most \( \delta_A \lambda \tilde{r} p^{t+1} \), which by assumption is strictly smaller than \( c \), so that it does not pay off for \( A \) to exert effort in period \( t \). If \( p^t \leq \tilde{r} \) and \( p^{t+1} \in [0, \tilde{r}_{A0}) \), all inexperienced consumers trade with \( A \) in period \( t + 1 \), regardless of their outcome in period \( t \), so that it does not pay off for \( A \) to exert effort in period \( t \). This completes the proof.

Proposition 1 fully characterizes the scope for high-effort equilibria in our framework. Clearly, the larger is the share of inexperienced consumers \( \lambda \), the more customers \( A \) loses after exerting low effort. Effort incentives increase in \( \tilde{r} \) for two reasons. First, a larger \( \tilde{r} \) implies that more inexperienced consumers observe a good outcome when effort is high. Second, a larger \( \tilde{r} \) allows \( A \) to charge a higher price while young, inexperienced consumers are still ready to trade with \( A \). Both effects increase the loss in revenues after exerting low effort.

4 Passive Consumers

The result in Proposition 1 relies on an implicit contract between \( A \) and consumers that forces \( A \) to charge a certain price. This price is set such that in each period young, inexperienced consumers trade with \( A \), while old, disappointed ones do not. In this section, we examine under what circumstances a high-effort equilibrium exists that does not involve such an implicit contract. Throughout, we will assume that consumers behave “passively”: They trade with \( A \) if and only if the expected payoff from trade weakly exceeds 0, provided that \( A \) always exerts high effort. We show that for a large class
of distributions $F$ a high-effort equilibrium exists where consumers behave passively. In such an equilibrium, there are neither reputational concerns (that are based on uncertainty about $A$’s type), nor explicit or implicit contracts between $A$ and consumers. Consumer learning provides sufficient incentives for $A$ and therefore constitutes the simplest and cheapest possible solution to the moral hazard problem. In the last section, we discuss the implications of this result for product design and advertising content.

**Definitions.** Before we start, some formal definitions are in order. Consumers behave passively if they play according to the following strategy: in period $t$, a consumer trades with $A$ if and only if her (expected) type weakly exceeds $p^t$. $A$’s revenue in period $t$ when consumers behave passively is then given by

$$
\Pi(p^t, p^{t-1}, e^{t-1}) = 2(1 - \lambda)(1 - F(p^t))p^t
+ 1_{e^{t-1}=1}1_{p^{t-1} \leq \tilde{r}}(1_{p^t \leq \tilde{r}}A_0 \lambda(1 - \tilde{r}) + 1_{p^t \leq \tilde{r}}A_1 \lambda\tilde{r}))p^t
+ 1_{e^{t-1}=0}1_{p^{t-1} < \tilde{r}}(1_{p^t \leq \tilde{r}}A_0 \lambda + 1_{p^t \leq \tilde{r}}A_1 \lambda\tilde{r})p^t + 1_{p^{t-1} > \tilde{r}}1_{p^t < \tilde{r}}2\lambda p^t,
$$

(1)

where $1$ is the indicator function (we assume that $p^0 = e^0 = 1$). The first line on the right-hand side is $A$’s revenue from experienced consumers (recall that two cohorts are alive in each period); the second line is $A$’s revenue from inexperienced consumers if in the previous period it served young, inexperienced consumers and exerted high effort; the first term in the third line is the corresponding expression for the case of low effort, and the second term in the third line is $A$’s revenue from inexperienced consumers if in the previous period it did not serve young, inexperienced consumers. Next, define

$$
p_i(F, \lambda) = \min\{p \mid p \in \arg\max_{\tilde{p} \in [\tilde{r}_A, \tilde{r}]} \Pi(\tilde{p}, \tilde{r}, i)\}.
$$

(2)

When $p^{t-1} \leq \tilde{r}$ and $e^{t-1} = i$ the price $p_i(F, \lambda)$ is the smallest price that maximizes $A$’s period profit under the constraint that it must lie in the interval $[\tilde{r}_A, \tilde{r}]$. Below, we will impose constrains on $F$ and $\lambda$ which make sure that $p_i(F, \lambda)$ is well-defined. Note that $p_0(F, \lambda) \leq p_1(F, \lambda)$ (we will use this fact in the proof of Proposition 2 below) and $p_i(F, \lambda) = \tilde{r}$ if $\lambda$ is sufficiently large.

**Existence of high-effort equilibria where consumers behave passively.** Assume first that all consumers are experienced. When they behave passively, they do not react to bad outcomes and there cannot exist a high-effort equilibrium. We therefore need some inexperienced consumers to support high effort.

$A$ always exerts high effort in an equilibrium where consumers behave passively only if on the equilibrium path it is optimal to charge a price in the interval $[\tilde{r}_A, \tilde{r}]$ in each period. When $p^{t+1} \leq \tilde{r}_A$, $A$ serves all old, inexperienced consumers in period $t + 1$, when
regardless of their experience in period $t$. In this case, it does not pay off for $A$ to exert high effort in period $t$. When $p^t > \hat{r}$, then in period $t$ no young inexperienced consumer trades with $A$ so that the belief about her type in period $t + 1$ is independent from $A$’s effort in period $t$. Again, it does not pay off for $A$ to choose $e^t = 1$. In the following, we state two conditions on $F$ which guarantee that it is never optimal for $A$ to charge a price outside the interval $(\hat{r}_{A0}, \hat{r}]$.

The first condition makes sure that it is strictly better for $A$ to charge $p^t = \hat{r}$ instead of a price $\hat{p}^t \leq \hat{r}_{A0}$, regardless of all past and future decisions. It is given by

$$2\hat{r}_{A0} < 2(1 - \lambda)(1 - F(\hat{r}))(1 - \hat{r}) + \lambda \hat{r}.$$

The left-hand side of this inequality is $A$’s maximal revenue if the price is $\hat{p}^t \leq \hat{r}_{A0}$. The first term on the right-hand side is the revenue from experienced consumers when $p^t = \hat{r}$, and the second term is the revenue from young, inexperienced consumers when $p^t = \hat{r}$. Condition $L$ holds if $\hat{r}_{A0}$ is small compared to $\hat{r}$ and $\lambda$ is sufficiently large.

The second condition makes sure that in any period $t$ it is strictly better for $A$ to charge $p^t = \hat{r}$ and to exert high effort instead of charging a price $\hat{p}^t > \hat{r}$, regardless of all past and future decisions. It is given by

$$2(1 - \lambda)(1 - F(\hat{r}))(1 - \hat{r}) + 2\lambda(1 - \hat{r})\hat{r} < \lambda \hat{r} - c.$$

When $A$ exerts low effort in period $t$, it may be optimal to charge a price $\hat{p}^t > \hat{r}$, because young, inexperienced consumers will then not observe bad outcomes and maintain the belief $\hat{r}$ about their type in period $t + 1$ (instead of $\hat{r}_{A0}$). Hence, it must be profitable for $A$ to serve young, inexperienced consumers and to exert high effort. This is why $c$ shows up on the right-hand side of the inequality. Condition $H$ holds if $\hat{r}$ and $\lambda$ are sufficiently large relative to $c$.

Finally, we have to make sure that it is optimal for $A$ to always exert high effort provided that in each period the price lies in the interval $(\hat{r}_{A0}, \hat{r}]$. We cannot use the same incentive constraint as in Proposition 1, $c \leq \delta_A \lambda \hat{r}^2$, because it is not necessarily optimal for $A$ to charge the price $\hat{r}$. The smallest price that can be optimal for $A$ is given by $p_0(F, \lambda)$. Hence, the revenue $A$ loses after exerting low effort is at least $\lambda \hat{r}p_0(F, \lambda)$.

**Proposition 2** If $L$, $H$ and $c < \delta_A \lambda \hat{r}p_0(F, \lambda)$ hold, then there exists a high-effort equilibrium where consumers behave passively.

**Proof.** The proof proceeds by steps. **Step 1.** We show that in any period $t$ it is strictly better for $A$ to choose $p^t = \hat{r}$ and $e^t = 1$ than to choose $\hat{p}^t > \hat{r}$ and effort $\hat{e}^t \in \{0, 1\}$, regardless of all previous or future decisions. Note that decisions in period $t$ do not affect $A$’s continuation value after period $t + 1$. Hence, it suffices to compare
the total discounted payoffs in the periods \( t \) and \( t + 1 \) under alternative strategies. The discounted payoff in the periods \( t \) and \( t + 1 \) from charging \( \hat{p}^t > \hat{r} \) and exerting effort \( \hat{e}^t \) is at most

\[
\hat{\pi}_1 = 2(1 - \lambda)(1 - F(\hat{r})) + 1_{e^{t-1}=1}1_{p^{t-1} \leq \hat{r}}\lambda\hat{r} - 1_{e^{t}=1}c + \delta_A[2(1 - \lambda)(1 - F(p^{t+1}))p^{t+1} + 1_{p^{t+1} \leq \hat{r}}2\lambda p^{t+1} - 1_{e^{t+1}=1}c].
\]

(3)

If instead \( A \) chooses \( p^t = \hat{r} \) and \( e^t = 1 \), the discounted payoff in the periods \( t \) and \( t + 1 \) is at least

\[
\pi_1 = 2(1 - \lambda)(1 - F(\hat{r}))\hat{r} + \lambda\hat{r} + 1_{e^{t-1}=1}1_{p^{t-1} \leq \hat{r}}\lambda\hat{r}^2 - c + \delta_A[2(1 - \lambda)(1 - F(p^{t+1}))p^{t+1} + 1_{p^{t+1} \leq \hat{r}}\lambda p^{t+1} + 1_{p^{t+1} \leq \hat{r}}\lambda\hat{r}p^{t+1} - 1_{e^{t+1}=1}c].
\]

(4)

We calculate the difference

\[
\pi_1 - \hat{\pi}_1 = -2(1 - \lambda)(1 - F(\hat{r}))(1 - \hat{r}) + \lambda\hat{r} - 1_{e^{t-1}=1}1_{p^{t-1} \leq \hat{r}}\lambda\hat{r}(1 - \hat{r}) + 1_{e^{t}=1}c - \delta_A 1_{p^{t+1} \leq \hat{r}}\lambda(1 - \hat{r})p^{t+1}.
\]

(5)

The right-hand side of this equality is minimal if \( e^t = 0, e^{t-1} = 1, p^{t-1} \leq \hat{r}, p^{t+1} = \hat{r} \) and \( \delta_A = 1 \). Hence, \( H \) makes sure that \( \pi_1 - \hat{\pi}_1 > 0 \), regardless of \( e^{t-1}, e^t, p^{t-1}, p^{t+1} \) and \( \delta_A \), which implies the result. **Step 2.** We show that in any period \( t \) it is strictly better for \( A \) to choose \( p^t = \hat{r} \) than to choose \( \hat{p}^t \leq \hat{r}_{A0} \), regardless of all previous or future decisions. The discounted payoff in the periods \( t \) and \( t + 1 \) from charging \( \hat{p}^t \leq \hat{r}_{A0} \) and exerting effort \( \hat{e}^t \) is at most

\[
\hat{\pi}_2 = 2\hat{r}_{A0} - 1_{e^{t}=1}c + \delta_A \Pi(p^{t+1}, \hat{p}^t, \hat{e}^t) - 1_{e^{t+1}=1}c,
\]

(6)

while the discounted payoff in the periods \( t \) and \( t + 1 \) from charging \( p^t = \hat{r} \) and exerting effort \( \hat{e}^t \) is at least

\[
\pi_2 = 2(1 - \lambda)(1 - F(\hat{r}))\hat{r} + \lambda\hat{r} + 1_{e^{t-1}=1}\lambda\hat{r}^2 - 1_{e^{t}=1}c + \delta_A \Pi(p^{t+1}, \hat{r}, \hat{e}^t) - 1_{e^{t+1}=1}c.
\]

(7)

In both cases, young, inexperienced consumers trade with \( A \) in period \( t \), so that \( \Pi(p^{t+1}, \hat{p}^t, \hat{e}^t) = \Pi(p^{t+1}, \hat{r}, \hat{e}^t) \). We calculate the difference

\[
\pi_2 - \hat{\pi}_2 = -2\hat{r}_{A0} + 2(1 - \lambda)(1 - F(\hat{r}))\hat{r} + \lambda\hat{r} + 1_{e^{t-1}=1}\lambda\hat{r}^2,
\]

(8)

which is minimal if \( e^{t-1} = 0 \). Condition \( L \) makes sure that \( \pi_2 - \hat{\pi}_2 > 0 \), regardless of \( e^{t-1} \), which implies the result. **Step 3.** We show that if \( e^{t-1} = i \), then it is strictly better for \( A \) to choose \( p^t = p_i(F, \lambda) \) than to choose \( \hat{p}^t \in (\hat{r}_{A0}, p_i(F, \lambda)) \), regardless of all previous or future decisions. \( A \)'s continuation value after period \( t \) is the same for all prices in the interval \((\hat{r}_{A0}, \hat{r}]\). Hence, the continuity of \( F \) and the definition of \( p_i(F, \lambda) \)
imply the result. **Step 4.** We prove the existence of a high-effort equilibrium. Define by \( \Pi(\sigma^A) \) \( A \)'s total discounted payoff if it plays according to strategy \( \sigma^A \) and consumers behave passively. Let \( \hat{\sigma}_e^A \) be any effort strategy where \( A \) exerts high effort in period \( t \) if \( p^t \in (\tilde{r}_{A0}, \tilde{r}) \). By Step 1 to 3 and the continuity of \( F \), there exists a pricing strategy \( \hat{\sigma}_p^A \) where \( p^t \in [p_0(F, \lambda), \tilde{r}] \) in each period \( t \) after any history \( h_t \), and \( \Pi(\hat{\sigma}_p^A, \hat{\sigma}_e^A) \geq \Pi(\sigma^A_p, \sigma^A_e) \) for all \( \sigma^A_e \). Next, the assumption \( c < \delta_A \lambda \tilde{r} p_0(F, \lambda) \) ensures that there exists an effort strategy \( \hat{\sigma}_e^A \) where on the path of play of \( \hat{\sigma}_e^A = (\hat{\sigma}_p^A, \hat{\sigma}_e^A) \) \( A \) exerts high effort in each period and \( \Pi(\hat{\sigma}_p^A, \hat{\sigma}_e^A) \geq \Pi(\hat{\sigma}_p^A, \sigma^A_e) \) for all \( \sigma^A_e \). It remains to show that \( \Pi(\hat{\sigma}_e^A) \geq \Pi(\hat{\sigma}_e^A) \) for all \( \sigma^A \). Assume by contradiction that there exists a strategy \( \sigma^A \) with \( \Pi(\sigma^A) > \Pi(\hat{\sigma}_e^A) \). By Step 1 to 3, we then can find an alternative strategy \( \tilde{\sigma}_e^A \) where on the path of play the price is in the interval \( [p_0(F, \lambda), \tilde{r}] \) in each period and \( \Pi(\tilde{\sigma}_e^A) \geq \Pi(\sigma^A_e) \). However, \( c < \delta_A \lambda \tilde{r} p_0(F, \lambda) \) ensures that \( \Pi(\hat{\sigma}_e^A) \geq \Pi(\hat{\sigma}_e^A) \), a contradiction. 

Proposition 2 indicates how the consumer population should look like so that a high-effort equilibrium with passive consumers exists. First, there must be a large amount of type heterogeneity among consumers so that \( \tilde{r}_{A0} \) is small relative to \( \tilde{r} \). It then does never pay off for \( A \) to cut prices in order to serve disappointed, inexperienced consumers. Second, the expected type \( \tilde{r} \) and the share of inexperienced consumers \( \lambda \) must be large relative to \( c \) so that it is always optimal to serve young, inexperienced consumers and to exert high effort.

Below, we illustrate this result in two examples. To facilitate the discussion, we derive a simplified version of Proposition 2. If we set \( \lambda = 1 \), condition \( L \) becomes \( 2\tilde{r}_{A0} < \tilde{r} \), condition \( H \) becomes \( 2(1 - \tilde{r}) \tilde{r} < \tilde{r} - c \), and the incentive constraint becomes \( c < \delta_A \tilde{r}^2 \). We then obtain the following result.

**Corollary 1** If \( \tilde{r} > \max\{2\tilde{r}_{A0}, \frac{1}{2}\} \), \( c \) is sufficiently small and \( \lambda \) is sufficiently large, there exists a high-effort equilibrium where consumers behave passively.

[Figure 1 about here]

**Example 1.** Consider the distribution over types in the left graph of Figure 1 below. There are two groups of consumers in each cohort. The first group dislikes \( A \)'s good and derives in expectation relatively little utility from it. The type \( r \) of these consumers is uniformly distributed in the interval \([0, \frac{1}{2}]\) with density \( f \). The second group of consumers likes \( A \)'s good and derives in expectation relatively high utility from it. Their type is uniformly distributed in the interval \([\frac{3}{4}, 1]\) with density \( 4 - f \) (so that the mass of both groups together equals 1). Under what circumstances is there a high-effort equilibrium where consumers behave passively?
The expected type of inexperienced consumers equals \( \tilde{r} = \frac{7}{8} - \frac{3}{40} f \) (see the Appendix for details), while the expected type of inexperienced consumers who observed a bad outcome is \( \tilde{r}_{A0} = \frac{3}{8} \). We have \( \tilde{r} > \frac{1}{2} \) if \( f < 2 \) and \( \tilde{r} > 2\tilde{r}_{A0} \) if \( f > 1 \). Corollary 1 then tells us that there exists a high-effort equilibrium where consumers behave passively, if \( f \in (1, 2) \), sufficiently many young consumers are inexperienced, and \( c \) is small enough. So the share of consumers who dislike \( A \)'s good should neither be too high, nor too low.

**Example 2.** Next, consider a situation where the consumers are quite homogenous. Let their type be uniformly distributed in the interval \([x, y]\) with density \( \frac{1}{y-x} \) (as in the right graph of Figure 1). We show that there does not exist a high-effort equilibrium where consumers behave passively (regardless of \( c \) and \( \lambda \)) if \( y \) is too close to \( x \).

Assume that \( A \) exerts high effort in each period. This implies that \( A \) charges a price \( p^t \leq \tilde{r} \) in each period \( t \) (otherwise it would not serve young, inexperienced consumers and hence would have no effort incentives). \( A \)'s revenue in a period \( t > 1 \) by charging a price \( p^t > \tilde{r}_{A0} \) is given by

\[
\Pi(p^t, \tilde{r}, 1) = 2(1 - \lambda)(1 - F(p^t))\tilde{r} + \lambda\tilde{r} + \lambda\tilde{r}p^t, \quad (9)
\]

while by charging the price \( p^t = x \) it would serve all consumers and earn \( \Pi(x, \tilde{r}, 1) = 2x \). We now can show that \( \Pi(p^t, \tilde{r}, 1) < \Pi(x, \tilde{r}, 1) \) for all \( \lambda \) if

\[
y + \frac{1}{2}y(y + x) < 2x, \quad (10)
\]

which holds whenever \( y \) is sufficiently close to \( x \) (see the Appendix for details). In this case, it is not an optimal pricing strategy to charge \( p^t > \tilde{r}_{A0} \) in any period \( t \). Consequently, under an optimal pricing strategy \( A \) charges a price weakly below \( \tilde{r}_{A0} \) in each period so that it serves all inexperienced consumers. However, it then has no incentive to exert high effort.

### 5 A Brief Comparison of Different Repeat-Purchase Mechanisms

[Figure 2 about here]

To put our approach in perspective, we present a simple example that illustrates how it differs from other repeat-purchase mechanisms. For convenience, we assume in this section that \( A \)'s pricing strategy \( \sigma^A_p \) is fixed to \( p^t = \frac{1}{2} \) in each period \( t \) (imagine, for example, that the price is set by the government through regulation). How can consumers then create effort incentives?
Dynamic incentives under perfect monitoring. The simplest repeat-purchase mechanism is a grim-trigger strategy as suggested by Klein and Leffler (1981). Suppose that all consumers of a cohort have type $r = 1$ as in Figure 2(a). Moreover, assume that they can communicate information about outcomes among cohorts (so that each cohort knows the history of outcomes). Consider the following strategy: Consumers trade with $A$ as long as $A$ has exerted high effort in all previous periods, and they stop trading if $A$ ever deviated to low effort. $A$ is then willing to exert high effort in each period if and only if the long-term loss of a deviation weakly exceeds its short-term gain, i.e., if and only if $\frac{\delta_a}{1-\delta_a}(1-c) \geq c$. Given the consumers’ strategy, $A$ will not exert high effort once it has deviated to low effort. Hence, it is rational for consumers not to trade with $A$ whenever it has exerted low effort in a previous period.

Dynamic incentives under imperfect private monitoring. Next, assume that monitoring is imperfect. Each consumer privately observes her own outcomes from trade with $A$, but not the outcomes of other consumers. Nevertheless, consumers can generate dynamic incentives. Suppose that all consumers of a cohort have type $r = \frac{1}{2}$ as in Figure 2(b). Let them play the following strategy: They always trade with $A$ in the first period of their life; in the second period, they trade with $A$ if and only if they have observed a good outcome. Provided that $A$ always exerts high effort, consumers observe a good outcome with probability $\frac{1}{2}$ so that half of them continues trading with $A$. If each cohort follows this strategy, it is rational for $A$ to always exert high effort if and only if $\frac{\delta_a}{4} \geq c$. If $A$ always exerts high effort, consumers are always indifferent between trading or not. Hence, it is rational for them to play this strategy so that we get a high-effort equilibrium.

Uncertainty about $A$’s type (reputation). A drawback of the previous mechanism is that it relies on the fact that a positive mass of consumers is indifferent between trading with $A$ or not when $A$ always exerts high effort. If the distribution over types is continuous and consumers are experienced, there is no scope for a high-effort equilibrium.

This problem disappears if a good (bad) outcome signals to a consumer that the probability of a good (bad) outcome in the future is relatively high (low). Such learning occurs if $A$’s “type” is uncertain. Assume that when born a consumer knows that with probability $\mu < 1$ firm $A$ is a good (strategic) type who plays effort strategy $\sigma_e^A$, and with probability $1 - \mu$ firm $A$ is a bad type who never exerts effort. Let all consumers be experienced. Given that the good type always exerts effort, a consumer with type $r$ updates her belief about $A$ after a bad outcome to

$$\mu(r) = \frac{\mu(1-r)}{(1-\mu) + \mu(1-r)} < \mu. \quad (11)$$

Hence, her valuation for $A$’s good decreases after observing a bad outcome even if the
good type always exerts high effort. It then may be rational for her to stop trading with $A$. In this case, $A$ exerts effort in order to “convince” consumers of being the good type.

As an example, consider a uniform distribution $F$ as depicted in Figure 2(c). Suppose that the good type always exerts effort. It is then rational for a consumer with type $r$ to trade with $A$ if $\mu r - \frac{1}{2} \geq 0$ and to stop trading with $A$ after observing a bad outcome if $\mu r - \frac{1}{2} \leq 0$. When $\mu = \frac{4}{5}$, all consumers with type $r > \frac{5}{8}$ trade with $A$ in the first period of their life, and trade (do not trade) with $A$ after observing a good (bad) outcome. In Figure 2(c), this mass of consumers is represented by the gray rectangle. The mass of consumers $A$ loses if it exerts low effort equals $\int_{5/8}^{1} r f(r) dr = \frac{39}{128}$. Hence, a high-effort equilibrium exists if $\frac{\delta A}{2} \frac{39}{128} \geq c$. Consumers are not required to coordinate on a punishment (as in the first mechanism), or to be indifferent between trading or not with $A$ (as in the second mechanism).

The reputation mechanism requires substantial uncertainty about $A$’s type. Observe that $\mu(r) \to 1$ for $\mu \to 1$ so that consumers’ incentive to stop trading with $A$ after a bad outcome in a high-effort equilibrium vanishes when it becomes clear (through some mechanism outside the model) that $A$ is the good type. For this case, the literature has suggested several modeling assumptions that constantly replenish uncertainty. The type of a firm may change over time, because the environment changes so that skills that once were useful become obsolete (Benabou and Laroque 1992, Holmström 1999, Mailath and Samuelson 2001). Alternatively, the type changes due to a switch in the ownership of the firm (Tadelis 1999, 2002). Consumers may also have finite memory so that they are never really sure about the firm’s type (Monte 2012, Liu 2011). Hörner (2002) shows that in a competitive market, it can be rational for disappointed consumers to switch firms. The threat of losing the consumer base sustains effort incentives even when uncertainty about a firm’s type vanishes. However, the consumers’ strategy then forces good firms out of the market and requires continuous entry of new firms.

**Consumer learning.** In our model, consumers attribute a bad outcome to the fact that their match value with the firm is probably low (and not to fact that the firm’s type may be inept). As in the reputation example, suppose that consumers are distributed uniformly over types and that a share $\lambda > 0$ of consumers in each cohort is inexperienced. The expected types are $\tilde{r} = \frac{1}{2}$, $\tilde{r}_{A0} = \frac{4}{5}$ and $\tilde{r}_{A1} = \frac{2}{3}$. Provided that $A$ always exerts effort, it is rational for inexperienced consumers to trade with $A$ in the first period of their life (since $\tilde{r} = \frac{1}{2}$), and to stop trading with it after observing a bad outcome (since $\tilde{r}_{A0} < \frac{1}{2}$). The mass of consumers $A$ loses if it exerts low effort equals $\frac{1}{2} \lambda$. Hence, if $\frac{2}{3} \lambda \geq c$, a high-effort equilibrium exists. In this equilibrium, the moral hazard problem is solved without requiring consumers to coordinate (as in the first mechanism), without requiring a fraction of consumers to be exactly indifferent between trading or not with $A$ (as in the
second mechanism), and without uncertainty about the type of the long-lived firm (as in the third mechanism).

6 Implications

Our model demonstrates that consumers’ learning about their match value can solve the moral hazard problem between a firm and its customers. In this section, we discuss its implications for optimal advertising, product design and consumer education.

Advertising. In the industrial organization literature, advertising either changes the demand for a product through information (or persuasion), or it serves as a signal for quality (see Bagwell 2007 for a review). The rationale behind the latter is that high quality goods generate more repeat purchases than low quality goods. Firms that produce high quality can credibly signal this to consumers by investing large sums in dissipative advertising. In our model, advertising can simultaneously play a functional and an informational role: it may increase young, inexperienced consumers’ demand and thereby signal commitment to high quality.

Let us build a simple advertising extension for our framework that illustrates this idea. Young consumers are unaware of firm \( A \) at their birth, but \( A \) can invest into advertising to increase the fraction of young consumers who know about the availability of its good. Denote by \( \phi^t \in [0, 1] \) the reach of \( A \)’s campaign in period \( t \) (the fraction of young consumers who become informed about \( A \)’s existence), and by \( \frac{1}{2} \lambda (\phi^t)^2 \) the cost of reach \( \phi^t \) that must be borne by \( A \) in period \( t \). Old consumers always know about the existence of \( A \)’s good. \( A \) decides about \( \phi^t \) at the beginning of period \( t \). The distribution over types \( F \) and experience \( \lambda \) is the same in the population of informed and uninformed consumers. Informed consumers also observe the reach of the current advertising campaign.

Consider first what reach would be optimal for \( A \) given that consumers trade with \( A \) if and only if the price equals \( \bar{r} \) (so that young, inexperienced consumers purchase \( A \)’s good and become repeat customers if and only if they observe a good outcome). The additional profit from advertising in period \( t \) equals

\[
\phi^t(1 - \lambda)(1 - F(\bar{r}))\bar{r} + \phi^t \lambda \bar{r} - \frac{1}{2} \gamma (\phi^t)^2.
\]

Hence, the optimal reach in the absence of moral hazard would be given by

\[
\phi^{t_b} = \frac{1}{\gamma}((1 - \lambda)(1 - F(\bar{r}))\bar{r} + \lambda \bar{r}).
\]

\[\text{This idea was first articulated by Nelson (1974). Kihlstrom and Riordan (1984) and Milgrom and Roberts (1986) provided the first formal models.}\]
However, $A$ also has to attract sufficiently many young, inexperienced consumers in order to convince its customers that it will exert high effort, that is, we must have $\phi^t \delta_A \lambda \tilde{r}^2 \geq c$ in each period $t$. Hence, the minimal level of advertising $\phi^*$ that creates sufficient commitment to high effort is given by

$$\phi^* = \frac{c}{\delta_A \lambda \tilde{r}^2}. \quad (14)$$

Suppose that consumers with (expected) type weakly larger than $\tilde{r}$ trade with $A$ in period $t$ if and only if $p^t = \tilde{r}$ and $\phi^t \geq \phi^*$. The level of advertising that will be realized in the corresponding high-effort equilibrium is then given by $\max\{\phi^{fb}, \phi^*\}$. Note that $\phi^*$ does not depend on the costs parameter $\gamma$. For an intermediate range of values of $\gamma$ we have $\phi^* > \phi^{fb}$, which implies that $A$ chooses a larger advertising campaign than it would do in the absence of moral hazard.\(^7\) It needs this reach in order to credibly signal its commitment to high effort to consumers. For sufficiently small values of $\gamma$ we have $\phi^* \leq \phi^{fb}$. The advertising campaign then accomplishes two goals: it maximizes $A$’s profit from young consumers and assures to all customers that it will provide high quality.\(^8\)

**Product Design and Advertising Content.** A firm’s product design decisions and the content of advertising information influence the effectiveness of the described mechanism. The product design determines whether the consumers’ dispersion over types is large (as in Example 1) or small (as in Example 2). For example, a restaurateur’s decisions on the menu and the combination of ingredients for the meals influence how many customers will like the final outcome. The content of advertising information determines how much consumers know about their match value before they trade with the firm.

Consumer learning generates sufficient effort incentives (without further implicit contracts) if the share of inexperienced consumers $\lambda$ as well as consumers’ expected type $\tilde{r}$ are sufficiently large relative to the cost of effort, and the dispersion of types is large such that the expected type after a bad outcome $\tilde{r}_{40}$ is small relative to $\tilde{r}$. Hence, the following combination of product design and advertising information is conductive for effort incentives: the good is a niche product so that consumers have a very high or a very low type, and no information is provided that could help inexperienced consumers to evaluate to which group they belong.

\(^7\)Clearly, if $\gamma$ is sufficiently large, then a campaign that creates sufficient commitment is too costly for $A$ and there will be low effort in each period of an equilibrium.

\(^8\)A similar mechanism was pointed out to me by Simon Anderson. Assume that a fraction of consumers are “experts” who observe the quality of the good and punish the firm for low quality. Non-experts then trade with the firm only if its advertising campaign is large enough such that there are sufficiently many experts in the firm’s consumer base. Apart from that I am not aware of any model that interprets advertising as commitment device that solves a moral hazard problem.
Interestingly, this marketing mix is not an optimal one in the absence of moral hazard. Johnson and Myatt (2006) show that profits are maximal under one of the following two extreme marketing strategies: a mass-market product (most consumers like the product to some extent) with an advertising campaign that only highlights the existence of the good, but does not help consumers to learn their type (“pure hype”); or a niche product (consumers either love or hate the product) combined with informative advertising that allows consumers to learn their true type (so that high type consumers are willing to pay a high price). The marketing mix that is optimal to resolve the moral hazard problem in our framework is a combination of these two strategies: niche product design and pure hype marketing.

Our model offers a new explanation of why firms choose to provide no or only partial information on product characteristics. Anderson and Renault (2006) argue that precise information creates a hold-up problem: when all consumers know their match value with the firm and there is no commitment to a low price, the expected price would be so high such that it does not pay off to incur prior search costs. In our model, precise information on the match value destroys the credibility of punishment for bad outcomes.

**Consumer Education.** A nascent literature analyzes to what extent firms have an incentive to educate inexperienced consumers about their demand type (e.g. Bergemann and Välimäki 2006, Villas-Boas and Villas-Boas 2008, Kamenica et al. 2011). Consumer education is possible through a temporal reduction in prices (“sales”) or the provision of additional information (modern information processing technologies enable firms to gain superior knowledge about their customers’ needs). In our model, consumer education may decrease the share $\lambda$ of inexperienced consumers. Optimal education then solves the trade-off between the following two opposing effects. On the one hand, if the real type of a consumer is larger (smaller) than the firm’s price, this consumer should trade (not trade) with the firm. Hence, consumer education can enable individuals to make better decisions and therefore increase welfare. On the other hand, since it also decreases the fraction of consumers who stop trading after observing a bad outcome, consumer education may reduce effort incentives.

**Appendix**

**Omitted calculations from Example 1.** We can calculate $\bar{r}$ as follows:

$$
\bar{r} = \frac{1}{4} \int_0^1 r f \, dr + \int_{3/4}^1 r(4 - f) \, dr = \frac{1}{32} f + \frac{7}{32} (4 - f) = \frac{7}{8} - \frac{3}{16} f.
$$

(15)
The conditional expectation $\tilde{r}_{A0}$ is given by

$$\tilde{r}_{A0} = \int_0^{1/4} rf(r\mid A0)\,dr + \int_{3/4}^1 rf(r\mid A0)\,dr$$

(16)

with

$$f(r\mid A0) = \frac{f(r)(1-r)}{\int_0^{1/4} (1-r)f\,dr + \int_{3/4}^1 (1-r)(4-f)\,dr}.$$  

(17)

**Omitted calculations from Example 2.** We show that if (10) holds, then for all $\lambda$ we have

$$(1-\lambda)2(1-F(\tilde{p}\tilde{t}))\tilde{p}\tilde{t} + \lambda(\tilde{p}\tilde{t} + \tilde{r}\tilde{p}\tilde{t}) < 2x.$$  

(18)

First, we show that if (10) holds, then $2(1-F(\tilde{p}\tilde{t}))\tilde{p}\tilde{t} < 2x$. Rewrite $\tilde{p}\tilde{t} = x + \varepsilon$ and note that $F(\tilde{p}\tilde{t}) = \frac{\varepsilon}{y-x}$. Hence, the inequality becomes $2(1 - \frac{\varepsilon}{y-x})(x + \varepsilon) < 2x$, which can be simplified to $2x + \varepsilon > y$. This inequality is implied by (10). Next, we show that if (10) holds, then $\tilde{p}\tilde{t} + \tilde{r}\tilde{p}\tilde{t} < 2x$. Again, rewrite this inequality as $(x+\varepsilon)(1+\tilde{r}) < 2x$. We calculate that $\tilde{r} = \frac{1}{2} y + \frac{1}{2} x$. Since $\varepsilon < y - x$, the inequality is implied by $y(1 + \frac{1}{2} y + \frac{1}{2} x) < 2x$, which is identical to (10).

**References**


Figure 1: Distribution over types for a heterogeneous (left) and a homogenous (right) consumer population.

Figure 2: The distributions over types for the examples in Section 5. The gray columns in (a), (b) and (d) indicate mass points. The gray columns in (d) at $\tilde{x}_{A1}$ ($\tilde{x}_{A0}$) indicate the mass of inexperienced consumers who observe a good (bad) outcome if they trade with $A$ and $A$ exerts high effort.